Course Outline for A3 Dynamics and Kinematics (8 lectures)

Lecturer: Dr Amy Zavatsky **Michaelmas Term 2004**

Expanded syllabus:

Dynamics: Angular momentum (general definition), rigid body motion with rotation and translation. Kinematics: Velocity and acceleration; motion in rotating frames of reference. Mechanisms; instantaneous centres, calculation of velocity and acceleration (including velocity and acceleration diagrams), simple, compound and epicyclic gear trains (calculation of gear ratios and torques, effective inertia).

Lecture Synopsis:

1. *Dynamics of rigid bodies: plane kinematics*

Translation (rectilinear, curvilinear), rotation about a fixed axis, and general plane motion. Linear and angular velocity and acceleration. Absolute and relative velocity and acceleration. Instantaneous centre of rotation. Rate of change of a vector with respect to a rotating coordinate frame. General motion in a rotating frame of reference.

2. *Dynamics of rigid bodies: plane kinetics*

Force, mass, and acceleration. General equations of motion. Free-body diagram, equivalent force-couple, kinetic diagram. Angular momentum about a fixed point, about the mass centre, about an arbitrary point. Work and energy (translational and rotational), power, impulse and momentum (linear and angular). General problem formulation and review.

3. *Linkages and mechanisms: basic concepts*

Definitions: mechanism, linkage (or kinematic chain), types of motion transformation, open-loop versus closed-loop, lower and higher joints (or pairs), mobility (or degrees-of-freedom). Grübler's equation for mobility. The four-bar linkage (input, output, coupler, transmission angle) and Grashof's law. Kinematic inversion. Introduction to slider-crank mechanism and quick-return mechanism. Analysis versus synthesis.

4. *Linkages and mechanisms: position and velocity*

Relative velocity (link with revolute joint, link with revolute joint and slider). Velocity diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Velocity image. Instantaneous centres and centrodes. Kennedy's theorem for three instantaneous centres. Velocity pole method for finding angular velocities in linkages (four-bar linkage, slider-crank).

5. *Linkages and mechanisms: acceleration*

Relative acceleration (link with revolute joint, link with revolute joint and slider). Acceleration diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Acceleration image. Relative acceleration of coincident particles at the point of contact of rolling elements. Equivalent mechanisms.

6. *Linkages and mechanisms: computational analysis*

Kinematic analysis by complex numbers (link with revolute joint, link with revolute joint and slider). Loop closure equations (slider-crank, quick-return mechanism). Solution of the position problem using the Newton-Raphson method. Vector chains to describe motion of an arbitrary point on a mechanism. Introduction to generalised coordinates and constraint equations.

7. *Linkages and mechanisms: forces in linkages*

Review of two-force bodies, three-force bodies, and couples. Static forces (slider-crank, four-bar linkage). D'Alembert's principle (inertia force and concept of dynamic equilibrium) in contrast to direct "F=ma" approach. Dynamic analysis (four-bar linkage, slider-crank). Virtual work method to find forces and torques. Kinetic energy and equivalent inertia.

8. *Gear trains: simple, compound, and epicyclic gear trains*

Friction drives versus gears. Examples of spur gears, rack and pinion, bevel gears, and worm gears. Spur gear terminology. Fundamental law of gearing (circular gears). Gear trains: simple, compound, planetary. Equivalent inertia.

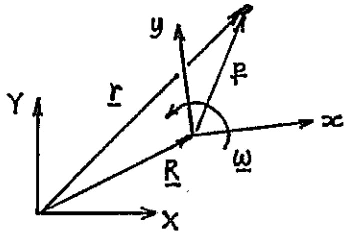
Examples Sheets: 3A3C, 3A3D

Learning Outcomes:

1. Ability to calculate absolute and relative positions, velocities, and accelerations for two-dimensional rigid body motions in fixed, translating, and rotating frames of reference.
2. Ability to calculate for a rigid body moving in two dimensions the angular momentum with respect to a fixed point, to the mass centre of the body, or to any arbitrary point.
3. Ability to solve dynamics problems for two-dimensional rigid body motions including both translation and rotation.
4. Ability to identify the type of motion produced by a mechanism and to calculate its mobility.
5. Ability to find the instantaneous centres for the various components of a mechanism.
6. Ability to calculate the relative positions, velocities and accelerations of the various components of a mechanism using vector methods, diagrammatic methods, and complex numbers / loop closure equations.
7. Ability to calculate the static and dynamic forces within a mechanism and the equivalent inertia for a mechanism.
8. Ability to calculate the gear ratios, torques, and effective inertia for simple, compound, and planetary gear trains.

