

Lecture 4

Stresses on Inclined Sections

Shear stress and shear strain.

Equality of shear stresses on perpendicular planes.

Hooke's law in shear.

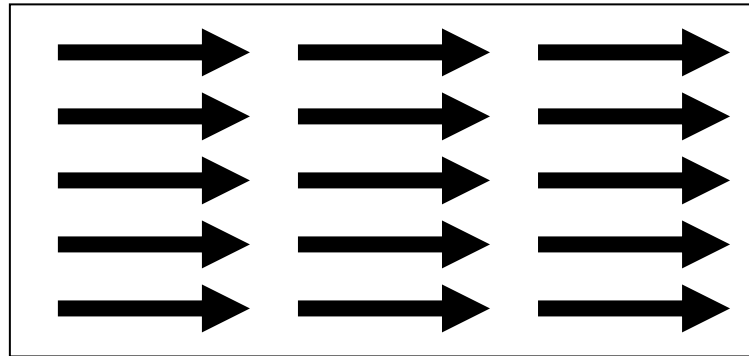
Normal and shear stresses on inclined sections.

Maximum stresses on a bar in tension.

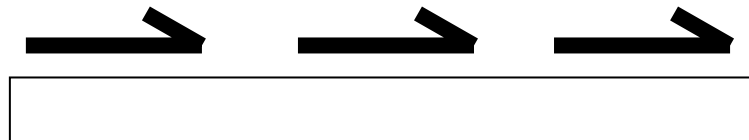
Introduction to stress elements.

Shear stress

Shear stress acts tangential to the surface of a material.



Top view



Side view

Average shear stress $\tau = V / A$

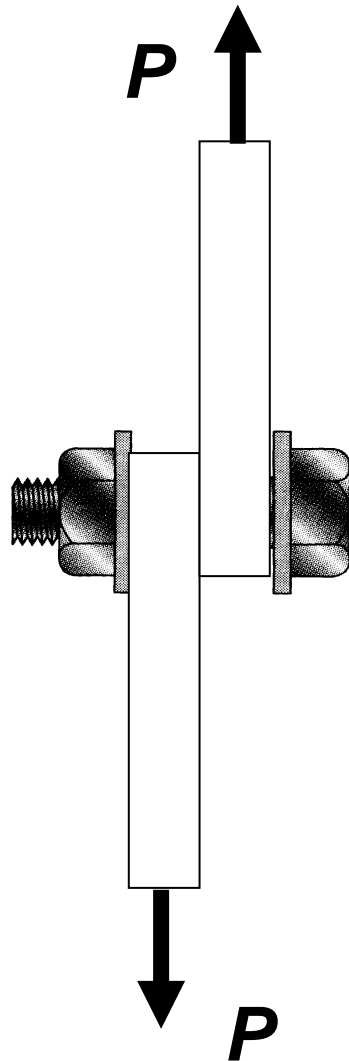
Greek letter τ (tau)

V = shear force

A = area on which it acts

(Disregard friction in the calculations to err on the conservative side.)

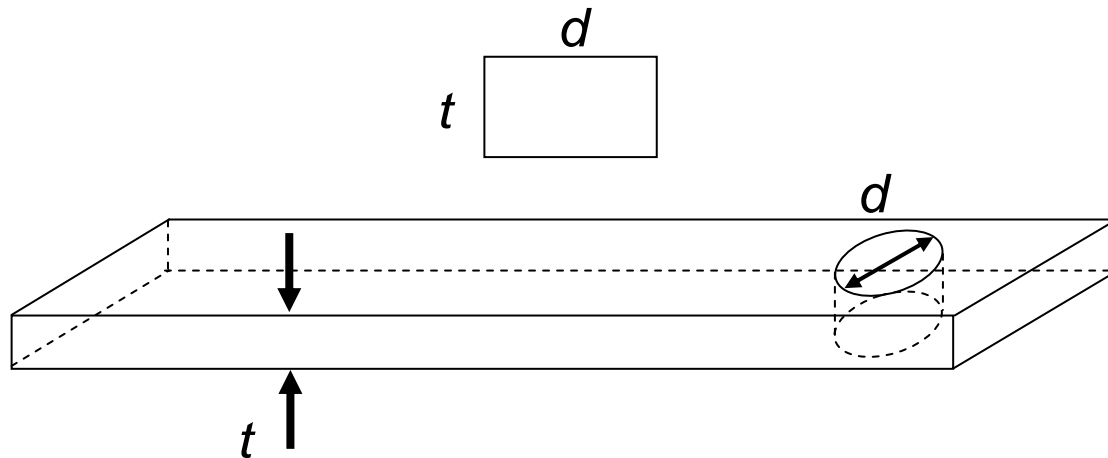
Bolts in “Single shear”

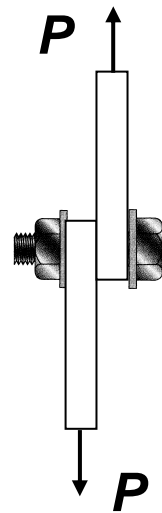


Average (normal) bearing stress σ_B

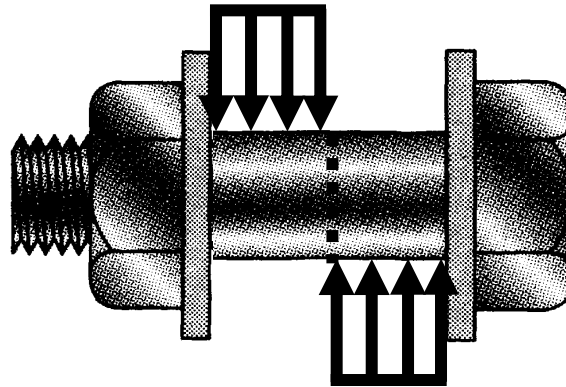
$$\sigma_b = \frac{\text{total bearing force } (P)}{\text{projected area of curved bearing surface } (A)}$$

$A = \text{plate thickness } t \times \text{bolt diameter } d$

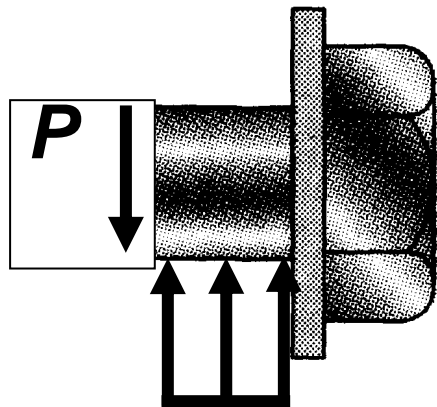




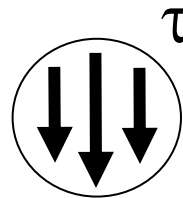
Forces applied to the bolt by the plates



These forces must be balanced by a shear force in the bolt.

