STRATEGIC VOTING INCENTIVES IN A THREE PARTY SYSTEM

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Updated February 2001

Abstract. I construct a new modelling framework for the analysis of strategic voting in plurality rule elections. Many existing formal models impose *statistical independence*: voters' decisions are drawn *independently* from a *commonly known* distribution. It follows that the only uncertainty over election outcomes is *idiosyncratic* — the decision of any particular individual is unknown. Departing from this literature, I introduce *constituency* uncertainty, where voters are uncertain of the constituency-wide support of the candidates. I prove that, for large constituencies, it is *only* constituency uncertainty that matters in determining voting behaviour, and hence conclude that existing analyses may be driven by the wrong factors. I proceed to show that, in contrast to the statistically independent case, the incentive to vote strategically is *finite*. It follows that multi-candidate support is perfectly consistent with optimal voting behaviour. Specialising to a three-candidate system, I build a microfoundation for voter beliefs based on the sampling of other constituents. This analysis generates a new and explicit strategic incentive variable for inclusion in empirical work. The behaviour of this strategic incentive variable may be surprising to some readers. It increases with both the distance from contention of a preferred candidate and the winning margin of the leading candidate. This contrasts with the informal hypothesis that the incentive to vote strategically should be larger in marginal constituencies.

1. Strategic Voting and Uncertainty

Social scientists have devoted considerable attention to the phenomenon of strategic voting. The plurality rule or "first past the post" electoral system is of particular interest. Individual constituents may vote, or perhaps consider voting, for someone other than their preferred candidate. This is a central component of the Duvergerian (1954) "psychological effect" and the hypothesised tendency toward two political parties in plurality systems.

Unfortunately, there is a conflict between many theoretical and empirical studies of strategic voting. A prominent class of theoretical analyses predict support for only two parties as the outcome of a plurality rule election with rational voters — equivalently, the degree of strategic voting is complete. Of course, applied researchers observe incomplete strategic voting. It is difficult, therefore, for applied authors to build their empirical models on suitable theoretical foundations.

My aim is to help resolve this conflict, by refining the existing theoretical approaches and incorporating suggestions drawn from empirical work. I argue that existing theoretical models, while providing important insights into the decision-making of a voter, are limited by their specification of uncertainty. I offer a revised formulation, that offers a more general model of voter beliefs. I am able to generate new and intuitive results, and some new comparative statics that differ from informal hypotheses. Perhaps most importantly, my analysis generates an explicit explanatory variable that may be used in empirical studies of strategic voting. My hope is that this work will allow applied researchers to base their statistical models on explicit theoretical foundations.

I must begin by establishing what it means to "vote strategically". Consider a single seat plurality rule election, with more than two competing candidates. Each voter casts a single vote, and the candidate receiving the largest number of votes is duly elected. *Sincere* voting (Farquharson 1969) occurs when a voter supports her most favoured candidate. Alternatively, such a voter may recognise that this candidate is poorly placed to win the election. In the parlance of Schattschneider (1942), she may be concerned that she will "waste her vote" on a candidate with limited viability. Mindful of a wasted vote, the voter may choose to support someone other than her preferred candidate, in the hope of exerting greater influence on the outcome of the election. The informal intuition is clear: A voter considers the tradeoff between her preference for candidates versus the likelihood of influencing the outcome of the election. This is the notion of strategic voting that I employ here. A voter is *strategic* if she bases her voting decision on both her relative preference for the different candidates and the relative likelihood of influencing the election outcome. She *votes strategically* if such considerations lead her to vote for someone other than her first preference. Equivalently, the focal voter is short term instrumentally rational — she cares only about the outcome of the election in question and pursues her objectives in a consistent way.

The theoretical literature is successful in highlighting the critical components of an instrumental voter's decision calculus. A central observation, which will be familiar to readers of McKelvey and Ordeshook (1972), Hoffman (1982) and Cox (1984) *inter alia*, is that an individual can only influence the outcome of an election when she possesses a *casting vote*. This requires a *pivotal* outcome, when there is a tie (or a near tie) for the lead. Only in this situation can a single vote alter the election outcome. In any other situation, a single vote has no effect, and hence such non-pivotal outcomes are of no interest to an instrumental voter. This is an important observation. I believe that it is equally critical, however, to understand the role played by *uncertainty*. A voter's decision will depend upon the relative likelihood of different pivotal outcomes. Her beliefs, therefore, deserve further scrutiny.

In many existing theoretical treatments, the specification of uncertainty includes the assumption of *statistical independence*. This is true of decision-theoretic analyses such as those of McKelvey and Ordeshook (1972) and Cox (1984) as well as the later game-theoretic treatments of Palfrey

(1989), Myerson and Weber (1993) and Cox (1994). From the perspective of a strategic voter, the decisions of the remaining electorate are all made *independently*.[1] This feature is *critical* to any subsequent results. To understand why, consider the decision calculus of a strategic voter. Entering her calculations will be not only her relative preference for the candidates, but also the likelihood of their involvement in a pivotal outcome. Allowing the constituency to grow large, it is of course true that the *absolute* probability of a pivotal event falls to zero. This is of no consequence to the instrumental voter. She cares only about the *relative* likelihood of pivotal events. Equivalently, she considers the probability of a particular tie *conditional* on a tie of any kind. When the decisions of the remaining electorate are made independently from a known distribution, the probability of a pivotal event involving the leading two candidates becomes infinitely larger than the probability of an event involving any other pair. That is, conditional on the occurence of a pivotal event, the tie will be between the leading two candidates with near certainty in a sufficiently large constituency. It follows that the strategic voter will support her favourite among the leading two contenders. In the words of Droop (1871) her vote will be "given in favour of one or the other parties between whom the election really lies". Moreover, she is certain of the identity of the leading two candidates. The same will be true of any other strategic voters sharing the same beliefs. All will vote for one of the leading two candidates — a strictly Duvergerian outcome.[2] The prediction, then, is that strategic voting in an instrumentally rational electorate leads to a two-candidate outcome — strategic voting is complete.

It is statistical independence, then, that drives the stark two-candidate result. Perhaps more importantly, this assumption leads to an effective *absence* of any real uncertainty. For larger constituencies, the Law of Large Numbers begins to bite. Individually idiosyncratic decisions will be averaged out in a large constituency, and hence a strategic voter may successfully predict the outcome of the election with near certainty. It follows that any significant uncertainty is removed. Indeed, this is reflected in the fact that the identity of the leading two candidates is, in fact, known. This unappealing feature is appropriately recognised by leading voting theorists. For instance, in his wide-ranging survey of strategic voting, Cox (1997) comments:

> "A fourth condition necessary to generate pure local bipartism is that the identity of trailing and front-running candidates is common knowledge ... If who trails is not common knowledge, then an extra degree of freedom is opened up in the model."

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[1]More accurately, the preferences of these individuals are all drawn *independently* from a (commonly) *known* distribution. Hoffman (1982) provides an exception. He specifies beliefs directly over the outcome space, and ensures that these beliefs are non-degenerate. His model, however, omits any foundation for these beliefs. I pursue the relationship with Hoffman's work later in this section.

[2]Of course, Duverger (1954) claimed that plurality voting yielded a *tendency* toward two-party systems. When discussing a "strictly Duvergerian" outcome I refer to the strict bipartism claims that arise from statistically independent formal models.