Linkages and Mechanisms:
Position and Velocity

Relative velocity

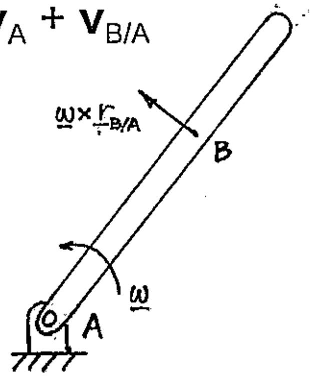


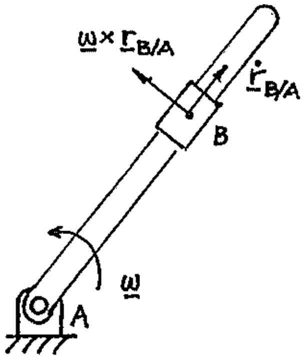
$\mathbf{r} = \mathbf{R} + \mathbf{p}$
$\dot{\mathbf{r}} = \dot{\mathbf{R}} + \dot{\mathbf{p}} + \boldsymbol{\omega} \times \mathbf{p}$

XYZ and xyz have same origin:
$\mathbf{R} = \mathbf{0}$, $\dot{\mathbf{R}} = \mathbf{0}$

radial component: $\dot{\mathbf{p}}$
tangential component: $\boldsymbol{\omega} \times \mathbf{p}$

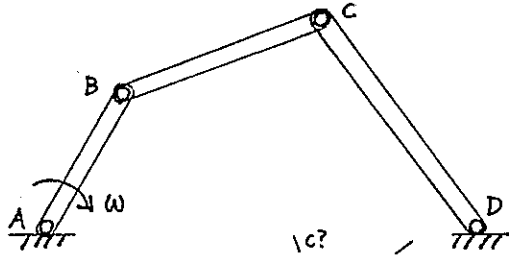
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

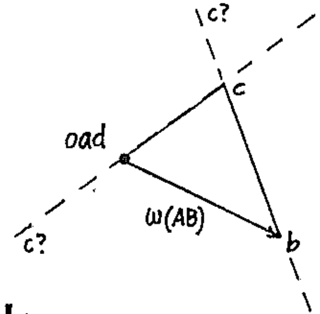




Velocity diagrams

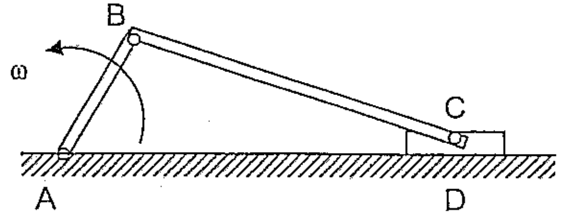
Four-bar linkage



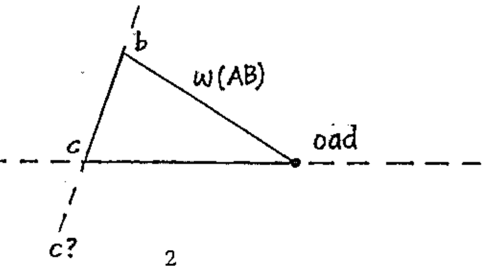


$V_B = \omega(AB)$
$V_C = \omega_{CD}(CD)$
$V_{C/B} = \omega_{BC}(BC)$

Slider-crank

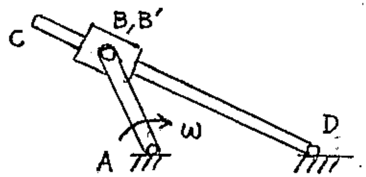


$V_B = \omega(AB)$
V_C
$V_{B/C} = \omega_{BC}(BC)$



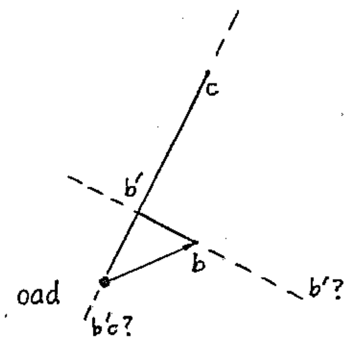
Velocity diagrams (continued)

