CHAPTER 9 | On the Emergence of Individuals in Physics

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I TAKE "INDIVIDUAL" TO mean an object that (i) persists, somehow, in time, and (ii) can be uniquely identified throughout the time that it persists. I take "object" (and interchangeably, "thing") to mean anything that can stand in predicate position, typically the value of a bound variable; in this I follow Quine.

The main task of this chapter is to show how individuals arise from classical and quantum indistinguishable particles—from objects that are *permutable*, meaning, whose descriptions are invariant under permutations. There is a prima facie difficulty with (ii): how can something be uniquely identified—at any time—if when interchanged with other things the overall description is unchanged? The problem arises, I shall argue, as much in classical physics as in quantum physics, although it takes a slightly different form in the two cases. In both cases (in the quantum case only in certain circumstances) we can identify something else as not being subject to interchange; for example, we can pass from talk of particles that have states to talk of the states themselves—to points of phase space or one-particle states in Hilbert space (one-dimensional subspaces). But as we shall see, this option has no real connection to the way we ordinarily refer to individuals in the laboratory, or the use of names in defining the state spaces of individuals as distinguishable things.

In this chapter I am interested in the question: what is the metaphysics appropriate to the way individuating reference actually goes, in the laboratory, consistent with the requirement of indistinguishability? This amounts to the question (or I shall take it as the question) of how permutation symmetry can be broken, at one level of description, whilst remaining intact at a more fundamental level of description.

9.1 Logic and Ontology

My starting point is the notion of object in first-order logic in simple declarative sentences. This would seem the most secure ground for relating representations in physics (phase space, Hilbert space) to things, via referring expressions in ordinary language. I see the detour through language as reflecting the fact that physical theories are primarily about *quantities*, rather than things, so we cannot simply consult our best physical theories to discover what there is. It may even be that the world is at bottom a mathematical structure, or "has" a mathematical structure; but in trying to be more precise as to what that involves, I see no safer way than to put questions of ontology into words, using simple declarative sentences and the standard apparatus of first-order quantifiers. So given the mathematical structure of a physical theory, if there are puzzles about what aspects of it are real, or correspond to reality, or what has purely mathematical as opposed to physical significance, we should first see what can be said about things in simple declarative terms—in terms of objects, identity, properties, and relations. Objectual structure in this sense I see as a coarse-graining of the mathematical structure of the world: the pegs and poles that gather its materials and most reliably tie them together.[1]

Evidently what is needed for this to work is a close association of predicates with physical quantities, on the one hand, and with the domain of quantification, on the other. My suggestion is that the "allowed" predicates be those that can be constructed from values (and changes in values) of physical quantities, and specifically quantities that are *invariant* under the exact symmetries of a theory. This plausibly includes all quantities that are actually measurable. And further, that allowed predicates be tied to the domain of quantification by the requirement that no more is admitted than are required by Leibniz's law.[2]

This needs some explanation. Let $\mathfrak{L}$ be a first-order language with a finite lexicon, including identity (what I shall call primitive identity). Suppose further that only certain (perhaps complex) predicates in a subset $\mathcal{P} \subset \mathfrak{L}$ are allowed (those corresponding to physically real properties, defined by invariant quantities). The principle is then: physical objects are values of variables (in the logical sense) that can be discerned by the allowed predicates. By this I mean, if s and t are terms for physical objects:

$$s = t \leftrightarrow \bigwedge_{F \in P} F\ldots s\ldots \leftrightarrow F\ldots t\ldots. \tag{9.1}$$

The implication from left to right follows from Leibniz's law (so the language is extensional). It is the implication from right to left that is controversial, enforcing, for physical objects, a version of the Principle of the

Identity of Indiscernibles (PII).[3] Of course, if primitive identity is itself an allowed predicate, (9.1) is a tautology.

Generalizing on free variables, one obtains from among the conjuncts of (9.1) not only sentences like

$$\forall y(Fsy) \leftrightarrow \forall y(Fty) \tag{9.2}$$

but also like

$$\forall y(Fsy \leftrightarrow Fty). \tag{9.3}$$

It is sentences of the form (9.3) that are false of the familiar supposed counterexamples to the PII (so by (9.1), $s = t$ is false as well, whereupon they *cease* to be counterexamples); those of the form (9.2) are all true. Thus consider Black's two iron spheres, exactly alike, one mile apart, in an otherwise empty Euclidean space. Suppose allowed predicates are those invariant under the symmetries of this space (translations and rotations). Every monadic predicate true of one sphere is true of the other, including complex monadic predicates with embedded quantifiers, as in (9.2). But taking F as the symmetric, irreflexive, and invariant (so clearly allowed) relation "is one mile apart from," (9.3) is false, so $s \neq t$.

As Quine (1976) showed (but using the terminology "discriminables"), identity construed in this way yields the following exhaustive classification: objects s and t are *absolutely discernible* if for some monadic predicate, Ps but not Pt; *relatively discernible* if for some binary predicate, Fst but not Fts; and *weakly discernible* if for some binary predicate, Fst and Fts, but not Ftt and not Fss. When I say an individual is *identifiable* (at a time or throughout a period of time), I mean it is absolutely discernible (at a time or throughout a period of time); thus individuals, in my sense, are always absolutely discernible. By "indistinguishables" I mean things that are at most weakly discernible, if discernible at all.[4]

Should the negation of identity itself be an allowed predicate? It is not, or not obviously, a relation definable in terms of the invariant values of any particular physical quantity (although, e.g., as suggested by causal set theory, a speculative program in quantum gravity, that might yet change). Rather, and more plausibly if physics is at bottom about quantities rather than things, it should be implicitly defined by them all, by the PII in the sense (9.1). So my proposed methodology counts against it. If it is allowed anyway, with identity taken as primitive, the PII in the form (9.1) is trivialized, and as an account of identity of physical things it loses its philosophical interest. But Quine's classification remains: it is just that among the things that are only weakly discerned, are those that are only discerned by the negation of identity. The essential notions for our purposes are absolute and weak discernibility,[5] categories that are defined whether or not primitive identity is allowed.

In summary: individuals are absolutely discernible. Indistinguishables, assuming they can be discerned at all, are at most weakly discernible. Since a permutation leaves their state-description unchanged, it should leave their predicative description—in terms of allowed predicates—unchanged as well. But that means reference to only one of a number of indistinguishables, by an allowed monadic predicate, however complex, is impossible: if it applies only to one indistinguishable, and not to any other, it would absolutely discern it, contrary to supposition. If it applies to one indistinguishable, it must apply to every; if it is a binary predicate and applies to one pair of indistinguishables, it must apply to every pair; and so on. Call such predicates *permutation invariant* (or invariant for short). Any talk of indistinguishables, if it is to respect permutation symmetry (if it is to be allowed, if it is to be invariant under permutation symmetry) must be conducted in terms of invariant predicates. I shall also talk of indistinguishable as *permutables*.

The contrast, evidently, is with predicates and function symbols (including names) that do not respect this symmetry—as are used in descriptions of laboratory systems and everyday things. And now there is an obvious difficulty. The descriptions (states) of electrons, protons, and neutrons are invariant under permutations; electrons, protons, and neutrons are therefore at most weakly discernible. In short, they are indistinguishable. Yet ordinary objects are constituted by electrons, protons, and neutrons, so reference to ordinary objects must break permutation symmetry. Or to turn the problem around: what, from the point of view of a symmetry-preserving description in terms of indistinguishables, is being described by a symmetry-breaking description in terms of individuals?

Call it the *paradox of constitution*: descriptions of macroscopic things may be singular; but specification of the electrons, protons, and neutrons of which they are composed is impossible without breaking permutation symmetry.

