Course Outline for A3 Dynamics and Kinematics (8 lectures)

Lecturer: Dr Amy Zavatsky **Michaelmas Term 2004**

Expanded syllabus:

Dynamics: Angular momentum (general definition), rigid body motion with rotation and translation. Kinematics: Velocity and acceleration; motion in rotating frames of reference. Mechanisms; instantaneous centres, calculation of velocity and acceleration (including velocity and acceleration diagrams), simple, compound and epicyclic gear trains (calculation of gear ratios and torques, effective inertia).

Lecture Synopsis:

1. *Dynamics of rigid bodies: plane kinematics*

Translation (rectilinear, curvilinear), rotation about a fixed axis, and general plane motion. Linear and angular velocity and acceleration. Absolute and relative velocity and acceleration. Instantaneous centre of rotation. Rate of change of a vector with respect to a rotating coordinate frame. General motion in a rotating frame of reference.

2. *Dynamics of rigid bodies: plane kinetics*

Force, mass, and acceleration. General equations of motion. Free-body diagram, equivalent force-couple, kinetic diagram. Angular momentum about a fixed point, about the mass centre, about an arbitrary point. Work and energy (translational and rotational), power, impulse and momentum (linear and angular). General problem formulation and review.

3. *Linkages and mechanisms: basic concepts*

Definitions: mechanism, linkage (or kinematic chain), types of motion transformation, open-loop versus closed-loop, lower and higher joints (or pairs), mobility (or degrees-of-freedom). Grübler's equation for mobility. The four-bar linkage (input, output, coupler, transmission angle) and Grashof's law. Kinematic inversion. Introduction to slider-crank mechanism and quick-return mechanism. Analysis versus synthesis.

4. *Linkages and mechanisms: position and velocity*

Relative velocity (link with revolute joint, link with revolute joint and slider). Velocity diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Velocity image. Instantaneous centres and centrodes. Kennedy's theorem for three instantaneous centres. Velocity pole method for finding angular velocities in linkages (four-bar linkage, slider-crank).

5. *Linkages and mechanisms: acceleration*

Relative acceleration (link with revolute joint, link with revolute joint and slider). Acceleration diagrams: four-bar linkage, slider-crank, quick-return mechanism, Geneva mechanism. Acceleration image. Relative acceleration of coincident particles at the point of contact of rolling elements. Equivalent mechanisms.

6. *Linkages and mechanisms: computational analysis*

Kinematic analysis by complex numbers (link with revolute joint, link with revolute joint and slider). Loop closure equations (slider-crank, quick-return mechanism). Solution of the position problem using the Newton-Raphson method. Vector chains to describe motion of an arbitrary point on a mechanism. Introduction to generalised coordinates and constraint equations.

7. *Linkages and mechanisms: forces in linkages*

Review of two-force bodies, three-force bodies, and couples. Static forces (slider-crank, four-bar linkage). D'Alembert's principle (inertia force and concept of dynamic equilibrium) in contrast to direct "F=ma" approach. Dynamic analysis (four-bar linkage, slider-crank). Virtual work method to find forces and torques. Kinetic energy and equivalent inertia.

8. *Gear trains: simple, compound, and epicyclic gear trains*

Friction drives versus gears. Examples of spur gears, rack and pinion, bevel gears, and worm gears. Spur gear terminology. Fundamental law of gearing (circular gears). Gear trains: simple, compound, planetary. Equivalent inertia.

Examples Sheets: 3A3C, 3A3D

Learning Outcomes:

1. Ability to calculate absolute and relative positions, velocities, and accelerations for two-dimensional rigid body motions in fixed, translating, and rotating frames of reference.
2. Ability to calculate for a rigid body moving in two dimensions the angular momentum with respect to a fixed point, to the mass centre of the body, or to any arbitrary point.
3. Ability to solve dynamics problems for two-dimensional rigid body motions including both translation and rotation.
4. Ability to identify the type of motion produced by a mechanism and to calculate its mobility.
5. Ability to find the instantaneous centres for the various components of a mechanism.
6. Ability to calculate the relative positions, velocities and accelerations of the various components of a mechanism using vector methods, diagrammatic methods, and complex numbers / loop closure equations.
7. Ability to calculate the static and dynamic forces within a mechanism and the equivalent inertia for a mechanism.
8. Ability to calculate the gear ratios, torques, and effective inertia for simple, compound, and planetary gear trains.

