Course Outline for A3 Dynamics and Kinematics (8 lectures)

Lecturer: Dr Amy Zavatsky **Michaelmas Term 2004**

Expanded syllabus:

Dynamics: Angular momentum (general definition), rigid body motion with rotation and translation. Kinematics: Velocity and acceleration; motion in rotating frames of reference. Mechanisms; instantaneous centres, calculation of velocity and acceleration (including velocity and acceleration diagrams), simple, compound and epicyclic gear trains (calculation of gear ratios and torques, effective inertia).

Lecture Synopsis:

1. *Dynamics of rigid bodies: plane kinematics*

Translation (rectilinear, curvilinear), rotation about a fixed axis, and general plane motion. Linear and angular velocity and acceleration. Absolute and relative velocity and acceleration. Instantaneous centre of rotation. Rate of change of a vector with respect to a rotating coordinate frame. General motion in a rotating frame of reference.

2. *Dynamics of rigid bodies: plane kinetics*

Force, mass, and acceleration. General equations of motion. Free-body diagram, equivalent force-couple, kinetic diagram. Angular momentum about a fixed point, about the mass centre, about an arbitrary point. Work and energy (translational and rotational), power, impulse and momentum (linear and angular). General problem formulation and review.

3. *Linkages and mechanisms: basic concepts*

Definitions: mechanism, linkage (or kinematic chain), types of motion transformation, open-loop versus closed-loop, lower and higher joints (or pairs), mobility (or degrees-of-freedom). Grübler's equation for mobility. The four-bar linkage (input, output, coupler, transmission angle) and Grashof's law. Kinematic inversion. Introduction to slider-crank mechanism and quick-return mechanism. Analysis versus synthesis.

4. *Linkages and mechanisms: position and velocity*

Relative velocity (link with revolute joint, link with revolute joint and slider). Velocity diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Velocity image. Instantaneous centres and centrodes. Kennedy's theorem for three instantaneous centres. Velocity pole method for finding angular velocities in linkages (four-bar linkage, slider-crank).

5. *Linkages and mechanisms: acceleration*

Relative acceleration (link with revolute joint, link with revolute joint and slider). Acceleration diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Acceleration image. Relative acceleration of coincident particles at the point of contact of rolling elements. Equivalent mechanisms.

6. *Linkages and mechanisms: computational analysis*

Kinematic analysis by complex numbers (link with revolute joint, link with revolute joint and slider). Loop closure equations (slider-crank, quick-return mechanism). Solution of the position problem using the Newton-Raphson method. Vector chains to describe motion of an arbitrary point on a mechanism. Introduction to generalised coordinates and constraint equations.

7. *Linkages and mechanisms: forces in linkages*

Review of two-force bodies, three-force bodies, and couples. Static forces (slider-crank, four-bar linkage). D'Alembert's principle (inertia force and concept of dynamic equilibrium) in contrast to direct "F=ma" approach. Dynamic analysis (four-bar linkage, slider-crank). Virtual work method to find forces and torques. Kinetic energy and equivalent inertia.

8. *Gear trains: simple, compound, and epicyclic gear trains*

Friction drives versus gears. Examples of spur gears, rack and pinion, bevel gears, and worm gears. Spur gear terminology. Fundamental law of gearing (circular gears). Gear trains: simple, compound, planetary. Equivalent inertia.

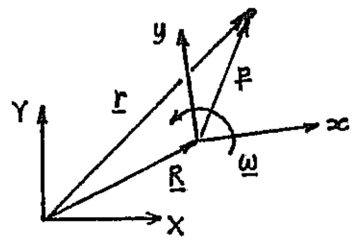
Examples Sheets: 3A3C, 3A3D

Learning Outcomes:

1. Ability to calculate absolute and relative positions, velocities, and accelerations for two-dimensional rigid body motions in fixed, translating, and rotating frames of reference.
2. Ability to calculate for a rigid body moving in two dimensions the angular momentum with respect to a fixed point, to the mass centre of the body, or to any arbitrary point.
3. Ability to solve dynamics problems for two-dimensional rigid body motions including both translation and rotation.
4. Ability to identify the type of motion produced by a mechanism and to calculate its mobility.
5. Ability to find the instantaneous centres for the various components of a mechanism.
6. Ability to calculate the relative positions, velocities and accelerations of the various components of a mechanism using vector methods, diagrammatic methods, and complex numbers / loop closure equations.
7. Ability to calculate the static and dynamic forces within a mechanism and the equivalent inertia for a mechanism.
8. Ability to calculate the gear ratios, torques, and effective inertia for simple, compound, and planetary gear trains.