9.2 Particles and Trajectories

The paradox seems stronger in the case of classical particles, where we think we know what we are talking about. But here there is an easy response: simply deny that classical particles *should* be treated as indistinguishable; insist that permutability of classical particles is simply unintelligible.

Since weak discernibles are permutable, and weak discernibility (as we have just seen) is a perfectly well-defined logical category of objects, this claim is hardly obvious. But it is supported by a simple argument: let the state-description pick out the exact motions of particles (so the state is not just a probability distribution); then the particles can be identified by their trajectories, *and are, therefore, not indistinguishable*.[6]

There is something right about this argument,[7] but there is something wrong with it too, for in certain circumstances the same can be said in quantum mechanics. In place of points in phase space, we have rays in Hilbert space. Let Π_N be the permutation group for N elements, and let c be a normalization constant. Consider a state of N particles of the form

$$|\Phi\rangle = c \sum_{\pi\in\Pi_N} |\phi_{\pi(a)}\rangle \otimes |\phi_{\pi(b)}\rangle \otimes \ldots \otimes |\phi_{\pi(c)}\rangle \otimes \ldots \otimes |\phi_{\pi(d)}\rangle, \tag{9.4}$$

where each summand is the tensor product of N pairwise-orthogonal one-particle vectors $|\phi\rangle$, elements of the one-particle Hilbert space $\mathcal{H}$ labeled by the symbols "a," "b," etc. (so there are no repetitions of these symbols). Such a state is totally symmetrized; as such it is invariant under permutations. It describes indistinguishable particles, specifically, bosons. If we allow for superpositions of states of the form (9.4), we obtain the entire Hilbert space of N bosons.[8] But restricting to states like (9.4), we can speak of one-particle states instead; and we may take it that each of these states, as orthogonal to any other, is absolutely discernible. It is true that in this case we do not have *trajectories* as such, but there is something just as good under a further restriction. Thus, let the unitary dynamics $\mathbf{U}$ factorize, so it is of the form

$$\mathbf{U}|\Phi\rangle = c \sum_{\pi\in\Pi_N} U|\phi_{\pi(a)}\rangle \otimes U|\phi_{\pi(b)}\rangle \otimes \ldots \otimes U|\phi_{\pi(c)}\rangle \otimes \ldots \otimes U|\phi_{\pi(d)}\rangle. \tag{9.5}$$

Then each one-particle state has a unique trajectory, namely, its orbit under U. The analogy with the classical case is complete. But now, if classically we can simply identify particles with trajectories—so that particles are *not* indistinguishable—then why not in quantum mechanics simply identify particles with the orbits of one-particle states? And if we can: does it follow that *quantum* particles are distinguishable after all?

Of course symmetrized N-particle states are *not* in general of the form (9.4),[9] whereupon it is no longer possible to identify (or replace) particles by one-particle states. And there is another difficulty in the case of fermions, in the analogous state to (9.4) (but antisymmetrized rather than symmetrized): in that case the state can also be rewritten in terms of *other* collections of N orthogonal one-particle states, as the singlet state of spin makes clear. Which is the right collection? Both difficulties show that something else is going on in quantum mechanics.

There surely is, but it seems to have very little to do with indistinguishability. The same problems arise for *distinguishable* quantum particles. The state space for N distinguishable particles is spanned by product states

$$|\Phi\rangle = |\phi_a\rangle \otimes |\phi_b\rangle \otimes \ldots \otimes |\phi_c\rangle \otimes \ldots \otimes |\phi_d\rangle \tag{9.6}$$

rather than by states of the form (9.4). For any state (9.6), each particle can be assigned a unique one-particle state (the kth in the sequence as specified by the tensor product)—a unique pairing of particles with states. But no such assignments of one-particle states to particles is possible for superpositions of states (9.6).[10] And as for the second problem, the ambiguity of the one-particle states in the case of fermions: it arises for the singlet state of spin, regardless of whether or not the two fermions are indistinguishable. (I shall come back to this question in section 9.6.)

The correct conclusion to draw, surely, is not that particle indistinguishability makes no sense in either classical *or* quantum mechanics;[11] it is that at least in some circumstances in quantum mechanics, and nearly always in classical mechanics, one can shift from a description in terms of indistinguishables (particles that have trajectories or that are in one-particle states) to a description in terms of distinguishables (the trajectories, the one-particle states). And notice, in this shift, we pass from a description in terms of all indistinguishable particles (in (9.4) and (9.5), of all N particles), to a description of a particular trajectory, a particular one-particle state, apparently without any need to make reference to any other trajectory or any other one-particle state.

9.3 Indistinguishability in Ordinary Language

The same shift in ontology can be mimicked in ordinary language. Consider:

(i) Buckbeak the hippogriff flies higher than Pegasus the winged horse.

Permuting the expressions "Buckbeak the hippogriff" with "Pegasus the winged horse" would give an entirely different sentence, one that contradicts (i). But suppose we omit proper names and make do with descriptive predicates instead, for example, "is Buckbeak-shaped" and "is Pegasus-shaped," where being Buckbeak-shaped includes being a hippogriff and so on, as descriptivists about proper names recommend. Assume for convenience that these are the only two mythical creatures (the only two things in our domain of discourse), so we do not have to worry about uniqueness. In that case (i) is equivalent to (dropping quantifiers)

(ii) there is x and there is y such that x is Buckbeak-shaped and y is Pegasus-shaped and x flies higher than y.

Now consider the complex predicate

(iii) x is Buckbeak-shaped and y is Pegasus-shaped and x flies higher than y, or y is Buckbeak-shaped and x is Pegasus-shaped and y flies higher than x.

Evidently (iii) is invariant under permutation of x and y. Taken as a single complex predicate, and assuming it is the only allowed predicate (or that there are others but they are all equally permutation-invariant), then x and y are only weakly discernible (note that (iii) is irreflexive). Under this constraint, it is impossible to make reference to x or y singly; yet (iii) under existential quantifiers conveys the same information as (ii).

In terms of properties, we have a way of understanding properties of permutables as disjunctive properties. It carries over to quantum mechanics, where properties are represented by projection operators. Sums of orthogonal projectors correspond to disjunctions of the corresponding properties. Since they all sum to one, to obtain the negation of a property subtract it from one. Then for N indistinguishable quantum particles, the projector corresponding to the fact that there is exactly one particle that is an A, with corresponding 1–particle projector P, is

(iv) $$P \otimes (I-P) \otimes \ldots \otimes (I-P) + (I-P) \otimes P \otimes (I-P) \\ \ldots \otimes (I-P) + \ldots + (I-P) \otimes \ldots \otimes (I-P) \otimes P.$$

where there are N factors in each term of the summation, and $\binom{N}{1} = N$ summands (it is clear how this generalizes to properties shared by $k \leq N$ particles). (iv) is not a property that one of the N particles has, and none of the others: it is a property of the collective. It is the property that exactly one of N indistinguishable things is an A, or has the corresponding property P.

In the predicate calculus the parallel construction is

(v) $$(Ax_1 \wedge \neg Ax_2 \wedge \ldots \wedge \neg Ax_N) \vee (\neg Ax_1 \wedge Ax_2 \wedge \neg Ax_3 \ldots \wedge \neg Ax_N \vee \\ \ldots \vee (\neg Ax_1 \wedge \ldots \wedge \neg Ax_{N-1} \wedge Ax_N).$$

As with (iii), (v) is permutation-invariant, and as with (iv), it is a complex N-ary predicate that is true of all N particles. Of course there are permutation-invariant predicates with arity $n < N$, but they only report what is true of every sub-collection of n particles out of N; of what is true of every particle (n = 1), of every pair of particles (n = 2), and so on. Thus, "there is an x that is a Buckbeak shape or a Pegasus shape, that does not fly higher than itself" exhausts what can be said of one of the two (hence of both). The more informative predications are those that include all their relations. In this sense, permutability forces a kind of structuralism: it is the global ascriptions of properties and relations that are the most informative.

