

Reproducing Harnau and Nielsen (2016) using the apc package

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Bent Nielsen Department of Economics, University of Oxford
& Nuffield College
& Institute for Economic Modelling
bent.nielsen@nuffield.ox.ac.uk
<http://users.ox.ac.uk/~nuff0078>

Contents

1	Introduction	1
2	Table 1: The data	1
3	Table 2: Deviance analysis	1
4	Table 3: Estimates	3
5	Table 4: Forecasts	8

1 Introduction

The purpose of this vignette is to use the `apc` package version 1.2.3 to reproduce some the result in Harnau and Nielsen (2016): *Asymptotic theory for over-dispersed age-period-cohort and extended chain-ladder models*. This generalises the theory presented in Martínez Miranda, Nielsen and Nielsen (2015), from a Poisson model to an over-dispersed Poisson model. There is also a vignette available for that paper. The `apc` package builds on the identification analysis and the forecast theory in Kuang, Nielsen and Nielsen (2008a,b), the development of deviance analysis for general data arrays in Nielsen (2014). The package is discussed in Nielsen (2015).

2 Table 1: The data

The data set is taken from Table 1 of Verrall (1991), who attributes the data to Taylor and Ashe (1983). The data consists of a reserving triangle.

The data are available in the `apc` package. They can be called with the command

```
> library(apc)
> data <- data.loss.TA()
> data$response
```

	1	2	3	4	5	6	7	8	9	10
1	357848	766940	610542	482940	527326	574398	146342	139950	227229	67948
2	352118	884021	933894	1183289	445745	320996	527804	266172	425046	NA
3	290507	1001799	926219	1016654	750816	146923	495992	280405	NA	NA
4	310608	1108250	776189	1562400	272482	352053	206286	NA	NA	NA
5	443160	693190	991983	769488	504851	470639	NA	NA	NA	NA
6	396132	937085	847498	805037	705960	NA	NA	NA	NA	NA
7	440832	847631	1131398	1063269	NA	NA	NA	NA	NA	NA
8	359480	1061648	1443370	NA	NA	NA	NA	NA	NA	NA
9	376686	986608	NA	NA	NA	NA	NA	NA	NA	NA
10	344014	NA	NA	NA	NA	NA	NA	NA	NA	NA

3 Table 2: Deviance analysis

The deviance table can be reproduced by the following commands. The first call has the APC model as reference. The second call has the AC model as reference. The third call has the Ad model as reference. For an overview of the models, see Nielsen (2014). The output is wide, so only selected columns are shown.

```
> apc.fit.table(data, "od.poisson.response")[,c(1,2,4,6)]
```

	deviance	df.residual	F vs.APC	prob(>F)
APC	1395518	28	NaN	NaN
AP	1780577	36	0.966	0.482

AC	1903014	36	1.273	0.297
PC	6862733	36	13.712	0.000
Ad	2269756	44	1.096	0.403
Pd	7990746	44	8.271	0.000
Cd	7807867	44	8.041	0.000
A	2474053	45	1.273	0.278
P	9765797	45	9.879	0.000
C	8597579	45	8.500	0.000
t	8897725	52	6.272	0.000
tA	9096181	53	6.180	0.000
tP	10655658	53	7.432	0.000
tC	9674925	53	6.645	0.000
1	10699464	54	7.180	0.000

```
> apc.fit.table(data,"od.poisson.response","AC")[,c(1,2,4,6)]
```

	deviance	df.residual	F vs.AC	prob(>F)
AC	1903014	36	NaN	NaN
Ad	2269756	44	0.867	0.552
Cd	7807867	44	13.963	0.000
A	2474053	45	1.200	0.325
C	8597579	45	14.071	0.000
t	8897725	52	8.270	0.000
tA	9096181	53	8.004	0.000
tC	9674925	53	8.648	0.000
1	10699464	54	9.245	0.000

```
> apc.fit.table(data,"od.poisson.response","Ad")[,c(1,2,4,6)]
```

	deviance	df.residual	F vs.Ad	prob(>F)
Ad	2269756	44	NaN	NaN
A	2474053	45	3.960	0.053
t	8897725	52	16.061	0.000
tA	9096181	53	14.704	0.000
1	10699464	54	16.341	0.000

