

# INTRODUCTION TO LOGIC

## Lecture 4

### The Syntax of Predicate Logic

Dr. James Studd

I counsel you, dear friend, in sum,  
That first you take collegium logicum.  
Your spirit's then well broken in for you,  
In Spanish boots laced tightly to,  
That you henceforth may more deliberately keep  
The path of thought and straight along it creep,  
And not perchance criss-cross may go,  
A- will-o'-wiping to and fro.  
Then you'll be taught full many a day  
What at one stroke you've done away,  
Like eating and like drinking free,  
It now must go like: One! Two! Three!

Goethe, Faust I

# Outline

- 1 Introduction to  $\mathcal{L}_2$
- 2 Straightforward predicate formalisation
- 3 The syntax of  $\mathcal{L}_2$

Recall argument 2 from week 1.

## Argument 2

- (1) Zeno is a tortoise.
  - (2) All tortoises are toothless.
- Therefore, (C) Zeno is toothless.

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Valid

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(1)  $T$

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## Propositional Formalisation

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- (2)  $A$

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But  $T, A \not\models L$ .

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## Propositional Formalisation

Not Valid

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(2)  $A$   
(C)  $L$

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The language we use is the **language of predicate logic**:  $\mathcal{L}_2$

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## $\mathcal{L}_2$ constants

- $a, b, c$ , etc.

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- John opened the bottle with his keyring

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A predicate's **arity** is the number of occurrences of designators it takes to make a whole sentence again.



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- $P^1, Q^1, R^1, P_1^1, Q_{36}^1$ , etc

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- $P^1, Q^1, R^1, P_1^1, Q_{36}^1$ , etc
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Examples:  $P^1 a$

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Examples:  $P^1a, R^2ab, R^2aa, P_{23}^4abca$

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(1) Alice paid Beatrice.

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Tom likes Miranda Fitzwilliam-Carter and she likes him.

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Tom likes Miranda Fitzwilliam-Carter and *she* likes him.

The pronouns each refer to a particular person

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Distinguish two uses of pronouns: *lazy* and *quantificational*.

## Lazy use

Tom likes Miranda Fitzwilliam-Carter and *Miranda Fitzwilliam-Carter* likes *Tom*.

The pronouns each refer to a particular person

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In English, pronouns are expressions used instead of nouns

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The pronoun cannot be replaced by the noun to which it refers back (without changing the meaning).

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If a girl likes Tom, Tom likes *her*.

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The pronoun 'her' does not refer to a single person: it is used to make a general claim.

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We could instead express this as:

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# Formalising general sentences

**Formalise**

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## Formalise

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Dictionary:  $M$ : ... has mass.

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Dictionary:  $M$ : ... has mass.

Note: it's fine to omit the arity index when formalising.

## More examples

- (1) Everything has mass.
- (2) Something has mass.
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- (1) Everything is such that: it has mass.

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- (1) Everything is such that: it has mass.  
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- Everything is such that: it has mass.  
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# Common variants

$\mathcal{L}_2$	standard connective	some other formulations
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## Examples

(1) A dog barked.

(2) A dog barks.

Dictionary.  $D$ : ... is a dog.  $B_1$ : ... barked.  $B_2$ : ... barks

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## Paraphrase (of the most natural readings)

- (1) Some dog barked



**Examples****Formalisations**

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$\exists x(Dx \wedge B_1x)$

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**Paraphrase (of the most natural readings)**

(1) Some dog barked

Some  $x$  is such that ( $x$  is a dog and  $x$  barked).

**Examples****Formalisations**

(1) A dog barked.

$\exists x(Dx \wedge B_1x)$

(2) A dog barks.

Dictionary.  $D$ : ...is a dog.  $B_1$ : ...barked.  $B_2$ : ...barks

**Paraphrase (of the most natural readings)**

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## Formalisations

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Dictionary.  $D$ : ... is a dog.  $B_1$ : ... barked.  $B_2$ : ... barks

## Paraphrase (of the most natural readings)

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Some  $x$  is such that ( $x$  is a dog and  $x$  barked).

(2) Every dog barks

Every  $x$  is such that (if  $x$  is dog, then  $x$  barks).

# Quantifiers at the end of sentences

## Examples

- (1) Everyone loves Zuleika
- (2) Zuleika loves everyone

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(1) Everyone loves Zuleika

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- (1) Every  $x$  is such that  
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# Quantifiers at the end of sentences

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(1) Everyone loves Zuleika

$\forall x(Px \rightarrow Lxa)$

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We use the same trick to deal with multiple quantifiers.

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We use the same trick to deal with multiple quantifiers.

### Example

### Formalisation

Someone loves everyone

$$\exists x(Px \wedge \forall y(Py \rightarrow Lxy))$$

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# The syntax of $\mathcal{L}_2$

Here's the official syntax of  $\mathcal{L}_2$

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## Definition (Predicate letters)

$P_n^k$ ,  $Q_n^k$ , or  $R_n^k$  are **predicate letters**, where  $k$  and  $n$  are either missing (no symbol) or a numeral '1', '2', '3', ...

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$$P^1 \quad Q^1 \quad R^1 \quad P_1^1 \quad Q_1^1 \quad R_1^1 \quad P_2^1 \quad \dots$$

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$P^1$	$Q^1$	$R^1$	$P_1^1$	$Q_1^1$	$R_1^1$	$P_2^1$	...	Unary predicates
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**Definition (atomic formulae of  $\mathcal{L}_2$ )**

If  $Z$  is a predicate letter of arity  $n$  and each of  $t_1, \dots, t_n$  is a variable or a constant, then  $Zt_1 \dots t_n$  is an **atomic formula** of  $\mathcal{L}_2$ .

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- 3 If  $v$  is a variable and  $\phi$  is a formula, then  $\forall v \phi$  and  $\exists v \phi$  are formulae of  $\mathcal{L}_2$ .

**Worked example: which of these are  $\mathcal{L}_2$ -formulae?**

- (i)  $Pa$
- (ii)  $Q^3axy$
- (iii)  $P$
- (iv)  $\forall v\phi$

(No bracketing conventions, etc. have been applied.)

**Worked example: which of these are  $\mathcal{L}_2$ -formulae?**

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## Formulae vs Sentences

An  $\mathcal{L}_2$  formula  $\phi$  is a **sentence** if every occurrence of each variable  $v$  in  $\phi$  is ‘bound’ by a quantifier  $\exists v$  or  $\forall v$ .

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## Examples

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# Formulae vs Sentences

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- But you should stick to letters early in the alphabet for constants and letters late in the alphabet (start with:  $x, y, z$ ) for variables.

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