

A Primer on Formal Metaphysics¹

1. Introduction

Ontology is the study of what there is. Often this is taken to include the project (not shared by everyone) of categorizing entities into various kinds: objects, properties, events, or what have you, and exposing the various dependencies which hold between these kinds. Some philosophers believe there is a ‘primitive ontology’, an ontology which has the property that for any given kind, it can be reduced to a more basic ‘primitive’ kind. For example, when using a computer we seem to be committed to the existence of files and data. However talk of files can be reduced to talk about hard disks, USB drives, floppy disks, CD’s. As you can see there is a high degree of multiple realisability here preventing any kind of reductive explanation. However commitment to files isn’t a commitment to anything *new* over and above material stuff. This seems to be a genuine ontological reduction. Two examples from the philosophical literature which differ in their primitive ontology are absolute and relational theories of space and time. Absolute theories of space-time will often attempt to reduce objects to space-time points and properties distributed over them, thus taking space-time points and properties as primitive. Relational theories will attempt to reduce space and time to objects, events and relations between them, thus taking objects, events and relations as primitive.

From such metaphysical theories arises the need for a certain degree of formal apparatus. Relationists can construct instants of time out of events and the simultaneity relation, by taking sets of simultaneous events (equivalence classes). It then turns out you can linearly order these instants using the ‘earlier than’ relation between events. Similarly absolute space-time theorists can take objects to be sets of space-time points. There are various formal frameworks which allow us to locate this brand of structure including set theory, category theory, topology and mereology. Here I shall talk briefly about set theory and topology but will concentrate mainly on mereology as it is the most distinctly philosophical and most nominalistically acceptable of the mentioned frameworks. As with all the aforementioned theories, it has a rich catalogue of philosophical applications.

2. Historical Background

Mereology is a collection of formal systems designed to capture the notion of ‘parthood’ – the relation of a part to its whole. I say a collection because there are variations in how mereology can be formulated. These depend on what primitives you choose², which axioms they include (this usually depends on philosophical disposition) and whether you

¹ This introduction is intended to prepare the reader for the technical aspects of the paper ‘Endurantism and Temporal Gunk’ (this issue). The paper is self-contained in that it includes a minimal amount of exposition, however this introduction is recommended for those readers with little or no acquaintance with formal mereology.

² Whether you take your system to be explaining the relation ‘is a part of’, as opposed to say ‘overlaps with’ or ‘is disjoint from’. You can define each of these relations in terms of the other, so for logical purposes it does not matter which relation you take as primitive.

formulate your theory using first or second order logic³. The study of the parthood relation has presocratic roots, however it was not until Brentano's work that mereology made its mark on philosophy. It was still later that mereology became the rigorous formal theory that we know it as today. In the hands of the famed Warsaw School of philosophical logicians it became a powerful tool in the study of formal metaphysics, foundational issues in mathematics not to mention its use in the theoretical computer and information sciences. Particularly important names in the history of mereology are Leśniewski and Tarski from the Warsaw school, then Whitehead, Leonard and Goodman for bringing mereology into the mainstream analytic tradition. Finally, more recent writers on mereology include Lewis, Simons and Varzi. For further reading on these writers see the bibliography.

3. The relation of part to whole

If you have done any formal set theory you will be familiar with the technique of capturing the inferences involving a particular relation by giving a set of axioms which govern those inferences. In the case of set theory that relation is the membership relation, written as ' \in ' to be read as 'x is a member of y'. In mereology the relation is the parthood relation, written ' \leq ' and read as 'x is a part of y'. To get a good grasp of what we mean by 'part', here are a few examples involving the parthood relation:

1. My hand is part of my body
2. The dustbin lid is part of the dustbin
3. That slice is part of the pizza.
4. Wales is a part of Great Britain
5. The second movement was my favourite part of the symphony
6. 'The Empire Strikes Back' was the worse part of the trilogy
7. The whole numbers are only part of the rational numbers.
8. Being pedantic is part of being a good logician

There is quite a diversity of examples here. **2** is an example of a part of something which needn't be spatially connected to the rest of it, **4** demonstrates a geographical part and a constitutional part, **5** is an example of a temporal part and **7** of parthood between abstracta. Finally in **8** it is controversial whether the parthood relation is being used at all – I put this in to make it clear that not *all* uses of the word 'part' should necessarily fall under the treatment we are considering here.