My aim, therefore, is to open up this degree of freedom and analyse the implications. In moving to a wider specification of uncertainty, however, I also wish to draw upon the suggestions of empirical authors. Contributors to the empirical literature have also observed that voters may be unsure of the ranking of the various candidates. In their critique of the Niemi *et al* (1992) measure of strategic voting, Heath and Evans (1994) make the following observation:

> "[The Niemi *et al* measure] does not allow for the possibility that some people may intend to avoid wasting their vote, but may be mistaken in their perceptions of the likely chances of the various parties winning the constituency."

To further my aim, I allow voters to be unsure of the constituency-wide distribution of voter intentions. I offer a formal model in Section 2 incorporating two sources of uncertainty. The *constituency support* for a candidate is the independent probability that a randomly selected voter supports that candidate. This leads to *idiosyncratic* uncertainty — even with full knowledge of constituency support, individual voting decisions are probabilistic and hence unknown. Importantly, however, the constituency support of each candidate is also unknown to a strategic voter. This I define as *constituency* uncertainty. Together, these two sources of uncertainty combine to yield a voter's beliefs over election outcomes. Notice that the statistical independence assumption has been removed. Indeed, the voting decision of one individual reveals information about the constituency support of candidates, and hence the probability that another individual will vote in a particular way. Hence constituency uncertainty leads to the interdependence of voter decisions.

What happens to a voter's beliefs as the constituency size grows large? I present a formal analysis in Section 3. The results are briefly as follows. As anticipated above, the idiosyncratic uncertainty is averaged out. The constituency uncertainty remains. It follows that beliefs over election outcomes are *entirely* determined by constituency uncertainty. From the perspective of a strategic voter, any idiosyncratic uncertainty is irrelevant. Of course, statistically independent models omit any constituency uncertainty, and so their results are determined by idiosyncratic uncertainy. When both are present, it is only constituency uncertainty that matters. It is possible, therefore, that statistically independent models may be driven by the wrong factors.

This result has implications for the voting calculus. In Section 4 I characterise the behaviour of a strategic voter in a three candidate election, contingent on her beliefs over constituency support. In the familiar way, the optimal decision involves a tradeoff between her relative preference for her favourite two candidates and the relative likelihood of their involvement in different relevant pivotal events. When constituency uncertainty is present, all relevant pivotal events retain positive conditional probability. It follows that the likelihood ratios of pivotal events, and hence the incentive to vote strategically, remain finite. Strategic voting, therefore, is limited. Unless relative preferences are sufficiently bounded, there will be multi-candidate support — a non-Duvergerian outcome.

The removal of the statistical independence assumption and the introduction of constituency uncertainty serve to yield a tractable model of strategic incentives in a large electorate. Further results, however, require me to add additional structure to the model. In Section 5, I use the Dirichlet distribution to model a voter's beliefs over constituency support. This is an attractive specification. The Dirichlet arises from a microfoundational model of a voter observing the voting intentions of a sample of the electorate and updating a neutral prior. In short, the Dirichlet specification models a voter's posterior beliefs following an opinion poll. The additional structure helps to build a link between the classic analysis of Hoffman (1982) and the statistically independent Cox-Palfrey approach. Hoffman (1982) specified beliefs directly over the outcome space, avoiding a degenerate distribution but omitting a microfoundation for this approach. In contrast, Cox (1984) and Palfrey (1989) generate the outcome distribution from individual voting decisions, but impose independence on these decisions. My model is designed to incorporate the non-degeneracy of the former and the foundations of the latter.

Dirichlet beliefs are conveniently characterised by the modal belief of a voter — the most likely outcome — and the precision of beliefs. Varying the modal belief mirrors changes in constituency support levels, whereas a change in the precision of beliefs mirrors changes in the information available to voters, or the size of an available opinion poll. My first observation is that strategic incentive is positive whenever a voter's first preference trails her second preference. It is possible, therefore, for strategic voters to switch away from a second-placed candidate. My second observation is that the incentive is decreasing in the strength of a preferred candidate, and increasing in the strength of the remaining candidates. Of course, these comparative statics offer only limited insight. The support for each of three candidates cannot be varied independently. Indeed, the configuration of a three candidate constituency may be characterised using only two parameters.

Two leading candidates for parameters, drawn from empirical analysis, are the *winning margin* and the *distance from contention*. They may be applied when the preferred candidate trails in third place. Using these parameters, I find that the incentive to vote strategically increases with the distance from contention of the preferred candidate. This corresponds to informal hypotheses from the empirical literature. Critically, however, when the distance from contention is fixed, the strategic incentive is also increasing in the winning margin. This means that the incentive is *lessened* in constituencies that a *more* marginal. This runs against the informal hypothesis that incentives will be greater in marginal constituencies. Such an informal hypothesis is based on the following argument: A voter is more likely to be pivotal in a marginal constituency. But this logic is flawed. It is not the *absolute* pivotal probability that matters. Rather, it is the *relative* probability of different pivotal events. As the winning margin widens, a tie involving the preferred candidate becomes relatively less likely, hence enhancing the incentive.

The Dirichlet specification also allows me to consider the impact of varying information, or of differing opinion poll size. An increase in the precision of beliefs corresponds to a better informed voter. Increasing the precision of beliefs increases the strategic incentive whenever the preferred candidate is expected to trail in third place. In contrast, increasing precision reduces the strategic incentive when the preferred candidate lies in second place. As beliefs become infinitely precise, the strategic incentive becomes infinitely large in the former case, and vanishes in the latter. I believe that this clarifies the role played by the extant Cox-Palfrey statistically independent modelling framework. Those models correspond to the case of extremely precise information sources, rather than large electorates.

In summary, I begin with the observation that theoretical models of strategic voting might sometimes be driven by a statistical independence assumption. Removing this assumption, and carefully modelling voter beliefs, yields a prediction of finite strategic incentives, and subsequent interesting comparative statics. A possible conclusion is that multi-candidate support under the plurality rule is perfectly consistent with rational voting behaviour. This argument relies, however, on the decision-theoretic approach to the analysis. I do not, in this paper, explicitly consider a voter's consideration of strategic behaviour by others in a constituency. That issue is addressed in my companion paper Myatt (1999). The results of the companion analysis are perhaps surprising. Allowing for game-theoretic effects may *reduce* the degree of strategic voting rather than increase it. I present a brief discussion of these related results and their intuition in Section 6.

In the remainder of the paper, I present these results in more detail. The basic framework is described in Section 2. Sections 3 and 4 analyse the nature of a voter's beliefs and her optimal decision rule respectively. I expand the model using the Dirichlet specification in Section 5, and explore its comparative static properties. In conclude the paper in Section 6 with some comments on properties of the related game-theoretic work.

2. A Model of Plurality Voting

2.1. **The Election.** There are m candidates competing in a single-seat district election, indexed by $j \in \{1, 2, \ldots, m\}$. I wish the model to have direct applicability in important settings such as the English constituencies of the United Kingdom, where three parties are dominant. I will, therefore, focus on the $m = 3$ case throughout much of the paper.