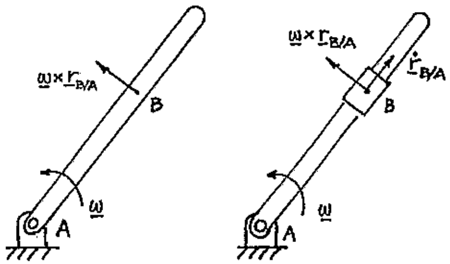
Linkages and Mechanisms: Acceleration

$\mathbf{r} = \mathbf{R} + \mathbf{p}$
$\dot{\mathbf{r}} = \dot{\mathbf{R}} + \dot{\mathbf{p}} + \omega \times \mathbf{p}$
$\ddot{\mathbf{r}} = \ddot{\mathbf{R}} + \ddot{\mathbf{p}} + \dot{\omega} \times \mathbf{p} +$
$\quad \omega \times (\omega \times \mathbf{p}) + 2\omega \times \dot{\mathbf{p}}$

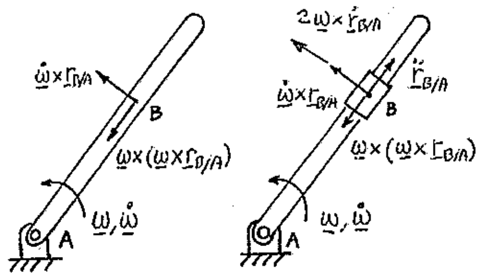


radial components: $\ddot{\mathbf{p}} + \omega \times (\omega \times \mathbf{p})$
tangential components: $\dot{\omega} \times \mathbf{p} + 2\omega \times \dot{\mathbf{p}}$

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

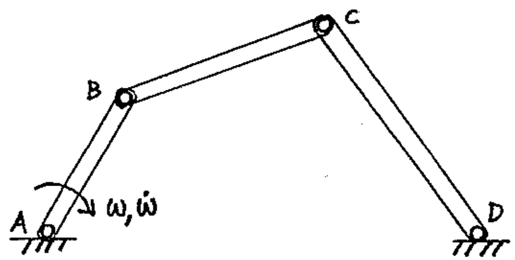


$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$



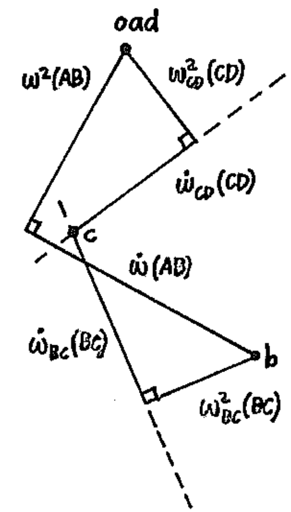
Acceleration diagrams

Four-bar linkage



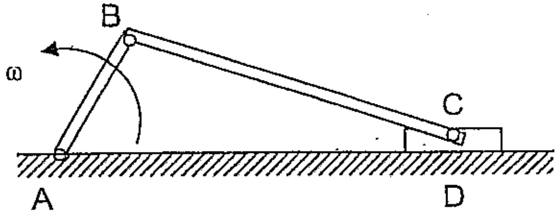
ω, $\dot{\omega}$ given
ω_{BC}, ω_{CD} found using velocity diagram, etc

radial components: $\cancel{\ddot{\mathbf{p}}}^{0} + \omega \times (\omega \times \mathbf{p})$
tangential components: $\dot{\omega} \times \mathbf{p} + 2\omega \times \cancel{\dot{\mathbf{p}}}^{0}$