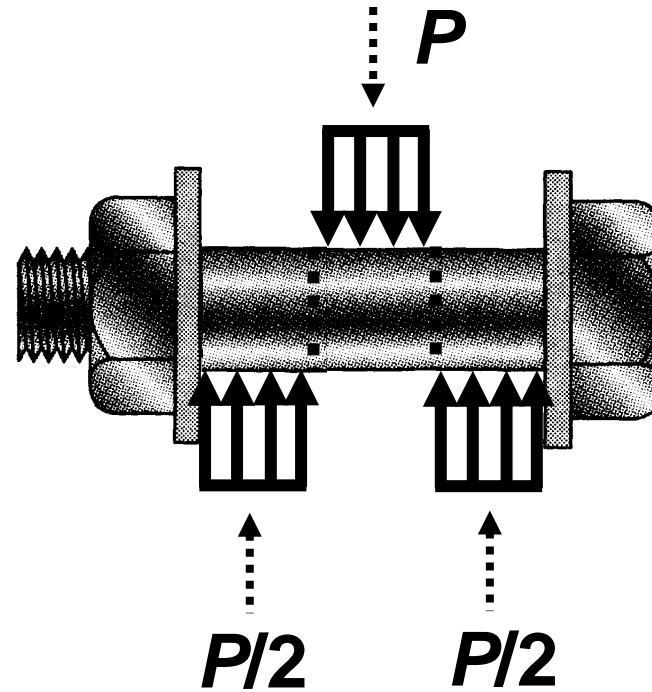
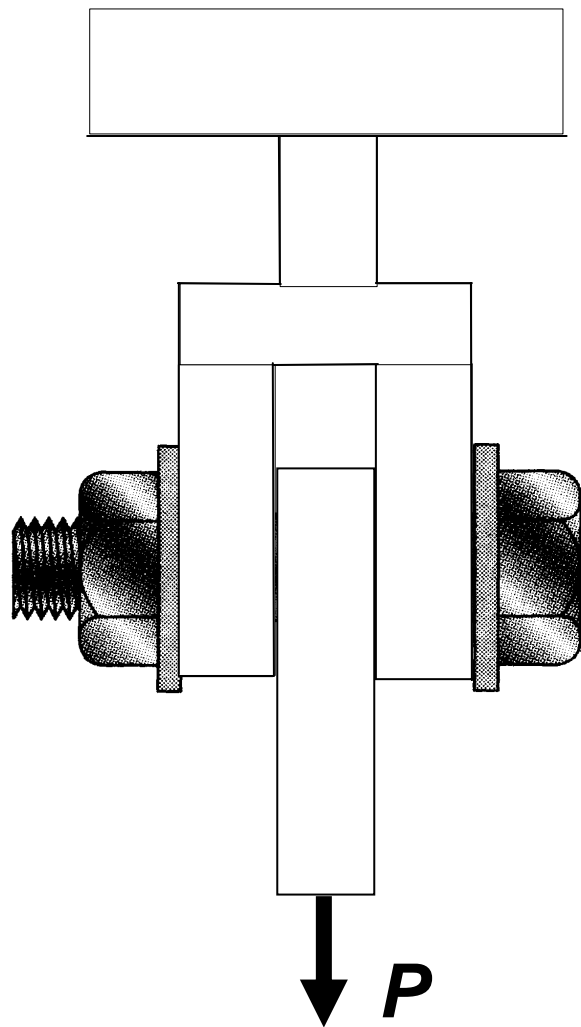


This results in a shear stress τ .



$$\tau = \frac{V}{A} = \frac{P}{(\pi d^2 / 4)}$$

Bolts in "Double shear"



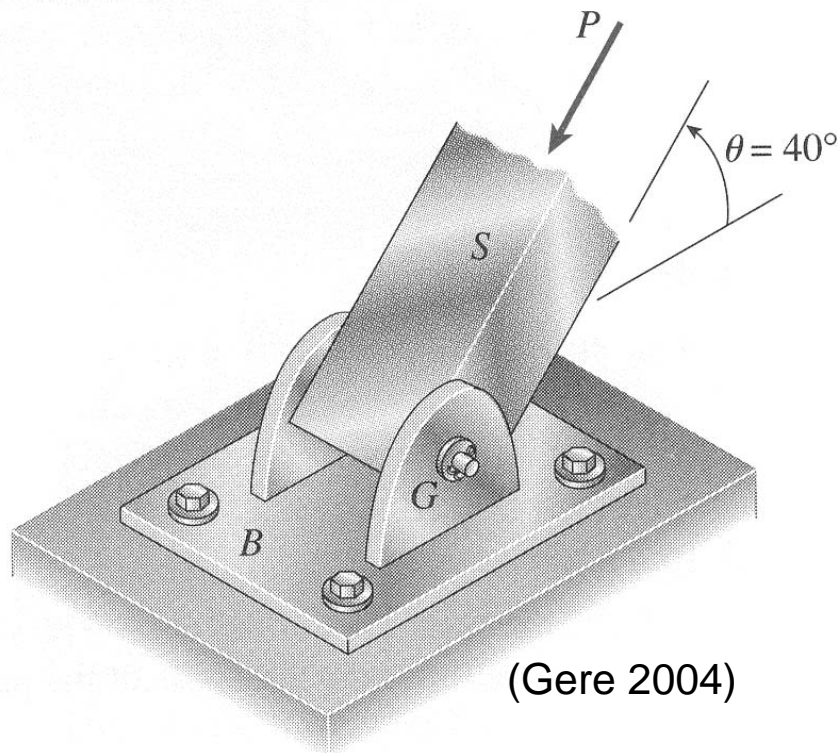
A cross-sectional view of the bolt, similar to the previous one, showing the downward load P and two upward reaction forces of $P/2$.

$$\tau = \frac{V}{A} = \frac{(P/2)}{(\pi d^2 / 4)}$$

Example of bearing stress and shear stress

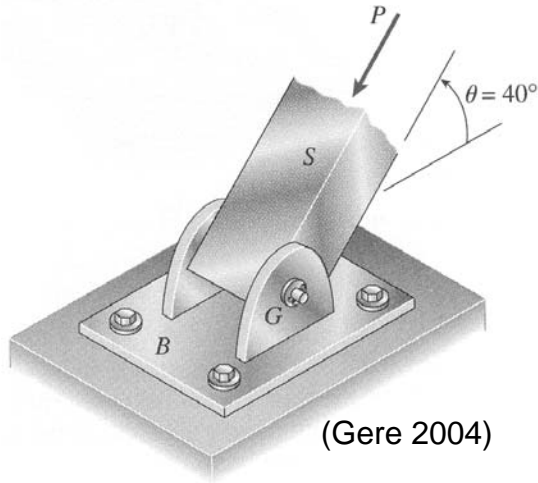
(based on Example 1-5, page 36, Gere, 6th ed. 2004)

A steel strut S is used as a brace for a boat hoist. It transmits a force P to the deck of a pier. The strut has a hollow square cross-section with wall thickness t . A pin through the strut transmits the compressive force from the strut to two gussets G that are welded to the base plate B .



Calculate:

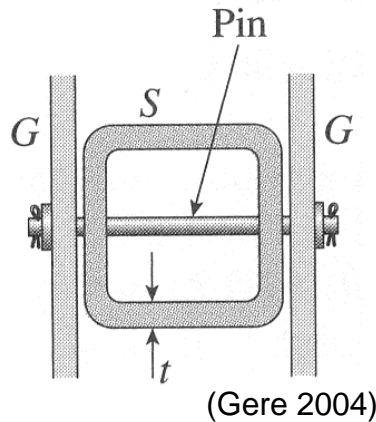
- the bearing stress between the strut and the pin
- the shear stress in the pin
- the bearing stress between the pin and the gussets
- the bearing stress between the anchor bolts and the base plate
- the shear stress in the anchor bolts



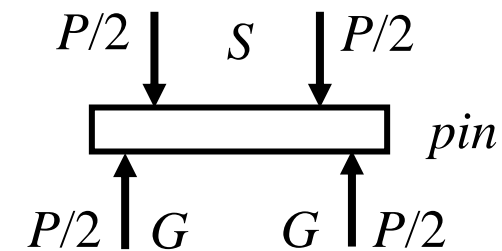
$P = 50 \text{ kN}$, $t = 10 \text{ mm}$
 diameter of pin, $d_{\text{pin}} = 20 \text{ mm}$
 thickness of gussets, $t_G = 15 \text{ mm}$
 diameter of anchor bolts $d_{\text{bolt}} = 12 \text{ mm}$
 thickness of base plate $t_B = 10 \text{ mm}$