Barring example **8**, this is the intuitive notion of parthood that we shall be trying to formalise here. The diversity of the examples here indicates that the parthood relation is *topic neutral*. Topic neutrality is a desirable property among theories contending for the title of 'pure logic'. Some have said topic neutrality is the sign of the logical – we can clearly see there seems to be no domain over which we cannot quantify over and hence apply first order logic, similarly there are not many domains of which we cannot form

³ In first order logic your quantifiers only range over objects in the domain. If your logic is second order the quantifiers can also range over subsets of the domain, so you can get the effect of quantifying over properties and functions.

sets from⁴. Some philosophers, for example David Lewis, have taken this one step further and claimed that mereology should count among the purely logical theories (for example he argues that identity, a logical notion, is merely a limiting case of overlap, a mereological notion meaning ‘shares a part with’).

Finally there is a small caution about the way mereologists use the parthood relation. A mereologist will count the Eiffel Tower among the Eiffel Tower’s parts, whereas in ordinary English we would only count strictly smaller parts of the Eiffel Tower among its parts. This is for convenience only, the mereologist could, if she wanted, take strict parthood as primitive, and define loose parthood (parthood which treats objects as parts of themselves) in terms of it by saying x is loosely part of y iff x is strictly part of y or $x = y$. Since they are interdefinable we shall always mean loose parthood when we talk of parthood short of an adjunct.

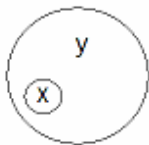
4. Some definitions

For sake of exposition, and for continuity with the literature, we shall take parthood as the primitive notion of mereology. As has been mentioned already, different relations can be used instead, for example ‘overlap’ or ‘disjoint from’. Here I define in terms of the parthood relation some common terminology used among mereologists. The symbol for parthood is ‘ \leq ’ and is to be read as ‘ x is a part of y ’.

Proper part

x is a proper part of y , written ‘ $x < y$ ’ iff x is a part of y and x is not the same as y .

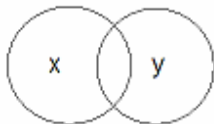
$$\circ \quad x < y \leftrightarrow [x \leq y \wedge \neg x = y]$$



Overlap

x overlaps with y , written ‘ $x \bullet y$ ’ iff x and y have a common part.

$$\circ \quad x \bullet y \leftrightarrow \exists z[z \leq x \wedge z \leq y]$$

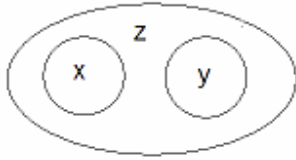


⁴ There are some exceptions. All the sets cannot be gathered into one set for example.

Underlap

x underlaps with y, written ' $x \cup y$ ' iff there is something of which x and y are both a part.

$$\circ x \cup y \leftrightarrow \exists z[x \leq z \wedge y \leq z]$$



Disjoint

x is disjoint from y, written ' $x \perp y$ ' iff x and y do not have a common part

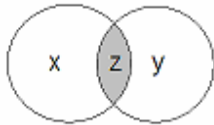
$$\circ x \perp y \leftrightarrow \neg x \bullet y$$



Product

If x and y overlap the product of x and y, written $x \times y$ is the object, z, whose parts are just the parts x and y have in common

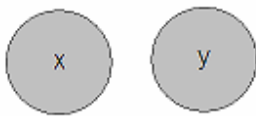
$$\circ x \times y =_{df} z \ \forall w(w \leq z \leftrightarrow (w \leq x \wedge w \leq y))$$



Sum

If x and y underlap the sum of x and y, written $x + y$ is the object, z, such that something overlaps with z just in case it overlaps with x or it overlaps with y.

