Definitions from *The Logic Manual*

AJ Gilbert

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1 Sets, Relations and Arguments

Binary relation: A set is a binary relation iff it contains only ordered pairs.

Types of binary relation: A binary relation \( R \) is

(i) **reflective** on a set \( S \) iff for all elements \( d \) of \( S \) the pair \( \langle d, d \rangle \) is an element of \( R \);
(ii) **symmetric** on a set \( S \) iff for all elements \( d, e \) of \( S \): if \( \langle d, e \rangle \in R \) then \( \langle e, d \rangle \in R \);
(iii) **asymmetric** on a set \( S \) iff for no elements \( d, e \) of \( S \): \( \langle d, e \rangle \in R \) and \( \langle e, d \rangle \in R \);
(iv) **antisymmetric** on a set \( S \) iff for no two distinct elements \( d, e \) of \( S \): \( \langle d, e \rangle \in R \) and \( \langle e, f \rangle \in R \), then \( \langle d, f \rangle \in R \).

Binary relations simpliciter: A binary relation \( R \) is

(i) **symmetric** iff it is symmetric on all sets;
(ii) **asymmetric** iff it is asymmetric on all sets;
(iii) **antisymmetric** iff it is antisymmetric on all sets;
(iv) **transitive** iff it is transitive on all sets.

Equivalence relation: A binary relation \( R \) is an equivalence relation on \( S \) iff \( R \) is reflexive on \( S \), symmetric on \( S \) and transitive on \( S \).

Function: A binary relation \( R \) is a function iff for all \( d, e, f \): if \( \langle d, e \rangle \in R \) and \( \langle d, f \rangle \in R \) then \( e = f \).

Domain, range, into:

(i) The **domain** of a function \( R \) is the set \( \{ d : \text{there is an } e \text{ such that } \langle d, e \rangle \in R \} \).
(ii) The **range** of a function \( R \) is the set \( \{ e : \text{there is a } d \text{ such that } \langle d, e \rangle \in R \} \).
(iii) \( R \) is a function into the set \( M \) iff all elements of the range of the function are in \( M \).

Function notation: If \( d \) is in the domain of a function \( R \) one writes \( R(d) \) for the unique object \( e \) such that \( \langle d, e \rangle \) is in \( R \).

\( n \)-ary relation: An \( n \)-place relation is a set containing only \( n \)-tuples. An \( n \)-place relation is called a relation of arity \( n \).

Argument: An argument consists of a set of declarative sentences (the premises) and a declarative sentence (the conclusion) marked as the concluded sentence.

Logical validity: An argument is logically valid iff there is no interpretation under which the premises are all true and the conclusion false.

Consistency: A set of sentences is logically consistent iff there is at least one interpretation under which all sentences of the set are true.

Logical truth: A sentence is logically true iff it is true under any interpretation.

Contradiction: A sentence is a contradiction iff it is false under all interpretations.

Logical equivalence: Sentences are logically equivalent iff they are true under exactly the same interpretations.
2 Syntax and Semantics of Propositional Logic

Sentence letters: P, Q, R, P₁, Q₁, R₁, P₂, Q₂, R₂ and so on are sentence letters.

Sentence of L₁:
(i) All sentence letters are sentences of L₁.
(ii) If φ and ψ are sentences of L₁, then ¬φ, (φ ∧ ψ), (φ ∨ ψ), (φ → ψ) and (φ ↔ ψ) are sentences of L₁.
(iii) Nothing else is a sentence of L₁.

Bracketing Convention:
1 The outer brackets may be omitted from a sentence that is not part of another sentence.
2 The inner set of brackets may be omitted from a sentence of the form ((φ ∧ ψ) ∧ χ) and analogously for ∨.
3 Suppose ⋄∈{∧, ∨} and ◦∈{→, ↔}. Then if (φ ◦ (ψ ⋄ χ)) or ((φ ⋄ ψ) ◦ χ) occurs as part of the sentence that is to be abbreviated, the inner set of brackets may be omitted.

L₁-structure: An L₁-structure is an assignment of exactly one truth-value (T or F) to every sentence letter of L₁.

