INTRODUCTION TO LOGIC

Lecture 3

Formalisation in Propositional Logic

Dr. James Studd

There is no other way to learn the truth
than through logic
Averroes

Outline

English connectives

Recall that connectives join one or more sentences together to make compound sentences.

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These correspond to the connectives of $\mathcal{L}_1$: $\neg$, $\wedge$, $\vee$, $\to$, $\leftrightarrow$

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More English connectives

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- ‘It must be the case that’
- ‘Pope Benedict XVI thought that’
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Only some English connectives can be captured in $\mathcal{L}_1$. None of these connectives can be.

Truth functionality

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A_1 V. Halbach is giving this lecture.

A_2 Two plus two equals five.

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NB: replacing non-direct subsentences may change the truth-value.

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Formalisation
$\boldsymbol{\neg R}$

Dictionary
R: Russell likes logic.

Formalise:

Russell likes logic and philosophers like conceptual analysis.

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Formalisation

$(\boldsymbol{R} \land \boldsymbol{P})$

Dictionary

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P: Philosophers like conceptual analysis.

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Formalisation: $\neg\boldsymbol{C}$

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Note: it's fine to use letters other than P, Q, R when formalising English sentences.

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Formalise:

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Formalise:

Neither Russell nor Whitehead likes logic.

Sometimes we need to paraphrase first.

Formalise:

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Formalisation: $\neg \boldsymbol{R}$
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Neither Russell nor Whitehead likes logic.

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Formalisation: $\neg \boldsymbol{R} \land \neg \boldsymbol{W}$

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Russell doesn't like logic

Paraphrase: It is not the case that Russell likes logic.
Formalisation: $\boldsymbol{\neg R}$
Dictionary: R: Russell likes logic.

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Neither Russell nor Whitehead likes logic.

Paraphrase: It is not the case that Russell likes logic and it is not the case that Whitehead likes logic.
Formalisation: $\boldsymbol{\neg R \wedge \neg W}$
Dictionary: R: Russell likes logic. W: Whitehead likes logic.

Common variants

Here are some of the most common variants of the standard connectives.

$\mathcal{L}_1$	standard connective	some other formulations
$\wedge$	and	but, although
$\vee$	or	unless
$\neg$	it is not the case that	not, none, never
$\rightarrow$	if … then	provided that, only if
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Rules of thumb for $\rightarrow$

Formalise:

(1) If John revised, [then] he passed. $R \rightarrow P$

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(1) **Formalisation**: $\boldsymbol{R \rightarrow P}$

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Formalise:

(1) If John revised, [then] he passed. $R \rightarrow P$

(2) John passed if he revised. '$P \leftarrow R$' i.e. $R \rightarrow P$

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Rules of thumb for $\to$

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(1) If John revised, [then] he passed. $R \to P$

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(3) John passed only if he revised.

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'If' mid-sentence corresponds to '$\leftarrow$'; 'only if' to $\rightarrow$.

Differences between $\rightarrow$ and 'if'

Differences between $\to$ and ‘if’

Formalise

If the lecturer hadn’t shown up last week, Plato would have given the lecture.

Differences between $\to$ and 'if'

Formalise

If the lecturer hadn't shown up last week, Plato would have given the lecture.

Consider: $\neg S \to P$.
Dictionary: S: The lecturer showed up last week.
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The English sentence appears to be false.

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See Sainsbury, *Logical Forms*, ch. 2 for further discussion.

Complex Sentences

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Formalise

If David folded or David didn't have the ace, Victoria won.

Complex Sentences

Formalise

If David folded or David didn't have the ace, Victoria won.

Paraphrase

If David folded or David didn't have the ace, Victoria won.

Complex Sentences

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Paraphrase

(If David folded or David didn't have the ace, Victoria won)

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Logical Form

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This is in (propositional) logical form.

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Formalisation: $((F \vee \neg A) \rightarrow W)$

Dictionary: F: David folded.

A: David had the ace.

W: Victoria won.

Sometimes the paraphrase may need to be quite loose.

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(and as before)

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David's hand was weak and Victoria was bound to win unless the Jack came up on the turn.

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In (1): $\lor$ has wider scope.

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In (1): $\vee$ has wider scope.
In (2): $\wedge$ has wider scope.

More on paraphrase

(1) Tom and Jerry are animals.

(2) Tom and Jerry are apart.

(3) Jerry is a white mouse.

(4) Jerry is a large mouse.

Worked example: are these acceptable paraphrases?

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Logical notions in English

Recall last week's definitions of tautology, contradiction and validity for $\mathcal{L}_1$ sentences and arguments.

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Logical notions in English

Recall last week's definitions of tautology, contradiction and validity for $\mathcal{L}_1$ sentences and arguments.
These properties of $\mathcal{L}_1$ can be transposed to English.

Definition

(1) An English sentence is a tautology if and only if its formalisation in propositional logic is a tautology.

(2) An English sentence is a propositional contradiction if and only if its formalisation in propositional logic is a contradiction.

(3) An argument in English is propositionally valid if and only if its formalisation in $\mathcal{L}_1$ is valid.

Worked Example

Show that the following argument is propositionally valid.

Unless CO_2-emissions are cut, there will be more floods. CO_2-emissions won't be cut. Therefore there will be more floods.

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C: CO_2 emissions will be cut.
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  Dictionary.
  C: CO_2 emissions will be cut.
  M: There will be more floods.
- Next, formalise the premisses and conclusion.
- Finally, check the formalised argument is valid.

Worked example (cont.)

P1 Unless CO_2-emissions are cut, there will be more floods.

P2 CO_2-emissions won't be cut.

C There will be more floods

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Formalise the argument:

P1 Paraphrase:
CO_2-emissions will be cut or there will be more floods.

Worked example (cont.)

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Formalise the argument:

P1 Paraphrase:
CO_2-emissions will be cut or there will be more floods.
Formalisation: $C \vee M$.

Worked example (cont.)

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Formalise the argument:

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It's not the case that CO_2-emissions will be cut.

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Formalisation: $\neg \boldsymbol{C}$.

Worked example (cont.)

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Formalise the argument:

P1 Paraphrase:
CO_2-emissions will be cut or there will be more floods.
Formalisation: $\boldsymbol{C \vee M}$.

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It's not the case that CO_2-emissions will be cut.
Formalisation: $\boldsymbol{\neg C}$.

C **Formalisation**: $\boldsymbol{M}$.

It remains to show.

$$(C \lor M), \neg C \models M$$

It remains to show.

$(C \vee M), \neg C \models M$

You know two ways to do this.

- Method 1: Forwards truth table.

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Backwards truth-table

C	M	$(C \vee M)$	$\neg C$	M

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C	M	$(C$	$\vee$	$M)$	$\neg$	C	M
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This shows there cannot be a line in the truth-table in which both premisses are true and the conclusion is false.

It remains to show.

$$(C \lor M), \neg C \models M$$

You know two ways to do this.

- Method 1: Forwards truth table.
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Backwards truth-table

C	M	$(C$	$\lor$	$M)$	$\neg$	C	M
		T_2	T	F_1	T	?	F

This shows there cannot be a line in the truth-table in which both premisses are true and the conclusion is false.
So, the English argument is propositionally valid.

http://logicmanual.philosophy.ox.ac.uk