(a) bearing stress between strut and pin

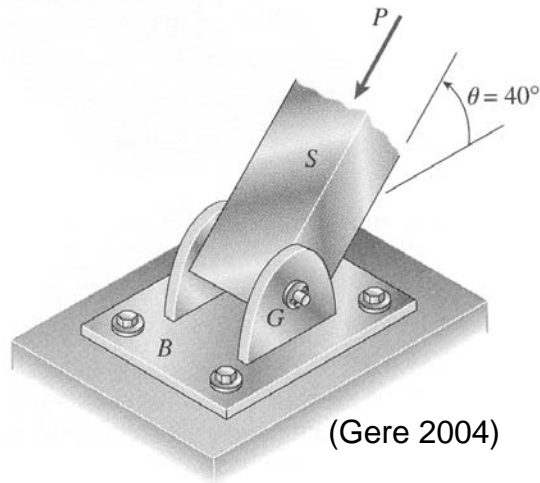
$$\sigma = \frac{P}{2(t d_{\text{pin}})} = 125 \text{ MPa}$$



(b) shear stress in pin



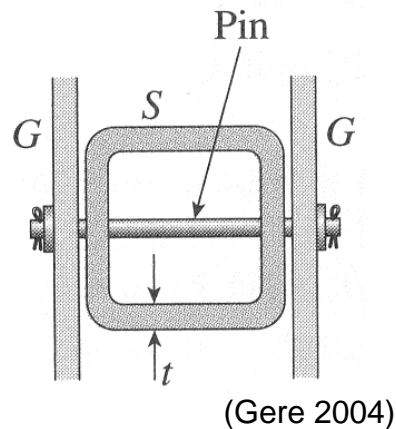
$$\tau = \frac{(P/2)}{(\pi d_{\text{pin}}^2 / 4)} = 79.6 \text{ MPa}$$



(c) bearing stress between pin and gussets

$$\sigma = \frac{P}{2(t_G d_{pin})} = 83.3 \text{ MPa}$$

(d) bearing stress between anchor bolts and base plate

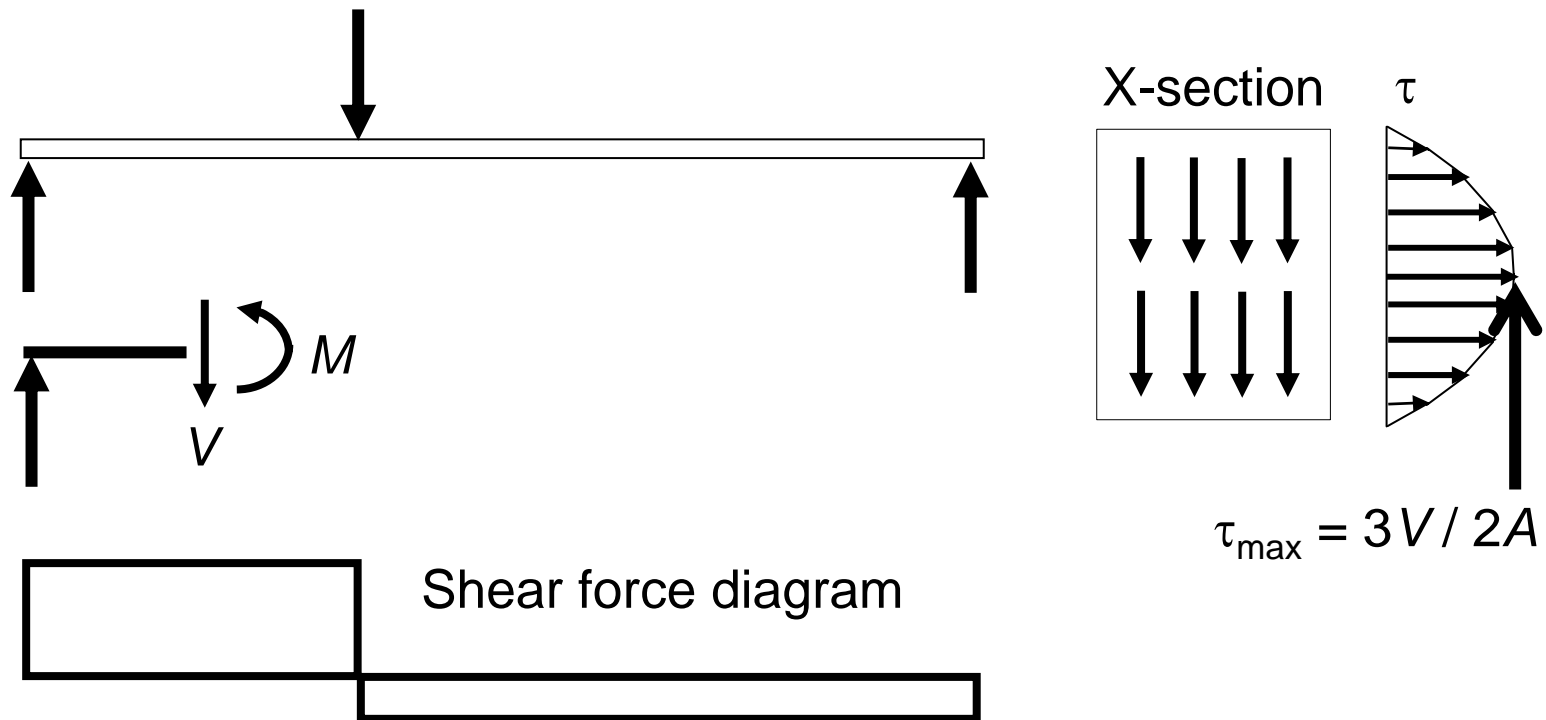


$$\sigma = \frac{P \cos \theta}{4(t_B d_{bolt})} = 79.8 \text{ MPa}$$

(e) shear stress in anchor bolts

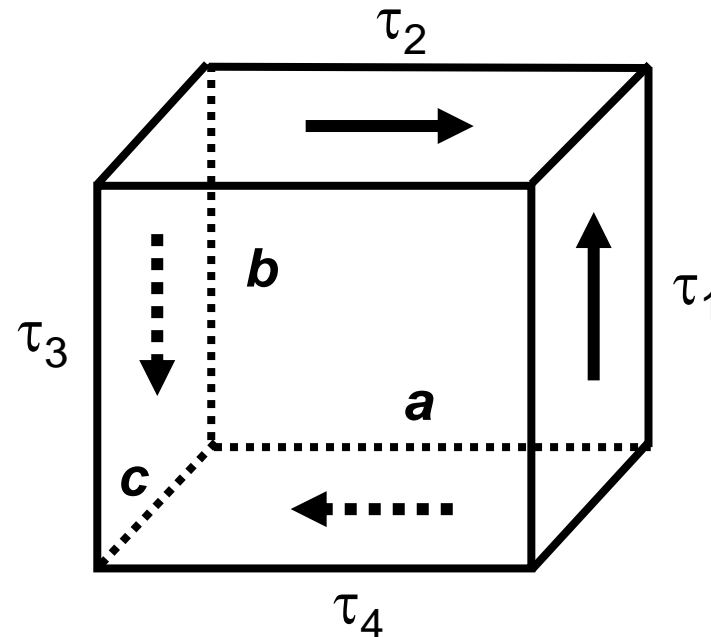
$$\tau = \frac{(P \cos \theta / 4)}{(\pi d_{bolt}^2 / 4)} = 84.7 \text{ MPa}$$

These are examples of “direct shear” -- the shear stresses are a result of the direct action of a shear force trying to cut through the material.