Acceleration diagrams (cont'd)

Slider-crank

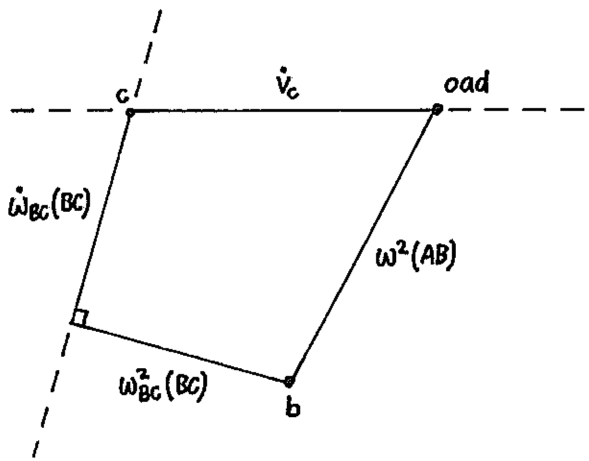


ω given, $\dot{\omega} = 0$

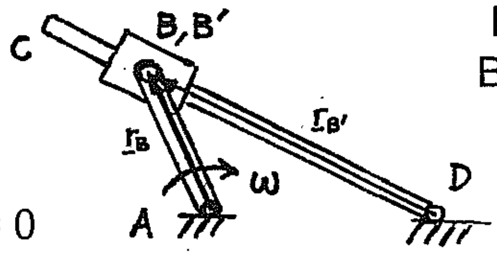
ω_{BC}, $\mathbf{v}_C$ found using velocity diagram, etc

radial components: $\ddot{\mathbf{p}} + \omega \times (\omega \times \mathbf{p})$

tangential components: $\dot{\omega} \times \mathbf{p} + 2\omega \times \dot{\mathbf{p}}$



Quick-return mechanism



B on AB
B' on CD

ω given, $\dot{\omega} = 0$

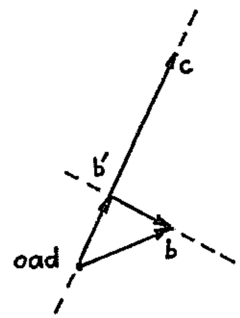
$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/B'}$

$\omega \times \mathbf{r}_B = \omega_{CD} \times \mathbf{r}_{B'} + \dot{\mathbf{r}}_{B/B'}$

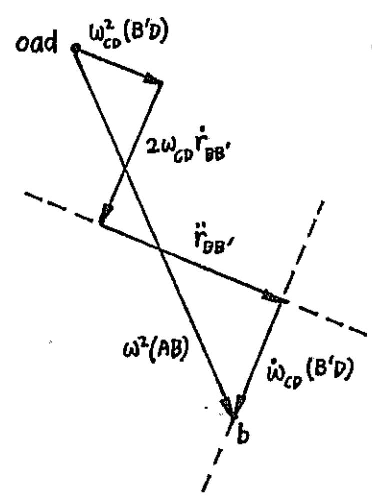
$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/B'}$

$\omega \times (\omega \times \mathbf{r}_B) = [\omega_{CD} \times (\omega_{CD} \times \mathbf{r}_{B'}) + \dot{\omega}_{CD} \times \mathbf{r}_{B'}] +$
$[\ddot{\mathbf{r}}_{B/B'} + 2\omega_{CD} \times \dot{\mathbf{r}}_{B/B'}]$