Linkages and Mechanisms:
Forces in Linkages

To design a mechanism for strength, it is necessary to know the forces and torques acting on each link and at each joint.

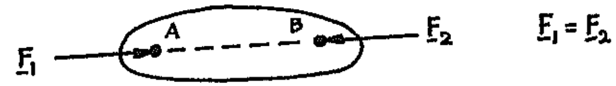
Forces in machines arise from various sources: forces due to the weight of the parts, forces of assembly, forces from applied loads, friction forces, inertia forces, spring forces, impact forces, and forces due to changes in temperature.

Forces associated with the principal function of the machine are usually known or can be assumed (gas force on piston, resistance of cutting tool in Whitworth quick-return mechanism). These forces are treated as static forces in machine analysis.

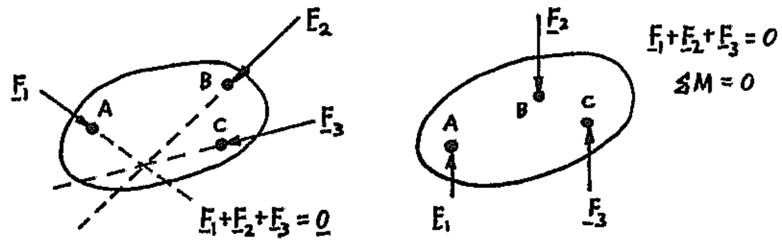
In mechanisms operating at high speed, the forces that produce acceleration of the links are often greater than the static forces related to the primary function of the machine.

Force Determination - Reminders

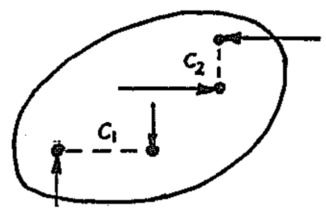
1. If a rigid body acted upon by two forces is in static equilibrium, then the two forces must have the same magnitude, the same line of action, and opposite directions.



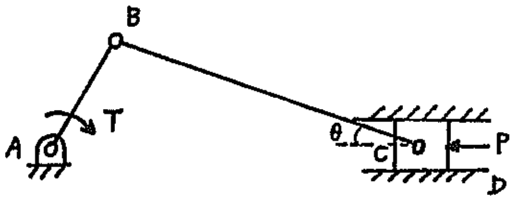
2. If a rigid body acted upon by three forces is in static equilibrium, the lines of action of the three forces must either be concurrent or parallel.



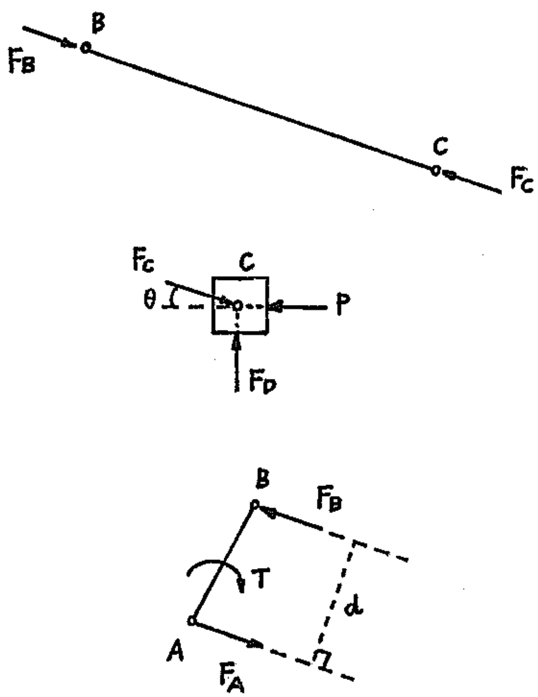
3. If a rigid body acted upon by a couple is in static equilibrium, then it must also be acted upon by another coplanar couple equal in magnitude and opposite in direction.



