Course Outline for A3 Dynamics and Kinematics (8 lectures)

Lecturer: Dr Amy Zavatsky **Michaelmas Term 2004**

Expanded syllabus:

Dynamics: Angular momentum (general definition), rigid body motion with rotation and translation. Kinematics: Velocity and acceleration; motion in rotating frames of reference. Mechanisms; instantaneous centres, calculation of velocity and acceleration (including velocity and acceleration diagrams), simple, compound and epicyclic gear trains (calculation of gear ratios and torques, effective inertia).

Lecture Synopsis:

1. *Dynamics of rigid bodies: plane kinematics*

Translation (rectilinear, curvilinear), rotation about a fixed axis, and general plane motion. Linear and angular velocity and acceleration. Absolute and relative velocity and acceleration. Instantaneous centre of rotation. Rate of change of a vector with respect to a rotating coordinate frame. General motion in a rotating frame of reference.

2. *Dynamics of rigid bodies: plane kinetics*

Force, mass, and acceleration. General equations of motion. Free-body diagram, equivalent force-couple, kinetic diagram. Angular momentum about a fixed point, about the mass centre, about an arbitrary point. Work and energy (translational and rotational), power, impulse and momentum (linear and angular). General problem formulation and review.

3. *Linkages and mechanisms: basic concepts*

Definitions: mechanism, linkage (or kinematic chain), types of motion transformation, open-loop versus closed-loop, lower and higher joints (or pairs), mobility (or degrees-of-freedom). Grübler's equation for mobility. The four-bar linkage (input, output, coupler, transmission angle) and Grashof's law. Kinematic inversion. Introduction to slider-crank mechanism and quick-return mechanism. Analysis versus synthesis.

4. *Linkages and mechanisms: position and velocity*

Relative velocity (link with revolute joint, link with revolute joint and slider). Velocity diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Velocity image. Instantaneous centres and centrodes. Kennedy's theorem for three instantaneous centres. Velocity pole method for finding angular velocities in linkages (four-bar linkage, slider-crank).

5. *Linkages and mechanisms: acceleration*

Relative acceleration (link with revolute joint, link with revolute joint and slider). Acceleration diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Acceleration image. Relative acceleration of coincident particles at the point of contact of rolling elements. Equivalent mechanisms.

6. *Linkages and mechanisms: computational analysis*

Kinematic analysis by complex numbers (link with revolute joint, link with revolute joint and slider). Loop closure equations (slider-crank, quick-return mechanism). Solution of the position problem using the Newton-Raphson method. Vector chains to describe motion of an arbitrary point on a mechanism. Introduction to generalised coordinates and constraint equations.

7. *Linkages and mechanisms: forces in linkages*

Review of two-force bodies, three-force bodies, and couples. Static forces (slider-crank, four-bar linkage). D'Alembert's principle (inertia force and concept of dynamic equilibrium) in contrast to direct "F=ma" approach. Dynamic analysis (four-bar linkage, slider-crank). Virtual work method to find forces and torques. Kinetic energy and equivalent inertia.

8. *Gear trains: simple, compound, and epicyclic gear trains*

Friction drives versus gears. Examples of spur gears, rack and pinion, bevel gears, and worm gears. Spur gear terminology. Fundamental law of gearing (circular gears). Gear trains: simple, compound, planetary. Equivalent inertia.

Examples Sheets: 3A3C, 3A3D

Learning Outcomes:

1. Ability to calculate absolute and relative positions, velocities, and accelerations for two-dimensional rigid body motions in fixed, translating, and rotating frames of reference.
2. Ability to calculate for a rigid body moving in two dimensions the angular momentum with respect to a fixed point, to the mass centre of the body, or to any arbitrary point.
3. Ability to solve dynamics problems for two-dimensional rigid body motions including both translation and rotation.
4. Ability to identify the type of motion produced by a mechanism and to calculate its mobility.
5. Ability to find the instantaneous centres for the various components of a mechanism.
6. Ability to calculate the relative positions, velocities and accelerations of the various components of a mechanism using vector methods, diagrammatic methods, and complex numbers / loop closure equations.
7. Ability to calculate the static and dynamic forces within a mechanism and the equivalent inertia for a mechanism.
8. Ability to calculate the gear ratios, torques, and effective inertia for simple, compound, and planetary gear trains.

Primary Reference 2 **Lectures 3-8**

Mechanisms and Dynamics of Machinery
HH Mabie and CF Reinholtz.
John Wiley & Sons, 1987 (4th ed.)

