Electron Beam Stabilization using Model Predictive Control

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Objective

The aim of this project is to design and implement a controller for electron beam stabilization at **Diamond Light Source**. The control algorithm will be based on **model predictive control** (MPC). The choice is motivated by its ability to meet the following control requirements:

- Treatment of **multiple actuator dynamics**
- Consideration of actuator **amplitude** and **slew-rate constraints**
- Handling of **actuator failures**

Synchrotron Light Source

A synchrotron is a special type of **particle accelerator** in which, typically, **electrons** travel around a circular path at relativistic speeds. Figure 1 depicts the path electrons follow after being accelerated up to the **speed of light**. When the electrons are bent around the storage ring, they lose kinetic energy and emit it in the form of light, often referred to as **synchrotron light**. The synchrotron light spans the electromagnetic spectrum from infrared to ultra-violet light and X-rays. The synchrotron light is channeled into **beamlines** and used as high-power light source for investigating e.g. the molecular structure of a wide range of materials.

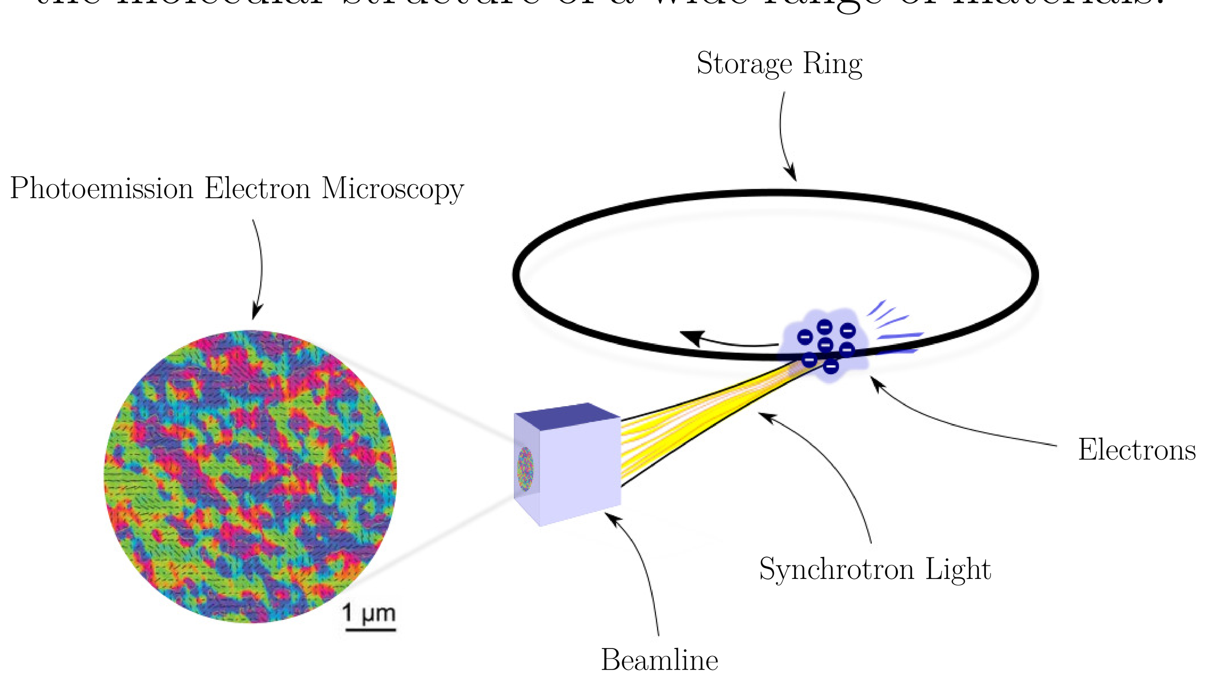

Figure 1: Illustration of a synchrotron. The image on the left-hand side depicts the magnetic profile of a nanotech sample; taken from *"Diamond in Action", Beamline I06, InsideDiamond, January 2013, Issue 1, Page 7.*

Fast Orbit Feedback

Numerous magnets serving different purposes are spread around the ring (Figure 2). In order to guarantee qualitative properties of the synchrotron light, **deviations** from the ideal orbit path must be **minimized**. The **corrector magnets** are responsible for curtailing deviations in the horizontal and vertical planes perpendicular to the direction of motion to about **10μm** and **0.6μm**, respectively.