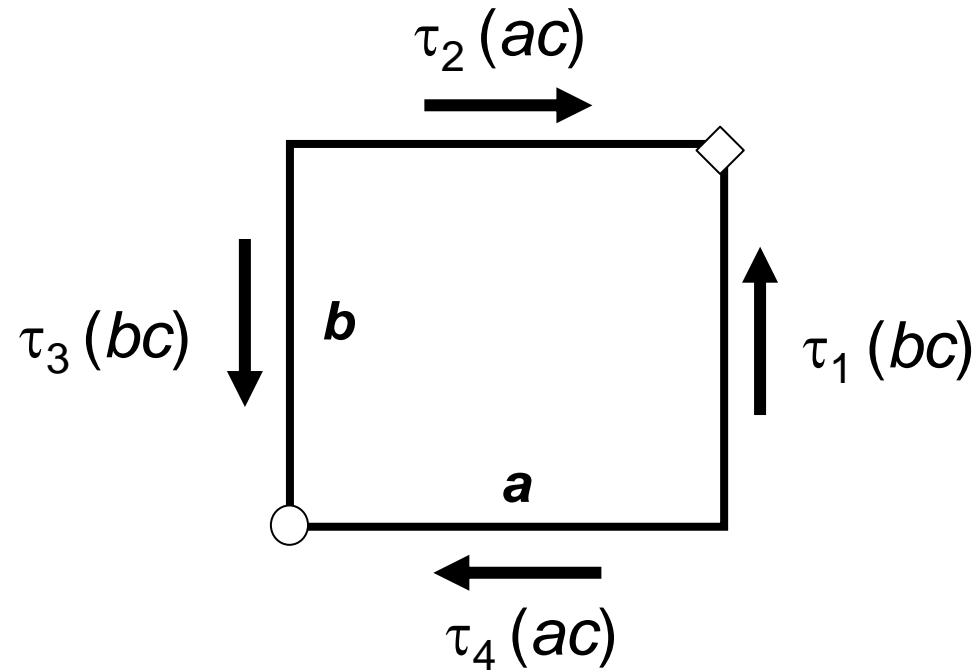
Shear stresses can also arise in an “indirect” manner – during tension, torsion, and bending.

Equality of shear stresses on perpendicular planes



Forces Vertical direction: $\tau_1 (bc) = \tau_3 (bc)$
Horizontal direction: $\tau_2 (ac) = \tau_4 (ac)$

So $\tau_1 = \tau_3$ and $\tau_2 = \tau_4$



Moments about \circ $[\tau_1(bc)](a) = [\tau_2(ac)](b)$

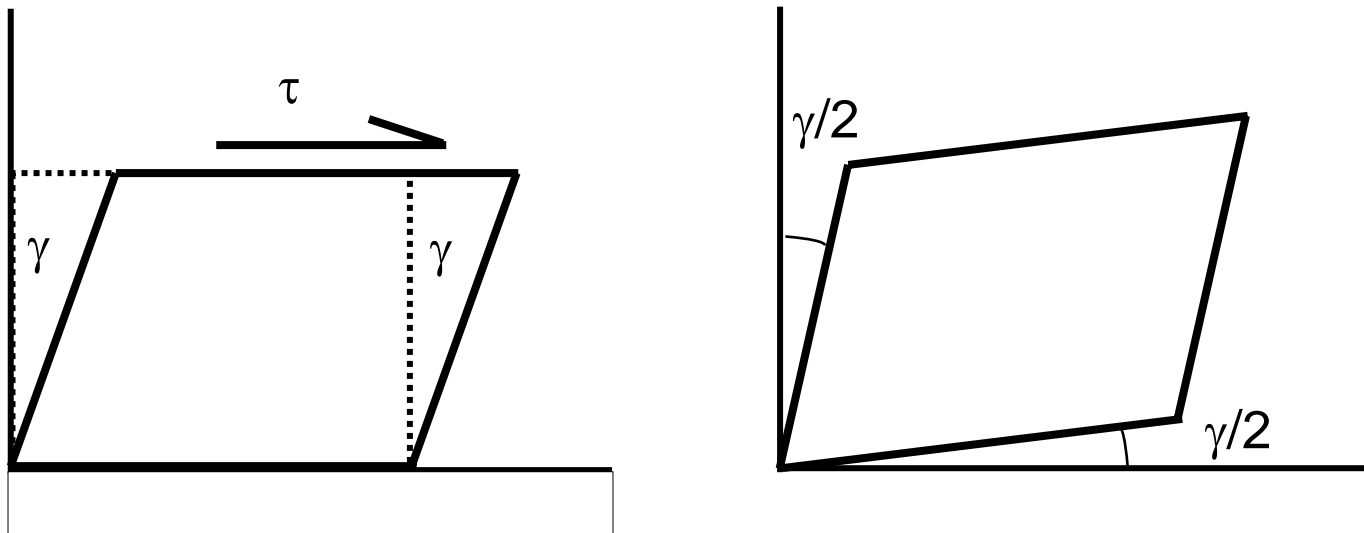
about \diamond $[\tau_3(bc)](a) = [\tau_4(ac)](b)$

So $\tau_1 = \tau_2$ and $\tau_3 = \tau_4$

Combining this with the previous result gives $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau$,
which called “pure shear”.

Shear strain

Shear stresses have no tendency to elongate or shorten; instead they produce a change in shape.



This change in shape is quantified by the angle γ , the shear strain. The angle is measured in radians, **not** degrees.

Greek letter γ (gamma)

Hooke's Law in shear

$$\tau = G \gamma$$

G is the shear modulus of elasticity (or “modulus of rigidity”).
Units are $\text{N} / \text{m}^2 = \text{Pa}$.

It can be shown that the elastic constants E and G are related by:

$$G = \frac{E}{2(1+\nu)}$$

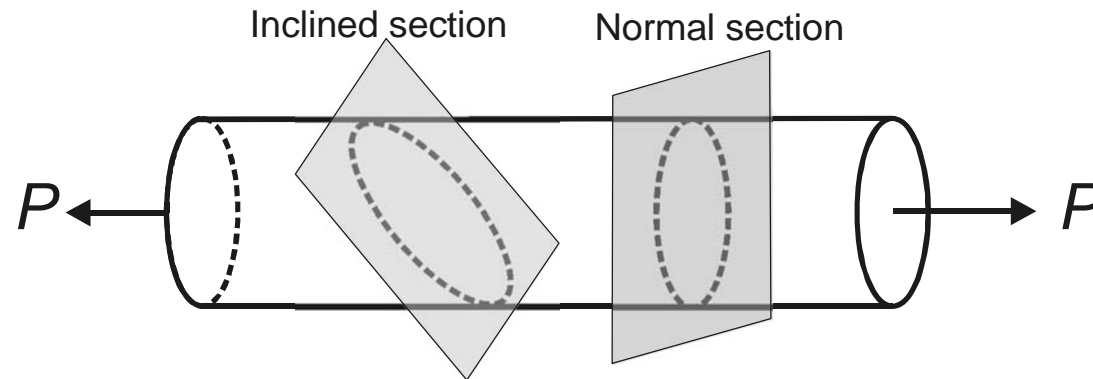
Since $0 < \nu < 0.5$ for most materials, G is typically one-third to one-half E .

For mild steel, $E = 210 \text{ GPa}$ and $G = 81 \text{ GPa}$.

For aluminium alloy, $E = 72 \text{ GPa}$ and $G = 28 \text{ GPa}$.

Normal and shear stresses on inclined sections

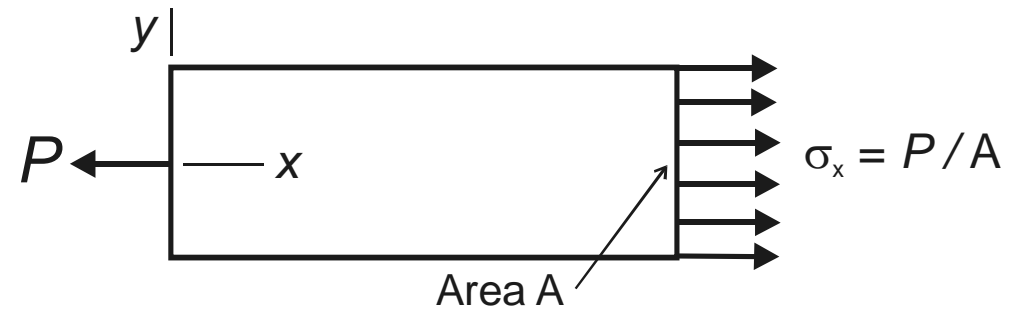
To obtain a complete picture of the stresses in a bar, we must consider the stresses acting on an “inclined” (as opposed to a “normal”) section through the bar.