Thus, Table 2 in the paper is constructed as follows.

```
> Table2 <- apc.fit.table(data,"od.poisson.response")[c(1:3,5,8),c(2,1,4,6)]
> Table2 <- cbind(Table2,rbind(matrix(nrow=3,ncol=2),apc.fit.table(data,"od.poisso
> Table2 <- cbind(Table2,rbind(matrix(nrow=4,ncol=2),apc.fit.table(data,"od.poisso
> colnames(Table2)<-c("df","Dsub","F_sub,apc","p","F_sub,ac","p","F_sub,ad","p")
> Table2
```

	df	Dsub	F_sub,apc	p	F_sub,ac	p	F_sub,ad	p
APC	28	1395518	NaN	NaN	NA	NA	NA	NA

AP	36	1780577	0.966	0.482	NA	NA	NA	NA
AC	36	1903014	1.273	0.297	NA	NA	NA	NA
Ad	44	2269756	1.096	0.403	0.867	0.552	NA	NA
A	45	2474053	1.273	0.278	1.200	0.325	3.96	0.053

4 Table 3: Estimates

The table of estimates can be reproduced by the following commands. The two first calls are for an APC design, the last two calls are for an AC call.

The first and the third call are for a Poisson response model, which is inappropriate here. The second and the fourth call are for an overdispersed response model. The point estimates are the same, but the standard deviations and p-values differ.

There was previously a bug in the calculation of the standard errors. This bug is corrected in version 1.3.2. Thus, the standard errors reported in the published paper are incorrect as they are based on version 1.2.3. The following code gives both the incorrect numbers in the published paper and the correct values.

```
> apc.fit.model(data, "poisson.response", "APC")$coefficients.canonical
```

	Estimate	Std. Error	z value	Pr(> z)
level	12.787863594	NA	NA	NA
age slope	0.697763921	0.0019497418	357.875035	0.000000e+00
cohort slope	0.111481669	0.0020135573	55.365530	0.000000e+00
DD_age_3	-0.895632465	0.0009858174	-908.517576	0.000000e+00
DD_age_4	0.013570514	0.0009117808	14.883528	4.216670e-50
DD_age_5	-0.642054188	0.0010298524	-623.442928	0.000000e+00
DD_age_6	0.258903877	0.0014025258	184.598298	0.000000e+00
DD_age_7	0.256459103	0.0017973054	142.690889	0.000000e+00
DD_age_8	-0.294147370	0.0022282682	-132.007165	0.000000e+00
DD_age_9	0.705787632	0.0028645524	246.386703	0.000000e+00
DD_age_10	-1.759462290	0.0047534360	-370.145359	0.000000e+00
DD_period_3	0.046442718	0.0026689076	17.401396	8.051629e-68
DD_period_4	0.213821686	0.0018875907	113.277570	0.000000e+00
DD_period_5	0.211836483	0.0015128207	140.027484	0.000000e+00
DD_period_6	-0.405308334	0.0012639987	-320.655662	0.000000e+00
DD_period_7	0.354415338	0.0012145608	291.805356	0.000000e+00
DD_period_8	-0.559003985	0.0011424331	-489.310018	0.000000e+00
DD_period_9	0.556712364	0.0011954761	465.682542	0.000000e+00
DD_period_10	-0.075721211	0.0011024668	-68.683437	0.000000e+00
DD_cohort_3	-0.365436915	0.0011273315	-324.161004	0.000000e+00
DD_cohort_4	-0.025435276	0.0011204241	-22.701472	4.332418e-114
DD_cohort_5	-0.009240882	0.0011665838	-7.921318	2.350053e-15
DD_cohort_6	0.114695160	0.0012454096	92.094327	0.000000e+00
DD_cohort_7	0.053026763	0.0012911471	41.069499	0.000000e+00
DD_cohort_8	0.050815892	0.0013498121	37.646640	0.000000e+00

```
DD_cohort_9 -0.408218405 0.0015891640 -256.876190 0.000000e+00
DD_cohort_10 0.101509160 0.0025485309 39.830460 0.000000e+00
```

```
> apc.fit.model(data,"od.poisson.response","APC")$coefficients.canonical
```