FINALS A3

1 Introduction
2 Linkages and Mechanisms
8 Velocity and Acceleration Analysis
9 Force Analysis of Machinery
7 Gear Trains

FINALS B1

3 Cams
4 Spur Gears
10 Balance of Machinery

[5 Nonstandard Spur Gears]
[6 Bevel, Helical, and Worm Gearing]
[11 Introduction to Synthesis]
[12 Spatial Mechanisms and Robotics]

Linkages and Mechanisms: Basic Concepts

Definitions

Mechanism:
a mechanical device that has the purpose of transferring motion and/or force from a source to an output

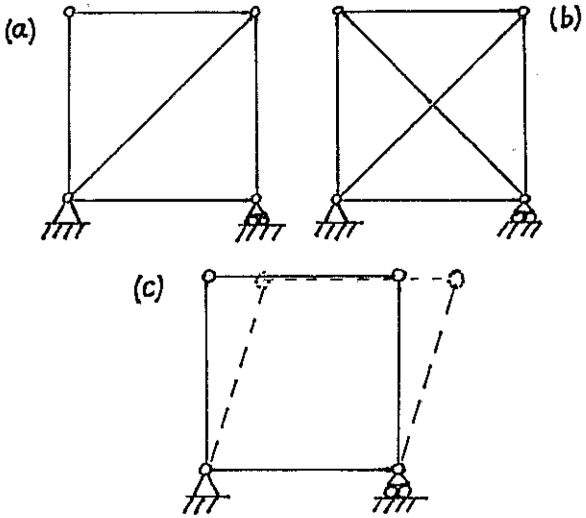
Linkage or kinematic chain:
a construction consisting of links (or bars), generally considered to be rigid, which are connected by joints (or pairs), to form open or closed chains (or loops).

Kinematic chains, with at least one link fixed, become (1) *mechanisms* if at least two other links retain mobility, or (2) *structures* if no mobility remains.

Statical determinacy of plane trusses

Unknowns: bar forces (b) + reactions (r)

Equations: equilibrium in x, y directions at each joint ($2j$)



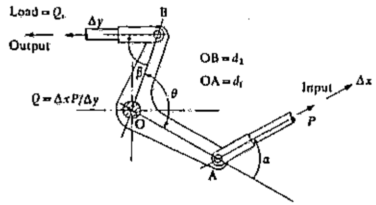
(a) $b + r = 5 + 3 = 8$ $\qquad 2j = 2(4) = 8$
May be statically determinate.

(b) $b + r = 6 + 3 = 9$ $\qquad 2j = 2(4) = 8$
Redundant.

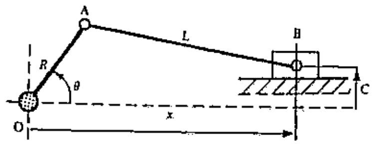
(c) $b + r = 4 + 3 = 7$ $\qquad 2j = 2(4) = 8$
Mechanism.

Motion Transformation (Grosjean)