There are $n + 1$ individuals in the electorate, indexed by $i \in \{0, 1, \ldots, n\}$. The focal voter $i = 0$ will consider the voting behaviour of the remaining n voters. Among these n individuals, I will use x_j to denote the number of votes cast for candidate j, and the vector $x = [x_j]_{j=1}^m$ to denote the election outcome, where $x \in X$ and the outcome space is:

$$X = \left\{ x \in \mathbb{Z}_+^m \mid \sum_{j=1}^m x_j = n \right\}$$

Clearly, each individual casts a single vote, and there are no abstentions. The candidate receiving the highest number of votes wins the election — this is a plurality rule or "first past the post" electoral mechanism. Of course, a tied outcome may obtain, where a number of candidates share a common and leading vote total. In this case, and following British electoral convention, such ties are broken at random.[3]

2.2. **Electorate Behaviour.** The model is decision-theoretic, and hence I exogenously specify behaviour of the n individuals indexed by $i > 0$. Their behaviour is summarised by the parameter $p \in \triangle$, representing the *constituency support* for the m candidates. Here, $\triangle$ is the $m-1$ dimensional unit simplex:

$$\triangle = \left\{ p \in \mathbb{R}_+^m \mid \sum_{j=1}^{m} p_j = 1 \right\}$$

A randomly selected individual $i > 0$ votes for candidate j with probability $p_j \in [0, 1]$. Voting decisions for the n individuals $i > 0$ are independent. It follows that the outcome x is a draw from the multinomial distribution with parameters p and n.

Two features are worthy of note. First, the parameter p represents the voting *intentions* of the electorate, and not necessarily their true underlying preferences. Indeed, their behaviour may already reflect strategic switching. As I hope to make clear, this is largely irrelevant to the decision-making of the focal voter $i = 0$. This voter is interested in electoral preferences only insofar as they affect voting intentions, and hence the election outcome vector x. I do not, therefore, assume that the n individuals $i > 0$ vote sincerely — hence the applicability of the framework may be wider than initial expectations would suggest. The second issue is the independence of voting decisions — does the model suffer from statistical independence? The answer is no. Conditional on p, voting decisions are indeed independent. But, as I will make clear, this parameter is unknown to the focal voter $i = 0$. It follows that, from this voter's perspective, the decisions of the remaining electorate are *not* independent. Indeed, the decision of one individual reveals information about p, and hence about the likely decisions of others. Voting decisions are thus independent from the viewpoint of an omniscient analyst only, and not from viewpoint of participating agents in the model.

2.3. **The Focal Voter.** It remains for me to specify the preferences and beliefs of the focal voter $i = 0$. She is short term instrumentally rational, caring about the outcome of the election process only insofar as it influences the winning candidate. I will denote her von Neumann-Morgenstern utility for a win by candidate j as u_j. Without loss of generality, I index the candidates so that $u_1 > u_2 > \cdots > u_m$.

The parameter p, representing the constituency wide support for the m candidates, is unknown to the focal voter $i = 0$. It follows that she is uncertain of the election outcome x. I represent

[3]More accurately, in the United Kingdom the returning office has a casting vote in such situations. By convention, the officer employs a coin toss to determine the winner.

uncertainty over p by the density $f(p)$, where:

$$f(p) : \triangle \mapsto \mathbb{R}_+ \quad \text{and} \quad \int_\triangle f(p)\,dp = 1$$

I assume that $f(p)$ continuous and strictly positive on the interior of $\triangle$. Given her beliefs $f(p)$ and preferences u, voter $i = 0$ maximises her expected utility. The density $f(p)$ represents the *constituency uncertainty* in the model.

3. Election Outcomes and Constituency Uncertainty

The focal voter $i = 0$ is only interested in influencing the identity of the winning candidate. She wishes to maximise her expected utility by casting her vote in the most effective way. This requires her to consider the different possible election outcomes in terms of vote counts, and the probabilities of such outcomes. I characterise optimal voting behaviour in Section 4. My purpose here is to clarify the nature of outcome probabilities, and the relative importance of constituency and idiosyncratic uncertainty.

3.1. **Idiosyncratic and Constituency Uncertainty.** I begin by conditioning on the constituency support parameter p. Absent the focal voter, the election outcome x follows a multinomial distribution with parameters p and n:

$$\Pr[x\,|\,p] = \frac{n!}{\prod_{j=1}^m x_j!}\prod_{j=1}^m p_j^{x_j} = \frac{\Gamma(n+1)}{\prod_{j=1}^m \Gamma(x_j+1)}\prod_{j=1}^m p_j^{x_j} \tag{1}$$

For later convenience, I have replaced the factorial notation $x_j!$ with the gamma function representation $\Gamma(x_j + 1)$. Equation 1 represents *idiosyncratic* uncertainty; even when the parameter p is known and votes are drawn independently, the outcome x remains uncertain. Of course, p is unknown to the focal voter. To compute the unconditional probability, I integrate over $f(p)$ to obtain:

$$\Pr[x] = \int_\triangle f(p)\Pr[x\,|\,p]\,dp \tag{2}$$

Thus the *constituency* uncertainty $f(p)$ combines with the idiosyncratic uncertainty to yield beliefs over election outcomes.

3.2. **Large Constituencies.** I envisage a typical constituency, with a large electorate. I thus allow the constituency size n to grow larger. Substituting in $\Pr[x|p]$, Equation 2 becomes:

$$\Pr[x] = \frac{\Gamma(n+1)}{\prod_{j=1}^m \Gamma(x_j+1)}\int_\triangle f(p)\prod_{j=1}^m p_j^{x_j}\,dp \tag{3}$$

As n grows large, it is clear that such probabilities vanish to zero. For an instrumental voter, however, it will not be the *absolute* probability of particular outcomes that are at issue. Rather, the *relative* probability of different outcomes determines behaviour — I make this logic explicit in Section 4. This leads me to a re-examination of Equation 3. Notice that the second term in the integrand will certainly vanish to zero as $n \to \infty$. This expression, however, is maximised at x/n. Formally:

$$\gamma = \frac{x}{n} = \arg\max_{p \in \triangle} \prod_{j=1}^{m} p_j^{x_j}$$

For larger n, all mass in this function congregates around the maximum. Intuitively, then, all significant weight in this function lies in a locality of the maximiser γ. The continuous density $f(p)$ is approximately equal to $f(\gamma)$ around this value. The claim, then, is that it is valid to replace $f(p)$ with $f(\gamma)$ in Equation 3. I state this formally in the following lemma.

Lemma 1. *For any $\gamma \in \triangle$:*

$$\lim_{n\to\infty} \frac{\int_\triangle f(p) \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp}{\int_\triangle f(\gamma) \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp} = 1$$

Proof. See Appendix 7.1. □

Notice that the ratio in Lemma 1 is continuous in γ. The limit is also (trivially) continuous in γ. Since γ is drawn from the compact set $\triangle$, it follows that the convergence is uniform. In other words, for a sufficiently large constituency size the ratio is arbitrarily close to 1 for *any* $\gamma \in \triangle$. Before reflecting this fact as a corollary, I find it useful to define the following notation.