Truth in an L₁-structure: Let A be some L₁-structure. Then |...|_A assigns either T or F to every sentence of L₁ in the following way.
(i) If φ is a sentence letter, |φ|_A is the truth-value assigned to φ by the L₁-structure A
(ii) |¬φ|_A = T iff |φ|_A = F
(iii) |φ ∧ ψ|_A = T iff |φ|_A = T and |ψ|_A = T
(iv) |φ ∨ ψ|_A = T iff |φ|_A = T or |ψ|_A = T
(v) |φ → ψ|_A = T iff |φ|_A = F or |ψ|_A = T
(vi) |φ ↔ ψ|_A = T iff |φ|_A = |ψ|_A

Truth tables:

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<tr>
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Logical truth etc. (L₁ version):
(i) A sentence φ of L₁ is logically true iff φ is true in all L₁-structures.
(ii) A sentence φ of L₁ is a contradiction iff φ is not true in any L₁-structures.
(iii) A sentence φ and a sentence ψ of L₁ are logically equivalent iff φ and ψ are true in exactly the same L₁-structures.

Validity (L₁ version): Let Γ be a set of sentences of L₁ and φ a sentence of L₁. The argument with all sentences in Γ as premisses and φ as conclusion is valid iff there is no L₁-structure in which all sentences in Γ are true and φ is false.
Counterexamples: An $L_1$-structure is a counterexample to the argument with $\Gamma$ as the set of premisses and $\phi$ as the conclusion iff for all $\gamma \in \Gamma$ we have $|\gamma|_A = T$ but $|\phi|_A = F$.

Semantic Consistency: A set $\Gamma$ of $L_1$-sentences is semantically consistent iff there is an $L_1$-structure $A$ such that for all sentence $\gamma \in \Gamma$ we have $|\gamma|_A = T$. A set $\Gamma$ of $L_1$-sentences is semantically inconsistent iff $\Gamma$ is not semantically consistent.

3 Formalization in Propositional Logic

Truth-functionality: A connective is truth-functional iff the truth-value of the compound sentence cannot be changed by replacing a direct subsentence with another sentence having the same truth-value.

Scope of a connective in $L_1$: The scope of an occurrence of a connective in a sentence $\phi$ of $L_1$ is the occurrence of the smallest subsentence of $\phi$ that contains this occurrence of the connective.

Logical truth etc. (propositional version):
(i) An English sentence is a tautology iff its formalization in propositional logic is logically true.
(ii) An English sentence is a contradiction iff its formalization in propositional logic is a contradiction.
(iii) An set of English sentences is propositionally consistent iff the set of all their formalizations in propositional logic is semantically consistent.

Propositional validity: An argument in English is propositionally valid iff its formalization in $L_1$ is valid.

4 The Syntax of Predicate Logic

Predicate letters: All expressions of the form $P_k^k$, $Q_k^k$, $R_n^k$ are predicate letters where $k$ and $n$ are either missing or a numeral ‘1’, ‘2’ . . . .

Arity: The value of the upper index of a predicate letter is called its arity. If a predicate letter does not have an upper index its arity is 0.

Constants: $a$, $b$, $c$, $a_1$, $b_1$, $c_1$, $a_2$, $b_2$, $c_2$, ... are constants.

Variables: $x$, $y$, $z$, $x_1$, $y_1$, $z_1$, $x_2$, $y_2$, $z_2$, ... are variables.

Atomic formulae of $L_2$: If $Z$ is a predicate letter of arity $n$ and each of $t_1$, ..., $t_n$ is a variable or constant, then $Zt_1\ldots t_n$ is an atomic formula of $L_2$.

Quantifier: A quantifier is an expression $\forall v$ or $\exists v$ where $v$ is a variable.

Formulae of $L_2$:
(i) All atomic formulae of $L_2$ are formulae of $L_2$.
(ii) If $\phi$ and $\psi$ are formulae of $L_2$ then $\neg \phi$, $(\phi \land \psi)$, $(\phi \lor \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are formulae of $L_2$. 

5
(iii) If $v$ is a variable and $\phi$ is a formula then $\forall v \phi$ and $\exists v \phi$ are formulae of $L_2$.
(iv) Nothing else is a formula of $L_2$.

**Free occurrence of a variable:**
(i) All occurrences of variables in atomic formulae are free.
(ii) The occurrences of a variable that are free in $\phi$ and $\psi$ are also free in $\neg \phi$, $\phi \land \psi$, $\phi \lor \psi$, $\phi \to \psi$, and $\phi \leftrightarrow \psi$.
(iii) In a formula $\forall v \phi$ or $\exists v \phi$ no occurrence of the variable $v$ is free; all occurrences of variables other than $v$ that are free in $\phi$ are also free in $\forall v \phi$ and $\exists v \phi$.

An occurrence of a variable is bound in a formula iff it is not free.
A variable occurs freely in a formula iff there is at least one free occurrence of the variable in the formula.