Quick-return mechanism

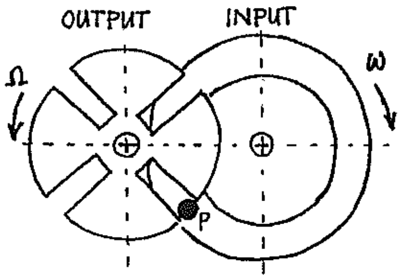


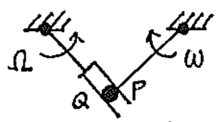
B on AB
B′ on CD

$V_B = \omega(AB)$
$V_{B'} = \omega_{CD}(B'D)$
$V_C = \omega_{CD}(CD)$
$V_{B/B'}$

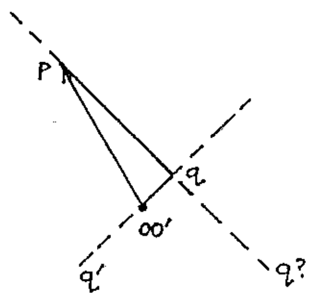


Geneva mechanism





$V_P = \omega(OP)$
$V_Q = \Omega(OQ)$



1. The input link AB of the four-bar linkage in Figure 6 has a uniform angular velocity $\omega = 30\ \text{rad s}^{-1}$ (clockwise).

   (a) For the linkage position shown, draw the velocity diagram and determine, by accurate drawing or otherwise, the velocity v_C of point C. Relevant lengths are shown in Figure 6.

   (b) Find the angular velocities ω_{BEC} and ω_{CD}.

   (c) Determine also the velocity v_E of point E (magnitude and direction), and show v_E on your velocity diagram.

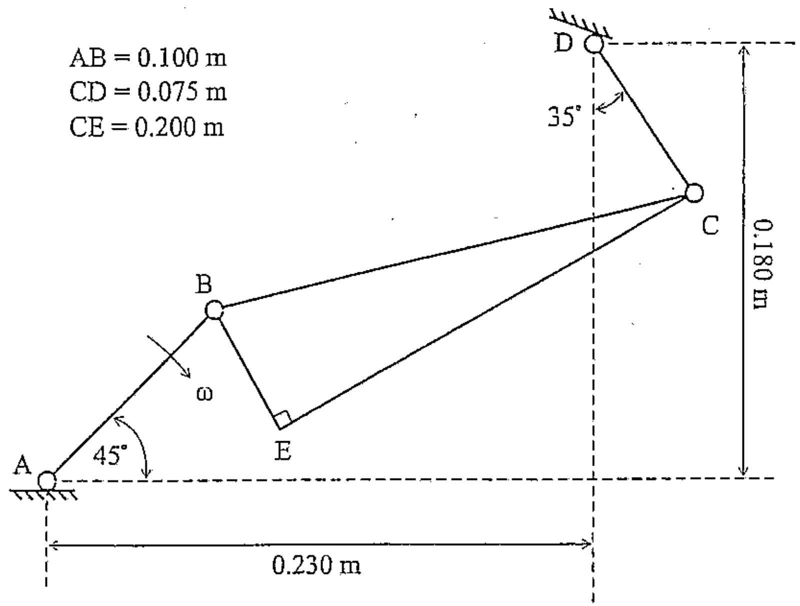
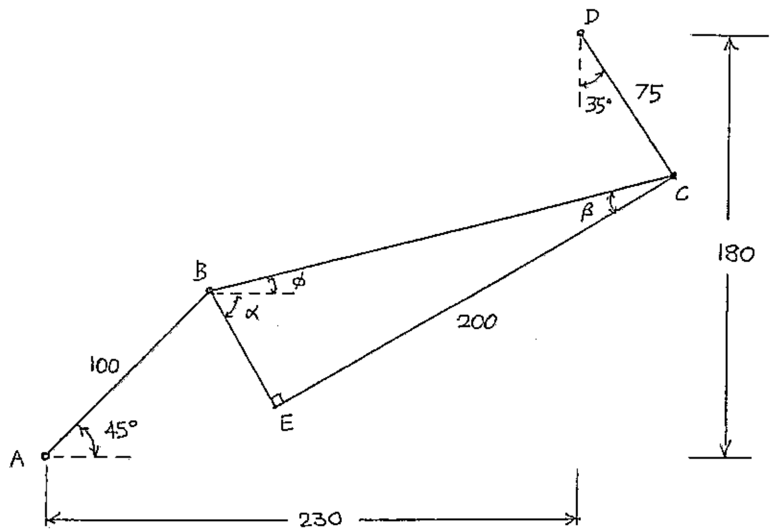