Are they informative enough? It seems so, at least to the extent that sentences in the predicate calculus about collections of N things are informative enough *without* the restriction to permutation symmetry. For it is a theorem that any categorical first-order sentence (hence a sentence whose models all have the same finite cardinality) describing N objects is logically equivalent to one of the form

$$\exists x_1 \exists x_2 \ldots \exists_N F x_1 \ldots x_N \tag{9.7}$$

where F is totally symmetric (Saunders 2006a). And here I am relaxed about the restriction to finitely many things, because states on classical phase space and in Hilbert space (supposing it is separable) are likewise restricted to descriptions of finite numbers of particles.

Having understood how to go from ordinary descriptive sentences to descriptions invariant under permutations, it is obvious how to go back again. From (iii), use the "that which" construction instead of quantifiers and variables in each disjunct, to obtain

(vi) That which is Buckbeak-shaped flies higher than that which is Pegasus-shaped, or that which is Buckbeak-shaped flies higher than that which is Pegasus-shaped.

The disjunction is then redundant; at the same time, there is no question of interchanging "that which is Buckbeak-shaped" with "that which is Pegasus-shaped" *salva veritate*, for "flies higher" is asymmetric. And from (vi), pass to "the Buckbeak shape," "the Pegasus shape," and then to "Buckbeak" and "Pegasus," like passing from "that which is butter" to "butter." We obtain the sentence (9.1) that we started with. The predicates "is Buckbeak shaped" and "is Pegasus shaped" function as what I shall call *individuating predicates*.

Evidently there are two steps involved: first, find an individuating predicate; second, make use of it as a mass noun, in object position in predicates, without any requirement of permutability. But to be serviceable—to be available in a wide variety of states of affairs—the individuating predicate should not include too much. It should be stable in time, if it is to serve as criterion that can actually be used, whilst allowing for plenty of change. "Animal shape" is reasonably robust in this sense, but only when understood in terms of general anatomy, not in terms of the precise shape that an animal has at a particular moment in time. It should not be too generic, either, if it is to absolutely discern one out of a collection of things at one time. It is a Goldilocks property, that is just right as a referring expression for a general context of use. Considerations like these are familiar to descriptivist theories of names.

9.4 Reasons for Permutation Invariance

Agreed that we can see how the trick is done; what is the point of it? The lengthy disjunctive properties corresponding to permutation-invariant predicates seem rather contrived. Why restrict to properties like this, or equivalently, to predicates like this?

One answer is that in quantum statistical mechanics and field theory, there are important *empirical* consequences of permutability. But while there are advantages to symmetrizing in classical statistical mechanics, in that case it leads to no directly testable consequence. It is unlikely to be of practical use in formal logic, either, so let me put the pragmatic answer to one side. The question remains: why symmetrize?

An obvious line of thought is that the symmetry arises because it doesn't matter which particle has which trajectory (or one-particle state), because the particles involved are "simples"; they all have exactly the same intrinsic properties (the same mass, charge, and spin). The various permutations of these particles (with everything else unchanged) yield observationally indistinguishable states of affairs, so they should not be conceptually distinguished, either. But it is not obvious how this is to work at the level of everyday language. In the case of Buckbeak and Pegasus, it invites us to picture a realm of things, all with the same intrinsic properties, each of which can have one or other of a number of animal-shapes. What are these things, exactly?

It is the wrong picture. It may be the wrong picture in quantum mechanics too, where—for example in string theory—the intrinsic (state-independent) properties of simples are in danger of disappearing altogether. If none of the intrinsic properties of elementary particles turn out to be state-independent (not mass, charge, or spin),[12] what, precisely, remains?

An alternative picture is that the redundancy attaches not to a choice among physical particulars that are intrinsically the same, but to something else; to a referential device, for example, or perhaps to something more metaphysical.[13] In formal logic: to values of variables as elements of some class $\mathcal{D}$, over and above their function as relative pronouns and the expression of pluralities. In phase space and Hilbert space: to values of particle labels, over and above their function of keeping track of sequence position (in terms of ordered sequences of 6-tuples or coordinates, for phase space points, and tensor products of one-particle states, for rays in

Hilbert space). The redundancy in each case is eliminated by passing to permutation-invariant descriptions. Let us take each case in turn.

9.4.1 Logic and Model Theory

The suggestion is that permutation symmetry may function in logic and model theory in roughly the way that it functions in a physical theory: it corresponds to a certain kind of redundancy of representation that can be eliminated by passing to an invariant description. How might this work?

Recall that an interpretation $\mathfrak{U}$ of a first-order language $\mathfrak{L}$ consists of a universe of discourse $\mathcal{D}$, an assignment of relations $P^{\mathfrak{U}}$ on $\mathcal{D}$ for each $\mathfrak{L}$-predicate *P*, and of functions $f^{\mathfrak{U}}:\ \mathcal{D}\times\ldots\times\mathcal{D}\rightarrow\mathcal{D}$ (*n* factors) to function symbols *f* (*n* arguments) in $\mathfrak{L}$. Proper names are 0-ary function symbols, assigned designated elements of $\mathcal{D}$. A valuation σ on $\mathfrak{L}$ is a mapping of variables $x\rightarrow x^{\sigma}\in\mathcal{D}$, inducing a mapping to truth values as: *Px* is true if and only if $x^{\sigma}\in P^{\mathfrak{U}}$, with obvious extensions to quantified variables. A model of an $\mathfrak{L}$-sentence (with no free variables) is an interpretation under which it is true for every valuation.

A model, therefore, comes equipped with relations on $\mathcal{D}$. How are they defined? Proper names are assigned designated elements of $\mathcal{D}$. Designated how? Predicates are interpreted by their extensions, for monadic predicates, by subclasses of $\mathcal{D}$. What are those elements, and how are they specified? The primary role of variables in syntax, apart from generality, is that the same variable may be repeated in a sentence: this their function as relative pronouns (the "that which" construction). But under a valuation of $\mathfrak{U}$, they are also assigned elements of $\mathcal{D}$. These elements are given in advance. They are, perhaps, abstract particulars. How are they related to physical particulars?

There are the usual suspects: by way of proper names; by the intentions of the users of the language; by indexicals; by way of identity (the model just is the world); by an antecedent understanding of the referents of proper names and of the extensions of predicates—take your pick. Or, returning us to our topic, it doesn't matter which element of $\mathcal{D}$ is associated with which physical particular. It is the structure of the model as a whole that represents the world.

Indeed, there is a puzzle here that has long been an embarrassment to philosophers. It was first stated by Quine, as one of a number of arguments for his doctrine of "ontological relativity."[14] According to Quine, at bottom, values of variables are no more than "neutral nodes" that can be shuffled among one another without any linguistic effect. They look very much like permutables.