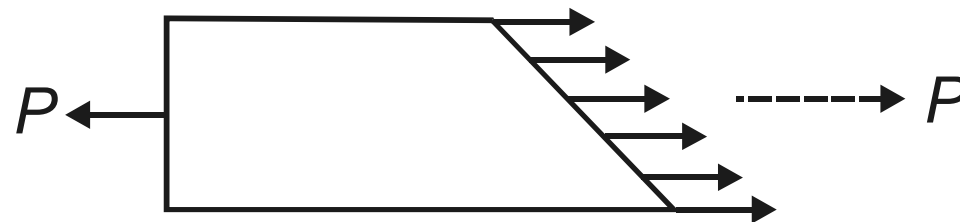
Because the stresses are the same throughout the entire bar, the stresses on the sections are uniformly distributed.



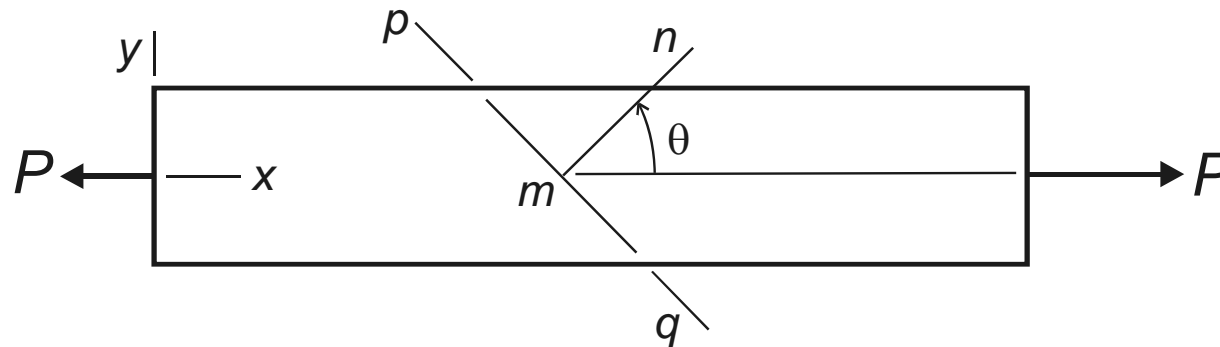
2D view of the normal section
(but don't forget the thickness perpendicular to the page)



2D view of the inclined section



Specify the orientation of the inclined section pq by the angle θ between the x axis and the normal to the plane.

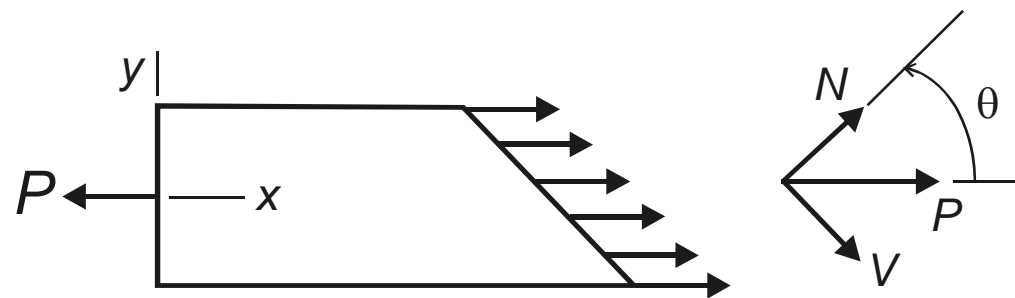


“Normal section” $\theta = 0^\circ$

Top face $\theta = 90^\circ$

Left face $\theta = 180^\circ$

Bottom face $\theta = 270^\circ$ or -90°

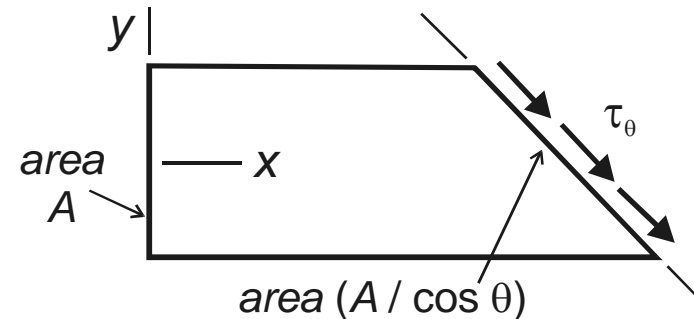
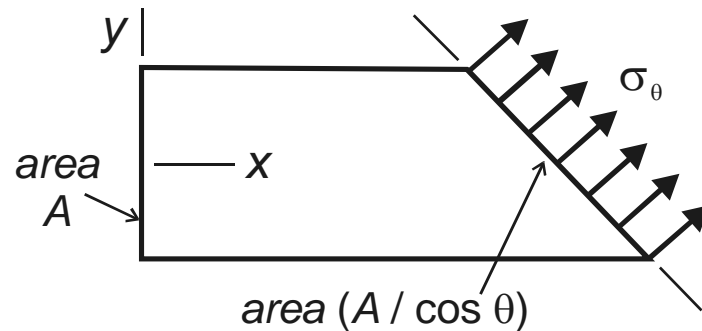


The force P can be resolved into components:

Normal force N perpendicular to the inclined plane, $N = P \cos \theta$

Shear force V tangential to the inclined plane $V = P \sin \theta$

If we know the areas on which the forces act, we can calculate the associated stresses.



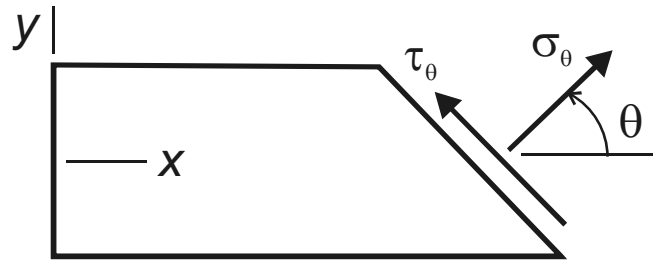
$$\sigma_{\theta} = \frac{\text{Force}}{\text{Area}} = \frac{N}{\text{Area}} = \frac{P \cos \theta}{A / \cos \theta} = \frac{P}{A} \cos^2 \theta$$

$$\sigma_{\theta} = \sigma_x \cos^2 \theta = \frac{\sigma_x}{2} (1 + \cos 2\theta) \quad \star$$

$$\tau_{\theta} = \frac{\text{Force}}{\text{Area}} = \frac{-V}{\text{Area}} = \frac{-P \sin \theta}{A / \cos \theta} = -\frac{P}{A} \sin \theta \cos \theta$$

$$\tau_{\theta} = -\sigma_x \sin \theta \cos \theta = -\frac{\sigma_x}{2} (\sin 2\theta) \quad \star$$

Sign convention

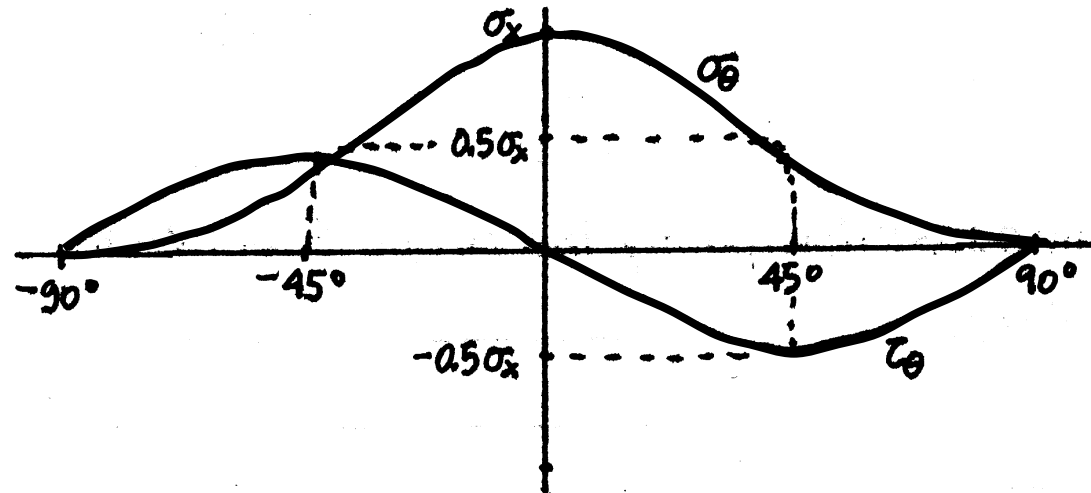


Normal stresses σ_θ positive for tension.