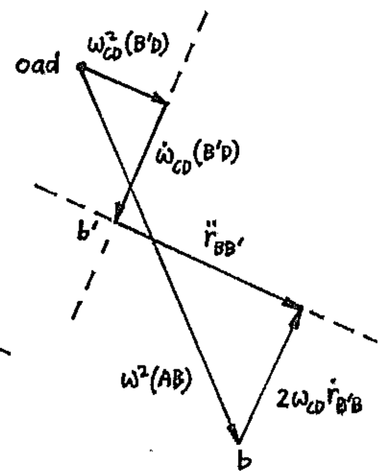
Velocity



Acceleration

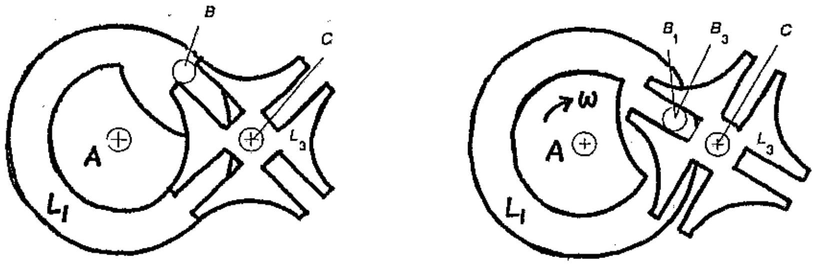


Acceleration



Acceleration diagrams (cont'd)

Geneva mechanism

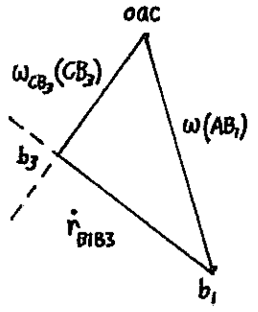


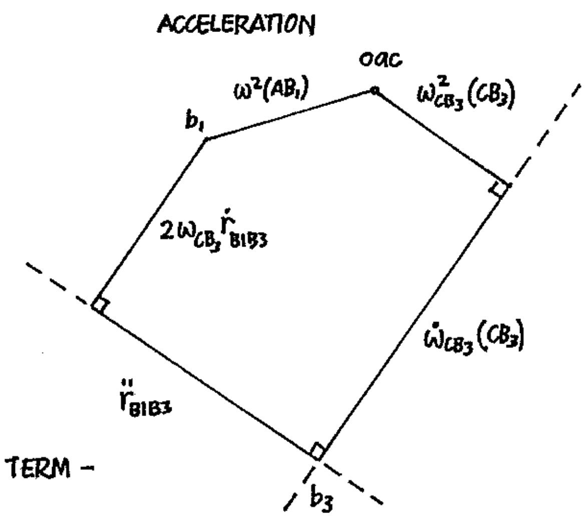
ω given, $\dot{\omega} = 0$

ω_{B3C}, $\mathbf{v}_C$ found using velocity diagram, etc

radial components: $\ddot{\mathbf{p}} + \omega \times (\omega \times \mathbf{p})$

tangential components: $\dot{\omega} \times \mathbf{p} + 2\omega \times \dot{\mathbf{p}}$





NOTE MAGNITUDE OF CORIOLIS TERM – NOT INSIGNIFICANT!

Figure 7 shows a quick-return mechanism which is driven by a torque applied to the crank at A. At the instant shown the angular velocity ω of the crank is 6 rad s^{-1} and the angular acceleration $\dot{\omega}$ of the crank is 5 rad s^{-2}. A mass of 5kg is mounted at D, the mass of the slider is 1kg and the mass of the crank and rocker may be assumed to be negligible. Using a graphical approach, or otherwise:

(a) Show that the angular velocity of BD is 2.1 rad^{-1} (to 2 significant figures).

(b) Find the angular acceleration of BD.

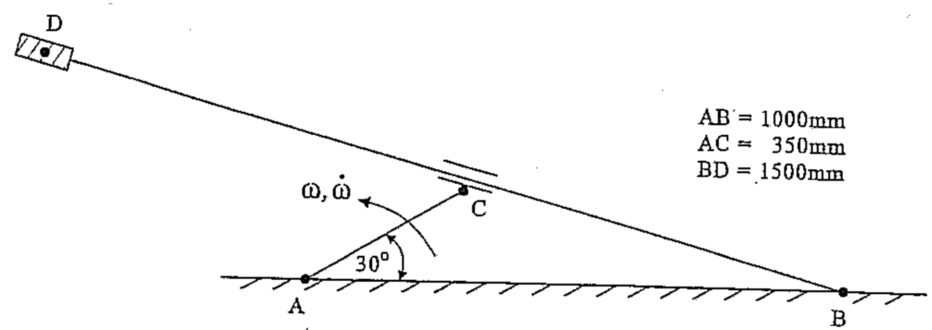
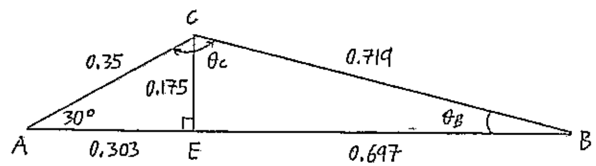