Quine argued as follows. Let $\lambda:\mathcal{D}\rightarrow\mathcal{D}$ be a bijection, what Quine called a "proxy function" on the universe of discourse, and consider, for any interpretation U, the interpretation U^* under which P is true of λ of what P was true of under U (with the obvious action of λ on sequences); and similarly for function symbols. It then follows that any $\mathfrak{L}$-sentence true under U will be true under U^* as well, even though it talks about quite different things (elements in D). In this way λ induces a new and "unintended" interpretation $\mathfrak{U}^*$ of $\mathfrak{L}$ (to put it in Putnam's terms). Quine viewed the matter as an extension of his doctrine of under-determination of meaning: reference was "relative to a manual of translation." But he also put the matter like this:

> Reference and ontology recede thus to the status of mere auxiliaries. True sentences, observational and theoretical, are the alpha and omega of the scientific enterprise. They are related by structure, and objects figure as mere nodes of the structure. What particular objects there may be is indifferent to the truth of observation sentences, indifferent to the support they lend to the theoretical sentences, indifferent to the success of the theory in its predications. (Quine 1990, 31)

In the case of finite models, the parallel with permutability is hard to ignore. But it cannot be the same: Quine's argument applies to any first-order language, any sentence, any interpretation, whether or not its allowed predicates are permutation invariant. It is also clearly paradoxical—and was seen as such by Putnam.[15] On the other hand, the method of section 9.3 *also* applies to any first-order language, any sentence, any interpretation, so long as it describes a finite number of things. And indeed, permutation-invariant sentences like (iii) are indifferent between U and U^* as their intended model; for them there is no paradox. Permutability is not the same as ontological relativity; it is the cure for it. If we mimic the procedure used in physics, the problem is solved.

There remains, however, the peculiarity that on passing to a description in terms of permutation-invariant predicates, singular reference to any of these "neutral nodes"—to anything less than the entire state of affairs—is impossible. Quine, on the reading I am giving, was half-right in his diagnosis: in classical and quantum mechanics, true sentences, the alpha and omega of ontology, are related by structure, and values of variables serve as mere nodes of the structure, tying it together, but not tying it to anything: it is the structure as a whole that is instantiated in the world. Only if this structure is sufficiently variegated is there a passage to singular reference, and that proceeds quite differently: it is reference to qualitative features of this structure, whether using proper names, Fregean senses, Russellian descriptions, causal chains, or ostension.

If, further, these qualitative features are robust (they admit variation) and are stable (they persist in time), then they are individuating properties. They are the values of variables and referents of proper names, and relations on them are the extensions of predicates, where now there is no requirement of permutation symmetry. Indeed, permuting one qualitative feature into another, altogether different, with all else unchanged, not only yields a distinct state of affairs, but is likely to take us out of the space of physically possible states of affairs altogether.

But can't we just run Quine's argument all over again, in talk of these qualitative features? Of course we can, but then there are other responses to the puzzle, one being that we have an antecedent understanding of what the elements of the universe of discourse are, of which of them are the referents of proper names, and of which of them lie in the extension of one predicate, rather than another—in short, of what are the "perfectly natural properties." All of that is expressed, or represented, in the ground-level representation, in terms of permutables and invariant predicates of the structure as a whole.

9.4.2 Phase Space and Hilbert Space

A point $\overrightarrow{\langle \boldsymbol{q}, \boldsymbol{p} \rangle} \in \Gamma^N$ is specified by $2N$ triples of numbers, where each triple is indexed by a particle label $k = 1, \ldots, N$, thus:

$$\overrightarrow{\langle \boldsymbol{q}, \boldsymbol{p} \rangle} = \langle q_1, p_1; .. ; q_k, p_k; .. ; q_N, p_N \rangle \in \Gamma^N. \tag{9.8}$$

Given that the Hamiltonian is a symmetric function of the N particles—the sequence position of the arguments of the Hamiltonian does not matter—the labels become irrelevant to the dynamics. The permutations are thus symmetries. The phase space point $\overrightarrow{\langle \boldsymbol{q}, \boldsymbol{p} \rangle}$ yields the same set of N trajectories in μ-space as the initial data

$$\pi\overrightarrow{\langle \boldsymbol{q}, \boldsymbol{p} \rangle} = \left\langle q_{\pi(1)}, p_{\pi(1)}; .. ; q_{\pi(k)}, p_{\pi(k)}; .. ; q_{\pi(N),}\, p_{\pi(N)} \right\rangle \tag{9.9}$$

for $\pi \in \Pi_N$. If there are no repetitions of phase space coordinates, there will be $N!$ distinct sequences of the form (9.8), (9.9), each corresponding to a set of N one-particle trajectories in μ-space, differing only in which trajectory is assigned which particle label (passive view), or which is assigned which particle (active view).

The distinction is as real, no more and no less, as the distinction between which element in $\mathcal{D}$ is assigned to which extension $P^{\mathfrak{U}}$ of each predicate P. Just as permutation-invariant predicates are oblivious to such distinctions, invariant functions on phase space (and in particular the Hamiltonian)—functions invariant under permutations of particle labels—are blind to them too.

Similar remarks apply to particle labels in quantum mechanics, and property ascriptions (in terms of totally symmetrized projectors, of the form (iv)) that are indifferent to distinctions as to which particle is in which one-particle state, for states of the form (9.6). And as in quantum mechanics, it is clear how to proceed to a new universe of discourse, in which the distinctions, previously ignored, no longer arise at all. Classically, points of Γ^N related by permutations can be simply identified: that is, Γ^N can be replaced by the quotient space Γ^N/Π_N of Γ^N under Π_N. This is *reduced phase space*. In place of N! equivalent points in Γ^N, there is a single point, denote $\widetilde{\langle \boldsymbol{q},\boldsymbol{p}\rangle} \in \Gamma^N/\Pi_N$. Here $\widetilde{\langle \boldsymbol{q},\boldsymbol{p}\rangle}$ is the same set of N pairs of configuration space and momentum space coordinates as in (9.8) and (9.9), yielding the same N trajectories in μ-space as all the $\overrightarrow{\langle \boldsymbol{q},\boldsymbol{p}\rangle}$s, but expressed as an unordered set:

$$\widetilde{\langle \boldsymbol{q},\boldsymbol{p}\rangle} = \{\langle q_a;p_a\rangle,\langle q_b;p_b\rangle,\ldots,\langle q_c;p_c\rangle,\ldots,\langle q_d;p_d\rangle\} \in \tilde{\Gamma}^N. \tag{9.10}$$

Assuming there are no repetitions,[16] then just as with (9.4), we may speak directly of the one-particle values of position and momenta (or phase-space coordinates) $\langle q_a;p_a\rangle,\langle q_b;p_b\rangle$, and so on, as points or point-like regions of μ-space, rather than of particles that have those coordinates or those trajectories; or equivalently, just use the word "particle" to denote such coordinates, or values of position and momentum, or point-like regions of μ-space, or properties.

Isn't this just to revert to particle labels, and won't the same considerations apply as before? No: particle labels were defined by sequence-position in $\overrightarrow{\langle \boldsymbol{q},\boldsymbol{p}\rangle}$, but $\widetilde{\langle \boldsymbol{q},\boldsymbol{p}\rangle}$ is not a sequence. In place of $\langle q_a;p_a\rangle,\langle q_b;p_b\rangle$, etc. we could just as well have written $\langle q;p\rangle,\langle q';p'\rangle$ and so on. Likewise in (9.4): in $|\phi_a\rangle,|\phi_b\rangle$, etc. "*a*" and "*b*" are not labeling particles, but orthogonal one-particle states: they are distinguishable.

With no use of the machinery of sequences and particle labels, there is no reference to values of labels; so no redundancy either. Predications true of one trajectory, or one one-particle state, will no longer be true if that trajectory or one-particle state is substituted for another—indeed, will in general be an out-right mathematical impossibility. With no restriction to permutation-invariant predicates, singular reference to particulars—so long as there are no repetitions—is straightforward, or as straightforward as it ever is in the use of coordinates to define positions and momenta (or velocities) of particles. And notice that in passing to reduced state space, and making no use or mention of particle labels or names (but only of coordinates $\langle q_a;p_a\rangle,\langle q_b;p_b\rangle$ and thereby of places *a, b* in one-particle phase space), we are implementing the reductionist ploy recommended by Quine:

> Those results [in quantum statistics] seem to show that there is no difference even in principle between saying of two elementary particles of a given kind that they are in the respective places *a* and *b* and that they are oppositely placed, in *b* and *a*. It would seem then not merely that elementary particles are unlike bodies; it would seem that there are no

such denizens of space-time at all, and that we should speak of places *a* and *b* merely as being in certain states, indeed the same state, rather than as being occupied by two things. (Quine 1990, 35).

