

INTRODUCTION TO LOGIC

Lecture 3

Formalisation in Propositional Logic

Dr. James Studd

There is no other way to learn the truth
than through logic
Averroes

Outline

- ① Truth-functionality
- ② Formalisation
- ③ Complex sentences
- ④ Ambiguity
- ⑤ Validity of English arguments

English connectives

Recall that connectives join one or more sentences together to make compound sentences.

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These correspond to the connectives of \mathcal{L}_1 : \neg , \wedge , \vee , \rightarrow , \leftrightarrow

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More English connectives

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Only some English connectives can be captured in \mathcal{L}_1 .
None of these connectives can be.

Truth functionality

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T	F
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A_1 V. Halbach is giving this lecture.

A_2 Two plus two equals five.

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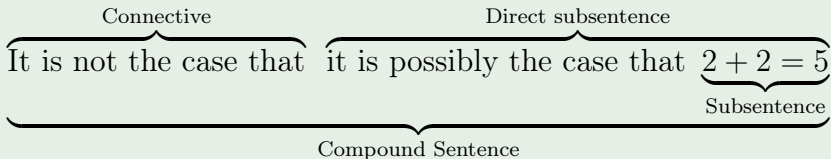
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Characterisation: truth-functional (p. 54)

A connective is **truth-functional** if and only if the truth-value of the compound sentence cannot be changed by replacing a direct subsentence with another having the same truth-value.