	Estimate	Std. Error	t value	Pr(> t)
level	12.787863594	NA	NA	NA
age slope	0.697763921	0.4352771	1.60303368	0.1201497769
cohort slope	0.111481669	0.4495239	0.24799944	0.8059446052
DD_age_3	-0.895632465	0.2200824	-4.06953304	0.0003486379
DD_age_4	0.013570514	0.2035538	0.06666795	0.9473198145
DD_age_5	-0.642054188	0.2299131	-2.79259495	0.0093198443
DD_age_6	0.258903877	0.3131119	0.82687324	0.4153002025
DD_age_7	0.256459103	0.4012459	0.63915691	0.5279155530
DD_age_8	-0.294147370	0.4974578	-0.59130118	0.5590624974
DD_age_9	0.705787632	0.6395073	1.10364274	0.2791431307
DD_age_10	-1.759462290	1.0611980	-1.65799628	0.1084815489
DD_period_3	0.046442718	0.5958299	0.07794627	0.9384251530
DD_period_4	0.213821686	0.4214020	0.50740550	0.6158438289
DD_period_5	0.211836483	0.3377351	0.62722669	0.5355923324
DD_period_6	-0.405308334	0.2821859	-1.43631653	0.1619920281
DD_period_7	0.354415338	0.2711490	1.30708703	0.2018161859
DD_period_8	-0.559003985	0.2550466	-2.19177189	0.0368752879
DD_period_9	0.556712364	0.2668884	2.08593707	0.0462147549
DD_period_10	-0.075721211	0.2461242	-0.30765449	0.7606221130
DD_cohort_3	-0.365436915	0.2516752	-1.45201805	0.1576107937
DD_cohort_4	-0.025435276	0.2501331	-0.10168696	0.9197298671
DD_cohort_5	-0.009240882	0.2604382	-0.03548205	0.9719471107
DD_cohort_6	0.114695160	0.2780360	0.41251916	0.6831005515
DD_cohort_7	0.053026763	0.2882468	0.18396307	0.8553682435
DD_cohort_8	0.050815892	0.3013437	0.16863102	0.8672991385
DD_cohort_9	-0.408218405	0.3547787	-1.15062842	0.2596172382
DD_cohort_10	0.101509160	0.5689560	0.17841303	0.8596832346

```
> apc.fit.model(data,"poisson.response","AC")$coefficients.canonical
```

	Estimate	Std. Error	z value	Pr(> z)
level	12.506404677	NA	NA	NA
age slope	0.912526274	0.0006490038	1406.041577	0.000000e+00
cohort slope	0.331272153	0.0006694427	494.847656	0.000000e+00
DD_age_3	-0.866221921	0.0009618684	-900.561768	0.000000e+00
DD_age_4	0.020862021	0.0009000823	23.177904	7.607769e-119
DD_age_5	-0.657887194	0.0010211274	-644.275319	0.000000e+00
DD_age_6	0.235501183	0.0013952621	168.786338	0.000000e+00
DD_age_7	0.268781621	0.0017901786	150.142348	0.000000e+00
DD_age_8	-0.301632720	0.0022207760	-135.823119	0.000000e+00

```

DD_age_9      0.791901153 0.0028547701 277.395766 0.000000e+00
DD_age_10    -1.793115320 0.0047435076 -378.014643 0.000000e+00
DD_cohort_3  -0.341425729 0.0011054800 -308.848408 0.000000e+00
DD_cohort_4  -0.005004999 0.0011101242  -4.508503 6.528651e-06
DD_cohort_5  -0.071485115 0.0011587242  -61.692949 0.000000e+00
DD_cohort_6   0.137404391 0.0012376357  111.021676 0.000000e+00
DD_cohort_7   0.051370708 0.0012814238   40.088772 0.000000e+00
DD_cohort_8   0.078993227 0.0013406177   58.923006 0.000000e+00
DD_cohort_9  -0.365523501 0.0015746744 -232.126403 0.000000e+00
DD_cohort_10 0.057497628 0.0025389348  22.646359 1.515286e-113

```

```
> apc.fit.model(data, "od.poisson.response", "AC")$coefficients.canonical
```

