INTRODUCTION TO LOGIC

Lecture 1

Validity

Introduction to Sets and Relations.

Dr. James Studd

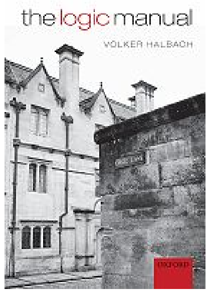
Pure logic is the ruin of the spirit.
Antoine de Saint-Exupéry

Outline

(1) Introductory
(2) Validity
(3) Course Overview
(4) Sets and Relations

Resources

- The Logic Manual
- **logicmanual.philosophy.ox.ac.uk**
  - Exercises booklet
  - Lecture slides
  - Worked examples
  - Past examination papers
    some with solutions



- Mark Sainsbury: *Logical Forms: An Introduction to Philosophical Logic*, Blackwell, second edition, 2001, chs. 1–2.

Why logic?

Logic is the scientific study of valid argument.

- Philosophy is all about arguments and reasoning.
- Logic allows us to rigorously test validity.
- Modern philosophy assumes familiarity with logic.
- Used in linguistics, mathematics, computer science,. . .
- Helps us make fine-grained conceptual distinctions.
- Logic is compulsory.

Validity 1/3

First approximation.

When an argument is valid, the truth of the premisses **guarantees** the truth of the conclusion.

An argument is valid if it ‘can’t’ be the case that all of the premisses are true and the conclusion is false.

- Validity does not depend on contingent facts.
- Validity does not depend on laws of nature.
- Validity does not depend on the meanings of subject-specific expressions.
- Validity depends purely on the ‘form’ of the argument.

Examples

Argument 1 — **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

The truth of the premiss does not provide a sufficiently strong guarantee of the truth of the conclusion

Argument 2 — **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

Validity 2/3

Characterisation (p. 19)

An argument is **logically valid** if and only if:
there is no interpretation under which:

(i) the premisses are all true, and
(ii) the conclusion is false.

Argument 1 revisited

Argument 1 — **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

Argument 1a — **Not valid**

Boris Johnson is a Conservative.
Therefore, Boris Johnson is a Liberal Democrat.

There is an interpretation under which:

(i) the premiss is true, and
(ii) the conclusion is false.

Argument 2 revisited

Argument 2 — **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

Argument 2a — **Valid**

Boris Johnson is a Conservative.
All Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

Argument 2b — **Valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, Radon is a chemical element.

Note: argument 2a is a valid argument with a false conclusion.

Validity 3/3.

Characterisation (p. 19)

An argument is **logically valid** if and only if:
there is no [**uniform**] interpretation [**of subject-specific expressions**] under which:

(i) the premisses are all true, and

(ii) the conclusion is false.

- Each occurrence of an expression interpreted in the same way
- Logical expression keep their usual English meanings.

Subject-specific versus logical expressions

Examples: logical terms

all, every, some, no.
not, and, or, unless, if, only if, if and only if.

Examples: subject-specific terms

Zeno, Boris Johnson, France, The North Sea, Radon, soap, bread, GDP, logical positivism, . . .
tortoise, toothless, Conservative, nobel gas, philosopher, chemical element, . . .
loves, owns, reacts with, voted for, . . .

Argument 2 revisited again

Argument 2 — **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

Argument 3 — **Not valid**

Boris Johnson is a Conservative.
No Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

Argument 4 — **Not valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, air is a chemical element.

Course overview

1: Validity; Introduction to Sets and Relations
2: Syntax and Semantics of Propositional Logic
3: Formalization in Propositional Logic
4: The Syntax of Predicate Logic

5: The Semantics of Predicate Logic
6: Natural Deduction
7: Formalization in Predicate Logic
8: Identity and Definite Descriptions

Sets 1/2

Characterisation
A **set** is a collection of zero or more objects.

- The objects are called **elements** of the set.
- $a \in b$ is short for 'a is an element of set b'.

Examples

- The set of positive integers less than 4: $\{1, 2, 3\}$ or $\{n : n$ is an integer between 1 and 3$\}$
- The set of positive integers: $\{1, 2, 3, 4, \ldots\}$ or $\{n : n > 0\}$
- The empty set: $\{\ \}$ or $\{x : x$ is a round square$\}$ or $\emptyset$

Sets 2/2

Fact about sets
Sets are identical if and only if they have the same elements.