Definition 1. *For functions $g(x,n)$ and $h(x,n)$ write $g(x,n) \underset{n\to\infty}{\approx} h(x,n)$ whenever, for any $\epsilon > 0$, there exists $\underline{n} \in \mathbb{N}$ such that for all $n \geq \underline{n}$, and for any $x \in X$:*

$$\left| \frac{g(x,n)}{h(x,n)} - 1 \right| < \epsilon$$

If $g(x,n) \underset{n\to\infty}{\approx} h(x,n)$ then they are asymptotically equivalent *in large constituencies.*

This notation allows me to state the following corollary.

Corollary 1. *For larger constituencies, outcome probabilities satisfy:*

$$\int_\triangle f(p) \prod_{j=1}^{m} p_j^{x_j}\, dp \underset{n\to\infty}{\approx} f\left(\frac{x}{n}\right) \int_\triangle \prod_{j=1}^{m} p_j^{x_j}\, dp$$

Proof. Application of Lemma 1 and Definition 1. See Appendix 7.1. □

Combining Corollary 1 with Equation 3 allows an asymptotic expression for $\Pr[x]$. Recognising that the remaining integrand is the kernel of the Dirichlet density yields to central results.

Proposition 1. *For large constituencies,* $\Pr[x]$ *behaves as $f(x/n)$. Formally:*

$$\Pr[x] \underset{n\to\infty}{\approx} \frac{1}{n^{m-1}} f\left(\frac{x}{n}\right)$$

Proof. See Appendix 7.1. □

Corollary 2. *For large constituencies, likelihood ratios are determined entirely by $f(p)$:*

$$\frac{\Pr[x]}{\Pr[\tilde{x}]} \underset{n\to\infty}{\approx} \frac{f(x/n)}{f(\tilde{x}/n)}$$

I wish to highlight two features of these results. First, as I suggested in my earlier informal argument, only constituency uncertainty matters — by inspection, limiting likelihood ratios are determined entirely by $f(p)$. Secondly, the limiting likelihood ratios of different outcomes are *finite*. This is *not* the case when p is known with certainty.

4. Voting Behaviour

I now consider the behaviour of the focal voter $i = 0$, and restrict to the case of $m = 3$. Since $u_1 > u_2 > u_3$, she will never find it optimal to vote for candidate 3. Her choice is therefore a binary one, between candidates 1 and 2. In Table 1(a) I give an exhaustive list of the situations in which she is pivotal. I also display the relevant payoffs for a vote for candidate 1 or 2, as well as the difference in payoffs between these two options. Calculating the probabilities of these various events, it is optimal for the voter $i = 0$ to support candidate 1 whenever the expected difference in payoffs is positive.

As Table 1(a) reveals, there are a large number of pivotal events, even when limiting to a three candidate election. Fortunately, the number of relevant pivotal events is somewhat smaller. In Table 1(b) I categorise the pivotal events in an appropriate way. Clearly, the probability of such events vanishes to zero as the constituency size grows large. This fact, however, is largely irrelevant to the voting calculus. Rather, it is the *relative* probabilities of different events that is key in determining a voter's behaviour. Furthermore in the limit, only two way ties matter. I state these claims formally in the following lemmata. First, consider three-way and near three-way ties.

Lemma 2. *Three way and near three-way ties are asymptotically equivalent, and satisfy:*

$$\Pr[x_1 = x_2 = x_3 + 1] \underset{n\to\infty}{\approx} \frac{1}{n^2} f\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \underset{n\to\infty}{\approx} \Pr[x_1 = x_2 = x_3]$$

where the first equivalence applies for $(n-1)/3 \in \mathbb{N}$ and the second for $n/3 \in \mathbb{N}$.

The qualification at the end of the lemma ensures that the events of interest exist.

Proof. Application of Proposition 1. □

Lemma 3. *Two-way and near two-way ties are asymptotically equivalent, and satisfy:*

$$\Pr[x_1 = x_2 > x_3] \underset{n\to\infty}{\approx} \frac{\int_{1/3}^{1/2} f(z, z, 1-2z)\, dz}{6n}$$

Proof. See Appendix 7.2. □

Event	u(Vote 1)	u(Vote 2)	u(Vote 1) $-$ u(Vote 2)
$x_1 = x_2 = x_3$	u_1	u_2	$(u_1 - u_3) - (u_2 - u_3)$
$x_1 = x_2 > x_3$	u_1	u_2	$(u_1 - u_3) - (u_2 - u_3)$
$x_1 = x_3 > x_2 + 1$	u_1	$\frac{u_1+u_3}{2}$	$\frac{u_1-u_3}{2}$
$x_2 = x_3 > x_1 + 1$	$\frac{u_2+u_3}{2}$	u_2	$-\frac{u_2-u_3}{2}$
$x_1 = x_3 = x_2 + 1$	u_1	$\frac{u_1+u_2+u_3}{2}$	$\frac{u_1-u_2-u_3}{2}$
$x_2 = x_3 = x_1 + 1$	$\frac{u_1+u_2+u_3}{2}$	u_2	$\frac{u_1+u_3-u_2}{2}$
$x_1 - 1 = x_2 > x_3$	u_1	$\frac{u_1+u_2}{2}$	$\frac{(u_1-u_3)-(u_2-u_3)}{2}$
$x_1 - 1 = x_3 > x_2$	u_1	u_1	0
$x_1 - 1 = x_2 = x_3$	u_1	$\frac{u_1+u_2}{2}$	$\frac{(u_1-u_3)-(u_2-u_3)}{2}$
$x_2 - 1 = x_1 > x_3$	$\frac{u_1+u_2}{2}$	u_2	$\frac{(u_1-u_3)-(u_2-u_3)}{2}$
$x_2 - 1 = x_3 > x_1$	u_2	u_2	0
$x_2 - 1 = x_1 = x_3$	$\frac{u_1+u_2}{2}$	u_2	$\frac{(u_1-u_3)-(u_2-u_3)}{2}$
$x_3 - 1 = x_1 > x_2$	$\frac{u_1+u_3}{2}$	u_3	$\frac{u_1-u_3}{2}$
$x_3 - 1 = x_2 > x_1$	u_3	$\frac{u_2+u_3}{2}$	$-\frac{u_2-u_3}{2}$
$x_3 - 1 = x_1 = x_2$	$\frac{u_1+u_3}{2}$	$\frac{u_2+u_3}{2}$	$\frac{(u_1-u_3)-(u_2-u_3)}{2}$

(a) All Pivotal Events and Payoffs

Category	Example
Three Way Tie	$x_1 = x_2 = x_3$
Near Three Way Tie	$x_1 = x_3 > x_2 + 1$
Two Way Tie	$x_1 = x_2 > x_3$
Near Two Way Tie	$x_1 - 1 = x_2 > x_3$

(b) Categorisation of Pivotal Events

Event	u(Vote 1)	u(Vote 2)	u(Vote 1) $-$ u(Vote 2)
$x_1 = x_2 > x_3$	u_1	u_2	$(u_1 - u_3) - (u_2 - u_3)$
$x_1 = x_3 > x_2 + 1$	u_1	$\frac{u_1+u_3}{2}$	$\frac{u_1-u_3}{2}$
$x_2 = x_3 > x_1 + 1$	$\frac{u_2+u_3}{2}$	u_2	$-\frac{u_2-u_3}{2}$
$x_1 - 1 = x_2 > x_3$	u_1	$\frac{u_1+u_2}{2}$	$\frac{(u_1-u_3)-(u_2-u_3)}{2}$
$x_2 - 1 = x_1 > x_3$	$\frac{u_1+u_2}{2}$	u_2	$\frac{(u_1-u_3)-(u_2-u_3)}{2}$
$x_3 - 1 = x_1 > x_2$	$\frac{u_1+u_3}{2}$	u_3	$\frac{u_1-u_3}{2}$
$x_3 - 1 = x_2 > x_1$	u_3	$\frac{u_2+u_3}{2}$	$-\frac{u_2-u_3}{2}$

(c) Relevant Pivotal Outcomes and Voter Payoffs

TABLE 1. Pivotal Events and Voter Payoffs

These lemmata confirm that the probability of a pivotal outcome vanishes to zero as $n \to \infty$. Notice, however, that the probability of a three-way tie vanishes at rate n^{-2}, whereas the probability of a two-way tie vanishes at rate n^{-1}. It follows that only two-way ties are of importance.