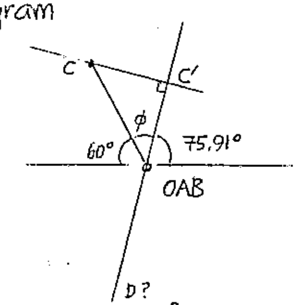
Figure 7

(a) geometry of the mechanism in its current position



$\sin\theta_B = \dfrac{0.175}{0.719}$ $\quad\theta_B = 14.09°$

velocity diagram



$V_C = \omega_{AC}\, AC$

$V_C = 6\,(0.35) = 2.1$

$\phi = 44.09°$

$V_{CC'} = V_C \sin\phi = 1.46$

$V_{C'} = V_C \cos\phi = 1.51$

$V_{C'} = \omega_{BD} BC' \qquad \omega_{BD} = 1.51/0.719 = 2.1 \text{ rad/s}$ $\qquad \underline{\omega}_{BD}$ directed into the page.

(b) Accelerations

Remember $\ddot{\underline{r}} = \ddot{\underline{R}} + \ddot{\underline{\rho}} + \underbrace{\dot{\underline{\omega}} \times \underline{\rho}}_{r\alpha} + \underbrace{\underline{\omega} \times \underline{\omega} \times \underline{\rho}}_{\omega^2 r} + \underbrace{2\underline{\omega} \times \dot{\underline{\rho}}}_{\text{Coriolis}}$

$\underline{\underline{AC}}$ $\quad \dot{\underline{\omega}} \times \underline{\rho} \qquad AC\,\dot{\omega} = 1.75 \qquad \perp$ to AC and ↖

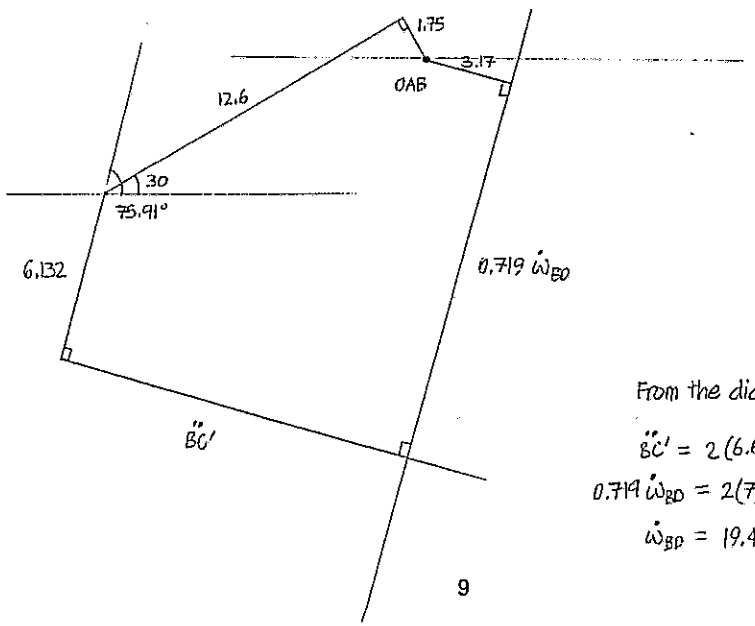
$\qquad \underline{\omega} \times \underline{\omega} \times \underline{\rho} \qquad AC\,\omega^2 = 12.6 \qquad$ along AC and ↙

$\underline{\underline{BC'}}$ $\quad \dot{\underline{\omega}} \times \underline{\rho} \qquad \dot{\omega}_{BD}(BC') = 0.719\,\dot{\omega}_{BD} \qquad \perp$ to BD