Example
The following sets are all identical:

- {Lennon, McCartney, Harrison, Ringo}
- {Ringo, Lennon, Harrison, McCartney}
- {Ringo, Ringo, Ringo, Lennon, Harrison, McCartney}
- $\{x : x$ is a Beatle$\}$
- $\{x : x$ sang lead vocals on an Abbey Road track$\}$

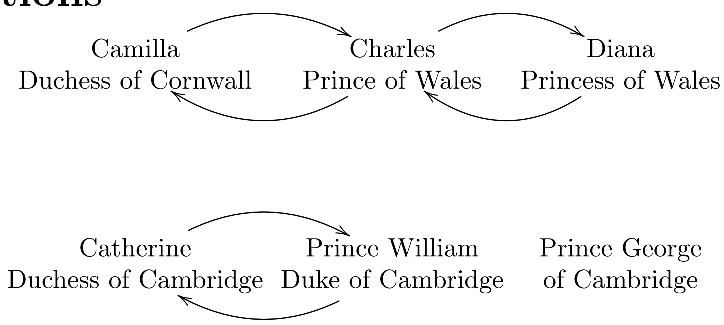
Ordered pairs

Characterisation
An <u>ordered</u> **pair** comprises two components in a given order.

- $\langle d, e\rangle$ is the ordered pair whose first component is d and whose second component is e, <u>in that order</u>.

Example
$\langle$London, Munich$\rangle \neq \langle$Munich, London$\rangle$
{London, Munich} = {Munich, London}

Relations



The relation of *having married*

$$\{\langle \text{Charles, Diana}\rangle, \langle \text{Diana, Charles}\rangle,$$
$$\langle \text{Charles, Camilla}\rangle, \langle \text{Camilla, Charles}\rangle,$$
$$\langle \text{Kate, William}\rangle, \langle \text{William, Kate}\rangle, \dots\}$$

Worked example

Write down the following relation as a set of ordered pairs.
Draw its arrow diagram.

The relation of *being countries in GB sharing a border*

Relations

Definition (p. 8)

A set R is a **binary relation** if and only if it contains only ordered pairs.

Informally: $\langle d, e\rangle \in R$ indicates that d stands in R to e.

Example

- The relation of *having married*.
  - $\{\langle \text{Kate, William}\rangle, \langle \text{Charles, Camilla}\rangle, \cdots\}$
  - $\{\langle d, e\rangle : d \text{ married } e\}$.
- The empty set: $\emptyset$

Properties of relations 1/3

Definition (p. 9)

A binary relation R is **reflexive on a set** S iff:

- for all d in S: the pair $\langle d, d\rangle$ is an element of R.

Informally: every member of S bears R to itself.

Example — Reflexive on the set of human beings

- The relation of *being the same height as*

Example — Not reflexive on this set

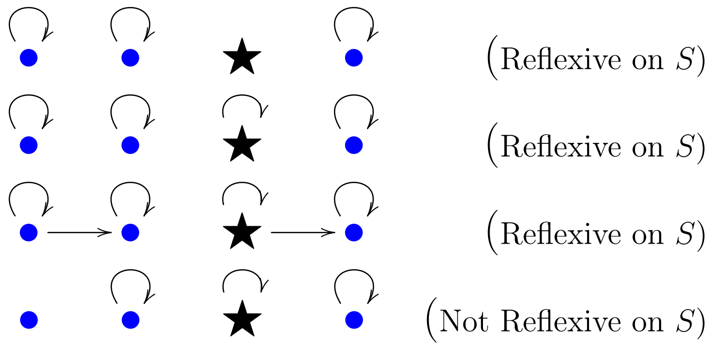
- The relation of *being taller than*

Example — Not reflexive on $\{1, 2, 3\}$

- $\{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 1, 3\rangle\}$ — Reflexive on $\{1, 2\}$

Reflexivity on S

Every point in S has a "loop".



Key: Member of S: ●
Non-member of S: ★

Properties of relations 2/3

Definition (p. 9)

A binary relation R is **symmetric on set** S iff:

- for all d, e in S: if $\langle d, e\rangle \in R$ then $\langle e, d\rangle \in R$.