Places, Quine is recommending, should take object position, and predications should be made of them. The state they are both in is the one-particle state. Note, however, that in making no mention of place in *momentum* space, Quine makes it seem easier than it is to pass to an ontology of places. It is places in μ-space (one-particle phase space) that replace particles, not physical space. It is not a reduction of material bodies to regions of space: it is a reduction of permutable particles to point-like regions of μ-space, and in quantum mechanics, to one-particle states in Hilbert space.

Can we think of either of these as properties? Classically, if strictly point-like, this is a property represented by a delta-function (a distribution) on μ-space, whereas properties are ordinarily associated with functions (characteristic functions). But in quantum mechanics it is purely a matter of terminology: there states are rays, and rays are "regions" of $\mathcal{H}$—meaning subspaces—and subspaces are properties, denoted by the associated projectors $P_{|\phi\rangle}$. The reduction, then, in the quantum case, consists in passing from a global description of *N* indistinguishables, in terms of the totally symmetrized projection

$$P_{|\phi_a\rangle} \otimes P_{|\phi_b\rangle} \otimes .. \otimes P_{|\phi_c\rangle} \otimes .. \otimes P_{|\phi_d\rangle} + \text{all permutations} \tag{9.11}$$

acting on $\mathcal{H} \otimes \ldots \otimes \mathcal{H}$ (*N* factors), to talk of distinguishable properties, the projections $P_{|\phi_a\rangle}, P_{|\phi_b\rangle}$, and so on, each acting on $\mathcal{H} -$ or equally, to talk of one-particle states. And when the unitary evolution factorizes (as in (9.5)), we can talk of the orbits of these one-particle states as well, or sequences of properties; just as we speak of trajectories in classical mechanics, or sequences of positions in μ-space.

But whether point-like regions of phase space, or one-particle states, can really function as individuating properties is another question. Given the dynamics, they may imply a kind of persistence in time (pass to the trajectories, where available); but the property itself, of having such-and-such position and momentum or such-and-such a one-particle state, does not persist, and cannot function as a state-independent property, even (or rather especially) when such properties are carefully identified in the laboratory (say by a sufficiently precise state-preparation device, whereby a particle is produced in a definite state)—the investigation, experimental and theoretical, of particles in such a state consists exactly in seeing how the particle evolves in time, how its state changes in time. Whatever characterizes it as a state cannot be a state-independent property. We must look elsewhere for individuating properties that can be used as names to define a state-space of distinguishable particles.

We have explained the need for permutation symmetry; we can see, roughly, why it requires global descriptions, and how, contra Quine, they are descriptions of definite things nonetheless. We have an answer of sorts to the paradox of constitution in the classical case, at least at one instant of time. But not even that is available, in general, in quantum mechanics, and the answer is in any case unsatisfactory. So what are the state-independent properties that can function as names for distinguishable things?

9.5 Demarcating Properties

I return to the notion of stable and robust properties that are fine-grained enough to stand in for single objects, but not so fine-grained as to have no interesting state space; Goldilocks properties, properties that are just right. But by now it should be evident that to solve the paradox of constitution we do not need uniqueness: it is enough to speak of relatively small numbers of particles, entangled or otherwise, as apart from all particles. Permutation symmetry only has to be broken enough to get down to a definite collective, stable in time; the description of the collective itself may be in terms of permutables. Or for another way of putting it: it is enough to show how a total symmetrized state-space can be replaced by a Cartesian product of two state-spaces, each totally symmetrized.

In fact, it is enough to show how this goes in a sufficiently good approximation, in some regime of energy and scale of interest in the laboratory (or for that matter that of everyday things). By our general ansatz, we should look for properties shared by some but not all permutables, properties that are robust and stable in time for the dynamical regime in question. Call them *demarcating* properties; individuating properties arise as the limiting case, where the "small number" is unity. Thus, in place of properties like (9.11) that specify everything, or, like (iv), the property of there being exactly one thing with some property, we need a more coarse-grained projector—say a demarcating property P that applies to n out of N particles. In that case the collective has the totally symmetrized property:

$$\overbrace{P\otimes\ldots\otimes P}^{n\text{ factors}}\otimes\overbrace{(I-P)\otimes\ldots\otimes(I-P)}^{N-n\text{ factors}}+\text{permutations.} \tag{9.12}$$

Because demarcating properties will only be available in a certain dynamical regime, and for limited periods of time, we shall say they are *emergent* properties, rather than fundamental ones. We then use the method of section 9.3, and use P in object position—but as a mass term, like "butter," or better, a natural kind term, like "metal." Thus, if the demarcating property is the projection onto bound states of the electron and proton, it is "hydrogen." We can equally construe *P* as the name of a plurality, and speak of *P*s, or of instances of *P*, or of particles of kind *P*. There will now be a range of dynamical behavior available to *P*s (consistent with the fact that they are *P*s, that it is hydrogen), ensured by the fact that the demarcating property is stable and robust. *P* can thus function as a label in state space, hosting a non-trivial dynamics, that applies to *P* particles, as opposed to not-*P* particles. And evidently the interchange of *P*s with not-*P*s will not be a symmetry; typically a transformation like that will take us out of the state-space altogether. But the *P*s are still permutable among themselves, and the not-*P*s are still permutable among themselves.

The idea, then, to return to the classical case, is to refer to more coarse-grained regions of μ-space than the point-like places of section 9.4.2: in effect, to find regions of phase space that can function like natural kinds. Let us see how this works in the simplest case of two particles. Because there are just two, the demarcating property will in fact be an individuating property, but the example extends easily to the *N*-body case.

Let μ-space be coarse-grained with respect to two independent degrees of freedom *h, v* ("horizontal" and "vertical" respectively), each into two regions, as in figure 9.1. Thus each particle can be either in *A* or *B*, and, independently, in *U* or *D* ("up" or "down"). There are four cells in all, denote *AU, AD, BU* and *BD*. This induces a coarse-graining of the two-particle phase space. If the particles are distinguishable, this is the space Γ^2: it is partitioned into 16 regions, as illustrated in figure 9.2a. If the particles are indistinguishable, it is the space Γ^2/Π_2. It is partitioned into 10 regions, as shown in figure 9.2b. (There are just 10 distinct descriptions of the two particles in terms of permutation-invariant predicates; 10 ways of distributing two indistinguishable particles over the four cells of figure 9.1.)

The Quine redux is to pass from talk of things being in regions *AU, AD, BU* and *BD*, to talk of regions being in the occupied or unoccupied states—in the 0-particle state, the 1-particle state, and the 2-particle state. However, there are correlations among these states: if *AU* is in the 2-particle state, then *AD, BU*, and *BD* are in the 0-particle state; and so on. The constraint is conservation of total particle number.

