Figure 7 shows a quick-return mechanism. Link 2 is the driver, and it rotates counter-clockwise at a constant 50 r.p.m. P_2 and P_4 are points on links 2 and 4 which are coincident at the instant shown.

(a) The velocities of P_2 and P_4, $\mathbf{v}_{P_2}$ and $\mathbf{v}_{P_4}$, are related by the vector equation:

$$\mathbf{v}_{P_2} = \mathbf{v}_{P_4} + \mathbf{v}_{P_2P_4}$$

where $\mathbf{v}_{P_2P_4}$ is the velocity of P_2 relative to P_4. Draw a velocity diagram based on this equation for the position of the mechanism shown in Figure 7. Hence find the magnitude of each of the three terms in the equation, along with the angular velocity of link 4.

[5 marks]

(b) The accelerations of P_2 and P_4, $\boldsymbol{a}_{P_2}$ and $\boldsymbol{a}_{P_4}$, are related by the vector equation:

$$\boldsymbol{a}_{P_2} = \boldsymbol{a}_{P_4} + \boldsymbol{a}_{P_2P_4}$$

where $\boldsymbol{a}_{P_2P_4}$ is the acceleration of P_2 relative to P_4. This equation can be re-written in terms of normal (n), tangential (t), and Coriolis components as follows:

$$\left(\boldsymbol{a}^n_{P_2} + \boldsymbol{a}^t_{P_2}\right) = \left(\boldsymbol{a}^n_{P_4} + \boldsymbol{a}^t_{P_4}\right) + \left(\boldsymbol{a}^n_{P_2P_4} + \boldsymbol{a}^t_{P_2P_4} + 2\boldsymbol{\omega}_4 \times \mathbf{v}_{P_2P_4}\right) .$$

Draw an acceleration diagram based on this equation for the position of the mechanism shown in Figure 7. Hence find the magnitude of each of the seven terms in the equation, along with the angular acceleration of link 4.

[15 marks]

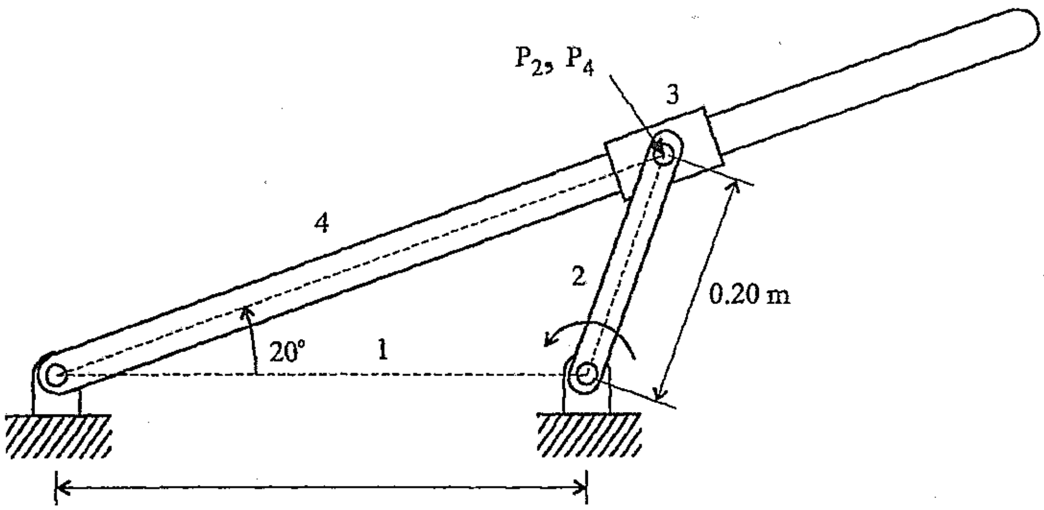
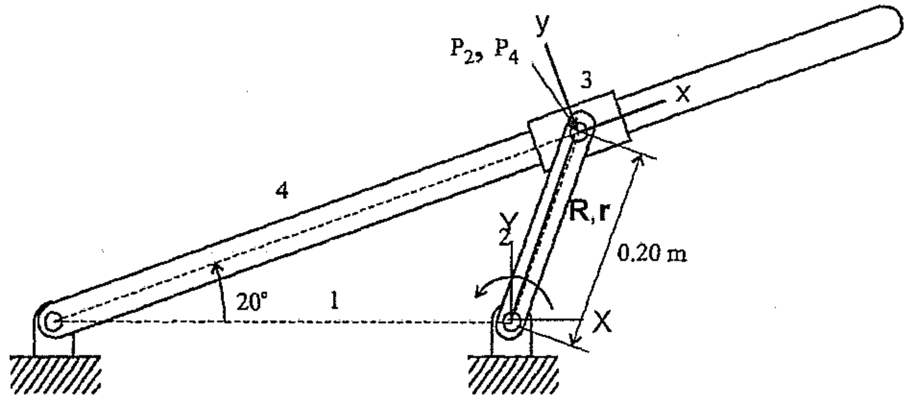
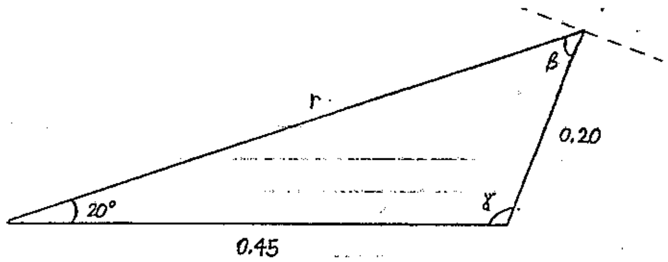