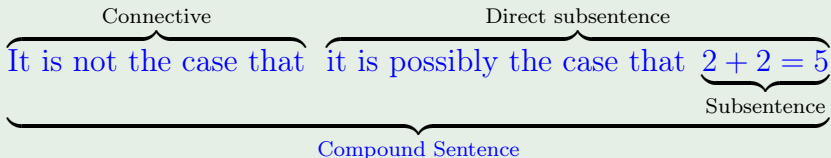
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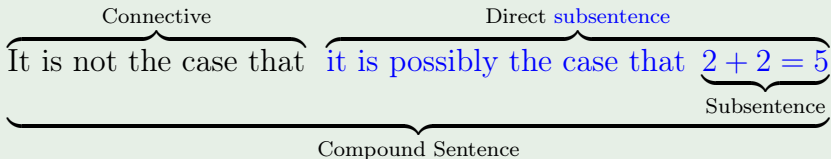
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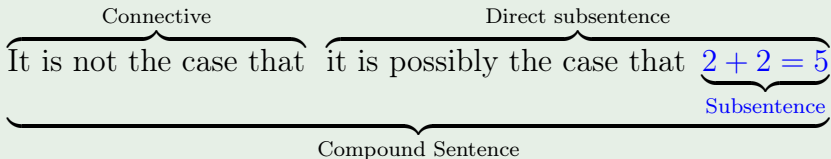
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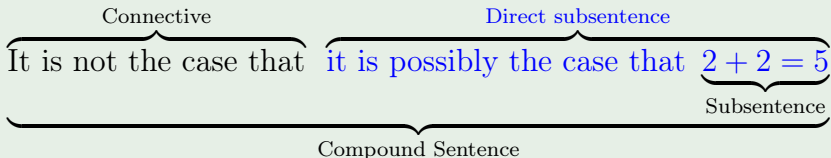
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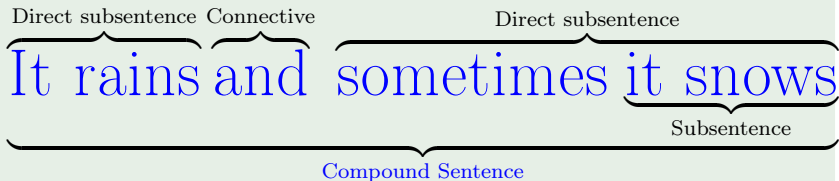
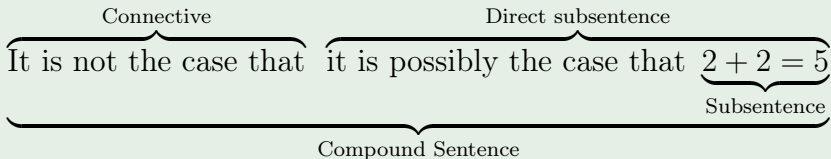
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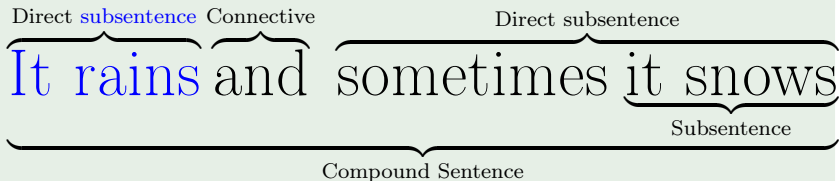
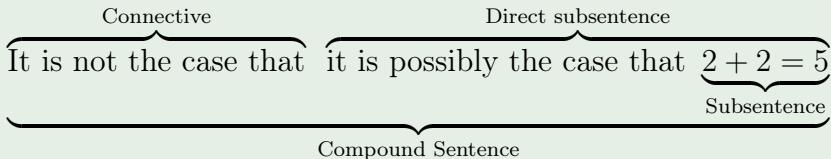
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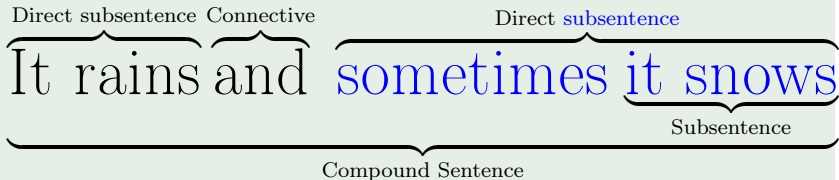
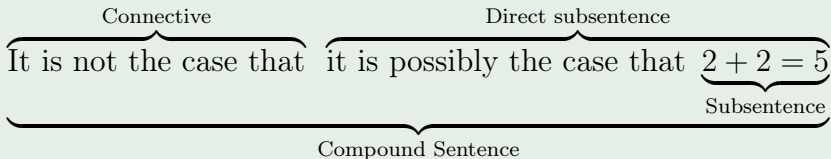
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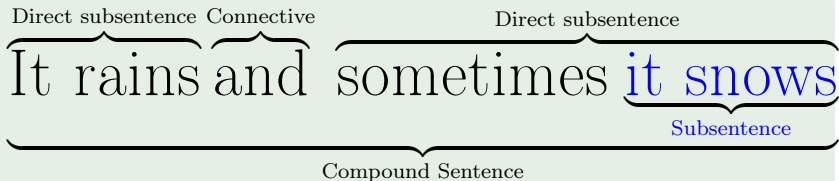
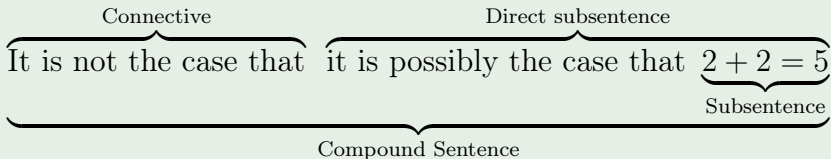
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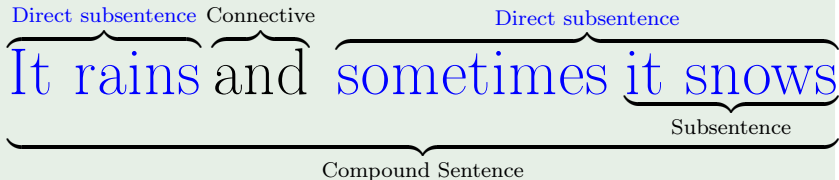
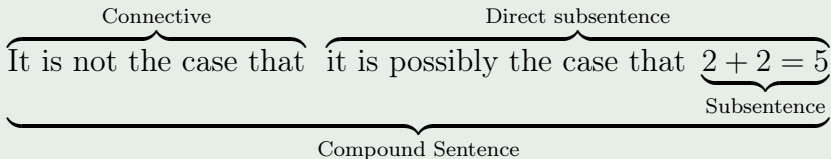
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Formalisation

$\neg R$

Dictionary

R : Russell likes logic.