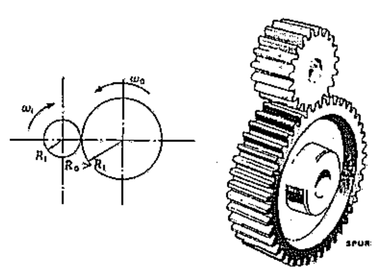
(a) Simple lever



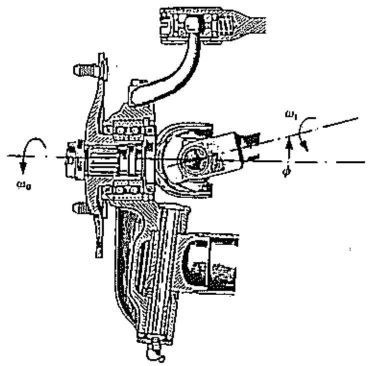
(d) Slider-crank



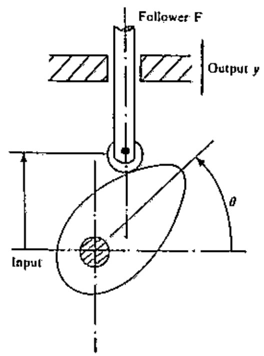
(b) Contacting cylinders; gears



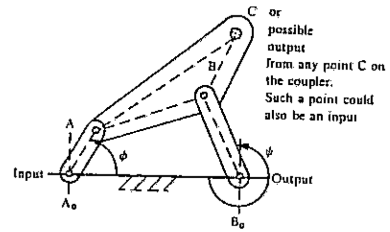
(e) Hooke's joint



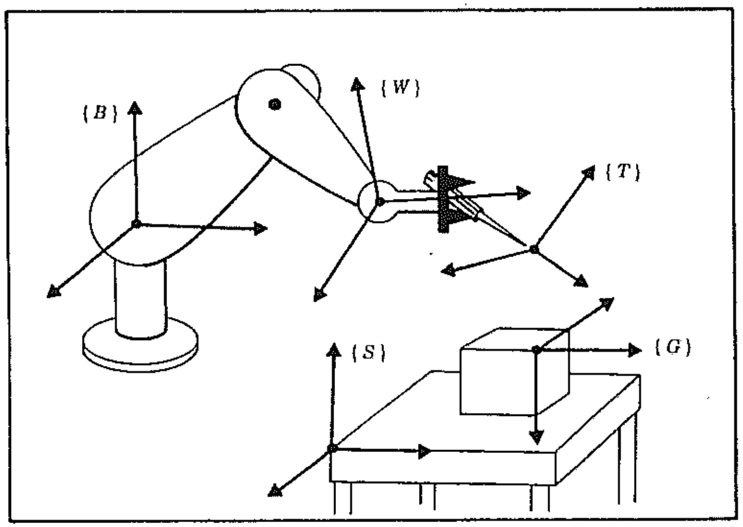
(c) Cam



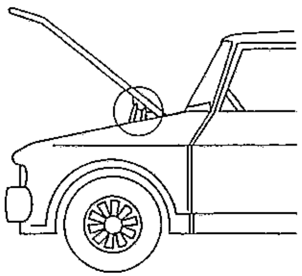
(f) Four-bar linkage

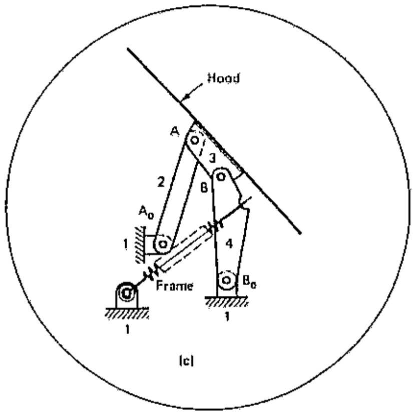


Open-loop versus Closed-loop



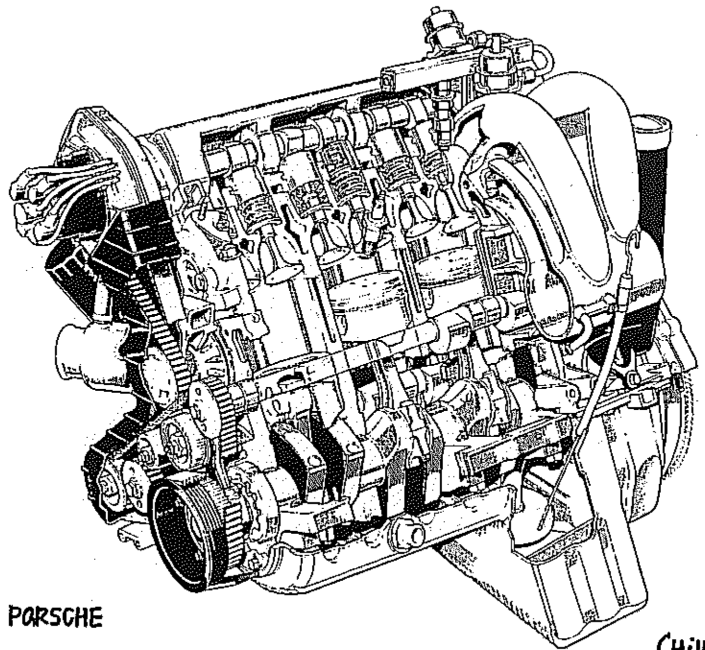
(Craig)





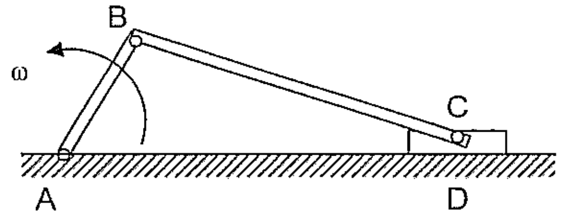
(Erdman & Sandor)

It may be difficult to discern the basic kinematic device when looking at a photograph or drawing of a complete machine. For this reason, kinematic diagrams are used.



