INTRODUCTION TO LOGIC

Lecture 1

Validity

Introduction to Sets and Relations.

Dr. James Studd

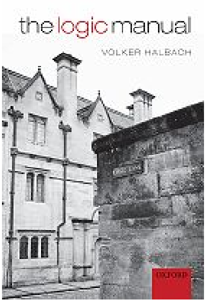
Pure logic is the ruin of the spirit.

Antoine de Saint-Exupéry

Outline

Resources

- The Logic Manual
- **logicmanual.philosophy.ox.ac.uk**
  - Exercises booklet
  - Lecture slides
  - Worked examples
  - Past examination papers
    some with solutions



- Mark Sainsbury: *Logical Forms: An Introduction to Philosophical Logic*, Blackwell, second edition, 2001, chs. 1–2.

Why logic?

Logic is the scientific study of valid argument.

- Philosophy is all about arguments and reasoning.
- Logic allows us to rigorously test validity.
- Modern philosophy assumes familiarity with logic.
- Used in linguistics, mathematics, computer science,...
- Helps us make fine-grained conceptual distinctions.
- Logic is compulsory.

Validity 1/3

First approximation.

When an argument is valid, the truth of the premisses **guarantees** the truth of the conclusion. 10

An argument is valid if it '**can't**' be the case that all of the premisses are true and the conclusion is false.

- Validity does **not** depend on contingent facts.
- Validity does **not** depend on laws of nature.
- Validity does **not** depend on the meanings of subject-specific expressions.
- Validity depends purely on the 'form' of the argument.

Examples

Argument 1 **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

The truth of the premiss does not provide a sufficiently strong guarantee of the truth of the conclusion

Argument 2 **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

Validity 2/3

Characterisation (p. 19)

An argument is **logically valid** if and only if:
there is <u>no</u> interpretation under which:

(i) the premisses are all true, and

(ii) the conclusion is false.

Argument 1 revisited

Argument 1 **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

Argument 1a **Not valid**

Boris Johnson is a Conservative.
Therefore, Boris Johnson is a Liberal Democrat.

There is an interpretation under which:

(i) the premiss is true, and
(ii) the conclusion is false.

Argument 2 revisited

Argument 2 **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

Argument 2a **Valid**

Boris Johnson is a Conservative.
All Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

Argument 2b **Valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, Radon is a chemical element.

Note: argument 2a is a valid argument with a false conclusion.

Validity 3/3.

Characterisation (p. 19)

An argument is **logically valid** if and only if:
there is no [uniform] interpretation [of subject-specific expressions] under which:

(i) the premisses are all true, and

(ii) the conclusion is false.

- Each occurrence of an expression interpreted in the same way
- Logical expression keep their usual English meanings.

Subject-specific versus logical expressions

Examples: logical terms

all, every, some, no.
not, and, or, unless, if, only if, if and only if.

Examples: subject-specific terms

Zeno, Boris Johnson, France, The North Sea, Radon, soap, bread, GDP, logical positivism, ...
tortoise, toothless, Conservative, nobel gas, philosopher, chemical element, ...
loves, owns, reacts with, voted for, ...

Argument 2 revisited again

Argument 2 **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

Argument 3 **Not valid**

Boris Johnson is a Conservative.
No Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

Argument 4 **Not valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, air is a chemical element.

Course overview

1: Validity; Introduction to Sets and Relations
2: Syntax and Semantics of Propositional Logic
3: Formalization in Propositional Logic
4: The Syntax of Predicate Logic

5: The Semantics of Predicate Logic
6: Natural Deduction
7: Formalization in Predicate Logic
8: Identity and Definite Descriptions

Sets 1/2

Characterisation

A **set** is a collection of zero or more objects.

- The objects are called **elements** of the set.
- $a \in b$ is short for 'a is an element of set b'.

Examples

- The set of positive integers less than 4:
  $\{1, 2, 3\}$ or $\{n : n \text{ is an integer between 1 and 3}\}$
- The set of positive integers:
  $\{1, 2, 3, 4, \ldots\}$ or $\{n : n > 0\}$
- The empty set:
  $\{\ \}$ or $\{x : x \text{ is a round square}\}$ or $\emptyset$

Sets 2/2

Fact about sets

Sets are identical if and only if they have the same elements.