The similarity with the occupation-number formalism of nonrelativistic quantum field theory is obvious:[17] the dynamics, as in that theory, consists in variations among integer-valued states of distinguishable things (modes of the field, labeled, typically, by wavelengths or frequencies), subject to conservation of total particle number. But now in addition each individual has a smooth "internal" degree of freedom—thus, in the case of *AU*,

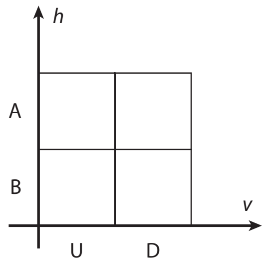

FIGURE 9.1 μ-space with degrees of freedom h and v, coarse-grained into regions A and B, and U and D, respectively

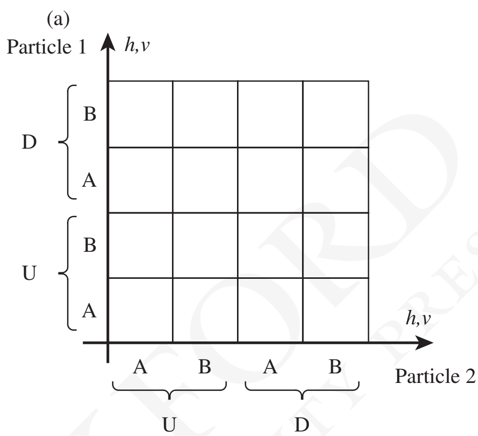

FIGURE 9.2A Γ-space for two distinguishable particles each with two degrees of freedom h and v. Each axis represents both degrees of freedom, coarse-grained into region A, B and U, D respectively

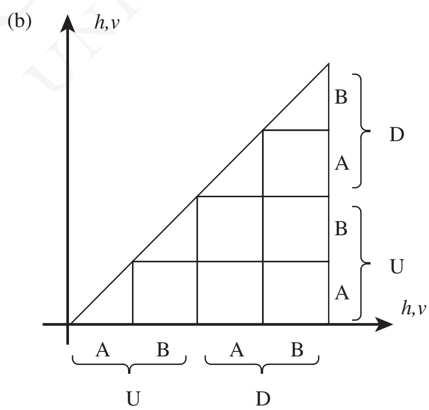

FIGURE 9.2B Reduced phase space for two indistinguishable particles, with the same coarse-graining of μ-space as figure 9.1

variation with respect to values of h restricted to A, denote h_A, and of v restricted to U, denote v_U; and similarly for AD, BU, and BD.

But we are looking for a correspondence with a classical Hamiltonian evolution for distinguishable particles, not with quantum theory. To that end, let us just treat regions defined by the coarse-graining of the horizontal degree of freedom as things, that is, call them A and B respectively; leave the vertical degrees of freedom as common to each. In addition, A and B have the internal degrees of freedom h_A and h_B respectively. As before, A and B are to be found in one of the three states, the 0-particle, the 1-particle, and the 2-particle state, subject to constraints. In general, as A and B change in time, this description has the further peculiarity than the *numbers* of degrees of freedom of A and B change as well. Thus when one of them (say A) is in the 0-particle state, it no longer has any degrees of freedom, whereas B has four: two horizontal (in this case, both h_B) and two degrees of freedom v. But still, peculiar as they are, with these rules we can pass from the coarse-grained description of two indistinguishable particles to the coarse-grained description of two distinguishable regions of phase space.

Notice now that the peculiarities disappear in the special case that A and B are each in the 1-particle state, *and the dynamics is such as to keep them there*. In that case, effectively, certain regions in phase space will not be accessible, the regions shaded in figure 9.3a. In this regime, particle number is frozen as a degree of freedom of A and B, and it can be left out of the effective description. So long as the dynamics acts in this way, A and B are always in the one-particle state; they can each be called "particles." The only degrees of freedom remaining are the two internal ones and the vertical degree of freedom. The accessible (unshaded) region of Γ^2/Π_2 has the structure of a phase space Γ^2 for two *distinguishable* particles, figure 9.3b, where the effective Hamiltonian lives. It is an effective dynamics—it gives a good approximation to the underlying dynamics of the permutables—only so long as the underlying dynamics keeps the shaded regions inaccessible; so long as the properties A and B are stable and robust over time.

How does our toy model generalize? It is obvious how to add additional degrees of freedom; what about additional particles? Evidently, if we have unique demarcating properties for each extra particle, then we have individuating properties, whereupon we pass to a description of N distinguishable particles, completely breaking permutation symmetry.[18] But more typical, when N is large and the particles are microscopic, the dynamics only freezes out a small number of coarse-grained properties. Thus, for two such properties (as in our toy model), the N particles may be divided into N_A permutables confined to region A, and N_B permutables confined to region B, by properties of the form (9.12), where $n = N_A$ (and expressed in words as disjunctions of predicates related by permutations,

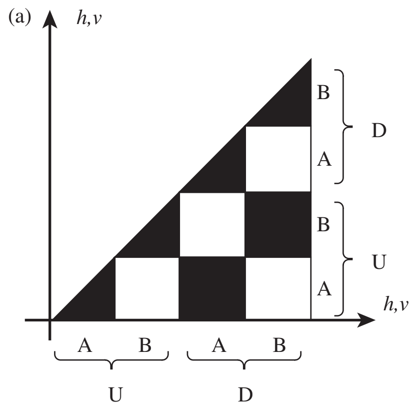

FIGURE 9.3A Shaded areas represent dynamically inaccessible regions of reduced phase space

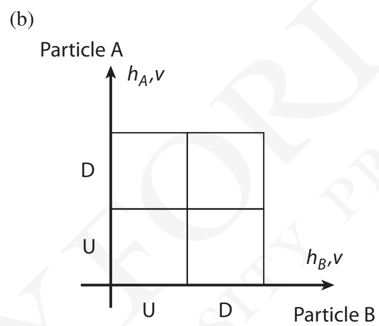

FIGURE 9.3B Γ-space for distinguishable particles, labeled by individuating properties A and B, with degree of freedom v, coarse-grained into regions U and D. Particles A and B also have internal degrees of freedom h_A and h_B respectively

as in (v)). We can then pass to an effective phase-space structure, but now using "*A*" and "*B*" as mass terms rather than proper names. The result is a phase space for N_A indistinguishable *A*-particles interacting with N_B indistinguishable *B*-particles, each permutable only among themselves.[19]

In real physics, of course, these demarcating properties are often only statable in a technical language. They do not line up with ordinary words. But the border is porous; talk of air easily goes over to talk of nitrogen, oxygen, and carbon dioxide. It may be that to talk of "spin" as a mass term seems odd, but less so with "charge" (and "mass," after all, is a mass-term par excellence). Mass, various kinds of charge, and spin are expected to suffice in a grand unified theory. Specify these (the combination of these demarcating properties) and you specify quarks of one flavor rather than another, or of quarks rather than leptons, and so on. In the Standard Model, demarcating properties are bound states of quarks: one bound state is a proton, another is a neutron, and so on through the hadrons. In all these cases we can see what I have been calling "internal" degrees of freedom clearly do arise, and can be experimentally probed (for example in the case of longer-lived hadrons, in elastic scattering experiments). In nuclear physics demarcating properties are bound states of protons and neutrons, yielding the 118 known nuclei. Internal degrees of freedom concern variations in nuclear structure that keep the nucleus intact. But typically, in effective theories, these "internal" degrees of freedom are neglected altogether: the whole point of an effective theory is to provide a simplified description at the appropriate regime of energy and scale.

However demarcating properties like these do not on their own provide an answer to the paradox of constitution; nor by themselves can they yield individuating properties—this time, not through lack of robustness, but through lack of uniqueness. However complex a bound state of a large number of molecules, or protons and neutrons, if it is stable, it is unlikely to be instantiated just the once in the universe. This much was right about our earlier solution to the constitution puzzle in terms of points in of μ-space: what is needed in addition are *spatial* demarcating properties, of being of definite spatial extent at a given time, in spatiotemporal relation to various other things, likewise localized in space and time. How, exactly, do these arise? This has nothing to do with questions about the physical meaning of coordinate systems, or inertial or freely falling frames. I mean, *given* all that: how do localized systems arise, in a way that it is stable over time?