Figure 6



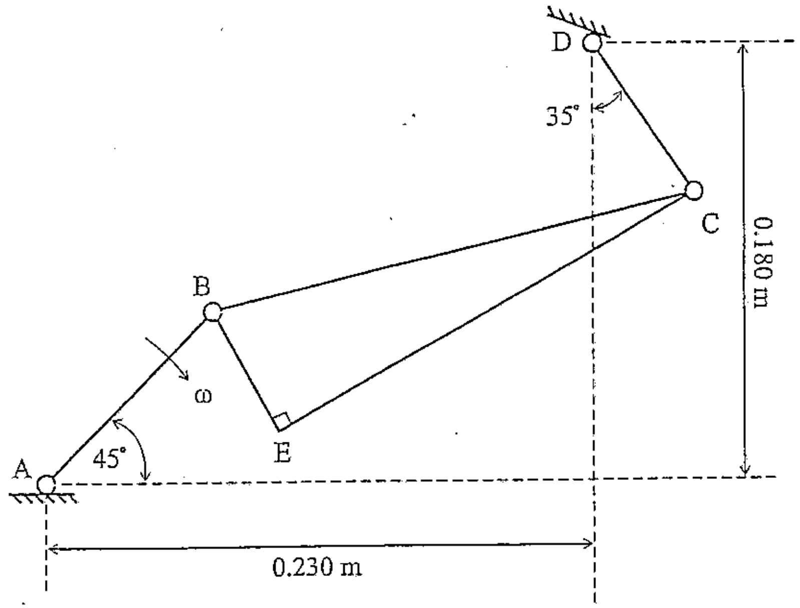
$$AB \cos 45 + BC \cos\phi - DC \sin 35 = 230$$
$$AB \sin 45 + BC \sin\phi + DC \cos 35 = 180$$

$$BC \cos\phi = 230 - AB\cos 45 + DC \sin 35 = 202.31$$
$$BC \sin\phi = 180 - AB \sin 45 - DC \cos 35 = 47.85$$

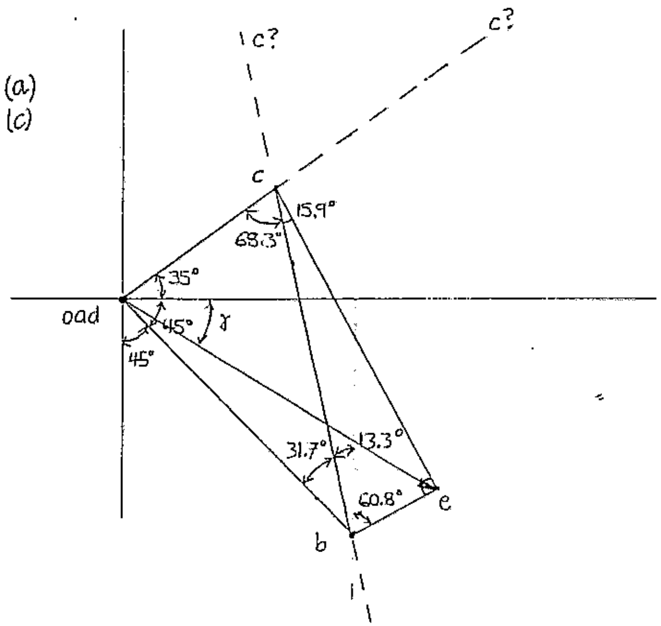
$$\tan\phi = 0.2365$$
$$\phi = 13.3° \quad \text{and} \quad BC = 208$$

$$BE^2 + CE^2 = BC^2$$
$$BE^2 = BC^2 - CE^2 = (208)^2 - (200)^2 = 3264 \qquad BE = 57.1$$

$$BC \cos(\phi+\alpha) = BE$$
$$\cos(\phi+\alpha) = BE/BC = 57.1/208$$
$$\phi+\alpha = 74.1°$$
$$\alpha = 60.8° \quad \text{and} \quad \beta = 90 - 74.1 = 15.9°$$



(a)
(c)