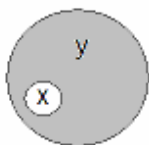
$$\circ x + y =_{df} z \ \forall w(w \bullet z \leftrightarrow (w \bullet x \vee w \bullet y))$$



Remainder

If x isn't a part of y the remainder of y from x, written $x - y$ is the object whose parts are just those parts of x which are disjoint from y.

$$\circ x - y =_{df} z \ \forall w(w \leq z \leftrightarrow (w \leq x \wedge w \perp y))$$



5. Axioms

It is now time to give some of the standard axioms of mereology. A few words on the need for explicit formal axioms are deserved. So far we have been talking about parthood intuitively, and whenever a formal definition of a term has been given it has always been accompanied by an equivalent in English. Why do we need to get formal at all? Part of the reason is that it allows us to get a handle on some infamously slippery notions, and apply logical techniques to test for the consistency and completeness of various theories based on them. Secondly it is all too easy to fall into paradox without due care. For example, some early versions of set theory allowed you create sets by using a property to define its members. Similarly an obvious way to give an object would be to give its parts (I am my arms legs torso etc..). We might say that for any property F under which at least one things falls, there is an object x such that for any y, y is a part of x just in case y has the property F. So now let's consider the object whose parts are just those things which are not a part of themselves. (Here I'm using part in the laymans sense - mereologists say everything is a part of itself). If this object isn't a part of itself it then it falls among the collection of things which consistutes its parts, i.e. it is a part of itself. If it is a part of itself then it has the property used to define it, namely that it is not a part of itself, contradiction. Although this is less of a paradox than the analogue for naïve set theory, using a formal theory to make things explicit avoids linguistic confusions such as this.

So here are the axioms. Our language is first order and the only non-logical symbol is '≤'.

Reflexivity

Everything is a part of itself

- $\forall x x \leq x$

Anti-symmetry

If x and y are parts of each other, they are the same

- $\forall x \forall y [(x \leq y \wedge y \leq x) \rightarrow x = y]$

Transitivity

If x is a part of y and y a part of z, then x is a part of z

- $\forall x \forall y \forall z [(x \leq y \wedge y \leq z) \rightarrow x \leq z]$

Supplementation

If x isn't a part of y then there is an object whose parts are just those parts of x which are disjoint from y (e.g. take $z = x - y$)

- $\forall x \forall y [\neg y \leq x \rightarrow \exists z [z \leq y \wedge z \perp x]]$

Product

If x and y overlap then there is a unique object, z, whose parts are just the parts x and y have in common (i.e. $z = x \cdot y$).

- $\forall x \forall y [x \cdot y \rightarrow \exists z \forall w [w \leq z \leftrightarrow [w \leq x \wedge w \leq y]]]$

Sum

If x and y overlap then there is a unique object, z , such that something overlaps with z just in case it overlaps with x or it overlaps with y (i.e. $z = x+y$).

- $\forall x \forall y [x \cup y \rightarrow \exists z \forall w [w \bullet z \leftrightarrow [w \bullet x \vee w \bullet y]]]$

The first three axioms simply say that parthood is a partial order. This is no surprise – the notion of a **partial** order derives from the parthood relation. Reflexivity is a consequence of mereologists quirky terminology (see section 3). It should be noted that any mereology which has reflexivity and anti-symmetry as axioms will have the consequence that x and y are identical just in case they have the same parts. We call this extensionality.

- $\forall x \forall y [x = y \leftrightarrow \forall z [z \leq x \leftrightarrow z \leq y]]$