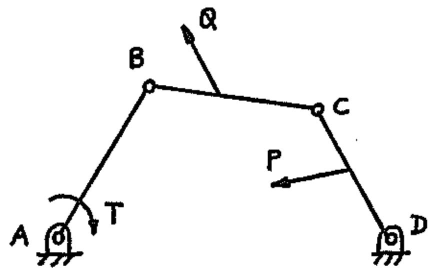
Static Forces: Slider-crank



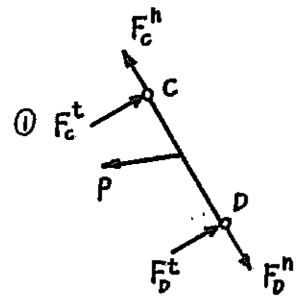
A force P is applied to the piston. For equilibrium, a torque T must be exerted on AB by the shaft at A. Find T.

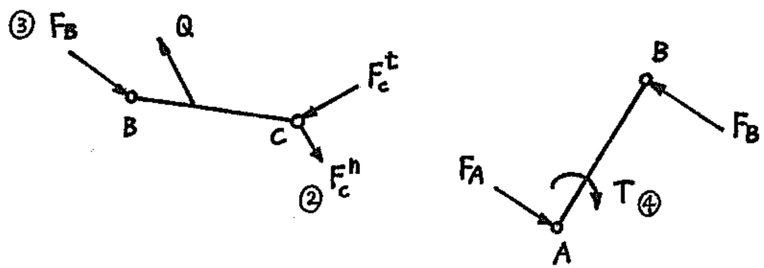


Static Forces: Four-bar linkage



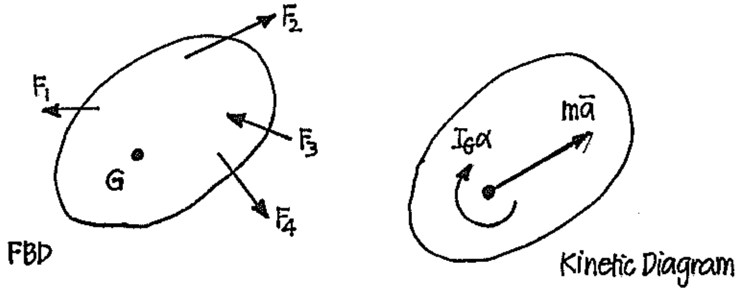
Force P is applied to link CD; force Q is applied to link BC. For equilibrium, a torque T must be exerted on AB by the shaft at A. Find T.





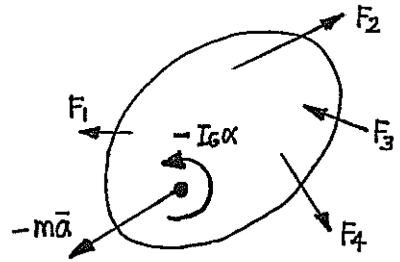
Equations of motion (2d)

$\Sigma \mathbf{F} = m\,\bar{\mathbf{a}} \qquad \Sigma M_G = I_G\alpha$



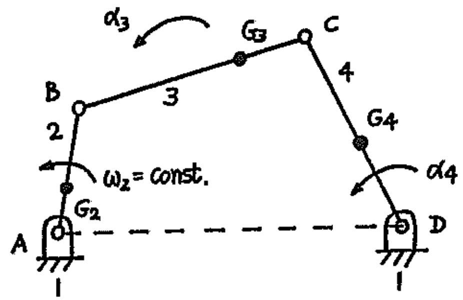
D'Alembert's Principle

$\Sigma \mathbf{F} - m\,\bar{\mathbf{a}} = \mathbf{0} \qquad \Sigma M_G - I_G\alpha = 0$



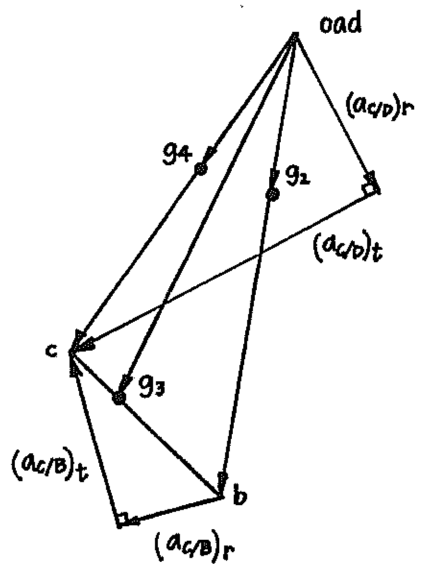
- *Traité de Dynamique 1743*
- "inertia force", "dynamic equilibrium"
- apparent transformation of a problem in dynamics to one in statics

Dynamic Analysis: Four-bar linkage



FIND THE TORQUE T WHICH THE SHAFT AT A MUST EXERT ON LINK 2 TO GIVE THE DESIRED MOTION.