Example

The following sets are all identical:

- {Lennon, McCartney, Harrison, Ringo}
- {Ringo, Lennon, Harrison, McCartney}
- {Ringo, Ringo, Ringo, Lennon, Harrison, McCartney}
- $\{x : x \text{ is a Beatle}\}$
- $\{x : x \text{ sang lead vocals on an Abbey Road track}\}$

Ordered pairs

Characterisation

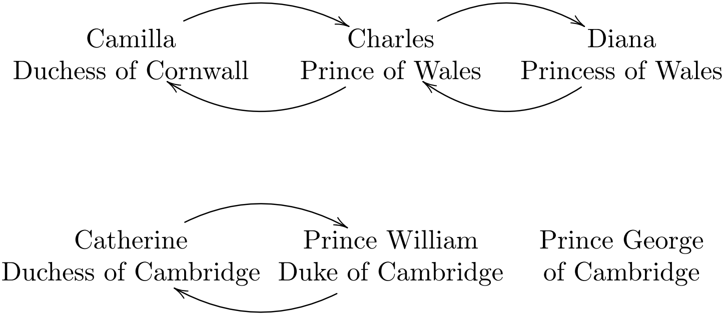
An **ordered pair** comprises two components in a given order.

- $\langle d, e \rangle$ is the ordered pair whose first component is d and whose second component is e, **in that order**.

Example

$\langle$London, Munich$\rangle \neq \langle$Munich, London$\rangle$
$\{$London, Munich$\} = \{$Munich, London$\}$

Relations



The relation of *having married*

$$\{\langle \text{Charles, Diana}\rangle, \langle \text{Diana, Charles}\rangle,\\ \langle \text{Charles, Camilla}\rangle, \langle \text{Camilla, Charles}\rangle,\\ \langle \text{Kate, William}\rangle, \langle \text{William, Kate}\rangle, \ldots\}$$

Worked example

Write down the following relation as a set of ordered pairs.
Draw its arrow diagram. 40

The relation of *being countries in GB sharing a border*

$$\{\langle \text{England, Scotland} \rangle, \langle \text{Scotland, England} \rangle,$$
$$\langle \text{England, Wales} \rangle, \langle \text{Wales, English} \rangle\}$$

$$\begin{tikzcd}
& \text{Scotland} \arrow[d, bend left] \\
\text{Wales} \arrow[r, bend left] & \text{England} \arrow[u, bend left] \arrow[l, bend left]
\end{tikzcd}$$

Relations

Definition (p. 8)

A set R is a **binary relation** if and only if it contains only ordered pairs.

Informally: $\langle d, e\rangle \in R$ indicates that d stands in R to e.

Example

- The relation of *having married.*
  - $\{\langle \text{Kate, William}\rangle, \langle \text{Charles, Camilla}\rangle, \cdots\}$
  - $\{\langle d, e\rangle : d \text{ married } e\}$.
- The empty set: $\emptyset$

Properties of relations 1/3

Definition (p. 9)

A binary relation R is **reflexive on a set S** iff:

- for all d in S: the pair $\langle d, d\rangle$ is an element of R. 45

Informally: every member of S bears R to itself.

Example **Reflexive on the set of human beings**

- The relation of *being the same height as*

Example **Not reflexive on this set**

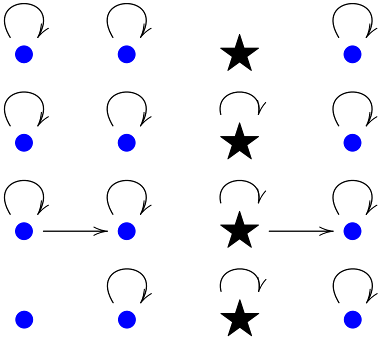
- The relation of *being taller than*

Example **Not reflexive on $\{1, 2, 3\}$**

- $\{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 1, 3\rangle\}$ Reflexive on $\{1, 2\}$

Reflexivity on S

Every point in S has a "loop".



(Reflexive on S)

(Reflexive on S)

(Reflexive on S)

(Not Reflexive on S)

Key: Member of S:
Non-member of S:

Properties of relations 2/3

Definition (p. 9)

A binary relation R is **symmetric on set S** iff:

- for all d, e in S: if $\langle d, e\rangle \in R$ then $\langle e, d\rangle \in R$.

Informally: any member of S bears R to a second only if the second bears R back to the first.