Lemma 4. *Three-way ties and near three-way ties are asymptotically irrelevant.*

Proof. Combination of Lemmata 2 and 3. □

Based on this lemma, the number of relevant events becomes rather smaller than the number displayed in Table 1(a). Inspecting the table, remove all three-way and near three-way ties. Next, notice that there is no payoff difference between candidates 1 and 2 for the events $x_1 - 1 = x_3 > x_2$ and $x_2 - 1 = x_3 > x_1$. I show the remaining events and payoff differences in Table 1(c). These payoff differences depend only on the preference for candidates 1 and 2 relative to candidate 3. It follows that that the optimal voting rule may be couched in terms of these relative payoffs.

Proposition 2. *Assuming without loss of generality that an indifferent voter $i = 0$ casts her vote in favour of candidate 1, the optimal voting rule is asympotically equivalent to:*

$$\textit{Vote 1} \quad \Leftrightarrow \quad \underbrace{\log \frac{u_1 - u_3}{u_2 - u_3}}_{\textit{Relative Preference}} \geq \underbrace{\log \frac{2\Pr[x_1 = x_2 > x_3] + \Pr[x_2 = x_3 > x_1]}{2\Pr[x_1 = x_2 > x_3] + \Pr[x_1 = x_3 > x_2]}}_{\textit{Strategic Incentive}} \equiv \lambda_n \tag{4}$$

Proof. Assemble the components of Table 1(c) to obtain the optimal voting criterion. This applies asympotically, since the three way ties have been removed. Impose the asymptotic equivalence of two-ties and near two-way ties, and re-arrange to obtain Equation 4. □

The interpretation is both clear and familiar. A voter balances her relative preference for candidates 1 and 2 against the relative likelihood of influencing the election outcome. For instance, a vote for candidate 1 rather than candidate 2 yields a payoff gain of $u_1 - u_3$ whenever there is a tie between candidates 1 and 3. Furthermore, whenever candidates 1 and 2 are tied, there is a gain of $u_1 - u_2 = (u_1 - u_3) - (u_2 - u_3)$. The probability of such a tie carries the coefficient 2, since the vote switch has twice the effect: there is the loss of a vote for candidate 2 and the gain of a vote for candidate 1. Assembling these effects yields the second term in Equation 4 as an appropriate measure of the strategic incentive. When this term is positive, there is an incentive for the voter to switch away from her preferred option and toward candidate 2. Of course, for Proposition 2 to hold, the strategic incentive must be finite. This is the content of the following proposition.

Proposition 3. *The strategic incentive is asymptotically finite, satisfying:*

$$\lambda_n \underset{n\to\infty}{\approx} \log \frac{p_{23} + 2p_{12}}{p_{13} + 2p_{12}} \equiv \lambda_\infty \quad \textit{where} \quad p_{12} = \int_{1/3}^{1/2} f(z, z, 1 - 2z)\, dz$$

with symmetric expressions for p_{13} and p_{23}.

Proof. Application of Lemma 3. □

The political scientific content of Proposition 3 is this: When a strategic voter is uncertain of the constituency-wide support for the different candidates, her incentive to vote strategically remains finite, even in an unboundedly large electorate. This opens the possibility for multi-candidate support. The analysis also generates an explicit strategic incentive variable, which varies with the structure of a voter's beliefs. It is thus possible to investigate the determinants of strategic incentives. This is the topic of Section 5.

Before proceeding, however, I wish to define an alternative representation of the strength of the strategic effect. The asymptotic strategic incentive λ_∞ is difficult to interpret, and takes values on the entire real line.

Definition 2. *Define the* strategic intensity $\Lambda \in [0,1]$ *whenever* $\lambda_\infty \geq 0$ *by:*

$$\Lambda \equiv \frac{e^{\lambda_\infty} - 1}{e^{\lambda_\infty}}$$

where λ_∞ *is the asymptotic* strategic incentive *from Proposition 3.*

Using this definition, Λ represents the proportional degree of payoff advantage for candidate 1 over candidate 2, both judged relative to candidate three, that is required to persuade a strategic voter to vote sincerely. In other words, a strategic voter is indifferent between candidate 1 and 2 when:

$$\Lambda = \frac{u_1 - u_2}{u_1 - u_3}$$

When $\Lambda = 1$, even a voter that is almost indifferent between candidates 2 and 3 will vote strategically. When $\Lambda = 0.25$, for instance, then any voter whose preference for a candidate 2 win is 0.75 or more of that for a candidate 1 win will vote strategically. The other extreme of $\Lambda = 0$ reveals the absence of any strategic incentive.

5. Modelling Constituency Uncertainty

5.1. **The Dirichlet Distribution.** I add structure to a voter's beliefs via the use of the Dirichlet distribution over the constituency support space $\triangle$. The Dirichlet may be characterised by a set of parameters $\{\beta_j\}_{j=1}^m$, where $\beta_j \geq 0$. The density function is then:

$$f(p) = \frac{\Gamma\left(\sum_{j=1}^m \beta_j\right)}{\Gamma(\beta_j)} \prod_{j=1}^m p_j^{\beta_j - 1} \quad \text{where} \quad \Gamma(\beta_j) = \int_0^1 \left[\log \frac{1}{t}\right]^{\beta_j - 1} dt$$

The relative values of β_j determine the expectation of p, and the precision of beliefs around this mean increase with the level of β_j. Formally $\mathrm{E}[p_j] = \beta_j / \left(\sum_{k=1}^m \beta_k\right)$. The special case of $\beta_j = 1$ for all j yields a uniform distribution over $\triangle$.