	Estimate	Std. Error	t value	Pr(> t)
level	12.506404677	NA	NA	NA
age slope	0.912526274	0.1492165	6.11545002	4.873380e-07
cohort slope	0.331272153	0.1539158	2.15229489	3.815328e-02
DD_age_3	-0.866221921	0.2211492	-3.91691154	3.839396e-04
DD_age_4	0.020862021	0.2069436	0.10081019	9.202605e-01
DD_age_5	-0.657887194	0.2347738	-2.80221693	8.119824e-03
DD_age_6	0.235501183	0.3207935	0.73412084	4.676299e-01
DD_age_7	0.268781621	0.4115912	0.65303050	5.178857e-01
DD_age_8	-0.301632720	0.5105926	-0.59075031	5.583786e-01
DD_age_9	0.791901153	0.6563582	1.20650767	2.354907e-01
DD_age_10	-1.793115320	1.0906097	-1.64414033	1.088534e-01
DD_cohort_3	-0.341425729	0.2541679	-1.34330808	1.875795e-01
DD_cohort_4	-0.005004999	0.2552356	-0.01960933	9.844633e-01
DD_cohort_5	-0.071485115	0.2664096	-0.26832787	7.899786e-01
DD_cohort_6	0.137404391	0.2845526	0.48287869	6.321055e-01
DD_cohort_7	0.051370708	0.2946202	0.17436247	8.625575e-01
DD_cohort_8	0.078993227	0.3082299	0.25628026	7.991933e-01
DD_cohort_9	-0.365523501	0.3620433	-1.00961269	3.194190e-01
DD_cohort_10	0.057497628	0.5837425	0.09849828	9.220831e-01

Thus, Table 3 with the correct standard errors is constructed as follows.

```

> Table3 <- apc.fit.model(data, "poisson.response", "APC")$coefficients.canonical[,c(1,2)]
> Table3 <- cbind(Table3, apc.fit.model(data, "od.poisson.response", "APC")$coefficients.canonical[,c(1,2)])
> Tab3 <- apc.fit.model(data, "poisson.response", "AC")$coefficients.canonical[,c(1,2)]
> Tab3 <- cbind(Tab3, apc.fit.model(data, "od.poisson.response", "AC")$coefficients.canonical[,c(1,2)])
> Tab3 <- rbind(Tab3[1:11,], matrix(nrow=8, ncol=3), Tab3[12:19,])
> Table3 <- cbind(Table3, Tab3)
> colnames(Table3) <- c("apc est", "apc se N", "apc se t", "ac est", "ac se N", "ac se t")
> Table3

```

	apc est	apc se N	apc se t	ac est	ac se N
level	12.787863594	NA	NA	12.506404677	NA

age slope	0.697763921	0.0019497418	0.4352771	0.912526274	0.0006490038
cohort slope	0.111481669	0.0020135573	0.4495239	0.331272153	0.0006694427
DD_age_3	-0.895632465	0.0009858174	0.2200824	-0.866221921	0.0009618684
DD_age_4	0.013570514	0.0009117808	0.2035538	0.020862021	0.0009000823
DD_age_5	-0.642054188	0.0010298524	0.2299131	-0.657887194	0.0010211274
DD_age_6	0.258903877	0.0014025258	0.3131119	0.235501183	0.0013952621
DD_age_7	0.256459103	0.0017973054	0.4012459	0.268781621	0.0017901786
DD_age_8	-0.294147370	0.0022282682	0.4974578	-0.301632720	0.0022207760
DD_age_9	0.705787632	0.0028645524	0.6395073	0.791901153	0.0028547701
DD_age_10	-1.759462290	0.0047534360	1.0611980	-1.793115320	0.0047435076
DD_period_3	0.046442718	0.0026689076	0.5958299	NA	NA
DD_period_4	0.213821686	0.0018875907	0.4214020	NA	NA
DD_period_5	0.211836483	0.0015128207	0.3377351	NA	NA
DD_period_6	-0.405308334	0.0012639987	0.2821859	NA	NA
DD_period_7	0.354415338	0.0012145608	0.2711490	NA	NA
DD_period_8	-0.559003985	0.0011424331	0.2550466	NA	NA
DD_period_9	0.556712364	0.0011954761	0.2668884	NA	NA
DD_period_10	-0.075721211	0.0011024668	0.2461242	NA	NA
DD_cohort_3	-0.365436915	0.0011273315	0.2516752	-0.341425729	0.0011054800
DD_cohort_4	-0.025435276	0.0011204241	0.2501331	-0.005004999	0.0011101242
DD_cohort_5	-0.009240882	0.0011665838	0.2604382	-0.071485115	0.0011587242
DD_cohort_6	0.114695160	0.0012454096	0.2780360	0.137404391	0.0012376357
DD_cohort_7	0.053026763	0.0012911471	0.2882468	0.051370708	0.0012814238
DD_cohort_8	0.050815892	0.0013498121	0.3013437	0.078993227	0.0013406177
DD_cohort_9	-0.408218405	0.0015891640	0.3547787	-0.365523501	0.0015746744
DD_cohort_10	0.101509160	0.0025485309	0.5689560	0.057497628	0.0025389348
	ac se t				
level	NA				
age slope	0.1492165				
cohort slope	0.1539158				
DD_age_3	0.2211492				
DD_age_4	0.2069436				
DD_age_5	0.2347738				
DD_age_6	0.3207935				
DD_age_7	0.4115912				
DD_age_8	0.5105926				
DD_age_9	0.6563582				
DD_age_10	1.0906097				
DD_period_3	NA				
DD_period_4	NA				
DD_period_5	NA				
DD_period_6	NA				
DD_period_7	NA				
DD_period_8	NA				
DD_period_9	NA				


```

DD_period_10      NA
DD_cohort_3      0.2541679
DD_cohort_4      0.2552356
DD_cohort_5      0.2664096
DD_cohort_6      0.2845526
DD_cohort_7      0.2946202
DD_cohort_8      0.3082299
DD_cohort_9      0.3620433
DD_cohort_10     0.5837425