Shear stresses τ_θ positive when they tend to produce counterclockwise rotation of the material.

Note that these equations are derived from **statics only** and are therefore independent of the material (linear or non-linear, elastic or inelastic).

Plot σ_θ and τ_θ versus θ .



$$\sigma_\theta = \sigma_x \text{ at } \theta = 0^\circ$$

$$\sigma_\theta = \sigma_x/2 \text{ at } \theta = \pm 45^\circ$$

$$\sigma_\theta = 0 \text{ at } \theta = \pm 90^\circ$$

This is σ_{\max} .

No normal stresses on sections cut parallel to the longitudinal axis.

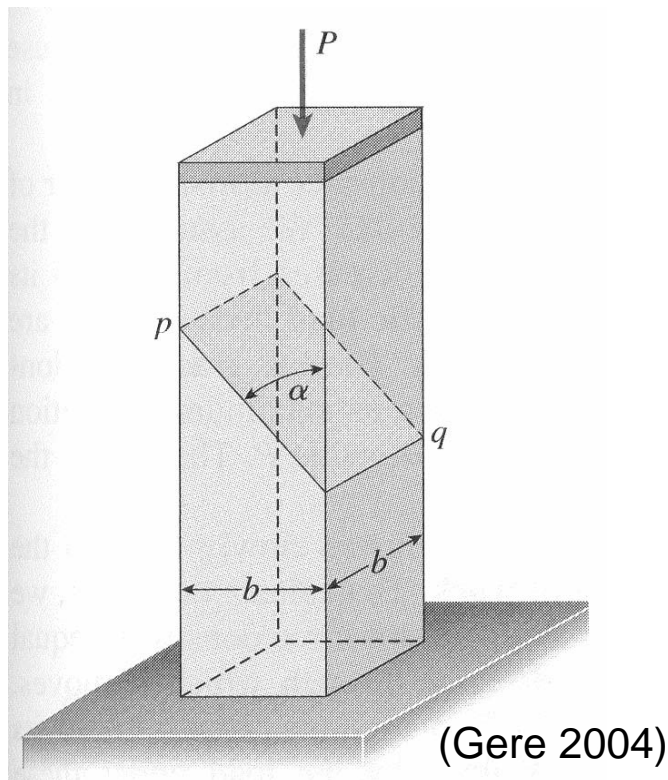
$$\tau_\theta = 0 \text{ at } \theta = 0^\circ, 90^\circ$$

$$\tau_\theta = \tau_{\max} = \pm \sigma_x/2 \text{ at } \theta = -/+ 45^\circ \quad |\tau_{\max}| = |\sigma_x/2|$$

Example of stresses on inclined sections

(based on Example 2-11, page 114, Gere, 6th ed. 2004)

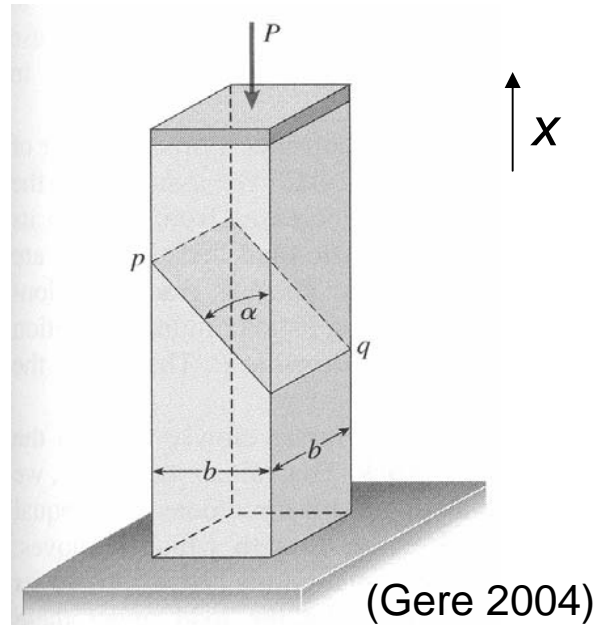
A compression bar with a square cross section of width b must support a load $P = 36$ kN. The bar is constructed from two pieces of material that are connected by a glued joint (known as a scarf joint) along plane pq which is at an angle $\alpha = 40^\circ$ to the vertical.



The material is a structural plastic with
 σ_{allow} (compression) = 7.6 MPa
 $\tau_{\text{allow}} = 4.1$ MPa

The glued joint has
 σ_{allow} (compression) = 5.2 MPa
 $\tau_{\text{allow}} = 3.4$ MPa

Determine the minimum width b of the bar.



$$\sigma_x = P / A = P / b^2$$

$$b = \sqrt{P / \sigma_x}$$

Smallest σ_x governs the design.

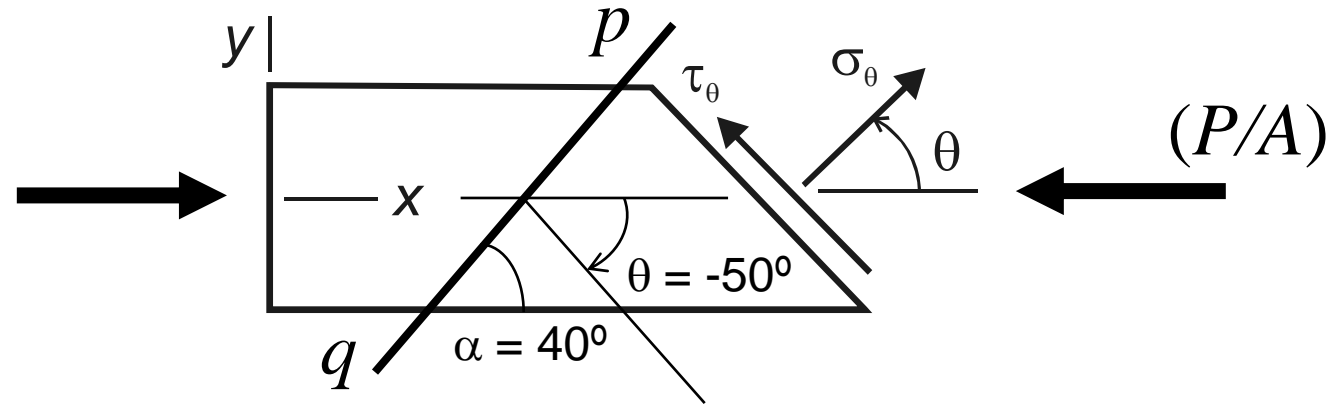
Values of σ_x based on allowable stresses in the plastic:

Maximum compressive stress is $-7.6 \text{ MPa} = \sigma_x$

Maximum shear stress is $4.1 \text{ MPa} = |\tau_{max}| = |\sigma_x/2|$ at $\theta = \pm 45^\circ$

This gives $\sigma_x = -2\tau_{max} = -8.2 \text{ MPa}$

Values of σ_x based on allowable stresses in the glued joint:



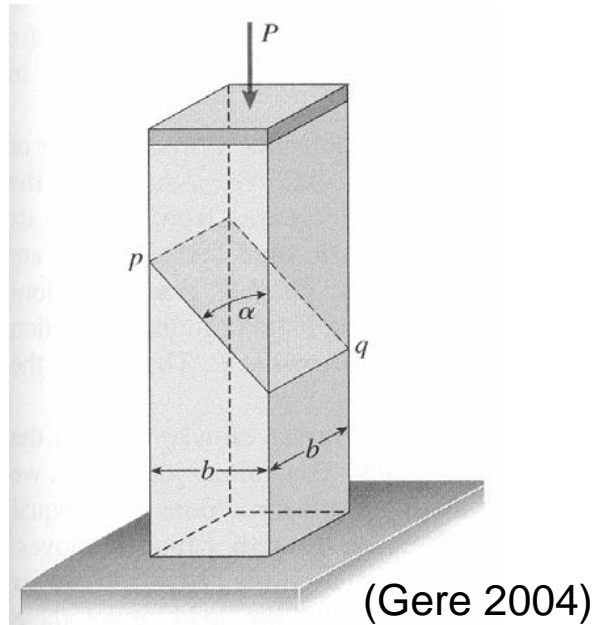
$$\sigma_\theta = \frac{\sigma_x}{2} (1 + \cos 2\theta) \qquad \tau_\theta = -\frac{\sigma_x}{2} (\sin 2\theta)$$

$$\sigma_x = \frac{2\sigma_\theta}{1 + \cos 2\theta} \qquad \sigma_x = -\frac{2\tau_\theta}{\sin 2\theta}$$

$$\sigma_x = \frac{2(-5.2)}{1 + \cos 2(-50)} \qquad \sigma_x = -\frac{2(-3.4)}{\sin 2(-50)}$$

$$\sigma_x = -12.6 \text{ MPa} \qquad \sigma_x = -6.9 \text{ MPa}$$

Smallest $\sigma_x = -6.9$ MPa (shearing of glue joint)



$$b = \sqrt{P / \sigma_x}$$

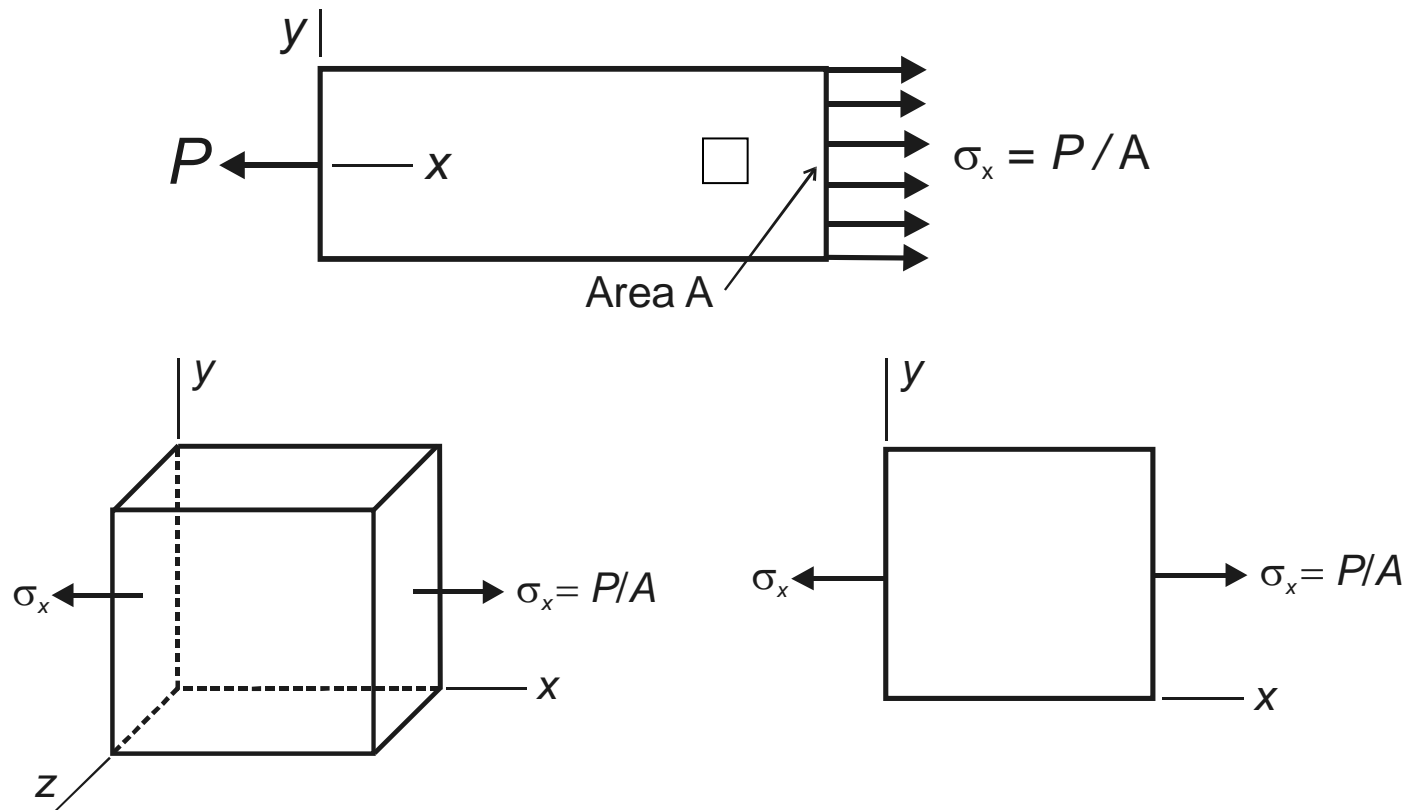
$$b = \sqrt{\frac{(-36 \times 10^3)}{(-6.9 \times 10^6)}}$$

$$b = 0.0722 \text{ m}$$

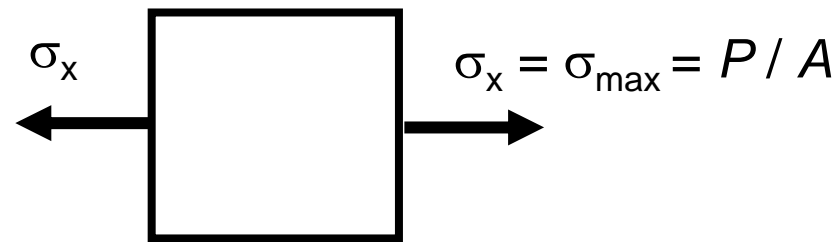
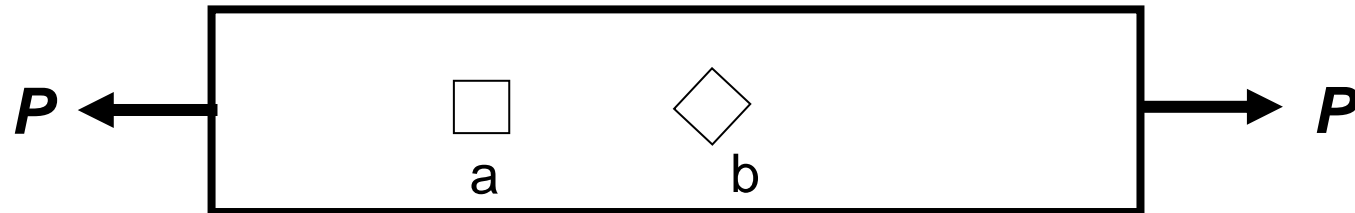
$$b = 72.2 \text{ mm}$$

Introduction to stress elements

Stress elements are a useful way to represent stresses acting at some point on a body. Isolate a small element and show stresses acting on all faces. Dimensions are “infinitesimal”, but are drawn to a large scale.