What justifies my application of the Dirichlet density? It offers a convenient conjugate to the multinomial distribution. Begin with a Dirichlet distribution with parameters $\{\beta_j\}$. Suppose now that a focal voter observes a random sample of voting intentions of size s. That is, for each of s randomly selected individuals, she observes their voting intention $j \in \{1, 2, \ldots, m\}$. I use the vector y with elements y_j to denote the number of individuals supporting candidate j, so that $\sum_{j=1}^{m} y_j = s$. Beginning with a Dirichlet prior, and updating following the observation y yields a posterior belief:

$$f(p) = \frac{\Gamma\left(s + \sum_{j=1}^{m} \beta_j\right)}{\Gamma(\beta_j + y_j)} \prod_{j=1}^{m} p_j^{\beta_j + y_j - 1} \tag{5}$$

Hence posterior beliefs retain the Dirichlet form. In particular, suppose that a voter begins with a uniform prior. I define $\pi_j = y_j/s$, the fraction of the sample of voters who support candidate j. Assembling these into a vector, it follows that $\pi \in \triangle$. With this formulation and the uniform prior, posterior beliefs satisfy:

$$f(p) = \frac{\Gamma(m+s)}{\Gamma(1+\pi_j s)} \left[\prod_{j=1}^{m} p_j^{\pi_j}\right]^s \propto \tilde{f}(p) = \left[\prod_{j=1}^{m} p_j^{\pi_j}\right]^s$$

The leading multiplicative constant may be neglected, since only relative likelihoods are of interest to a strategic voter. Examining $\tilde{f}(p)$, notice that $\pi = \arg\max \tilde{f}(p)$. It follows that $\pi \in \triangle$ is the *modal belief* for the focal voter, and s indexes the *precision* of beliefs around this mode. Turn now to the leading case of interest — a three candidate system with $m = 3$.

Lemma 5. *For the Dirichlet with $m = 3$, the asymptotic strategic incentive satisfies:*

$$\lambda_\infty = \log \frac{\int_{1/3}^{1/2} \left[2g(z,\pi_3)^s + g(z,\pi_1)^s\right] dz}{\int_{1/3}^{1/2} \left[2g(z,\pi_3)^s + g(z,\pi_2)^s\right] dz} \quad \textit{where} \quad g(z,\pi) = z^{1-\pi}(1-2z)^{\pi} \tag{6}$$

Evaluating the integrals, this may be expressed as:

$$\lambda_\infty = \log \frac{B_{1/3}(1+\pi_3 s, 1+(1-\pi_3)s) + 2^{(\pi_1-\pi_3)s-1} B_{1/3}(1+\pi_1 s, 1+(1-\pi_1)s)}{B_{1/3}(1+\pi_3 s, 1+(1-\pi_3)s) + 2^{(\pi_2-\pi_3)s-1} B_{1/3}(1+\pi_2 s, 1+(1-\pi_2)s)}$$

where $B_{1/3}(\alpha_1, \alpha_2)$ is the incomplete Beta function evaluated at $1/3$ with parameters α_1 and α_2 — see, for instance, Larson (1982) for details.

Proof. See Appendix 7.3 □

This lemma generates an explicit strategic incentive term, and hence a possible explanatory variable for empirical work. Appendix 8 comments on its implementation.

5.2. **Comparative Statics.** When will the strategic incentive be positive?

Lemma 6. *For $m = 3$, the asymptotic strategic incentive λ_∞ is positive if and only if $\pi_2 > \pi_1$.*

Proof. See Appendix 7.3 □

Lemma 6 demonstrates that it is not necessary for a voter to expect her preferred candidate to trail in third place before the strategic incentive is positive — the configuration $\pi_2 > \pi_1 > \pi_3$ yields a positive incentive to vote strategically. Further comparative statics, however, are less clear. In a three candidate system, modal beliefs are conveniently summarised by $\pi = (\pi_1, \pi_2, \pi_3)$. Unfortunately, these parameters cannot be varied independently, since $\sum_{j=1}^{m} \pi_j = 1$. Analysts must therefore take great care in the use of "intuitive" measures such as the "distance from contention" or the "marginality" of an election. When varying any parameter, it is important to recognise which other parameters are being adjusted and which are fixed. Some progress may be made, however. Examining Equation 6, notice that, for $1/3 < z < 1/2$, $g(z, \pi)$ is decreasing in π — this fact is made explicit by Lemma 10 in Appendix 7.3. It follows that λ_∞ is increasing in π_2 and π_3, and decreasing in π_1, when the $\sum_{j=1}^{m} \pi_j = 1$ restriction is ignored. Using this information, I offer some immediate comparative statics in the following lemma.

Lemma 7. *Fixing π_3, λ_∞ is increasing in $\pi_2 - \pi_1$. Fixing π_2, λ_∞ is increasing in $\pi_3 - \pi_1$.*

Proof. By inspection of Equation 6. □

Hence the strategic incentive increases as the preferred candidate loses support, independent of where this support moves to. What Lemma 7 fails to reveal, however is the impact of a movement of support between candidates 2 and 3, whenever the strength of candidate 1 is fixed. Indeed, varying $\pi_2 - \pi_3$ while fixing π_1 yields a non-monotonic relationship, as illustrated in Figure 1.

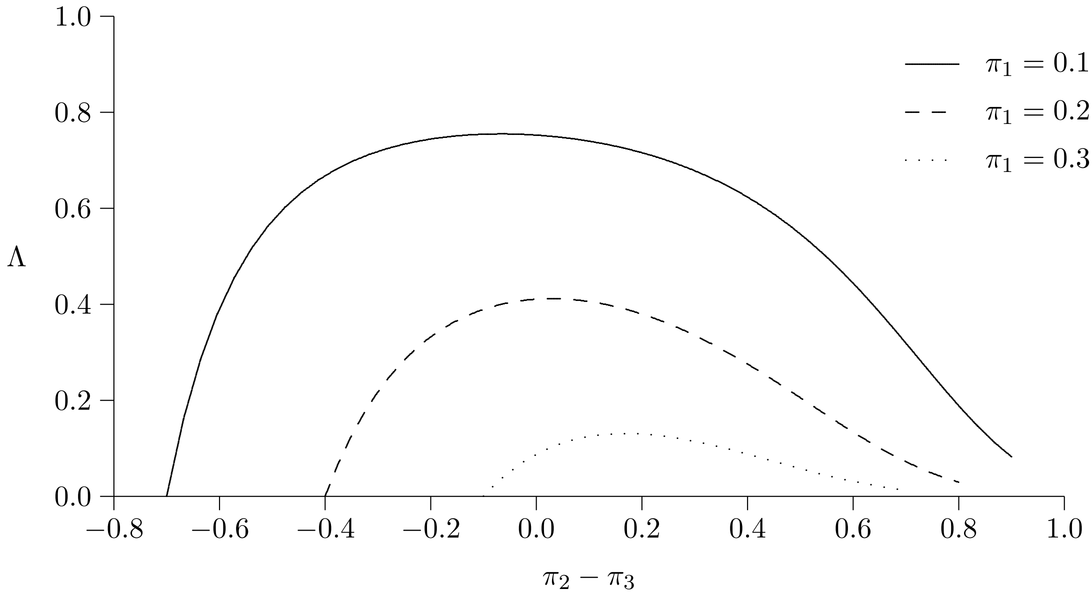
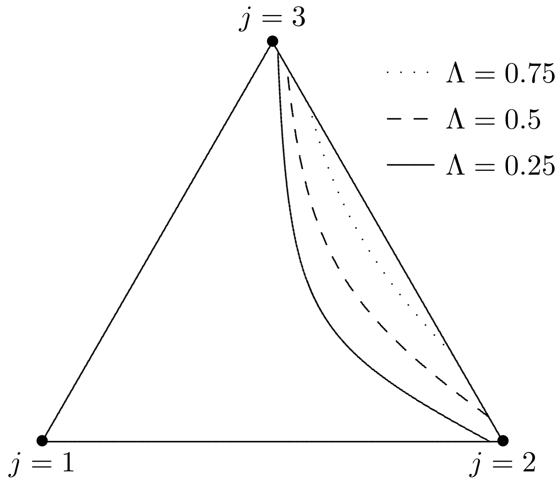
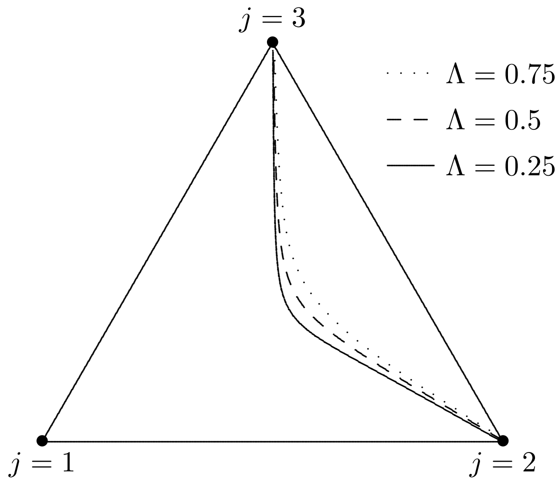