```

Thus, Table 3 in the paper with the incorrect standard errors is constructed as follows.

```

> Table3 <- apc.fit.model(data,"poisson.response","APC",replicate.version.1.3.1=TR
> Table3 <- cbind(Table3,apc.fit.model(data,"od.poisson.response","APC",replicate.
> Tab3 <- apc.fit.model(data,"poisson.response","AC",replicate.version.1.3.1=TRUE)
> Tab3 <- cbind(Tab3,apc.fit.model(data,"od.poisson.response","AC",replicate.versi
> Tab3 <- rbind(Tab3[1:11,],matrix(nrow=8,ncol=3),Tab3[12:19,])
> Table3 <- cbind(Table3,Tab3)
> colnames(Table3) <- c("apc est","apc se N","apc se t","ac est","ac se N","ac se t
> Table3

```

	apc est	apc se N	apc se t	ac est	ac se N
level	12.787863594	NA	NA	12.506404677	NA
age slope	0.697763921	0.0010034992	0.2240298	0.912526274	0.0005121711
cohort slope	0.111481669	0.0011224663	0.2505890	0.331272153	0.0005731092
DD_age_3	-0.895632465	0.0009858174	0.2200824	-0.866221921	0.0008821453
DD_age_4	0.013570514	0.0009117808	0.2035538	0.020862021	0.0009000816
DD_age_5	-0.642054188	0.0010298524	0.2299131	-0.657887194	0.0010211239
DD_age_6	0.258903877	0.0014025258	0.3131119	0.235501183	0.0013952616
DD_age_7	0.256459103	0.0017973054	0.4012459	0.268781621	0.0017901749
DD_age_8	-0.294147370	0.0022282682	0.4974578	-0.301632720	0.0022207471
DD_age_9	0.705787632	0.0028645524	0.6395073	0.791901153	0.0028546193
DD_age_10	-1.759462290	0.0047534360	1.0611980	-1.793115320	0.0047413068
DD_period_3	0.046442718	0.0020805252	0.4644743	NA	NA
DD_period_4	0.213821686	0.0018875907	0.4214020	NA	NA
DD_period_5	0.211836483	0.0015128207	0.3377351	NA	NA
DD_period_6	-0.405308334	0.0012639987	0.2821859	NA	NA
DD_period_7	0.354415338	0.0012145608	0.2711490	NA	NA
DD_period_8	-0.559003985	0.0011424331	0.2550466	NA	NA
DD_period_9	0.556712364	0.0011954761	0.2668884	NA	NA
DD_period_10	-0.075721211	0.0011024668	0.2461242	NA	NA
DD_cohort_3	-0.365436915	0.0011273315	0.2516752	-0.341425729	0.0010535291
DD_cohort_4	-0.025435276	0.0011204241	0.2501331	-0.005004999	0.0011101093
DD_cohort_5	-0.009240882	0.0011665838	0.2604382	-0.071485115	0.0011587235
DD_cohort_6	0.114695160	0.0012454096	0.2780360	0.137404391	0.0012376357

```

DD_cohort_7  0.053026763 0.0012911471 0.2882468  0.051370708 0.0012814185
DD_cohort_8  0.050815892 0.0013498121 0.3013437  0.078993227 0.0013405098
DD_cohort_9 -0.408218405 0.0015891640 0.3547787 -0.365523501 0.0015744312
DD_cohort_10 0.101509160 0.0025485309 0.5689560  0.057497628 0.0025285794
      ac se t
level      NA
age slope  0.1177565
cohort slope 0.1317671
DD_age_3    0.2028196
DD_age_4    0.2069434
DD_age_5    0.2347730
DD_age_6    0.3207934
DD_age_7    0.4115904
DD_age_8    0.5105859
DD_age_9    0.6563235
DD_age_10   1.0901037
DD_period_3      NA
DD_period_4      NA
DD_period_5      NA
DD_period_6      NA
DD_period_7      NA
DD_period_8      NA
DD_period_9      NA
DD_period_10     NA
DD_cohort_3  0.2422235
DD_cohort_4  0.2552322
DD_cohort_5  0.2664094
DD_cohort_6  0.2845526
DD_cohort_7  0.2946190
DD_cohort_8  0.3082050
DD_cohort_9  0.3619874
DD_cohort_10 0.5813616