That they arise, in this way, is not in doubt. And of course spatial localization is routinely studied—and assumed—in the physical sciences, whether in terms of boundary conditions and confining potentials, rigid bodies, crystals and lattices, or in terms of dynamical models (as for example in plasma physics and cosmology). There are countless questions about all of these uses and studies—and answers. But the burden of proof, at this point, is unclear. The original question was about permutation symmetry; the provenance of properties of spatial localization might be thought a larger question.

The paradox of constitution can therefore be solved: in terms of quantum demarcating properties, on the one hand (in terms of the formation of stable bound states, robust over long periods of time); and in terms of spatial demarcating properties, on the other (as used throughout the physical sciences).

9.6 The Problem of Measurement

You who are content with this answer, and are relaxed about the various options for solving foundational problems in quantum mechanics, can stop reading here. The puzzle as we have posed it, in terms of permutation symmetry, is solved. But others not so sure, or who are persuaded that the foundational problems in quantum mechanics exactly concern questions of ontology, read on.

A first point is that the dynamics, relevant to the formation of bound states and questions of stability over time, is the unitary dynamics; it is Schrödinger's equation. The definition and investigation of dynamics like this is more or less what quantum mechanics at low energy scales is about. But the fact that there are localized events at all, in quantum mechanics, is not without difficulty.

Why not make do with properties as defined by projectors—and specifically, projectors on localized regions Δ of configuration space or phase space? In place of the much too fine-grained properties $P_{|\phi\rangle}$ (projectors onto rays in $\mathcal{H}$), use coarser-grained projectors P_{Δ} instead. Thus, given (as before) a minimally entangled state (9.4), suppose that n of the N one-particle states $|\phi_a\rangle, |\phi_b\rangle, \ldots$ are localized in Δ, some small spatial region, with the remainder localized in the complement of Δ, and that the supports of these states (in configuration space) are changing slowly in time. In that case the N particles have the property (9.12), with $P = P_{\Delta}$, over some appreciable period of time. In that case too, the expectation value of any local dynamical quantity is the same, whether the state is totally symmetrized over all N particles, or the product of symmetrized states for n and N–n particles.

This is an important consistency check. It also has the supreme virtue that it works for superpositions of such states, too, as long as in all the states superposed, n of N permutables are located in Δ. This follows from linearity: any superposition of states each with property P itself has property P, meaning, is an eigenstate (with eigenvalue 1) of the associated projector. For the same reason, the problem of nonuniqueness of fermion one-particle states, briefly remarked on in section 9.2, is solved. That problem, recall, was that for fermions, even when minimally entangled, there is no unique set of N orthogonal one-particle states entering into the entanglement. But whether or not a state is an eigenstate of an operator is independent of the basis in which the state is written.

This point is perfectly familiar in special cases. I have already mentioned an example: the spherically symmetric singlet state of spin, which can be written with respect to z-component of spin, $|\psi_+^z\rangle$ and $|\psi_-^z\rangle$, as

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|\psi_+^z\rangle \otimes |\psi_-^z\rangle - |\psi_-^z\rangle \otimes |\psi_+^z\rangle).$$

It can equally well be written with respect to any other components of spin, say $|\psi_\pm^y\rangle$, as

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}(|\psi_+^y\rangle \otimes |\psi_-^y\rangle - |\psi_-^y\rangle \otimes |\psi_+^y\rangle).$$

It is the same state nonetheless. But then a similar non-uniqueness of composition applies to any state of this form. Thus for any orthogonal one-particle states $|\varphi_a\rangle, |\varphi_b\rangle$, the state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|\phi_a\rangle \otimes |\phi_b\rangle - |\phi_b\rangle \otimes |\phi_a\rangle) \tag{9.13}$$

can be written as

$$\frac{1}{\sqrt{2}}(|\phi_+\rangle | \otimes |\phi_-\rangle - |\phi_-\rangle | \otimes |\phi_+\rangle,$$

where

$$\left|\phi_+\right\rangle = \frac{1}{\sqrt{2}}(|\phi_a\rangle + i|\phi_b\rangle\rangle, \quad |\phi_-\rangle = \frac{1}{\sqrt{2}}(i|\phi_a\rangle + |\phi_b\rangle)$$

or, indeed in terms of $|\phi_1\rangle, |\phi_2\rangle$, where

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}(|\phi_a\rangle + |\phi_b\rangle\rangle, \quad |\phi_2\rangle = \frac{1}{\sqrt{2}}(|\phi_a\rangle - |\phi_b\rangle).$$

In the case of the singlet case of spin, this indeterminateness is explained in terms of rotational symmetry: no one direction in space is preferred. But the indeterminateness afflicts any two fermions in a minimally entangled state[20]—in fact, it afflicts any *N* fermions, in a minimally entangled fermion state. So it afflicts fermions, period, whatever their state. This method for passing to individuals, for special states of the form (9.4), as one-particle states, was pointless anyway, but now we see that in the case of fermons, failing a notion of preferred basis, it was completely illusory.

But the underdetermination of fermion one-particle states is irrelevant to the dynamical emergence of any demarcating property, if defined by the spectrum of a self-adjoint operator and written as a projection operator. The basis in which a state is written makes no difference to the subspace in which it lies. Quite generally, so long as the dynamics is unitary, dynamical considerations are indifferent to the basis used to represent the state: the choice of basis is like the choice of coordinates in classical spacetime theories. Of course, like a coordinate system in classical spacetime theories, a basis can be better or worse "adapted" to the dynamics and initial state; it may be more or less convenient (from the point of view of explicit equations and calculations) to use one basis rather than another.

Thus, if the initial state is spherically symmetric (and this is a symmetry respected by the Hamiltonian), use spherically symmetric coordinates.

The only obstacle, then, to defining spatial demarcating properties in quantum mechanics in a dynamically stable way is in understanding how states satisfying projectors like (9.12) for $P = P_\Delta$ arise in the first place. Such states describe systems of particles well-localized in Δ over appreciable periods of time.[21] How do they arise?

But *this* problem, we now see, is not at all trivial. For in quantum mechanics states like these do *not* readily arise under the purely unitary evolution. There is no point in simply positing them at much earlier times, either; quantum mechanical states, initially localized, tend to spread over time. Admittedly, if they are states of large numbers of bound particles (so of relatively large mass), this dispersion is slow, especially if they are left to themselves. Thus, a rock of mass 1 kg, prepared in a state localized to within a micron, freely evolving in time, will remain localized to within a few microns for the entire lifetime of the universe. But if it is subject to complex external forces, if, say, it is tumbling chaotically, in the way that Hyperon tumbles in its orbit about Saturn, it will become delocalized much sooner—much, much sooner. Hyperion has a mass of about 10^{20} kg, and diameter 10^5 m; according to the estimate of Zurek and Paz (1995), if initially well-localized, it will become completely delocalized over its entire orbit about Saturn in less than 10 years.

Of course if we are willing to apply quantum mechanics to macroscopic bodies, there are any number of ways in which initially localized quantum states become wildly delocalized on much smaller timescales: virtually any quantum experiment can be arranged to have this result. Rather than Schrödinger's cat being alive or dead, depending on the outcome of the experiment, have it end up in one corner of the laboratory rather than the other, if possible alive either way. Evidently we are up against *the quantum-mechanical problem of measurement*.