Formalise:

Russell likes logic and philosophers like conceptual analysis.

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Formalisation

$(R \wedge P)$

Dictionary

R : Russell likes logic.

P : Philosophers like conceptual analysis.

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Note: it's fine to use letters other than P, Q, R when formalising English sentences.

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Neither Russell nor Whitehead likes logic.

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Formalisation: $\neg R$

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Formalisation: $\neg R \wedge \neg W$

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Formalisation: $\neg R \wedge \neg W$

Dictionary: R : Russell likes logic. W : Whitehead likes logic.

Common variants

Here are some of the most common variants of the standard connectives.

\mathcal{L}_1	standard connective	some other formulations
\wedge	and	but, although
\vee	or	unless
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Rules of thumb for \rightarrow

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- | | |
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'If' mid-sentence corresponds to ' \leftarrow '; 'only if' to \rightarrow .

Differences between \rightarrow and 'if'

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Formalise

If the lecturer hadn't shown up last week, Plato would have given the lecture.

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See Sainsbury, *Logical Forms*, ch. 2 for further discussion.

Complex Sentences

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Formalise

If David folded or David didn't have the ace, Victoria won.

Complex Sentences

Formalise

If David folded or David didn't have the ace, Victoria won.

Paraphrase

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Complex Sentences

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Logical Form

(If ((David folded) or it is not the case that (David had the ace)), (Victoria won))

This is in (propositional) **logical form**.

- All connectives are standard connectives
- No sentence can be further formalised in \mathcal{L}_1 .

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Formalisation: $((F \vee \neg A) \rightarrow W)$

Dictionary: F : David folded.

A : David had the ace.

W : Victoria won.

Sometimes the paraphrase may need to be quite loose.

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(and as before)

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David's hand was weak and Victoria was bound to win unless the Jack came up on the turn.

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Scope of \vee

In (1): \vee has wider scope.

In (2): \wedge has wider scope.

More on paraphrase

- (1) Tom and Jerry are animals.
- (2) Tom and Jerry are apart.
- (3) Jerry is a white mouse.
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Worked example: are these acceptable paraphrases?

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Recall last week's definitions of tautology, contradiction and validity for \mathcal{L}_1 sentences and arguments.

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- (1) An English sentence is a **tautology** if and only if its formalisation in propositional logic is a tautology.
- (2) An English sentence is a **propositional contradiction** if and only if its formalisation in propositional logic is a contradiction.
- (3) An argument in English is **propositionally valid** if and only if its formalisation in \mathcal{L}_1 is valid.

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Show that the following argument is propositionally valid.

Unless CO₂-emissions are cut, there will be more floods.
CO₂-emissions won't be cut. Therefore there will be more floods.

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- Next, formalise the premisses and conclusion.
- Finally, check the formalised argument is valid.

Worked example (cont.)

P1 Unless CO₂-emissions are cut, there will be more floods.

P2 CO₂-emissions won't be cut.

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It's not the case that CO₂-emissions will be cut.

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Formalisation: $C \vee M$.

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Formalisation: $\neg C$.

C **Formalisation:** M .

It remains to show.

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Backwards truth-table

C	M	$(C \vee M)$	$\neg C$	M

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- Method 2: Backwards truth table.

Backwards truth-table

C	M	$(C \vee M)$	$\neg C$	M
		T ₂ T F ₁	T ?	F

This shows there cannot be a line in the truth-table in which both premisses are true and the conclusion is false.

It remains to show.

$$(C \vee M), \neg C \models M$$

You know two ways to do this.

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This shows there cannot be a line in the truth-table in which both premisses are true and the conclusion is false. So, the English argument is propositionally valid.

<http://logicmanual.philosophy.ox.ac.uk>