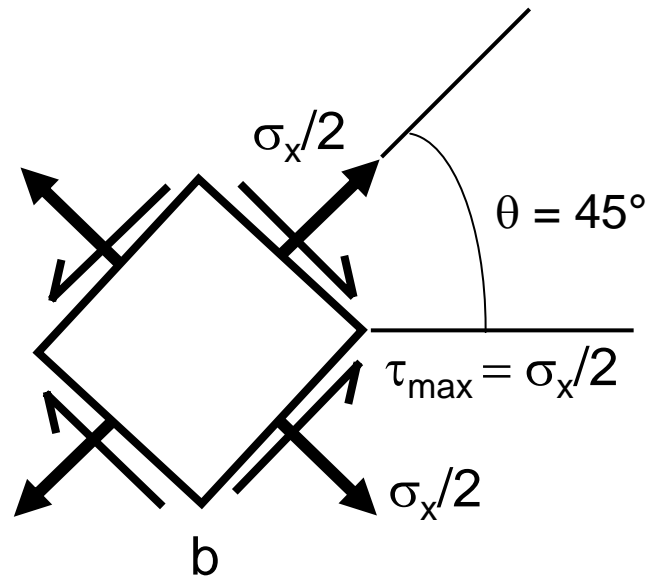
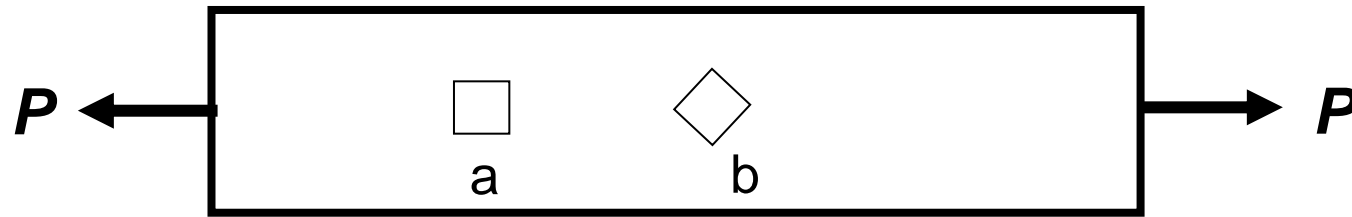


Maximum stresses on a bar in tension



a

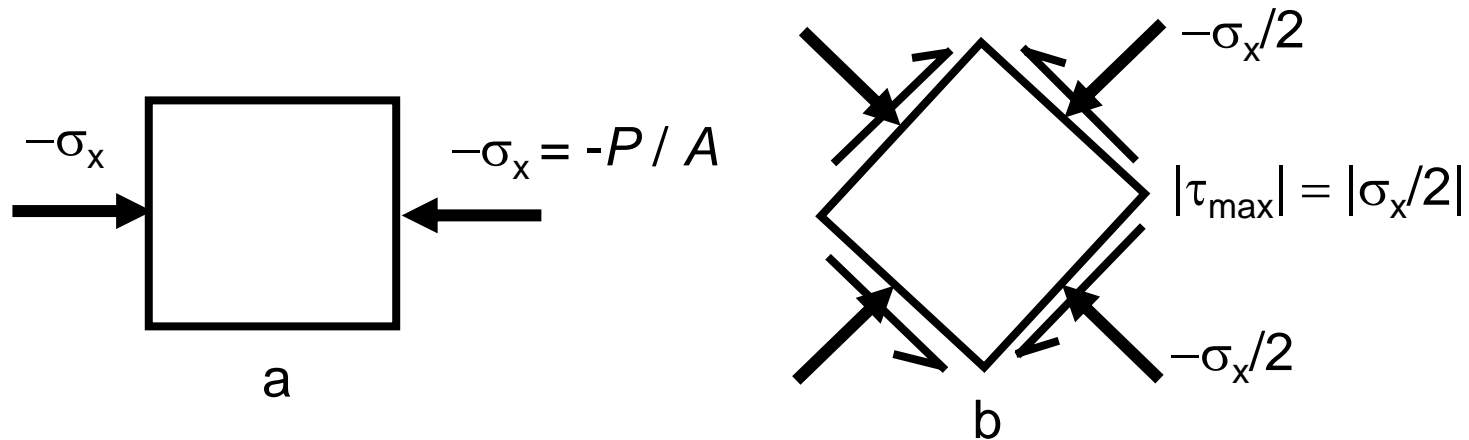
No shear stresses



Angle	σ_θ	τ_θ
$\theta = 45^\circ$	$\sigma_x/2$	$-\sigma_x/2$
$\theta = 135^\circ$	$\sigma_x/2$	$\sigma_x/2$
$\theta = -45^\circ$	$\sigma_x/2$	$\sigma_x/2$
$\theta = 225^\circ$	$\sigma_x/2$	$-\sigma_x/2$

In case b ($\theta = 45^\circ$), the normal stresses on all four faces are the same, and all four shear stresses have **equal** and **maximum** magnitude.

If the bar is loaded in compression, σ_x will have a negative value and stresses will be in the opposite directions.

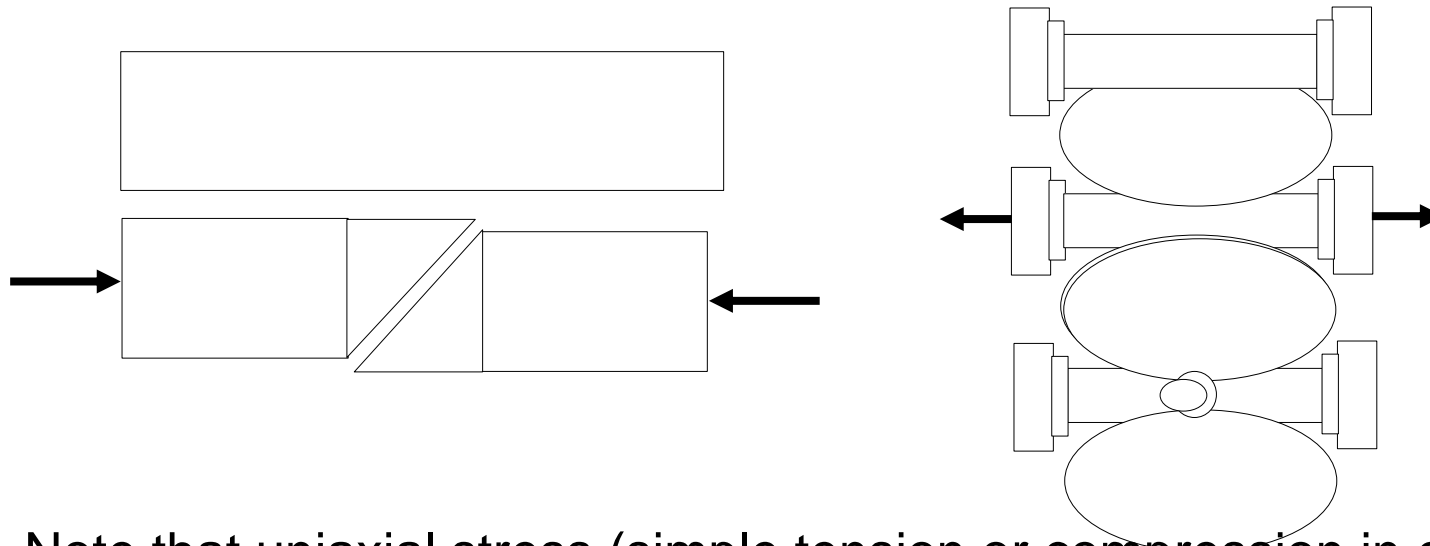


Even though the maximum shear stress in an axially loaded bar is only half the maximum normal stress, the shear stress may cause failure if the material is much weaker in shear than in tension.

Examples

Wood block in compression fails by shearing on 45° planes

Mild steel loaded in tension. Visible “slip bands” (Lüders bands) appear on the sides of the bar at approximately 45° to the axis of loading when the yield stress is reached. These indicate that the material is failing in shear along planes of maximum shear stress (cup-and-cone failure).



Note that uniaxial stress (simple tension or compression in one direction) is just a special case of a more general stress state known as “plane stress”.