(a) $V_B = 30(0.1) = 3$ m/s

$V_C = 1.7$ m/s
$V_{C/B} = 3.2$ m/s

$\Big\}$ measured on diagram

$$\frac{oc}{\sin\angle obc} = \frac{ob}{\sin\angle ocb} \qquad oc = 3\,\frac{\sin 31.7}{\sin 68.3} = 1.7\ \text{m/s as above } (v_c)$$

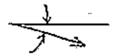
$$\frac{bc}{\sin\angle cob} = \frac{ob}{\sin\angle ocb} \qquad bc = 3\,\frac{\sin 80}{\sin 68.3} = 3.18\ \text{m/s as above } (v_{c/B})$$

(b) $$\omega_{BEC} = \frac{v_{c/B}}{BC} = \frac{3.2}{0.208} = 15.4\ \text{rad/s (ccw)}$$

$$\omega_{DC} = \frac{v_c}{DC} = \frac{1.7}{0.075} = 22.7\ \text{rad/s (ccw)}$$

(c) From "velocity image" on diagram,

$v_E = 3.35$ m/s

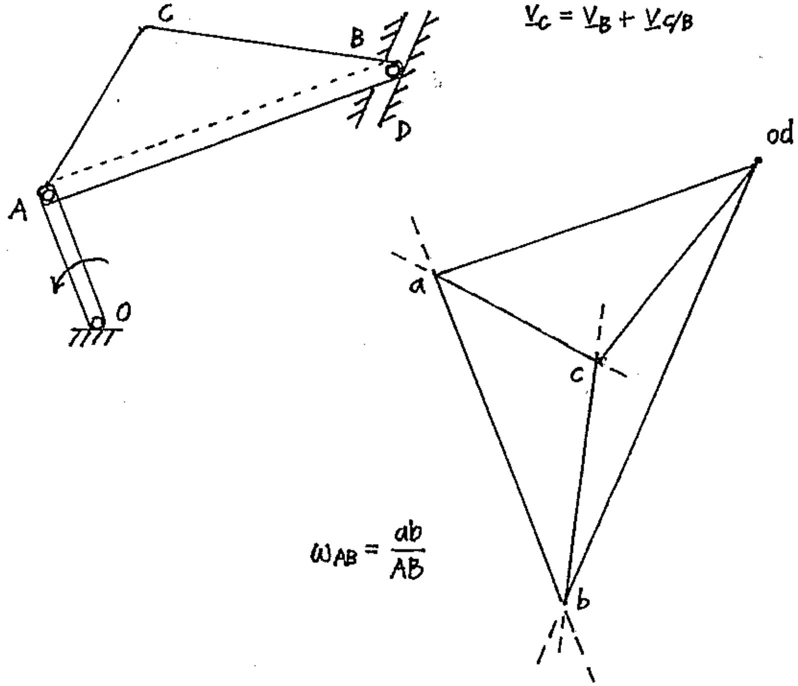


$\gamma = 31°$

Also, $v_{B/E} = 0.87$ m/s

Velocity image

$$\underline{v}_c = \underline{v}_A + \underline{v}_{c/A}$$
$$\underline{v}_c = \underline{v}_B + \underline{v}_{c/B}$$