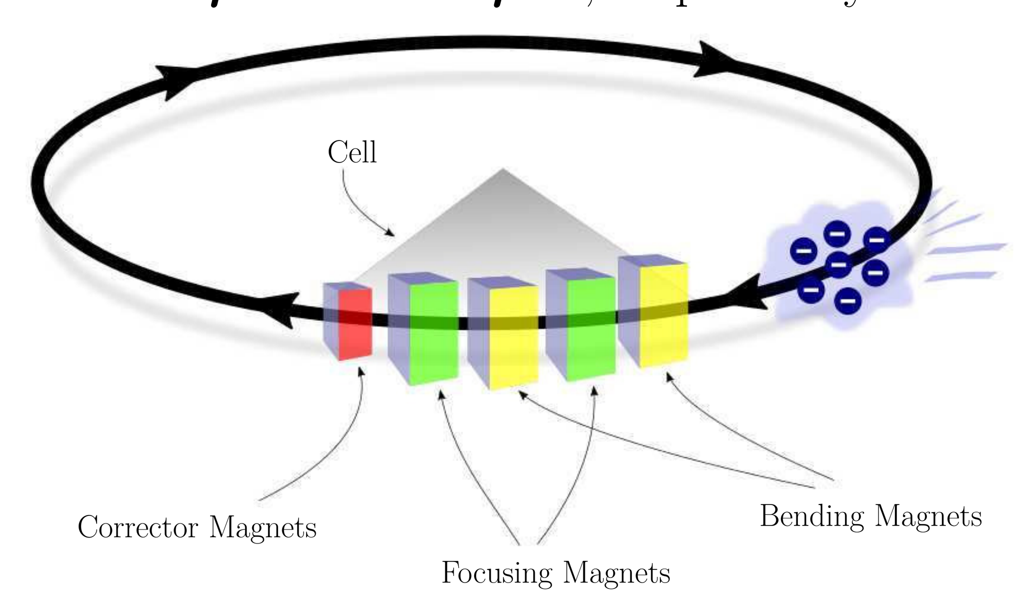

Figure 2: Magnets used to steer electrons around the ring.

The deviations are measured using **beam position monitors** and forwarded to the controller. Setpoints minimizing the deviations are calculated and routed back to the magnets (Figure 3). This feedback loop is operated at a frequency of **100kHz**. Most synchrotrons employ a PID controller with the exception of Diamond Light Source which uses Internal Model Control. Next generation control requires the consideration of actuator constraints.

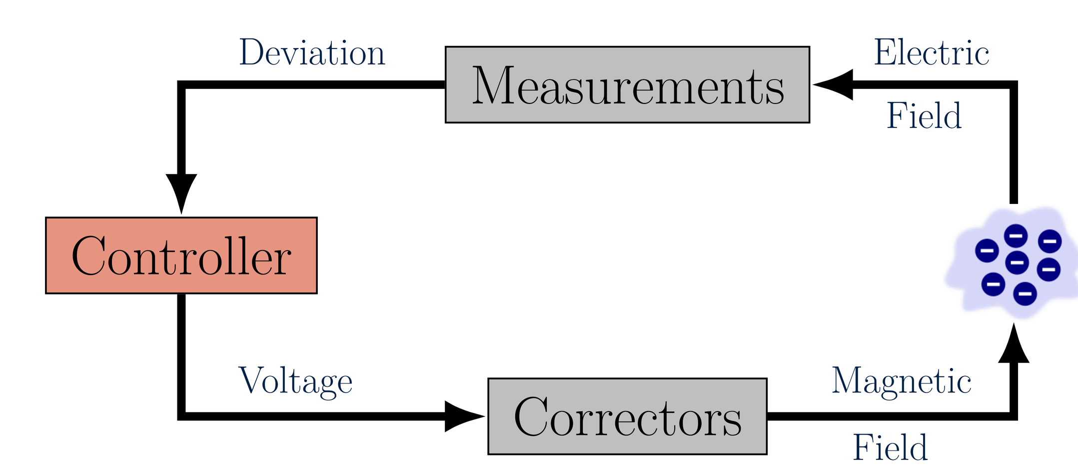

Figure 3: Fast Orbit Feedback for electron beam stabilization.

High-Frequency Real-Time Optimization

This project requires solving a **constrained quadratic program** with more than **200 decision variables** at a rate of **100,000 times per second**.

Model Predictive Control

Given a discrete-time linear dynamical system and an initial condition $x(t)$ at time t, MPC computes a control law by predicting the future evolution of the system and **minimizing** an objective function over a horizon H. This is achieved via repeated solution of the following **quadratic program**,

$$\min \sum_{k=0}^{H-1} x_k^{\mathrm{T}} R^{\mathrm{T}} R x_k + u_k^{\mathrm{T}} Q u_k + x_H^{\mathrm{T}} P x_H, \quad (1a)$$

$$s.t.\ x_{k+1} = A x_k + B u_k, \qquad x_0 = x(t), \quad (1b)$$

$$-u_{max} \leq u_k \leq u_{max}, \quad (1c)$$

$$-u_{rate} \leq u_k - u_{k-1} \leq u_{rate}, \quad (1d)$$

where x_k represents the current flowing through the corrector magnets, $R x_k$ the instantaneous effect of the magnetic field onto the beam position and u_k the input voltage to the magnets. For this problem, the main **advantage** of MPC lies in the consideration of **amplitude** (1c) and **rate constraints** (1d).