**Sentence of $L_2$:** A formula of $L_2$ is a sentence of $L_2$ iff no variable occurs freely in the formula.

## 5 The Semantics of Predicate Logic

**$L_2$-structure:** An $L_2$-structure is an ordered pair $\langle D, I \rangle$ where $D$ is some non-empty set and $I$ is a function from the set of all constants, sentence letters, and predicate letters such that
- the value of every constant is an element of $D$
- the value of every sentence letter is a truth-value $T$ or $F$
- the value of every $n$-ary predicate letter is an $n$-ary relation.

**Variable assignment:** A variable assignment over an $L_2$-structure $A$ assigns an element of the domain $D_A$ of $A$ to each variable.

**Satisfaction:** Assume $A$ is an $L_2$-structure, $\alpha$ is a variable assignment over $A$, $\phi$ and $\psi$ are formulae of $L_2$, and $v$ is a variable. For a sentence letter $\phi$ either $|\phi|_A^\alpha = T$ or $|\phi|_A^\alpha = F$ obtains. Formulae other than sentence letters receive the following semantic values.
(i) $|\Phi t_1 \ldots t_n|_A^\alpha = T$ iff $\langle |t_1|_A^\alpha, \ldots, |t_n|_A^\alpha \rangle \in |\Phi|_A^\alpha$, where $\Phi$ is an $n$-ary predicate letter for $n \geq 1$ and each of $t_1, \ldots, t_n$ is either a variable or a constant
(ii) $|\neg \phi|_A^\alpha = T$ iff $|\phi|_A^\alpha = F$
(iii) $|\phi \land \psi|_A^\alpha = T$ iff $|\phi|_A^\alpha = T$ and $|\psi|_A^\alpha = T$
(iv) $|\phi \lor \psi|_A^\alpha = T$ iff $|\phi|_A^\alpha = T$ or $|\psi|_A^\alpha = T$
(v) $|\phi \to \psi|_A^\alpha = T$ iff $|\phi|_A^\alpha = F$ or $|\psi|_A^\alpha = T$
(vi) $|\phi \leftrightarrow \psi|_A^\alpha = T$ iff $|\phi|_A^\alpha = |\psi|_A^\alpha$
(vii) $|\forall v \phi|_A^\alpha = T$ iff $|\phi|_A^\beta = T$ for all variable assignments $\beta$ over $A$ differing from $\alpha$ in $v$ at most
(viii) $|\exists v \phi|_A^\alpha = T$ iff $|\phi|_A^\beta = T$ for at least one variable assignment $\beta$ over $A$ differing from $\alpha$ in $v$ at most

**Truth:** A sentence $\phi$ is true in an $L_2$-structure $A$ iff $|\phi|_A^\alpha = T$ for all variable assignments $\alpha$ over $A$.

**Logical truth etc. ($L_2$ version)**
(i) A sentence $\phi$ of $\mathcal{L}_2$ is logically true iff $\phi$ is true in all $\mathcal{L}_2$-structures.

(ii) A sentence $\phi$ of $\mathcal{L}_2$ is a contradiction iff $\phi$ is not true in any $\mathcal{L}_2$-structures.

(iii) Sentences $\phi$ and $\psi$ of $\mathcal{L}_2$ are logically equivalent iff both are true in exactly the same $\mathcal{L}_2$-structures.

(iv) A set $\Gamma$ of $\mathcal{L}_2$-sentences is semantically consistent iff there is an $\mathcal{L}_2$-structure $A$ in which all sentences in $\Gamma$ are true. A set of $\mathcal{L}_2$-sentences is semantically inconsistent iff it is not semantically consistent.

**Validity ($\mathcal{L}_2$ version):** Let $\Gamma$ be a set of sentences of $\mathcal{L}_2$ and $\phi$ a sentence of $\mathcal{L}_2$. The argument with all sentences in $\Gamma$ as premisses and $\phi$ as conclusion is valid iff there is no $\mathcal{L}_2$ structure in which all sentences in $\Gamma$ are true and $\phi$ is false. This is abbreviated as $\Gamma \models \phi$.