ACCELERATION DIAGRAM :

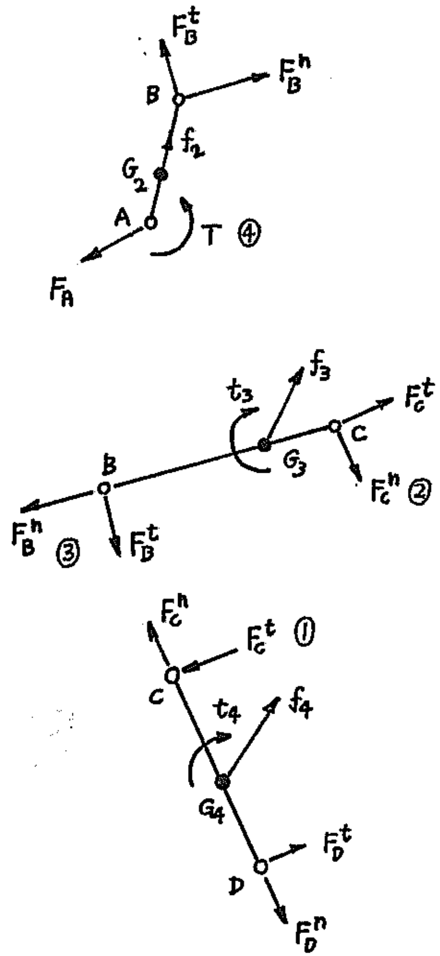


$F_4 = m_4 a_4$

"Inertia force" $f_4 = -m_4 a_4$

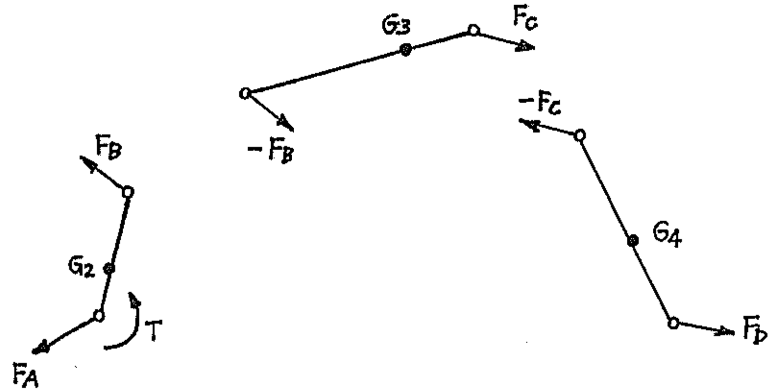
$T_4 = I_4 \alpha_4$

"Inertia torque" $t_4 = -I_4 \alpha_4$



"SHAKING FORCE" IS THE RESULTANT OF ALL THE FORCES ACTING ON THE FRAME OF A MECHANISM DUE TO INERTIA FORCES ONLY. THIS MAY SET UP TROUBLESOME VIBRATIONS IN THE FRAME.

MATRIX METHOD



Unknowns : $\underline{F}_A, \underline{F}_B, \underline{F}_C, \underline{F}_D, T$ (9)

Equations : 9 equations of motion

Assume forces due to weights of links are negligible.