PORSCHE

(Hillier)



Joints (or Pairs)

Lower pair: joint between two bodies (links) which have surface contact

Higher pair: joint between two bodies (links) which have line or point contact

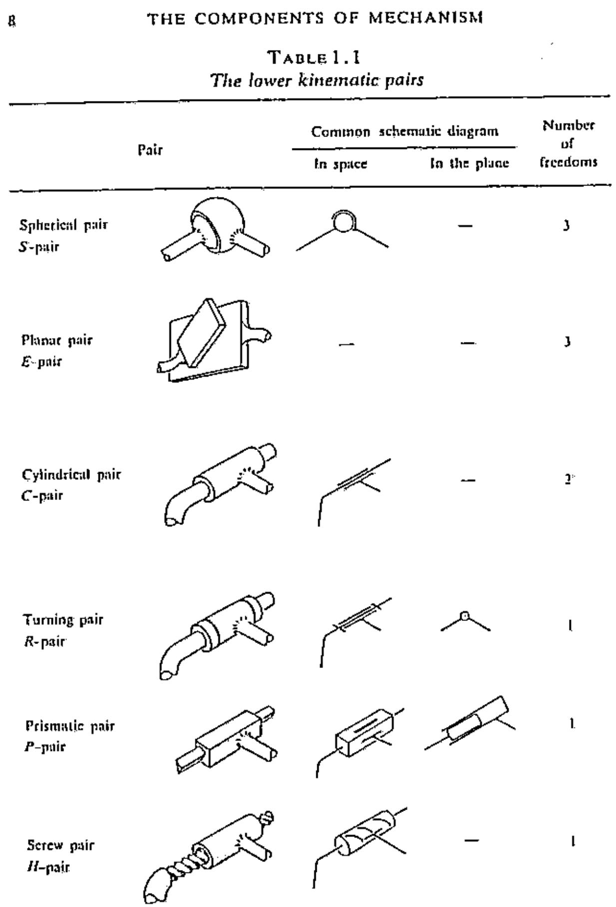

8 THE COMPONENTS OF MECHANISM

TABLE 1.1
The lower kinematic pairs

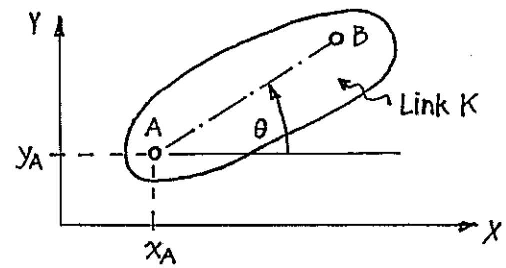
Pair		Common schematic diagram: In space	Common schematic diagram: In the plane	Number of freedoms
Spherical pair *S*-pair			—	3
Planar pair *E*-pair		—	—	3
Cylindrical pair *C*-pair			—	2
Turning pair *R*-pair				1
Prismatic pair *P*-pair				1
Screw pair *H*-pair			—	1

(Hunt)

Mobility *m* (degrees of freedom)

The number of independent inputs re-quired to determine the positions of all the links of the mechanism with respect to ground.

In planar motion, a rigid body (link) has three degrees of freedom (dof). n links have $3n$ dof.



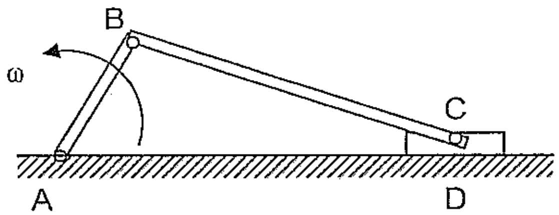
Each single dof joint removes two dof of relative motion between the links.