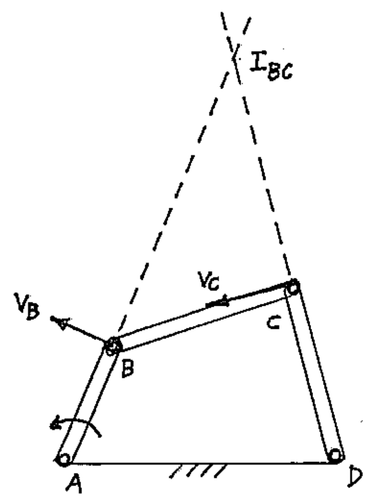
$$\omega_{AB} = \frac{ab}{AB}$$

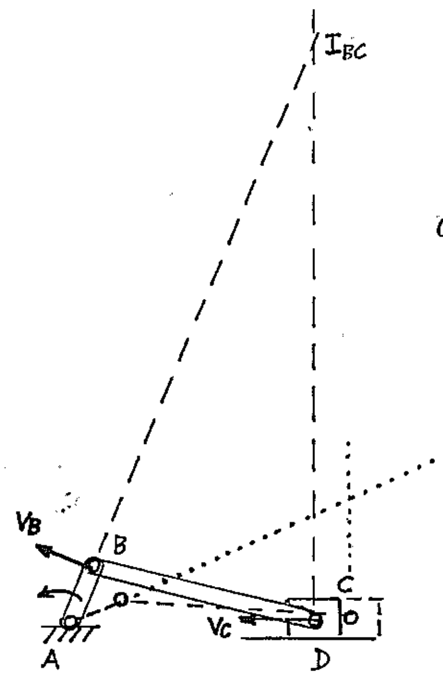
If C is any point on the link AB, its velocity is obtained by drawing a line through a perpendicular to AC and a line through b perpendicular to BC. The velocity v_c of C is given by oc to the scale of the diagram.

By virtue of the construction, we see that ΔABC is similar to Δabc, since each side of one is perpendicular to the other. The triangle abc is the *velocity image* of the triangle ABC.

Instantaneous centres

$V_B = \omega(AB)$
$V_B = \omega_{BC}(B I_{BC})$
$V_C = \omega_{BC}(C I_{BC})$
$V_C = \omega_{CD}(CD)$





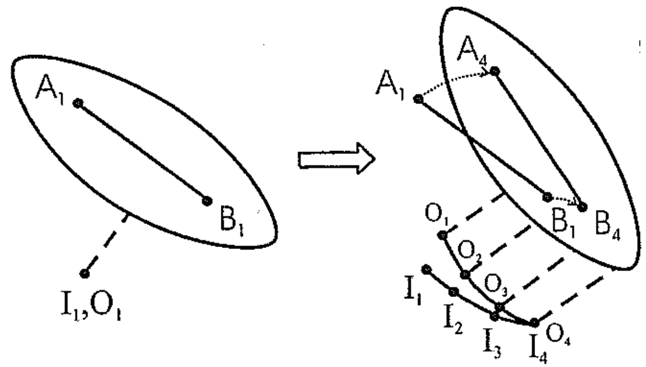
centrode - the smooth curve describing the path of a moving instantaneous centre

Centrodes

The locus of instantaneous centres (ICs) is called a **centrode**. If the locus of the ICs in space is drawn, it is called the **space-centrode**. If the locus on the ICs on the body (or "body extended") is drawn, it is called the **body-centrode**.

These two curves are generally of different shape, but at every instant they have one point in common, the IC of the body at that instant.

Consider two points, A and B, on a rigid body at four sequential body positions (i = 1,2,3,4). Points I_i are the locations of the ICs in space. Points O_i are the locations of the ICs on the body (or "body extended"). At the instant when the body is in position i, the space-centrode I_i and the body-centrode O_i coincide.

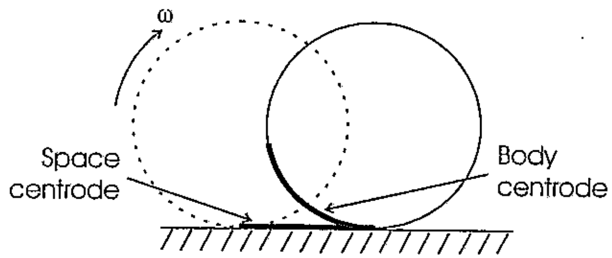


The curve passing through $I_1I_2I_3I_4$ is the space-centrode.
The curve passing through $O_1O_2O_3O_4$ is the body-centrode.

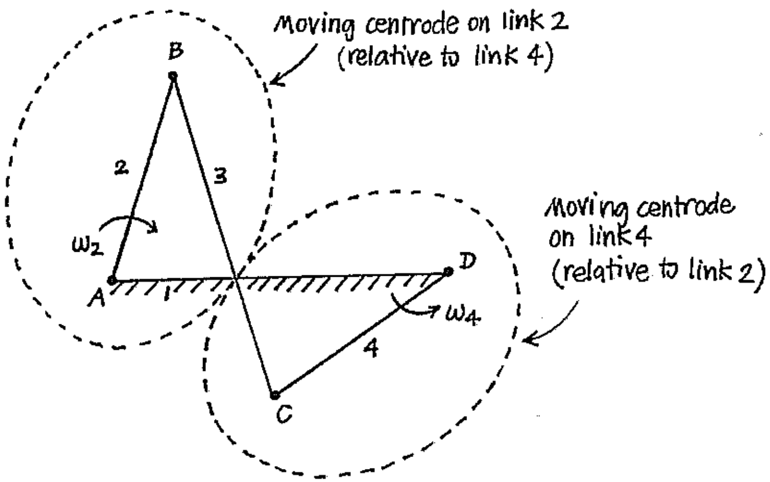
As the body moves from position 1 to position 4, the point of contact of the two centrodes will move from I_1 to I_4. The continuous motion of the body corresponds to a rolling of the body-centrode on the space-centrode.