Example **Symmetric on the set of human beings**

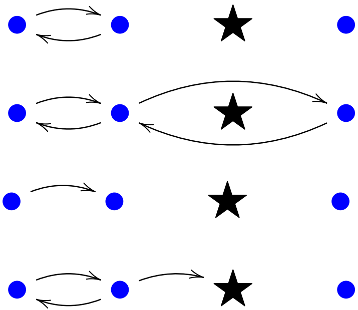
- The relation of *being a sibling of*

Example **Not symmetric on this set**

- The relation of *being a brother of*

Symmetry on S

Every “outward route” between points in S has a “return route”.



$\big($Symmetric on $S\big)$

$\big($Symmetric on $S\big)$

$\big($Not symmetric on $S\big)$

$\big($Symmetric on $S\big)$

Properties of relations 3/3

Definition

A binary relation R is **transitive on S** iff:

- for all d, e, f in S:
  if $\langle d, e\rangle \in R$ and $\langle e, f\rangle \in R$, then also $\langle d, f\rangle \in R$

Informally: if any member of S bears R to a second, and the second also bears R to a third, the first bears R to the third.

Example — **Transitive on the set of human beings**

- The relation of *being taller than*

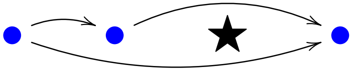
Example — **Not transitive on this set**

- The relation of *not having the same height* ($\pm$1cm)

Transitivity on S

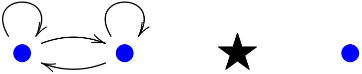
Every "double-step" between points in S has a "one-step shortcut".

(Not transitive on S)



(Transitive on S)

(Not transitive on S)



(Transitive on S)

Functions

Definition (p. 14)

A binary relation F is a **function** iff for all d, e, f:

- if $\langle d, e\rangle \in F$ and $\langle d, f\rangle \in F$ then $e = f$.

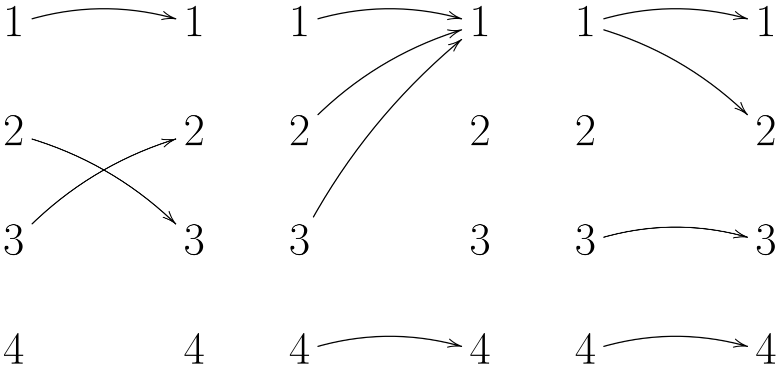
Informally, everything stands in F to at most one thing.

Example

- The function that squares positive integers.
  $\{\langle 1,1\rangle, \langle 2,4\rangle, \langle 3,9\rangle, \ldots\}$
  $\{\langle x,y\rangle : y = x^2,$ for x a positive integer$\}$

F is a function

Everything stands in F to at most one thing ("many-one" or "one-one")



"one-one" function

"many-one" function

"one-many" not a function

A "straightforward and elementary" example

(a) What is a binary relation?

(b) Consider the relation R of *sharing exactly one parent*:

$$R = \{\langle d, e\rangle : d \text{ and } e \text{ share exactly one of their parents}\}$$

Determine whether R is:

(i) reflexive on the set of human beings

(ii) symmetric on the set of human beings

(iii) transitive on the set of human beings

Explain your answers.

A straightforward and elementary example

(a) What is a binary relation?

A binary relation is a set of zero or more ordered pairs.

A straightforward and elementary example

(b) $R = \{\langle d, e\rangle : d \text{ and } e \text{ share exactly one of their parents}\}$

(i) Is R reflexive on the set of human beings? No.
I share two parents with myself, not one.

(ii) Is R symmetric on the set of human beings? Yes.
If human beings d and e share exactly one parent, clearly e and d—the very same people—share exactly one parent too.

(iii) Is R transitive on the set of human beings? No.
For example, my maternal half-sister Rachel and I share exactly one parent, and me and my paternal half-sister Debby share exactly one parent, but Rachel and Debby share no parents.

logicmanual.philosophy.ox.ac.uk