Grübler's equation: $m = 3(n-1) - 2j_1 - 1j_2$

n = number of links
$n - 1$ = number of mobile links
j_1 = number of single dof joints (pins, sliders)
j_2 = number of two dof joints (roll-slide)

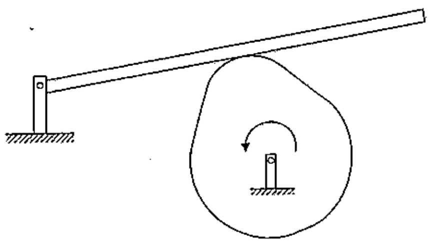
Example

Crank-slider mechanism



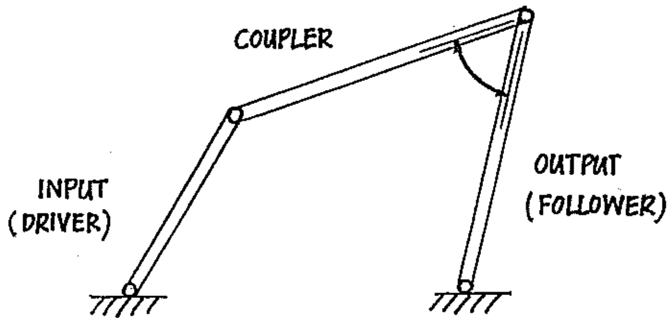
$n = 4,\ j_1 = 4,\ j_2 = 0$
$m = 3(4\text{-}1) - 2(4) - 0 = 1$

Cam-follower



$n = 3,\ j_1 = 2,\ j_2 = 1$
$m = 3(3\text{-}1) - 2(2) - 1 = 1$

The four-bar linkage



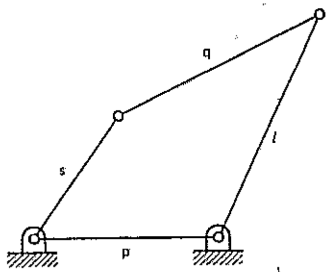
- one of the simplest and most useful mechanisms
- single degree of freedom (Grübler's equation)
- transmission angle: the acute angle between the connecting or coupler link and the output link

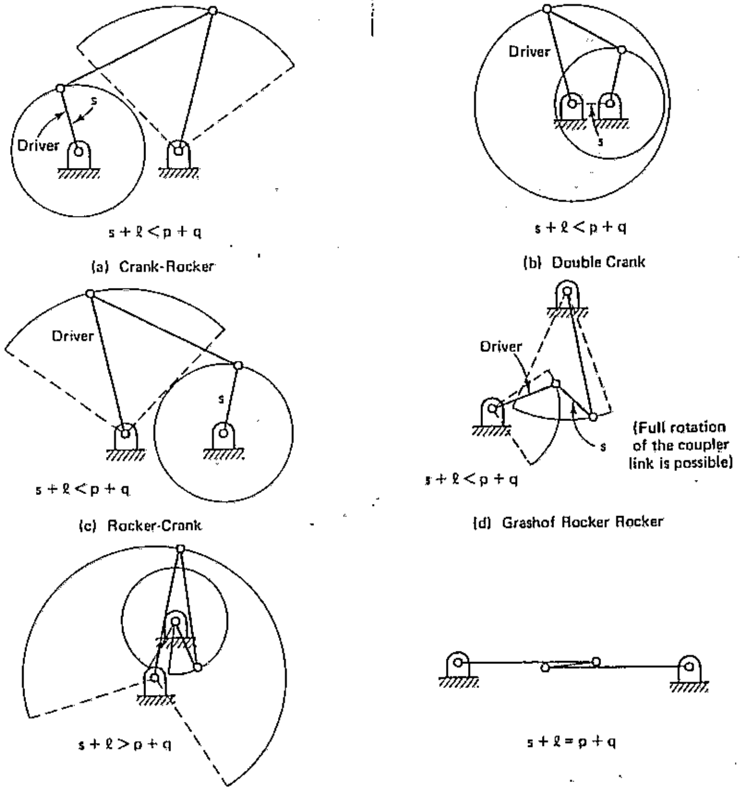
The four-bar linkage takes different forms depending on the *range of motion* of the two links connected to the ground link.

Grashof's Law

The sum of the shortest and longest links of a planar four-bar linkage cannot be greater than the sum of the remaining two links if there is to be continuous relative rotation between two links.

s = shortest link, l = longest link
intermediate length links p and q





$s + \ell < p + q$
(a) Crank-Rocker

$s + \ell < p + q$
(b) Double Crank

$s + \ell < p + q$
(c) Rocker-Crank

$s + \ell < p + q$
(d) Grashof Rocker Rocker

$s + \ell > p + q$
(e) Non-Grashof Rocker-Rocker (Triple Rocker)

$s + \ell = p + q$
(f) Change-Point Mechanism in Toggle Configuration

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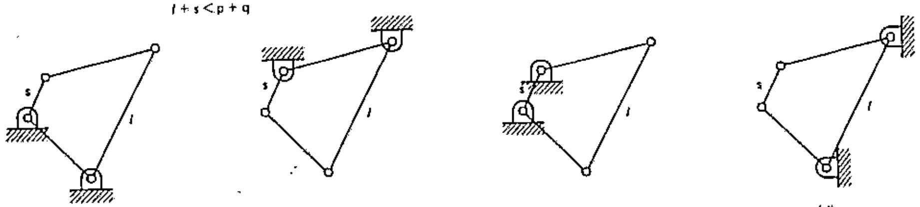
Kinematic inversion

The process of fixing different links of a chain to create different mechanisms.