Examples of centrodes

Cylinder rolling without sliding



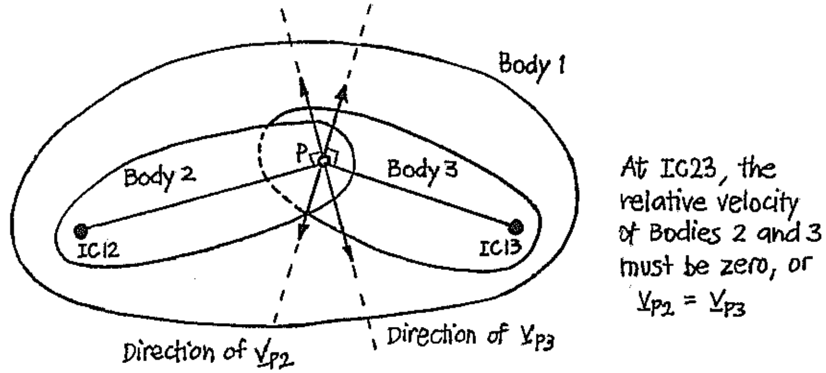
Crossed four-bar linkage

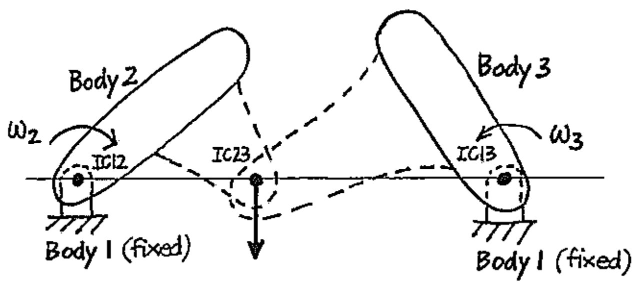


- Link 1 is fixed. Links 2, 3, and 4 can move.
- The instantaneous centre for motion of link 2 relative to link 4 (and vice versa) lies at the intersection of BC and AD.
- The paths of the instantaneous centres relative to links 2 and 4 are ellipses.
- Ellipses have rolling contact, hence elliptical gears.

Kennedy's theorem

Any **three** bodies having plane motion relative to one another have **three** instantaneous centres, and these lie on a <u>straight line</u>.





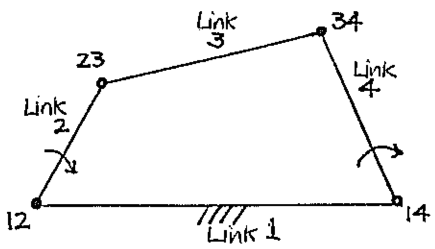
If the relative IC lies *between* the two absolute ICs, the angular velocity ratio is *negative*.

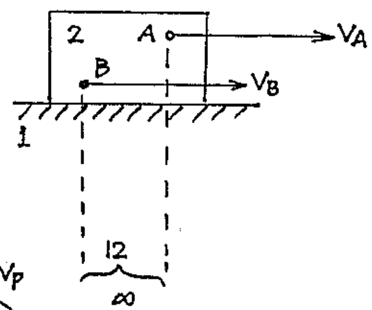
If the relative IC lies *outside* the other two, the angular velocity ratio is *positive*.

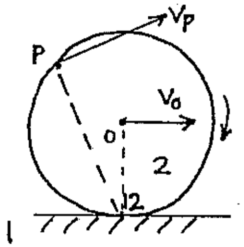
Any two links in a mechanism have motion relative to one another and thus have a common IC. Hence the number of ICs for a mechanism is equal to all the possible combinations of two from the total number of links. The number of ICs is therefore:

$$N = {}^{n}C_2 = n(n-1)/2$$

All ICs which can be found merely by inspection are called *primary ICs*. If these are found, then Kennedy's theorem can be used to find the others.