$\qquad \underline{\omega} \times \underline{\omega} \times \underline{\rho} \qquad \omega_{BD}^2(BC') = 3.17 \qquad$ along BD and ↘

$\qquad \ddot{\underline{\rho}} \qquad (\ddot{BC'})$ along BD

$\qquad 2\underline{\omega} \times \dot{\underline{\rho}} \qquad 2\omega_{BD}(\underset{\uparrow \, V_{cc'}}{\dot{BC'}}) = 2(2.1)(1.46) = 6.132 \qquad \perp$ to BD



From the diagram,

$\ddot{BC'} = 2(6.65) = 13.3 \text{ m/s}^2$

$0.719\,\dot{\omega}_{BD} = 2(7)$

$\dot{\omega}_{BD} = 19.47 \text{ rad/s}^2$

Using the geometry of the diagram,

$\ddot{BC'} = 3.17 + 1.75\cos(60-14.09) + 12.6\cos(30+14.09) = 13.44 \text{ m/s}^2$

$0.719\,\dot{\omega}_{BD} = 6.132 + 12.6\sin(30+14.09) - 1.75\sin(60-14.09) = 13.642$

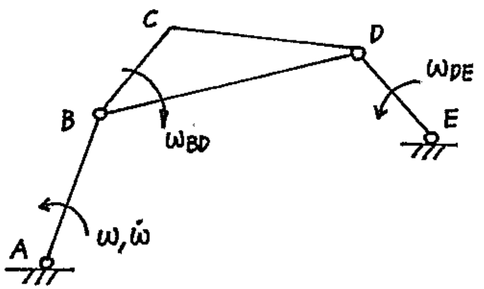
$\dot{\omega}_{BD} = 18.97 \text{ rad/s}^2 \qquad \dot{\underline{\omega}}_{BD}$ directed out of the page

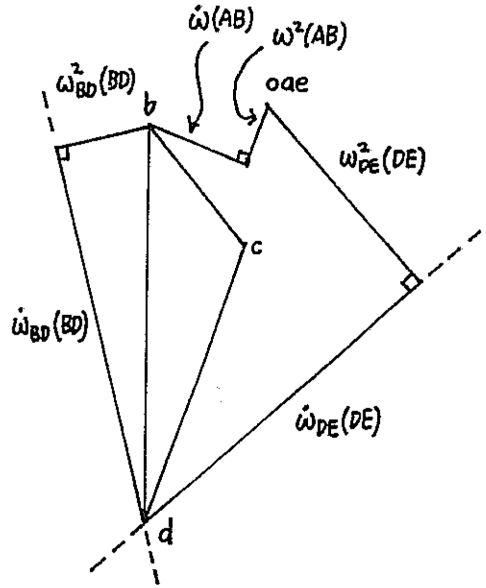
Acceleration image

$a_{C/B} = a_{(C/B)n} + a_{(C/B)t}$

$a_{C/B} = \{ [(BC)\omega^2]^2 + [(BC)\dot{\omega}]^2 \}^{1/2} = BC\{\omega^4 + \dot{\omega}^2\}^{1/2}$

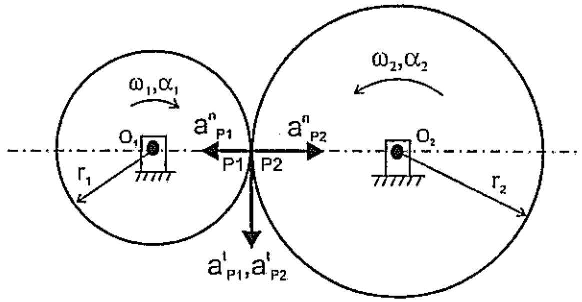
Since ω and $\dot{\omega}$ are properties of the whole link, this indicates that the relative acceleration is proportional to the distance between the points.





ΔBCD is similar to Δbcd
Care must be taken not to flip the image over. Since BCD are in clockwise order, bcd must be in the same order.