Figure 1. The effect of $\pi_2 - \pi_3$. Fixing π_1 at a variety of values, this chart shows the effect of $\pi_2 - \pi_3$ on the strength of the strategic incentive $\Lambda = (e^\lambda - 1)/e^\lambda$. This illustration fixes the precision level at $s = 10$.

Examining Figure 1, it is clear that a reduction in π_1, with support directed equally towards candidates 2 and 3, increases the strategic incentive. Note, however, that no clear message is available by varying $\pi_2 - \pi_3$. A possible hypothesis is that the strategic incentive is maximised when $\pi_2 = \pi_3$, corresponding to an expected tie between the disliked and second preference candidate. This is not the case.



(a) $s = 7$



(b) $s = 50$

FIGURE 2. Points of equal strategic incentive.

To some extent, the behaviour of the strategic incentive may be illustrated with reference to a simplex plot. In Figure 2 I illustrate lines of equal strategic incentive, for precision levels $s = 7$ and $s = 50$. It is clear once again that a loss in support for candidate 1 increases the strategic incentive. Moreover, Figure 2 shows the incentive increasing with s when $\pi_3 > \pi_1$, and decreasing with s when $\pi_3 < \pi_1$ — in Section 5.4 I pursue this issue further.

5.3. **Marginality and Distance from Contention.** Lemma 7 and Figure 1 shed some light on the issue of comparative statics. Informal ideas of strategic voting, however, often draw upon ideas of "marginality" and "distance from contention". These ideas are used when the preferred candidate trails in third place. The typical hypothesis is that the incentive to vote strategically will be higher in a "close" election or "marginal" constituency, where the winning margin provides a proxy for this measure. This is often coupled with the idea that the incentive increases with the "distance from contention" of the preferred candidate — usually the gap between the preferred candidate and second placed candidate. My aim in this section is to formalise these ideas, and examine their comparative static properties.

I begin by considering a configuration where the preferred candidate trails in third place, so either $\pi_2 > \pi_3 > \pi_1$ or $\pi_3 > \pi_2 > \pi_3$. An immediate candidate for the "distance from contention" of the preferred candidate is then $d = \min\{\pi_2, \pi_3\} - \pi_1$. An obvious candidate for the "winning margin" of the election is $|\pi_3 - \pi_2|$. This formulation, however, neglects to identify the expected winner. Hence the parameter employed here is $c = \pi_3 - \pi_2$, where $|c|$ is then the margin of victory.

Definition 3. *Define the* winning margin *and* distance from contention *by:*

$$\begin{aligned} \textit{Winning Margin} &= c = \pi_3 - \pi_2 \\ \textit{Distance from Contention} &= d = \begin{cases} \pi_2 - \pi_1 & \textit{if} \quad c > 0 \\ \pi_3 - \pi_1 & \textit{if} \quad c < 0 \end{cases} \end{aligned}$$

Using these definitions, and the requirement that $\pi_1 + \pi_2 + \pi_3 = 1$, I may define π in terms of c and d, and hence generate the strategic incentive. Solving the relevant equations linearly yields:

$$\pi_3 > \pi_2 \quad \Rightarrow \quad c > 0 \quad \Rightarrow \quad \pi = \frac{1}{3}\begin{bmatrix} 1 - 2d - c \\ 1 + d - c \\ 1 + d + 2c \end{bmatrix}$$

$$\pi_3 < \pi_2 \quad \Rightarrow \quad c < 0 \quad \Rightarrow \quad \pi = \frac{1}{3}\begin{bmatrix} 1 - 2d + c \\ 1 + d - 2c \\ 1 + d + c \end{bmatrix}$$

Notice that the behaviour of the strategic incentive with respect to the winning margin c changes at $c = 0$. Inspecting these solutions, and fixing the winning margin c, an increase in d lowers π_1 and simultaneously increases both π_2 and π_3. It follows unambiguously from Lemma 7 that λ_∞ rises. Comparative statics on c are less clear by inspection. Nevertheless, straightforward derivations allow me to establish the following.

Lemma 8. *Whenever $\pi_1 < \min\{\pi_2, \pi_3\}$, the strategic incentive λ_∞ is increasing in both the distance from contention and the size of the winning margin. Formally, λ_∞ is increasing in c when $c > 0$, and decreasing in c when $c < 0$.*

Proof. The behaviour with respect to d follows by inspection. Comparative statics with respect to c are straightforward but tedious, and hence relegated to Appendix 7.3. $\square$