6 Natural Deduction

Propositional Logic Rules

\[
\begin{array}{c}
\vdots \vdots
\end{array}
\]

\[
\frac{\phi}{\phi \land \psi} \landIntro
\]

\[
\begin{array}{c}
\vdots \vdots
\end{array}
\]

\[
\frac{\phi \land \psi}{\phi} \landElim_1 \quad \frac{\phi \land \psi}{\psi} \landElim_2
\]

\[
\begin{array}{c}
\vdots \vdots \vdots
\end{array}
\]

\[
\frac{\phi}{\phi \lor \psi} \lorIntro_1 \quad \frac{\phi \lor \psi}{\psi} \lorIntro_2
\]

\[
\begin{array}{c}
\vdots \vdots \vdots
\end{array}
\]

\[
\frac{\phi \lor \psi \chi \chi}{\chi} \lorElim
\]

\[
\begin{array}{c}
\vdots \vdots \vdots
\end{array}
\]

\[
\frac{\phi}{\phi \rightarrow \psi} \rightarrowIntro \quad \frac{\phi \rightarrow \psi}{\psi} \rightarrowElim
\]

\[
\begin{array}{c}
\vdots \vdots \vdots
\end{array}
\]

\[
\frac{\phi \psi \neg \psi \neg \psi}{\neg \phi} \negIntro \quad \frac{\psi \phi \neg \psi}{\phi} \negElim
\]

7
\[
\begin{align*}
\frac{[\phi] \quad [\psi]}{\phi \leftrightarrow \psi} & \quad \leftrightarrow \text{Intro} \\
\frac{\psi}{\phi \leftrightarrow \psi} & \quad \leftrightarrow \text{Elim}_1 \\
\frac{\phi \leftrightarrow \psi}{\psi} & \quad \leftrightarrow \text{Elim}_2
\end{align*}
\]

Predicate Logic Rules

\[
\frac{[\phi[t/v]]}{\forall \forall \phi} \quad \forall \text{Intro}
\]

provided that the constant \( t \) does not occur in \( \phi \) or in any undischarged assumption in the proof of \( \phi[t/v] \).

\[
\frac{\forall \forall \phi}{\phi[t/v]} \quad \forall \text{Elim}
\]

\[
\frac{\exists \forall \phi}{\psi} \quad \exists \text{Intro}
\]

\[
\frac{[\phi[t/v]]}{\exists \forall \phi} \quad \exists \text{Elim}
\]

provided that the constant \( t \) does not occur in \( \exists \forall \phi \) or in \( \psi \) or in any undischarged assumption other than \( \phi[t/v] \) in the proof of \( \psi \).

Identity Rules

\[
\frac{[t = t]}{\vdots} \quad = \text{Intro}
\]

\[
\frac{\phi[s/v]}{\phi[t/v]} \quad s = t \quad = \text{Elim}
\]

\[
\frac{\phi[s/v]}{\phi[t/v]} \quad t = s \quad = \text{Elim}
\]
7 Formalization in Predicate Logic

Syntactic consistency: A set $\Gamma$ of $\mathcal{L}_2$-sentences is syntactically consistent iff there is a sentence $\phi$ such that $\Gamma \not\vdash \phi$.

Scope of a quantifier or connective in $\mathcal{L}_2$: The scope of an occurrence of a quantifiers or a connective in a sentence $\phi$ of $\mathcal{L}_2$ is the occurrence of the smallest $\mathcal{L}_2$-formula that contains that occurrence of the quantifier or connective and is part of $\phi$.

Logical truth etc. (predicate version):
1. An English sentence is logically true in predicate logic iff its formalization in predicate logic is logically true.
2. An English sentence is a contradiction in predicate logic iff its formalization in predicate logic is a contradiction.
3. A set of English sentences is consistent in predicate logic iff the set of their formalizations in predicate logic is semantically consistent.

Validity (predicate version): An argument in English is valid in predicate logic iff its formalization in the language $\mathcal{L}_2$ of predicate logic is valid.

8 Identity and Definite Descriptions

Atomic formulae of $\mathcal{L}_=$: All atomic formulae of $\mathcal{L}_2$ are atomic formulae of $\mathcal{L}_=$. Furthermore, if $s$ and $t$ are variables or constants then $s = t$ is an atomic formula of $\mathcal{L}_=$.

Formulae of $\mathcal{L}_=$:
1. All atomic formulae of $\mathcal{L}_=$ are formulae of $\mathcal{L}_=$.
2. If $\phi$ and $\psi$ are formulae of $\mathcal{L}_=$ then $\neg \phi$, $(\phi \land \psi)$, $(\phi \lor \psi)$, $(\phi \rightarrow \psi)$ and $(\phi \leftrightarrow \psi)$ are formulae of $\mathcal{L}_=$.
3. If $v$ is a variable and $\phi$ is a formula then $\forall v \phi$ and $\exists v \phi$ are formulae of $\mathcal{L}_=$.
4. Nothing else is a formula of $\mathcal{L}_=$

Satisfaction in $\mathcal{L}_=$: As in the definition of satisfaction in $\mathcal{L}_2$ with the additional clause
5. $|s = t|_A^\alpha = T$ iff $|s|_A^\alpha = |t|_A^\alpha$