We may even be at the heart of it. For the three worked-out solutions to the measurement problem (worked out, at least, for nonrelativistic quantum mechanics) all take the quantum state as representative of something physically real, and they all engineer a process by which the state is localized, whether effectively (according to effective equations), or fundamentally (by tampering with the unitary equations). They all involve "wave-packet collapse," over length and timescales that are under theoretical control. In Everettian (many-worlds) quantum theory and in pilot-wave theory, according to which the universal state propagates unitarily at the fundamental level, the collapse is effective, as defined in decoherence theory (taking slightly different forms in the two cases). In dynamical collapse theories like the GRWP theory (due to Ghirardi, Rimini, Weber, and Pearle), the collapse is fundamental, and the equations at the fundamental level are no longer unitary. In all cases the collapse, effective or real, is associated with probabilistic events, so it is also inseparable from that other dimension to the problem of measurement, the role of the quantum state in determining probabilities.

But in all these theories, collapse, however it is achieved, is onto states of mesoscopic bodies that are sharply localized in position and momentum space. Any account of ontology based on the structure of the quantum state, subject to collapse onto well-localized states at a rate subject to theoretically control, stable in time, will ipso facto provide an account of how spatiotemporal demarcating properties can be dynamically defined. If defined by fundamental equations, as fundamental properties; if by effective equations, as emergent properties.

I plump for effective collapse and its explanation in terms of decoherence theory, preserving the unitary dynamics unchanged, without any additional hidden variables. The spatial localization of quantum states of ordinary things, as branches decohere, is then dynamically emergent. The states best adapted to this dynamics, for reasonably massive particles, are Gaussians, localized in position and momentum space. This is the "decoherence basis"—the right basis for resolving the ambiguity in (9.13), when $|\Phi\rangle$ has a decohering structure, consistent with the unitary dynamics.

In whatever way decoherence theory solves the measurement problem,[22] it solves the problem of spatiotemporal localization as well, the final piece of the jigsaw of how to break permutation symmetry at the effective level of description, consistent with permutation symmetry at the fundamental level. Decoherence theory defines spatial demarcating properties for molecules and larger structures, from decoherence scales and times on up, as dynamically emergent properties. Given a sufficiency of other demarcating properties, we obtain individuating properties too. And thereby we arrive at an account of how individuals arise at an emergent level in physics, and thence across the special sciences.[23]

Acknowledgments

Thanks to Adam Caulton and David Wallace for helpful discussions.

Notes

1. What more, precisely, of the mathematical structure of physical theories might be understandable as physical ontology (or the structure of physical ontology) is an open question. In the tradition of Russell, Carnap, and Quine, presumably, *all of it*—when reformulated in formal logic. But the ontology then is essentially made up of the members of sets in an elaborate construction in set theory, with no distinction

between mathematical and physical particulars. I am sceptical of this tradition, especially if cashed out in tems of the Ramsey sentence (as it so often is by structural realists): see Saunders and McKenzie 2014.

2. For an indication of the scope of this method in application to space-time points and classical particle and field theories, see my 2003a, 2003b, 2013a and, for mirror-symmetry, 2007.

3. This definition of identity was first proposed by Hilbert and Bernays (1934), based on the axiom schema for identity introduced by Gödel in 1930 in his proof of the completeness of the predicate calculus. It played a largely implicit role in Quine's early writings on identity (e.g. in his 1950), but it was explicit in Quine 1960.

4. Are elementary particles in quantum discernible at all? I have argued that fermions are, in my 2003a, 2006b, 2013b, and in Muller and Saunders 2008, but elementary bosons (as opposed to bosons that are multiplets of fermions) pose a special difficulty. It may this can be solved (see, for example, Muller and Seevink 2009), but elementary bosons, with the sole exception of the Higgs, are all gauge bosons, like photons, which we might well do better to talk of as excitations of modes of quantum fields. The Higgs, meanwhile, is exceptional for a number of reasons.

5. Why no particular role for *relative* discernibility? But there could be; for example, if we were to systematize talk of causal relations, or the before-after relation.

6. Assuming of course, their trajectories are absolutely discernible. That need not be the case (for example, the trajectories of Black's iron spheres, unchanging in time in an otherwise empty universe, are only weakly discernible).

7. It was explicit in Bach (1997 7), but it was surely implicit in a number of other criticisms of the notion of classical indistinguishables; see e.g. Van Kampen 1984.

8. Since quantum states are rays, rather than vectors, there is another possibility: the ray will still be invariant if permutations produce a change of overall phase. In fact the only consistent assignment of phase changes of this kind is alteration of sign for odd permutations, i.e. antisymmetrization (fermionic states, which therefore satisfy the Pauli exclusion principle). (Important as the distinction between states [rays] and vectors is, I shall not belabor it in what follows.)

9. Such minimal entanglements have been called "trivial" by Ghirardi et al. 2002, Ghirardi and Marinatto 2004 (see also Penrose 2004, 598).

10. As before, for "minimal" entanglements of the form (9.4), there is just enough entanglement to obliterate the correspondence between place in the tensor product and one-particle state, while still picking out a set of N one-particle states. This does not yield a determinate assignment of one-particle states to particles, however, when the latter are distinguishable.

11. Although that has been suggested; see Dieks and Lubberdink 2011.

12. There is a physically deep way of making, e.g., spin state-dependent (supersymmetry), but there is also a shallow way (as shown by Goldstein et al. 2005a, 2005b), which applies to any supposedly intrinsic property of elementary particles. The shallow method suffices for our purposes.

13. "Bare particulars," for example, or perhaps "haecceities." Huggett (1999) suggests that haecceitism is the culprit, a position endorsed by Albert (2000, 47–48) and elsewhere by me (Saunders 2013b, 356).

14. Quine 1968. Another concerned the Lowenheim-Skolem theorem, which I shall not consider here (restricted as we are to finitary theories).

15. It is otherwise known as "Putnam's paradox" (Lewis 1984). Lewis's solution, in terms of "perfectly natural properties," bears some relation to the one that I offer; see below.

16. Cases of repetitions are important to the observable differences between classical and quantum statistics. For a detailed discussion, see Saunders 2006a, 2013b.

17. Otherwise known as the Fock space representation: see any introductory text in quantum field theory; see also Teller (1995). I single out the nonrelativistic case because in relativistic quantum theory only conservation of total energy, not particle number, is ensured.

18. As can sometimes arise in statistical mechanics: in the case of colloids, for example (see Swendson 2002, 2006), or, say, stars in stellar nebulae.

19. This connects fairly directly to the Gibbs paradox, and the extensivity of the entropy, both in classical and quantum theory. See my 2006a and 2013b.

20. There is something to the explanation from spherical symmetry. The triple of operators $\{P_{|\phi_a\rangle} - P_{|\phi_b\rangle}, P_{|\phi_+\rangle} - P_{|\phi_-\rangle}, P_{|\phi_1\rangle} - P_{|\phi_2\rangle}\}$ satisfy the same commutation relations as the spin operators $\{S_x, S_y, S_z\}$, for orthogonal directions *x, y, z* in space. In this sense any state of the form (9.13) also has a kind of spherical symmetry.

21. It follows that they are states well-localized in momentum-space as well (subject of course to the constraints imposed by noncommutativity). They pick out—or rather define, assuming phase space is an emergent structure—small patches of phase space. There are no strictly point-like classical objects (points of μ-space), if quantum mechanics is true.

22. See Saunders 2010, Wallace 2012, chaps. 1–3 for more on emergence, decoherence theory, and many worlds. I do not believe decoherence theory in a one-world setting can solve the measurement problem, if the unitary dynamics is fundamental, for their still remains a multiplicity of decohering states, i.e., there are many worlds.

23. Notice that neither the trajectories of pilot-wave theory, nor the "flashes" or mass-densities of collapse theories, have any role to play in the emergent notion of individuals here in play. What need, then, for "primitive ontology" in these theories?

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