This is easy to prove. Suppose $x = y$, then something is a part of x just in case it is a part of y (this is an instance of the indiscernability of identicals). Conversely, suppose something is a part of x iff it is a part of y . Now x is a part of x by reflexivity, so x is a part of y from the assumption. By similar reasoning y is a part of x . So x is a part of y and y is a part of x , now applying anti-symmetry it follows that $x = y$. That this is a theorem of standard mereology is sometimes said to pose problems for endurantism. For suppose object x undergoes the loss of one of its inessential parts y (e.g. if x lost a fingernail). According to endurantism x will retain its identity despite this loss, so indexing our object with the times t and t' – before and after the loss - we get $x_t = (x-y)_t$. However by supplementation $(x-y)_t$ exists and by extensionality it follows that $(x-y)_t = (x-y)_{t'}$ since they both have the same parts. Finally by transitivity of identity we get that $x_t = (x-y)_{t'}$ which is a contradiction – $y_{t'}$ is a part of the left hand side but not the right hand side. Some endurantists have rejected extensionality on this basis and have developed intensional mereologies (cf Simons, 1987, [11]). Similar problems can be formulated modally instead of temporally, this fact can be construed as showing that these kinds of arguments are problematic for everyone, not just endurantists.

The last three axioms tell us that we can always subtract, take the product of or find the sum of entities. In particular, given any finite set of objects such that any pair of them overlaps (underlaps) we can take the product (sum) of them all simply by applying the respective axiom to the first and second object, then applying it again to this new product (sum) and the third entity, and so on. Arbitrary products and sums are not permitted – it will take a stronger axiom schema to allow the product and sum of infinite collections of objects. There are some philosophical issues here involving the summation axiom. According to this axiom, for any two objects there is another object, their sum. This axiom can be applied whatever the objects are, leading to fusions of objects which needn't be spatially connected. Consequently philosophers have objected for it commits us to the existence of some very weird objects, for example the trout-turkey (an example from Lewis – the sum of a trout and a turkey). This is a matter of philosophical intuition, often the philosopher sympathetic with mereology will reply that what really exists out there has nothing to do with the way the human mind slices up experience to make it manageable. However, mereology claims to capture only the facts about the parthood relation, the objector might reply that, in contrast to the first five axioms, commitment to mereological sums does not seem to be a fact about *parts* at all.

6. Further Axioms

The following are various axioms that may be added to standard mereology to strengthen it. These principles are not assumed without being explicitly stated as they can often rest on controversial philosophical assumptions.

Unrestricted Fusion

Given any consistent property, there is at least one object, y , such that something overlaps with y just in case it overlaps with something having that property.

First order version:

- $[\exists x\phi \rightarrow \exists y\forall z[z \bullet y \leftrightarrow \exists x[\phi \wedge x \bullet z]]]$

For any well formed formula ϕ with no free occurrences of y or z

Second order version

- $\forall X[\exists xXx \rightarrow \exists y\forall z[z \bullet y \leftrightarrow \exists x[Xx \wedge x \bullet z]]]$

Unique Fusion

Given any consistent property, there is exactly one object, y , such that something overlaps with y just in case it overlaps with something having that property.

First order version:

- $[\exists x\phi \rightarrow \exists!y\forall z[z \bullet y \leftrightarrow \exists x[\phi \wedge x \bullet z]]]$

For any well formed formula ϕ with no free occurrences of y or z

Second order version

- $\forall X[\exists xXx \rightarrow \exists!y\forall z[z \bullet y \leftrightarrow \exists x[Xx \wedge x \bullet z]]]$

Top

There is something of which everything is a part.

- $\exists t\forall x[x \leq t]$

Bottom

There is something which is a part of everything.

- $\exists b\forall x[b \leq x]$

Atoms

Everything has a part which has no proper parts.

- $\forall x\exists y[y \leq x \wedge \neg\exists z[z < y]]$

Gunk

Everything has proper parts

- $\forall x\exists y[y < x]$

The Unrestricted and Unique Fusion axioms allow us to take arbitrary fusions of objects, whereas Sum only allowed us to take finite fusions. These axioms have been objected to on similar grounds as the Summation axiom. There is also a choice as to whether we use a first order or a second order logic. The pros of using a first order language are that they are supposedly ontologically innocent. Second order theories are said to commit us to sets and other abstract objects. The first order formulation is an axiom schema, and is thus

actually infinitely many axioms (one for each choice of φ). The second order formulation, which allows quantification over properties, is only one axiom. On the down side, first order theories will always have unintended models. This is because, in a mereology with atoms, we expect the size of the universe to be 2^κ for some cardinal κ . If κ is finite so is the domain, and if κ is infinite, the domain is uncountable, so either way the domain is never countably infinite. If the mereology is gunky then the universe is always uncountable. But for first order languages there are always countable models if there are infinite models (due to the Löwenheim-Skolem theorem), so first order mereology will always have unintended models (cf Bacon, [1]). Second order mereology avoids this problem. Also the Fusion axioms only quantify over monadic (one place) properties. Since we can interpret monadic second order logic in terms of plural quantifiers (Boolos [2]) we have a nominalistically acceptable way to formulate these axioms.