Link 2 : $\Sigma \underline{F} = \underline{F}_A + \underline{F}_B = m_2 \underline{a}_2$

$\Sigma \underline{M}_{G2} = \underline{r}_{A_2} \times \underline{F}_A + \underline{r}_{B_2} \times \underline{F}_B + \underline{T} = I_{G2} \underline{\alpha}_2$

Link 3 : $\Sigma \underline{F} = -\underline{F}_B + \underline{F}_C = m_3 \underline{a}_3$

$\Sigma \underline{M}_{G3} = \underline{r}_{B_3} \times (-\underline{F}_B) + \underline{r}_{C_3} \times \underline{F}_C = I_{G3} \underline{\alpha}_3$

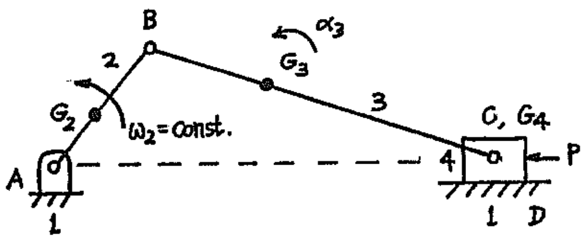
Link 4 : $\Sigma \underline{F} = -\underline{F}_C + \underline{F}_D = m_4 \underline{a}_4$

$\Sigma \underline{M}_{G4} = \underline{r}_{C4} \times (-\underline{F}_C) + \underline{r}_{D4} \times \underline{F}_D = I_{G4} \underline{\alpha}_4$

$\underline{r}_{Ai}, \underline{r}_{Bi}, \underline{r}_{Ci}, \underline{r}_{Di}$ point from G_i to the point at which $\underline{F}_A, \underline{F}_B, \underline{F}_C, \underline{F}_D$ are applied.

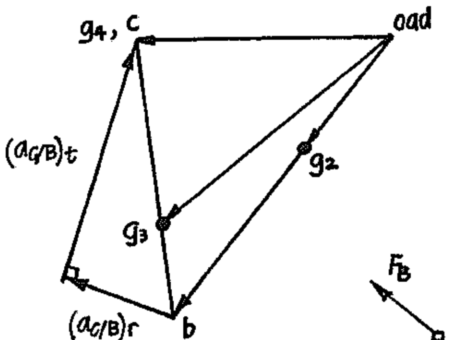
Write $\Sigma \underline{F}$ equations in terms of global xy coord system.

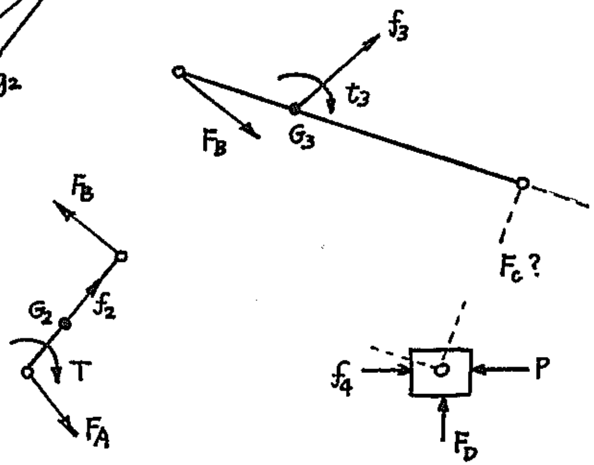
Dynamic Analysis: Slider-crank



P IS THE FORCE ON THE PISTON DUE TO GAS PRESSURE.
ω_2 IS KNOWN AND ASSUMED TO BE CONSTANT.
FIND THE TORQUE T WHICH CRANK 2 EXERTS ON THE CRANKSHAFT.

ACCELERATION DIAGRAM





TORQUE THAT CRANK EXERTS ON CRANKSHAFT IS EQUAL TO T BUT OPPOSITE IN SENSE.

Virtual Work

The method of force analysis just presented was based on the principle of equilibrium of forces. Another method applicable to linkages is that of *virtual work*.

The *principle of virtual work* states that if a system is in equilibrium under the action of forces and torques, the total work done by these forces and torques is equal to zero for small (virtual) displacements compatible with the constraints.