Relative acceleration of coincident particles at the point of contact of rolling elements



Consider the pitch circles of a pair of gears in mesh. Point P_1 on gear 1 and point P_2 on gear 2 are coincident at the point of contact. Rolling, but no sliding occurs.

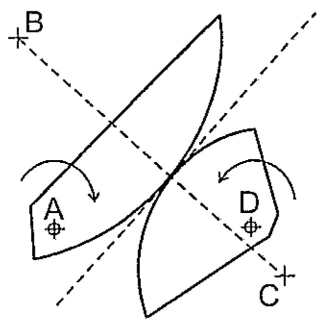
Tangential velocities are $v_{P1} = r_1\omega_1$ and $v_{P2} = r_2\omega_2$.
Since there is no sliding, $v_{P1} = v_{P2}$ and $v_{P1/P2} = 0$.

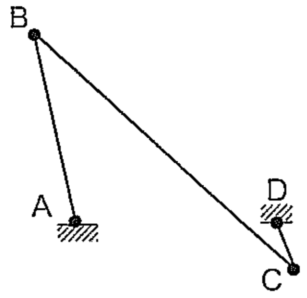
The tangential accelerations of P_1 and P_1 are $a^t_{P1} = r_1\alpha_1$ and $a^t_{P2} = r_2\alpha_2$.
Since there is no slipping at the point of contact, $a^t_{P1} = a^t_{P2}$ and $a^t_{P1/P2} = 0$.

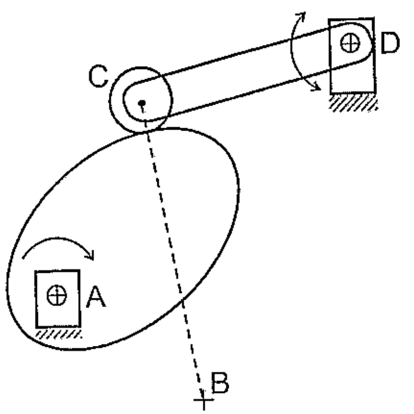
The normal acceleration of P_1 is $a^n_{P1} = (\omega_1)^2 r_1$ towards O_1, while the normal acceleration of P_2 is $a^n_{P2} = (\omega_2)^2 r_2$ towards O_2. Since $a^n_{P1} \neq a^n_{P2}$, $a^n_{P1/P2} \neq 0$.

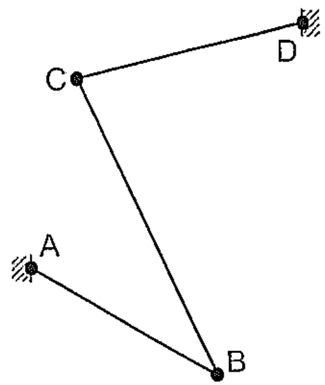
Equivalent mechanisms

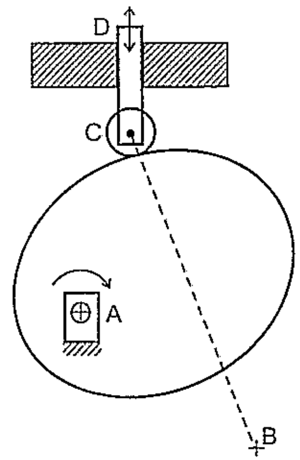
The calculation of velocities and accelerations of certain mechanisms with contact surfaces, such as cams, can be simplified by replacing these surfaces by four-bar linkages that are kinematically equivalent to them *at a particular instant.*

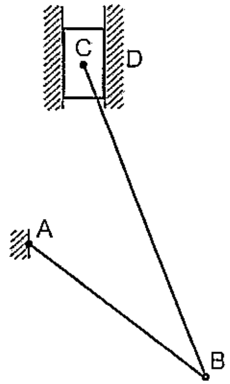












References

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Linkages and Mechanisms: Acceleration

Relative acceleration
(2 bodies or 2 links in a mechanism)

1 Acceleration diagrams (vectors)
Four-bar linkage
Slider-crank
Quick-return mechanism
Geneva mechanism

2 Acceleration image

3 Relative acceleration at point of contact

4 Equivalent mechanisms