Top states the existence of the ‘universe’ – everything is a part of it. Bottom on the other hand is a widely rejected principle of mereology (except, perhaps, in universes containing only one thing). It states the existence of a ‘null object’, something which is a part of everything much like the way the empty set is a subset of every set. For obvious reasons the existence of a null object is philosophically spurious. However the existence of such an object makes mereology equivalent to a Boolean algebra, and assuming the existence of this object can simplify many proofs.

Call something an atom iff it has no proper parts, and call something gunky iff all of its parts have proper parts. Atoms then says that everything is made from atoms: the basic building blocks of the universe. Note that an atom is not to be thought of as a chemistry atom. What people take to be the atoms depends on their ontology. A popular choice might be space-time points although this isn’t necessary (mereology doesn’t choose you’re ontology for you). Gunk on the other hand, says that there are no atoms, everything is made up of gunk, which in turn is made up of more gunk and so on and so forth - turtles all the way down. Gunk is thus, in some sense, infinitely divisible. However gunkiness is a stronger property than that – a line (of real numbers say) is also infinitely divisible, you can keep cutting it in half, but a line isn’t gunky since it is composed of points – it is a sequence of real numbers and each of these points has no proper parts. Gunkiness is a very bizarre property for something to have, it implies it has no basic parts. It would be very difficult to say what gunk was made of, since each part of it is made of more gunk, thus perpetually evading explanation.

7. Model Theory⁵

In this section we shall be concentrating on a particular collection of models for mereology. These are relevant to most of the metaphysical discussions involving mereology and should be enough to demonstrate the various dependencies between the axioms. If you understand this section you should be able to keep up with most of the philosophical literature on mereology.

⁵ A model can be thought of as a mathematical structure which satisfies a given set of axioms.

The most important concept we shall need to grasp is Euclidean space. Euclidean space is a mathematical abstraction which is supposed to model our intuitive idea of space (or space-time⁶). It can be thought of as a three dimensional graph each axis of which can be represented as a line of real numbers, written \mathbf{R} . Three dimensional Euclidean space is then written as \mathbf{R}^3 (or \mathbf{R}^n for more generality). We now introduce the idea of a metric space. A metric space is:

- A non-empty set S
- A function, d , such that
 - $d: S \times S \rightarrow \mathbf{R}$
 - $d(x, y) \geq 0$
 - $d(x, y) = 0$ iff $x = y$
 - $d(x, y) = d(y, x)$
 - $d(x, z) \leq d(x, y) + d(y, z)$

Here S is to be thought of as a set of points, in our case we take S to be \mathbf{R}^3 . The function d is supposed to represent the distance between points in our set S . The first constraint on d says that d takes pairs of elements (two elements) from S and gives us a real number which is to be thought of as the shortest distance between those two points. The second and third constraint says that this distance is never negative and is zero between a point and itself but never between two distinct points. The fourth constraint says that the distance from x to y is the same as the distance from y to x . The last constraint says that for any three points x , y and z the distance between x and z is always more than the distance between x and y plus the distance between y and x (this can be seen intuitively by drawing a triangle of 3 points and noting that any two of the sides will add up to more or as much as the remaining side). In the case of Euclidean space we define the function d as follows. Suppose x represents the three dimensional coordinate, (x_1, x_2, x_3) and y the coordinate (y_1, y_2, y_3) then

- $d(x, y) =_{\text{df}} \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$

This turns out to be a generalized version of Pythagoras's theorem, don't worry if this doesn't make any sense to you – all you need to know is that $d(x, y)$ represents the distance (as you would have intuitively thought of it) between x and y .