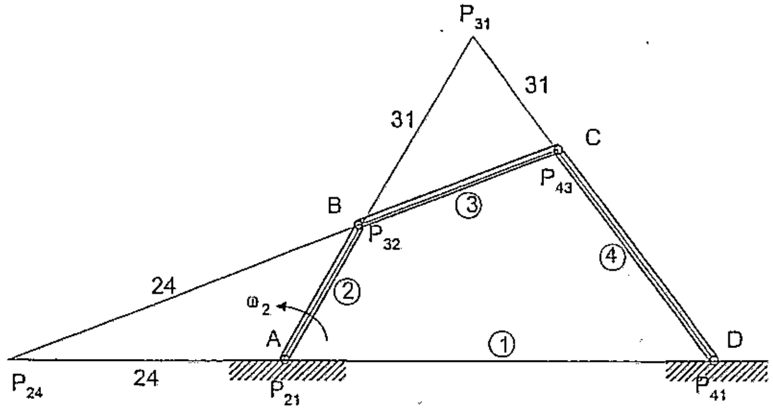






ICs are sometimes referred to as "velocity poles".

Example: four-bar linkage



Number of velocity poles $= \frac{n(n-1)}{2} = \frac{4(3)}{2} = 6$

12, 13, 14, 23, 24, 34

$P_{21}, P_{32}, P_{43}, P_{41}$ are primary ICs (poles) and are found by inspection.

P_{13}, P_{24} can be found using Kennedy's theorem.

At P_{24}, the velocities of links 2 and 4 are equal:

$\omega_4(P_{24}P_{41}) = \omega_2(P_{24}P_{21})$ $\qquad \omega_4 = \omega_2(P_{24}P_{21}/P_{24}P_{41})$

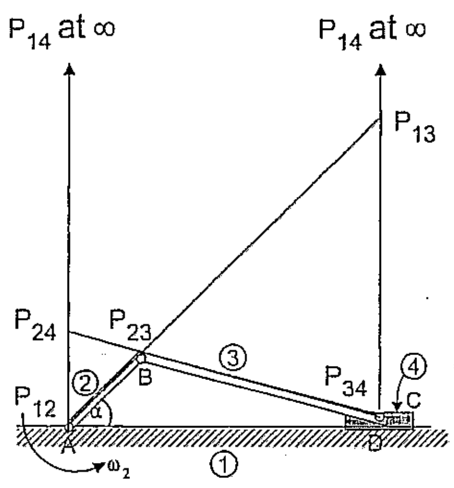
At P_{32}, the velocities of links 3 and 2 are equal:

$\omega_3(P_{31}P_{32}) = \omega_2(P_{21}P_{32})$ $\qquad \omega_3 = \omega_2(P_{21}P_{32}/P_{31}P_{32})$

Directions of ω_3, ω_4?
ω_4 same as ω_2, ω_3 in opposite direction

Compare velocity diagram approach.

Example: slider-crank



$P_{12}, P_{23}, P_{34}, P_{14}$ are primary poles.

P_{13}, P_{24} found by Kennedy's theorem.

$\omega_3 (P_{13}P_{23}) = \omega_2 (P_{12}P_{23})$ At P_{23} the velocities of links 2 and 3 are the same.

$V_C = \omega_2 (P_{12}P_{24})$ At P_{24} the velocities of links 2 and 4 are the same.

References

HD Eckhardt. *Kinematic design of machines and mechanisms.* McGraw Hill, New York, 1998.

AG Erdman and GN Sandor. *Mechanism design: analysis and synthesis (2nd ed.). Volume I.* Prentice Hall, Englewood Cliffs, NJ, 1991.

J Grosjean. *Kinematics and dynamics of mechanisms*. McGraw-Hill, London, 1991.

HH Mabie and CF Reinholtz. *Mechanisms and Dynamics of Machinery* (4th ed.) John Wiley & Sons, New York, 1987.

GH Martin. *Kinematics and dynamics of machines.* McGraw-Hill, New York, 1969.

JLM Morrison and B Crossland. *An introduction to the mechanics of machines*. Longmans, London, 1964.

Linkages and Mechanisms:
Position and Velocity

Relative velocity
(2 bodies or 2 links in a mechanism)

1 Velocity diagrams (vectors)
- Four-bar linkage
- Slider-crank
- Quick-return mechanism
- Geneva mechanism
- Velocity image

2 Instantaneous centres
(velocity poles)
- Centrodes
- Kennedy's theorem
- Examples