Informally: any member of S bears R to a second only if the second bears R back to the first.

Example **Symmetric on the set of human beings**

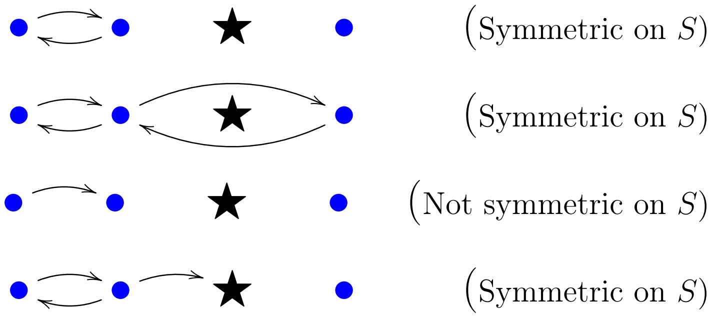
- The relation of *being a sibling of*

Example **Not symmetric on this set**

- The relation of *being a brother of*

Symmetry on S

Every "outward route" between points in S has a "return route".



Properties of relations 3/3

Definition

A binary relation R is **transitive on S** iff:

- for all d, e, f in S:
  if $\langle d, e\rangle \in R$ and $\langle e, f\rangle \in R$, then also $\langle d, f\rangle \in R$

Informally: if any member of S bears R to a second, and the second also bears R to a third, the first bears R to the third.

Example **Transitive on the set of human beings**

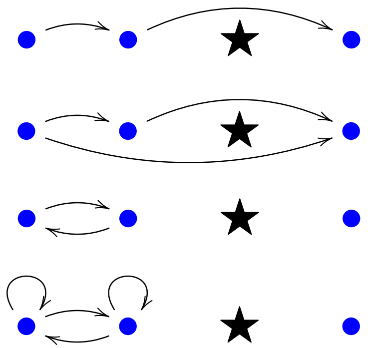
- The relation of *being taller than*

Example **Not transitive on this set**

- The relation of *not having the same height* ($\pm$1cm)

Transitivity on S

Every "double-step" between points in S has a "one-step shortcut".



(Not transitive on S)

(Transitive on S)

(Not transitive on S)

(Transitive on S)

Functions

Definition (p. 14)

A binary relation F is a **function** iff for all d, e, f:

- if $\langle d, e\rangle \in F$ and $\langle d, f\rangle \in F$ then $e = f$.

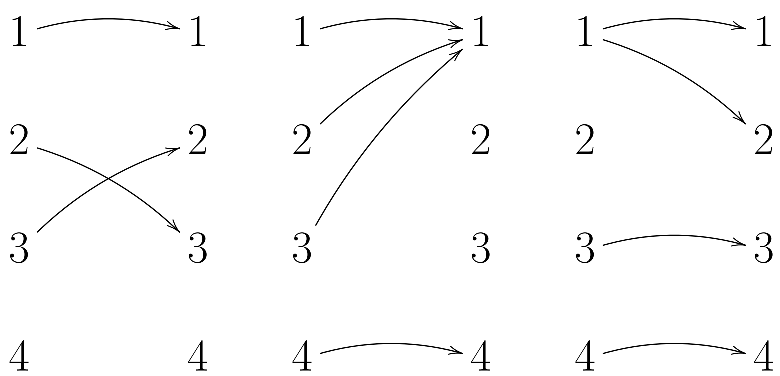
Informally, everything stands in F to at most one thing.

Example

- The function that squares positive integers.
  $\{\langle 1,1\rangle, \langle 2,4\rangle, \langle 3,9\rangle, \ldots\}$
  $\{\langle x,y\rangle : y = x^2, \text{ for } x \text{ a positive integer}\}$

F is a function

Everything stands in F to at most one thing ("many-one" or "one-one")



"one-one" function

"many-one" function

"one-many" not a function

A "straightforward and elementary" example

(a) What is a binary relation?

(b) Consider the relation R of *sharing exactly one parent*:

$$R = \{\langle d, e\rangle : d \text{ and } e \text{ share exactly one of their parents}\}$$

Determine whether R is:

(i) reflexive on the set of human beings
(ii) symmetric on the set of human beings
(iii) transitive on the set of human beings

Explain your answers.