Block Circulant Matrices

A **block circulant matrix** R of order n with blocks $R_i \in \mathbb{R}^{p\times q}$ is characterized by the following structure:

$$R = \begin{bmatrix} R_1 & R_2 & \dots & R_n \\ R_n & R_1 & & R_{n-1} \\ \vdots & & \ddots & \vdots \\ R_2 & R_3 & \dots & R_1 \end{bmatrix} \in \mathbb{R}^{np\times nq} \quad (2)$$

A fundamental property of block circulant matrices is that all block circulant matrices are **block diagonalized** by the same pair of matrices:

$$(F_n \otimes \mathbb{I}_p)^H R (F_n \otimes \mathbb{I}_q) = \mathrm{diag}\{\sigma_1, \dots, \sigma_n\}, \quad (3)$$

where $\sigma_i \in \mathbb{C}^{p\times q}$ are complex-valued blocks and F_n the Fourier matrix. For a synchrotron, the matrices present in (1a)-(1b) are block circulant matrices. This fact can be exploited to **decompose** the optimization problem (1) and significantly **speed up** the computation time required to solve (1) [1].

Results

Preliminary tests demonstrated that using the block circulant decomposition it is possible to solve problem (1) at the required frequency [1]. The controller is being implemented on a *Texas Instruments C6678* digital signal processor featuring **eight cores**. The **Operator Splitting Quadratic Program** solver (osqp.org) - a software suitable for solving (1) on embedded systems - is currently being ported to the digital signal processor and **parallelized**. For that purpose, a fast interprocessor communication scheme has been developed (github.com/kmpape/Fast-IPC-TIC6678).

Diamond Light Source

Diamond Light Source is the UK's national synchrotron facility and is located at the Harwell Science Campus in Oxfordshire. It features a storage ring with a circumference of 560m. Over 200 corrector magnets and beam position monitors are used to stabilize the electron beam. It is planned to test the control algorithm at Diamond Light Source in 2020.

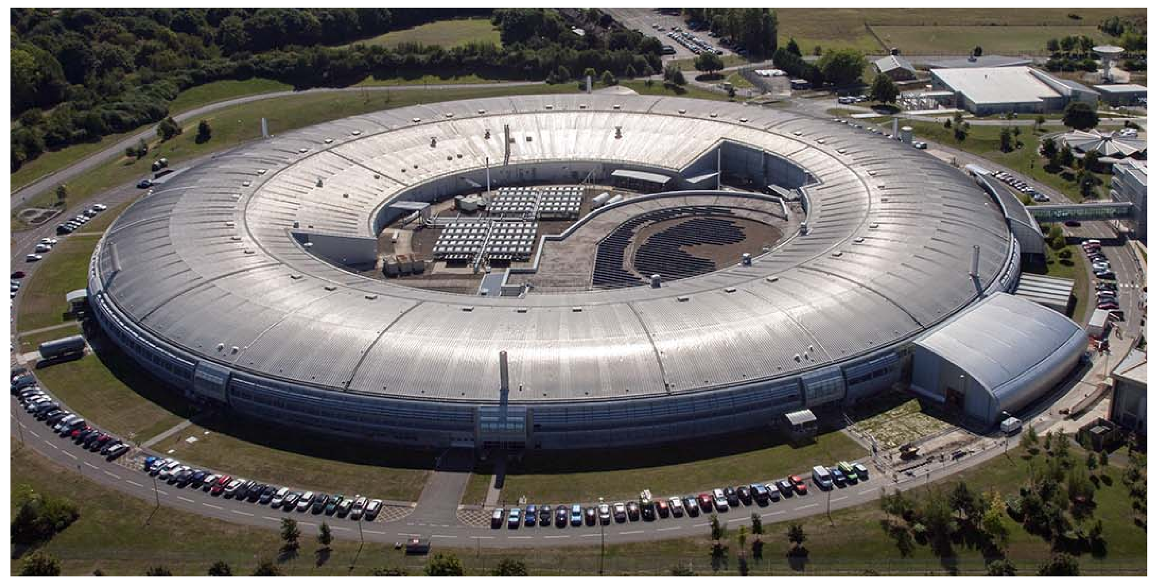

Figure 4: Diamond Light Source synchrotron.

[1] I. Kempf *et al.*, "Alternating direction of multipliers method for block circulant model predictive control," in *2019 58th IEEE Conference on Decision and Control (CDC)*, Nice, France, December 2019, accepted.