Figure 7



Geometry & Position analysis



$0.45 \sin 20 = 0.20 \sin \beta$ $\quad \beta = 50.31°$ $\qquad \gamma = 180 - 20 - \beta = 109.69°$

$r = 0.45 \cos 20 + 0.20 \cos \beta = 0.55$ $\qquad 180 - \gamma = 70.31$

$50 \frac{rev}{min} \cdot \frac{2\pi\, rad}{rev} \cdot \frac{min}{60s} = 5.24\ rad/s$

r locates P2
R locates oxy (same as P4)
$p = 0$

$$r = R + p$$

$$r_{P2} = r_{P4} + r_{P2/P4}$$

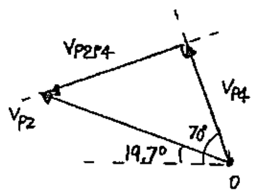
Velocity analysis

$$\dot{r} = \dot{R} + \dot{p} + \omega \times p$$

$$v_{P2} = v_{P4} + v_{P2/P4} = \dot{R} + \dot{p}$$

For relative velocity, think of P2 moving, P4 fixed

(a) $V_{P2} = \omega_2 l_2 = (5.24)(0.20) = 1.048\ m/s$



$V_{P4} = V_{P2} \cos(70 - 19.7) = 0.669\ m/s$

$V_{P4} = \omega_4 l_4 \quad \omega_4 = 0.669 / 0.55 = 1.216\ rad/s$

$V_{P2P4} = V_{P2} \sin(70 - 19.7) = 0.806\ m/s$

Acceleration analysis

$$\ddot{r} = \ddot{R} + \ddot{p} + \omega \times \omega \times p + \dot{\omega} \times p + 2\omega \times \dot{p}$$

$$a_{P2} = a_{P4} + a_{P2/P4}$$

$$(a^n_{P2} + a^t_{P2}) = (a^n_{P4} + a^t_{P4}) + (a^n_{P2/P4} + a^t_{P2/P4} + 2\omega_4 \times v_{P2/P4})$$

For relative acceleration, think of P2 moving, P4 fixed

$A^n_{P2} = \omega_2^2 \ell_2 = (5.24)^2 (0.20) = 5.492 \text{ m/s}^2$ ✓

$A^t_{P2} = 0$ since $\omega_2 =$ constant

$A^n_{P4} = \omega_4^2 \ell_4 = (1.216)^2 (0.55) = 0.813 \text{ m/s}^2$ ✓

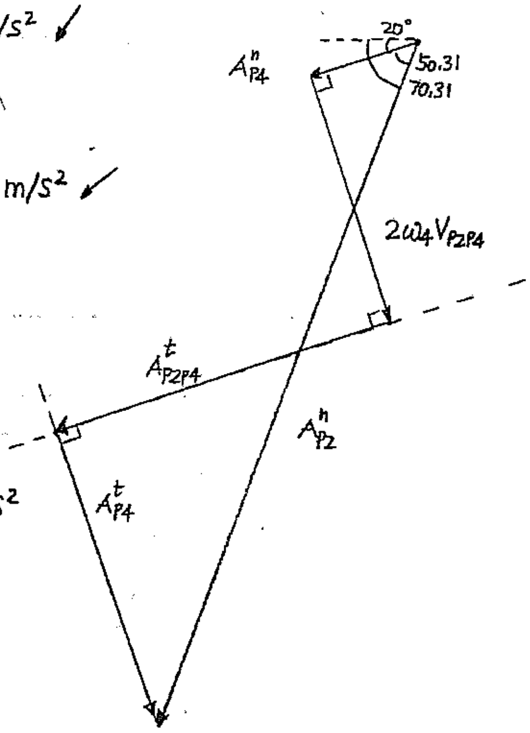
$A^t_{P4} = ?$

$A^n_{P2P4} = \frac{V_{P2P4}}{R} = 0$ since R is infinite (P2 relative to P4 follows a straight line)

$A^t_{P2P4} = ?$

$2\omega_4 V_{P2P4} = 2(1.216)(0.806) = 1.960 \text{ m/s}^2$

Direction of Coriolis $\underline{\omega_4} \times \underline{V_{P2P4}}$
$\underline{\omega_4}$ directed out of the page, $\underline{V_{P2P4}}$ as in velocity diagram



$A^t_{P2P4} = A^n_{P2} \cos 50.31 - A^n_{P4}$
$A^t_{P2P4} = 5.492 \cos 50.31 - 0.813 = 2.694 \text{ m/s}^2$

$A^t_{P4} = A^n_{P2} \sin 50.31 - 2\omega_4 V_{P2P4} = 5.492 \sin 50.31 - 1.960 = 2.266 \text{ m/s}^2$
$A^t_{P4} = \alpha_4 \ell_4 \qquad \alpha_4 = 2.266/0.55 = 4.12 \text{ rad/s}^2$