The next important concept we must tackle is the idea of a region of Euclidean space. We may think of a region of Euclidean space as a region of space as we would normally talk of it. However as all we have from the definition of Euclidean space is points and the notion of distance between points we must define a region of space to be a set of points. The region defined is simply to be thought of as the region of space which occupies just those points in the set. One important kind of region is the 'open ball'. An open ball should be thought of as a sphere minus its skin – a sphere without the spherical boundary surrounding it. Given a centre, a , and a radius, ϵ , we define the open ball around a of radius ϵ (the epsilon ball around a for short) as:

- $B_\epsilon(a) =_{\text{df}} \{x \in \mathbf{R}^3 \mid d(x, a) < \epsilon\}$

⁶ I shall talk only about space in three dimensions, but I will assume that this can be generalised to space-time and four dimensions if required (for example to discuss eternalism).

This ball is open because it does not contain its skin (by the ‘skin’ of a region I shall always mean the two dimensional surface which surrounds that region). In general we shall define an open region, X , as follows. Remember X is a set of points from \mathbf{R}^3 .

- A region X is open iff for every $x \in X$, there is an $\varepsilon > 0$, $\varepsilon \in \mathbf{R}$ such that $B_\varepsilon(x) \subseteq X$.

What this says intuitively is that, which ever point you take within the region (no matter how close to the edge of the region) you always have room to wiggle around in any direction and stay inside the region (this is expressed by saying that there is a ball small enough to fit inside X and contain your point). This is also equivalent to saying the region does not contain any of its skin, for if it did then there will be a point *on* the skin, in which wiggling away from it will always force you to leave the region.

With this machinery in hand we should now be in a position to give some models for the various axioms given in section 5. Remember that mereology uses only one non-logical symbol, \leq , so to provide a model we simply must specify the domain and give an interpretation for \leq . All this means is that we must specify a set of objects which the quantifiers of our theory range over, and a relation over our objects which is supposed to represent the parthood relation. For the six standard axioms with Unique Fusion, Top, Bottom and Atoms we shall take our domain to be regions of Euclidean space. Then to interpret \leq , we take the subset relation, \subseteq , between regions. Remember that regions are sets of points and thus a subset of a set of points will correspond to a subregion of that region. So under this interpretation subregions are parts of regions. It is left as an exercise to the reader to show that the six standard axioms come out true on this interpretation (for example, Product and Sum are guaranteed by the fact that two sets always have an intersection and a union).

To see that Unique Fusion is true in this model consider the set of points, S , which satisfy the first order definable property φ (or the property X in the second order case). Given the interpretation, x and y are supposed to overlap iff the intersection of x and y is non empty. Something intersects with S non-trivially iff it contains a point of S , i.e. iff it contains (and hence overlaps with) a point having the property φ . Thus S has the characteristic that something overlaps with S iff it overlaps with something having the property φ . That this is true regardless of our choice of φ shows us that Unrestricted Fusion is true. Proving this fusion is unique is left to the advanced reader.

That Top and Bottom are true in this model is fairly easy to see. For Top we simply take the set of all points in Euclidean space. All regions of Euclidean space will be subregions of the whole of Euclidean space. Similarly for Bottom, take the empty set of points. The empty set is a subset of all sets and, in this model, is thus a part of all regions of Euclidean space (it is easy to see, in this case, why Bottom is so philosophically controversial). It is trivial to modify our model so that both \neg Bottom and \neg Top come out true. For \neg Bottom we simply take our domain to be *non-empty* subsets of Euclidean space, and for \neg Top we simply consider the *proper subsets* of Euclidean space.

To see that Atoms is true in this model we note that Bottom has no proper parts and is thus an atom, and similarly is also a part of everything. This is less helpful since most

mereologists reject the existence Bottom, so let us give a model for Atoms and \neg Bottom along with the standard axioms. Here we simply take the non-empty subsets of Euclidean space as for \neg Bottom. To see Atoms is true take the regions consisting of one point (the so-called singleton sets – a set containing exactly one object) to be counted as the atoms. Each of these regions contains only one element, and since Bottom (the empty set) is discounted, it will have no proper parts. Notice also that every region is a set of points so every set of points will have a singleton set containing a point as a subset. Thus in this model the points are the atoms.