The work done by a force is $\delta U = \mathbf{F} \cdot \boldsymbol{\delta}\mathbf{r}$
The work done by a torque is $\delta U = \mathbf{T} \cdot \boldsymbol{\delta\theta}$

So, for a mechanism acted upon by forces and torques, $\delta U = \Sigma\, \mathbf{F} \cdot \boldsymbol{\delta}\mathbf{r} + \Sigma\, \mathbf{T} \cdot \boldsymbol{\delta\theta} = 0$

Since such small displacements take place during the same interval of time, the equation above can be divided by δt and the limit taken as $\delta t \to 0$, giving:
$\Sigma\, \mathbf{F} \cdot \mathbf{v} + \Sigma\, \mathbf{T} \cdot \boldsymbol{\omega} = 0$

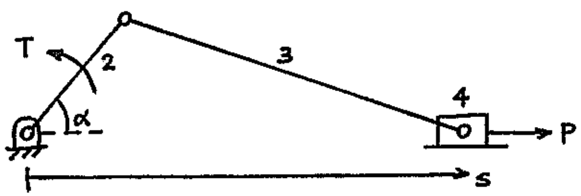
The previous equation can be modified to include all inertial forces and torques, giving:

$\Sigma \mathbf{F} \cdot \mathbf{v} + \Sigma \mathbf{T} \cdot \boldsymbol{\omega} + \Sigma \mathbf{F}_i \cdot \mathbf{v} + \Sigma \mathbf{T}_i \cdot \boldsymbol{\omega} = 0$, where

$\mathbf{F}_i = -m\mathbf{a}$ (inertial force) and $\mathbf{T}_i = -I\boldsymbol{\alpha}$ (inertial torque)
Note the sign!

Note that only the virtual work done by the external forces and torques appears in these equations. The internal forces between connecting links occur in pairs. These are equal in magnitude but opposite in direction, so their net work during any displacement is zero. Thus, the principle of virtual work cannot be used to evaluate bearing forces between links.

Slider-crank



In the simplest case, inertial effects may be neglected. Given P, T may be found using:

$P\,\delta s + T\,\delta\alpha = 0$ or $Pv + T\omega = 0$

Figure 7 shows a quick-return mechanism which is driven by a torque applied to the crank at A. At the instant shown the angular velocity ω of the crank is 6 rad s^{-1} and the angular acceleration $\dot{\omega}$ of the crank is 5 rad s^{-2}. A mass of 5kg is mounted at D, the mass of the slider is 1kg and the mass of the crank and rocker may be assumed to be negligible. Using a graphical approach, or otherwise:

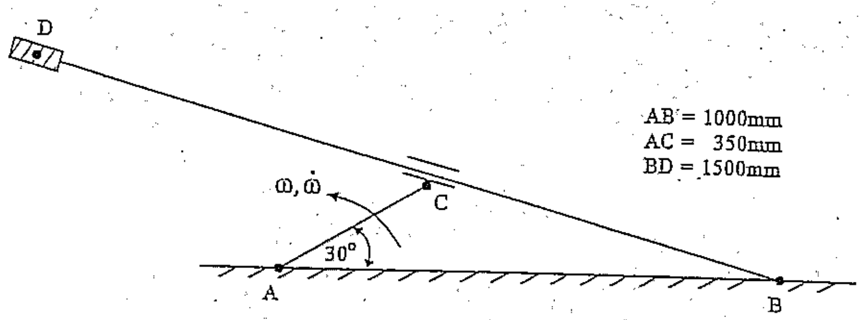
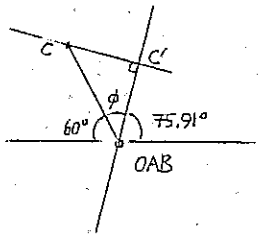


Figure 7

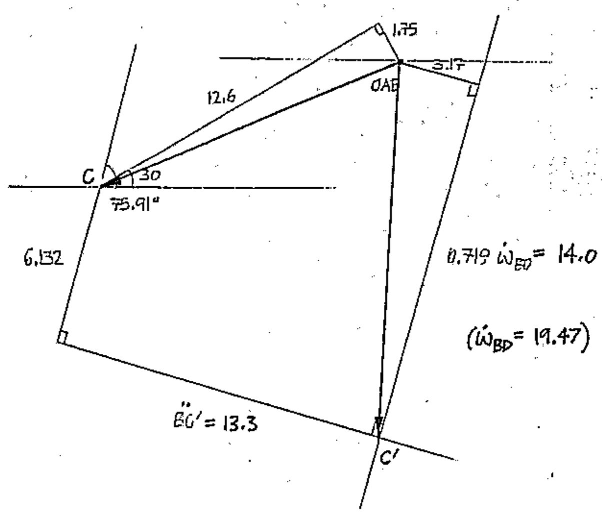
Evaluate the instantaneous torque, T, applied to the crank at A. Neglect the influence of frictional and gravitational forces.