Note that the relative motion between the links of a mechanism does not change in different inversions.

$l + s < p + q$



(Erdman & Sandor)

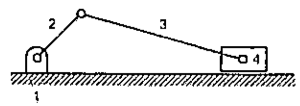


Figure 3.8 Slider-crank.

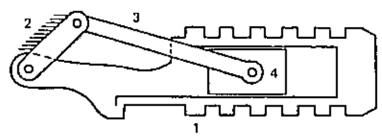


Figure 3.9 Inversion of the slider-crank: rotating slide.

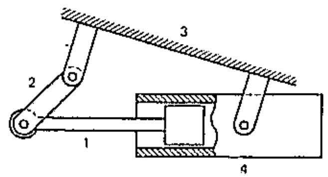


Figure 3.10 Inversion of the slider-crank: oscillating slide.

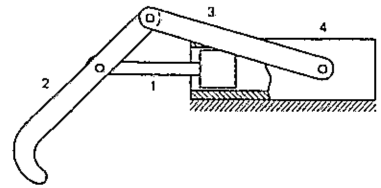
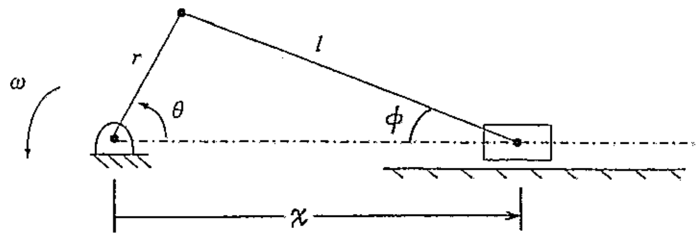


Figure 3.11 Inversion of the slider-crank: stationary slide.

Slider-crank mechanism



- widely used (internal combustion engine)
- flywheel needed to overcome the two dead points of the cycle

$x = r \cos \theta + l \cos \phi$

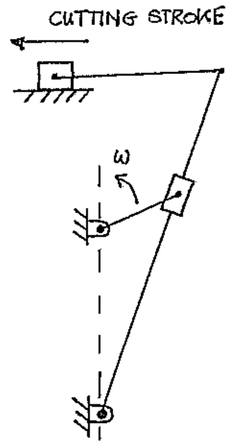
$l / \sin \theta = r / \sin \phi$

$\dot{x} = - r \sin \theta \, \dot{\theta} - (r^2/2l) \sin 2\theta \, \dot{\theta} [1 - (r^2/l^2) \sin^2\theta]^{-½}$

Expand the series.
Take $\dot{\theta}$ = constant, so $\ddot{\theta} = 0$.
Disregard higher order terms.

$\ddot{x} = - r \dot{\theta}^2 [\cos \theta + (r/l) \cos 2\theta]$

Quick-return mechanisms



- used on machine tools to give a slow cutting stroke and a quick return stroke for a constant angular velocity of the driving crank

- combinations of basic linkages such as the four-bar and the slider-crank (several types)

- ratio of crank angle for the cutting stroke (α) to that for the return stroke (β) is the *time ratio*

- to produce a quick return the time ratio must be greater than 1

Analysis versus Synthesis

The process of drawing kinematic diagrams and determining degrees of freedom are the first steps in the *kinematic analysis* and *kinematic synthesis*.

Kinematic analysis: a particular mechanism is investigated based on mechanism geometry plus possibly other known characteristics (such as input angular velocity, angular acceleration, etc.).

Kinematic synthesis: the process of designing a mechanism to accomplish a desired task. Decisions about the *type* and the *dimensions* of the mechanism are involved.

The ability to visualise relative motion, to reason why a mechanism is designed the way it is, and the ability to improve on a particular design are the marks of a successful kinematician. Although some of this ability comes in the form of innate creativity, much of it is a learned skill that improves with practice.

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References

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