Finally we shall give a model for Gunk. Gunk is the negation of Atoms and thus cannot be consistently added to a system already containing Atoms as an axiom. For this model we take the non-empty regular⁷ open sets of points in Euclidean space as our domain. So here we have restricted ourselves even further by discounting all regions which contain part of their skin. So points are not in our domain since a point does not contain any open ball around itself (because open balls always have non-zero radius) so singleton sets are not open. All parts of an open set will have further proper parts since you can show it contains an open ball which is strictly smaller.

If you have been following so far, and you know your completeness theorem for first order logic, it should be clear we can glean some independence results from the preceding remarks. An independence result simply says that a certain axiom is not already provable from some other axioms and hence isn't superfluous (which is a good thing). We have provided models for standard mereology + Unique Fusion + Top and for standard mereology + Unique Fusion + \neg Top. This means that, given standard mereology + Unique Fusion is consistent (which it is), Top cannot be proven from them. Similarly reasoning shows that Bottom is independent of standard mereology + Unique Fusion. Since Atoms and Gunk are mutually incompatible we can also show that these are independent from mereology because the model we provided for Gunk, non-empty open sets in Euclidean space, also satisfies standard mereology + Unique Fusion.

One very important model theoretic result about mereology, which just about trumps everything I've said so far, is the following:

Tarski's Theorem

Any model of Standard Mereology + Unique Fusion + Top + Bottom is a model of a complete Boolean algebra and vice versa.

Similarly any Boolean algebra with the bottom element deleted is a model for Standard Mereology + Unique Fusion + Top + \neg Bottom (which is the formulation of mereology most people use). It is not important that we know what a Boolean algebra is, but if you happen to know then that is great! The result is here for completeness. A lot of results have been proved about Boolean algebras, so this result is very useful because it says all these results apply to mereology too!

⁷ In topological jargon, an open set is said to be regular if it equals the interior of its closure. The 'closure' of an open region is simply that region plus its skin, the 'interior' of a region is that region minus its skin (if it has any). Surprisingly removing and then replacing a regions skin can result in a different region!

8. Further Stuff

Mereology as I've discussed it above accounts for lots of facts about objects and their parts. However there have been some occasional need to look at stronger systems which include some of the elements of mereology we have been discussing. Here is a very brief overview.

Intensional Mereology

The idea behind intensional mereologies was introduced by Peter Simons. The aim was to merge modal and temporal concerns with mereology. One of the problems with mereology as we have discussed it is that it has Extensionality as a theorem (recall that this meant x and y are identical iff they have the same parts). A consequence of this is that we seem to be committed to an ontology of 4D perduring objects (see section 5). Another consequence of this is mereological essentialism: an object necessarily has the parts it actually has. Intensional mereology mixes the parthood relation with elements of temporal and modal logic in an attempt to explain the problems surrounding extensionality. An important book to read on this is Simons, 1987, [11].

Mereotopology

Mereology deals with the parthood relation. Various other concepts can be defined in terms of parthood, such as overlap, sums and so on. However the notion of connectedness⁸ is not definable in terms of parthood – connectedness is a purely topological property. Mereotopology combines mereology with the idea of connection and has been used to solve various problems to do with boundaries, as well as being used extensively by computer scientists. Casati and Varzi, 1999, [3] cover mereotopology well. For the historical reading, Whitehead, 1929, [17] is probably its first appearance and see Clark, 1981, [4] for a rigorous version of Whitehead's system.

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⁸ A region, X , is connected iff given any two points in X , you can draw a continuous line from one point to the other without leaving X . In topological terms we say a region is connected iff it cannot be represented as the union of two disjoint open regions, but it will take a little thought to see how these definitions are equivalent. For an explanation the term 'open' see section 7.

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