```

5 Table 4: Forecasts

Table 4 with the correct standard errors is reproduced as follows.

```

> ac.fit <- apc.fit.model(data,"od.poisson.response","AC")
> ac.forecast <- apc.forecast.ac(ac.fit,quantiles=0.95)
> Table4 <- ac.forecast$response.forecast.per[,c(1,6)]
> Table4 <- rbind(Table4,ac.forecast$response.forecast.coh[,c(1,6)])
> Table4 <- rbind(Table4,ac.forecast$response.forecast.all[,c(1,6)])
> rownames(Table4)[19] <- "all"
> Table4

```

	forecast	t-0.950
per_11	5226535.83	6491431.2
per_12	4179394.44	5381287.5
per_13	3131667.52	4221849.2
per_14	2127271.92	2938174.2
per_15	1561878.91	2247271.8
per_16	1177743.69	1794299.1
per_17	744287.39	1242589.9
per_18	445521.29	870306.7
per_19	86554.62	269795.5
coh_2	94633.81	280973.2
coh_3	469511.29	835155.7
coh_4	709637.82	1151152.9
coh_5	984888.64	1498634.3
coh_6	1419459.46	2054154.9
coh_7	2177640.62	3016047.4
coh_8	3920301.01	5257277.2
coh_9	4278972.26	6050153.2
coh_10	4625810.69	7977049.4
all	18680855.61	23666264.4

Table 4 in the paper with the incorrect standard errors is reproduced as follows.

```
> ac.fit <- apc.fit.model(data,"od.poisson.response","AC",replicate.version.1.3.1=T)
> ac.forecast <- apc.forecast.ac(ac.fit,quantiles=0.95)
> Table4 <- ac.forecast$response.forecast.per[,c(1,6)]
> Table4 <- rbind(Table4,ac.forecast$response.forecast.coh[,c(1,6)])
> Table4 <- rbind(Table4,ac.forecast$response.forecast.all[,c(1,6)])
> rownames(Table4)[19] <- "all"
> Table4
```

	forecast	t-0.950
per_11	5226535.83	6429497.8
per_12	4179394.44	5323703.3
per_13	3131667.52	4171983.7
per_14	2127271.92	2906977.7
per_15	1561878.91	2224013.7
per_16	1177743.69	1774416.8
per_17	744287.39	1228946.0
per_18	445521.29	858592.0
per_19	86554.62	267850.3
coh_2	94633.81	280437.0
coh_3	469511.29	832650.6
coh_4	709637.82	1147493.1
coh_5	984888.64	1493914.5
coh_6	1419459.46	2047360.8

coh_7	2177640.62	3005099.3
coh_8	3920301.01	5232860.0
coh_9	4278972.26	6015633.6
coh_10	4625810.69	7862685.3
all	18680855.61	23297767.3

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