VELOCITY DIAGRAM



$V_C = 2.1$ m/s
$V_{C/C'} = 1.46$ m/s
$V_{C'} = 1.51$ m/s
$\omega_{BD} = 2.1$ rad/s

ACCELERATION DIAGRAM



Mass of crank and rocker assumed to be negligible.

Neglect influence of friction and gravity.

$\sum \underline{F}\cdot\underline{v} + \sum \underline{T}\cdot\underline{\omega} + \sum \underline{F_i}\cdot\underline{v} + \sum \underline{T_i}\cdot\underline{\omega} = 0$

$\sum \underline{F}\cdot\underline{v} = 0$ since no external forces.

$\sum \underline{T}\cdot\underline{\omega} = T(6)$

$\sum \underline{F_i}\cdot\underline{v_i} = \sum (-m_i \underline{a_i})\cdot\underline{v_i}$

For mass mounted at D

$m = 5$ kg

$v = \omega_{BD}(BD) = (2.1)(1.5) = 3.15$ m/s ↗ (see velocity diagram)

$a_r = (\omega_{BD})^2(BD) = (2.1)^2(1.5) = 6.615$ m/s^2 ↘ (see acceleration diagram)

$a_t = \dot{\omega}_{BD}(BD) = (19.47)(1.5) = 29.205$ m/s^2 ↙

$(-m\underline{a})\cdot\underline{v} = 5(29.205)(3.15) = 459.98$ N m/s

For slider

$m = 1$ kg

$v = 1.46$ m/s ↖ (see velocity diagram)

$a_r = 13.3 - 3.17 = 10.13$ m/s^2 ↖ (see acceleration diagram)

$a_n = 0.719\,\dot{\omega}_{BD} - 6.132 = 7.87$ m/s^2 ↙

$(-m\underline{a})\cdot\underline{v} = 1(-10.13)(1.46) = -14.79$ N m/s

So $\sum \underline{F_i}\cdot\underline{v_i} = 459.98 + (-14.79) = 445.19$ N m/s

$\sum \underline{T_i}\cdot\underline{\omega_i} = 0$

$6T + 445.19 = 0$ and $T = -74.2$ N m

Torque required to move the mechanism is in opposite direction,

so torque = 74.2 Nm

Kinetic Energy and Equivalent Inertia

If a link in a mechanism is in motion, then its kinetic energy is given by $T_i = \frac{1}{2} m_i v_i^2 + \frac{1}{2} I_i \omega_i^2$

Hence for a mechanism consisting of N elements, its total kinetic energy will be

$T = \sum_{i=1}^{N} T_i = \sum_{i=1}^{N} (\frac{1}{2} m_i v_i^2 + \frac{1}{2} I_i \omega_i^2)$

If ω_1 is the angular velocity of the input to the mechanism, then it is possible to write:

$v_i = k_i\,\omega_1$ and $\omega_i = K_i \omega_1$

The kinetic energy can then be expressed as:

$T = \frac{1}{2} I_e \omega_1^2$

where $I_e = \Sigma\, m_i k_i^2 + I_i K_i^2$

I_e is referred to as the equivalent inertia, reflected inertia, or generalised inertia. It represents the inertia of the mechanism as seen by the input. k_i and K_i depend on the configuration of the mechanism at a particular instant, so I_e varies during the cycle of operation of the mechanism.

References

J Grosjean. *Kinematics and dynamics of mechanisms*. McGraw-Hill, London, 1991.

EJ Haug. *Computer aided kinematics and dynamics of mechanical systems. Volume I: Basic Methods*. Allyn and Bacon, Boston, 1989.

HH Mabie and CF Reinholtz. *Mechanisms and Dynamics of Machinery* (4th ed.) John Wiley & Sons, New York, 1987.

GH Martin. *Kinematics and dynamics of machines*. McGraw-Hill, New York, 1969.