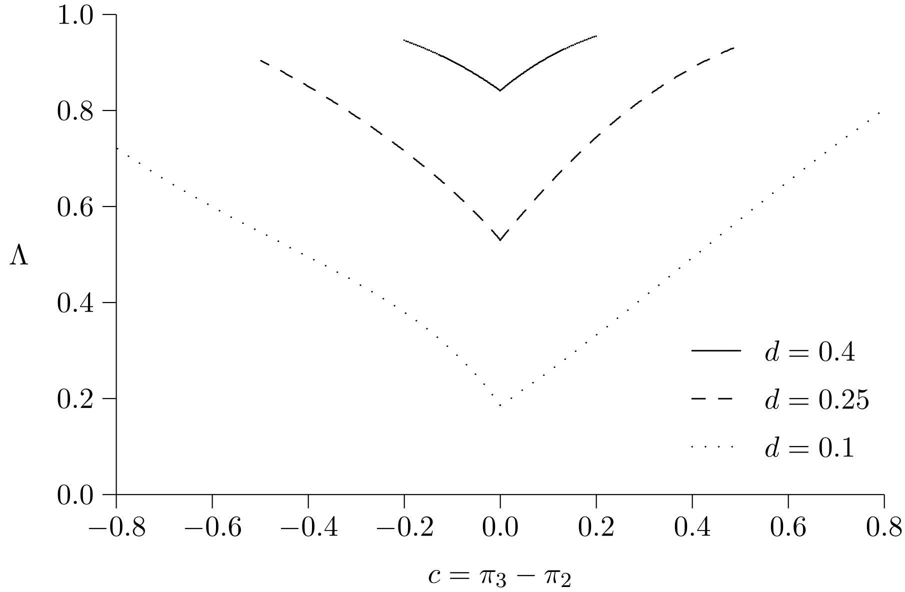
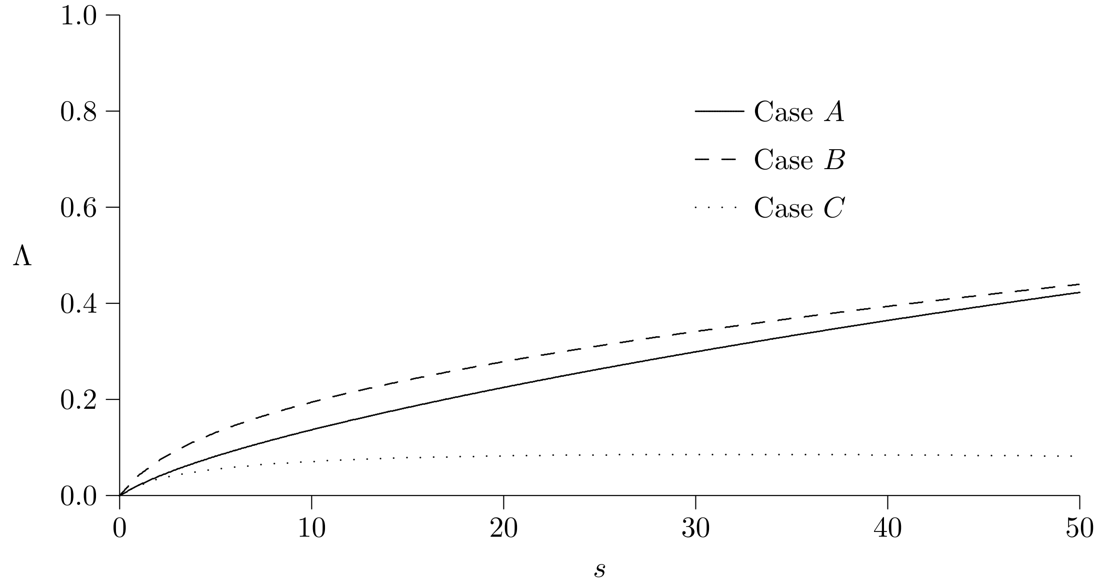


FIGURE 3. The effect of the winning margin. Fixing d, this chart shows the effect of c on the strategic intensity $\Lambda = (e^{\lambda_\infty} - 1)/e^{\lambda_\infty}$. This illustration fixes the precision level at $s = 10$.

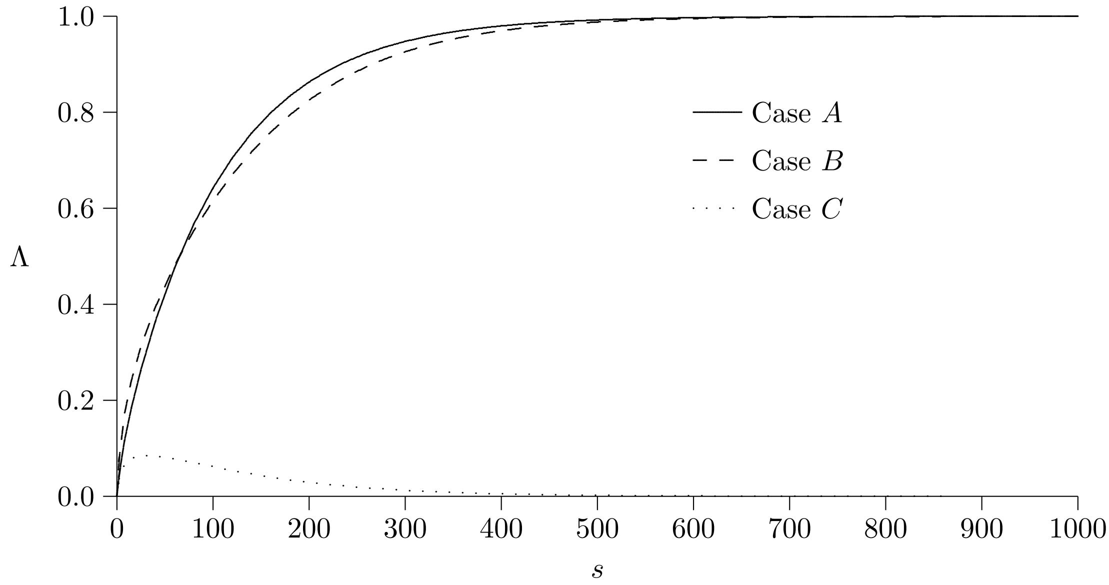
To illustrate the comparative static in the margin of victory c, I have plotted the strategic incentive against c for a variety of values of d in Figure 3. Note that my choice of c and d must ensure that all values of π_j remain positive, yielding the restriction $|c| < 1 - 2d$. It is clear that the strategic incentive increases with the (absolute) margin of victory — an opposite comparative static to that typically put forward in the informal literature. Moreover, notice that, due to the switch at $c = 0$, this effect is always strictly positive local to this value.

What generates this result? The informal argument is that in a marginal election — with a small margin of victory — there is a larger chance of influencing the outcome of the election. It is not the absolute probability of a tie that is relevant to a sophisticated voter, however. Rather, it is the relative probability of different ties. Fixing the distance from contention, a reduction in the margin of victory increases the likelihood of a 1-3 and 2-3 tie. Importantly, however, the probability of a 1-3 tie conditional on a tie occuring actually falls. Hence the strategic incentive falls.

5.4. **The Precision of Beliefs.** It remains for me to consider the impact of the precision of beliefs. The central innovation of my model is the introduction of constituency uncertainty. The degree of constituency uncertainty is parametrised by s. Indeed, setting $s = 0$ yields a uniform distribution over $\triangle$ and hence an absence of any strategic incentive. Moreover, allowing $s \to \infty$ corresponds to the absence of constituency uncertainty.



(a) $0 \leq s \leq 50$



(b) $0 \leq s \leq 1000$

Case	π_1	π_2	π_3
A	0.27	0.33	0.4
B	0.27	0.4	0.33
C	0.33	0.4	0.27

(c) Configurations

FIGURE 4. The effect of the precision of beliefs. These charts show the effect of s on the strategic incentive for the differing configurations of π.

In Figure 4 I illustrate the effect of s for a variety of configurations. It is clear that the strength of the strategic incentive increases with s whenever the preferred candidate is expected to trail in third place. Moreover, when the the preferred candidate is expected to hold second place, the strategic incentive dies away as s grows large. This is intuitive: For large s the strategic voter is increasingly sure of the election outcome, and hence the identity of the leading two candidates. She thus has an increased incentive to vote for her favourite from this pair. In fact, I am able to show the asymptotic behaviour of λ_∞ as $s \to \infty$.

Lemma 9. *If* $\pi_1 < \min\{\pi_2, \pi_3\}$ *then* $\lim_{s\uparrow\infty} \lambda_\infty = \infty$. *If* $\pi_3 < \pi_1 < \pi_2$ *then* $\lim_{s\uparrow\infty} \lambda_\infty = 0$.

Proof. See Appendix 7.3 □

From this Lemma, it follows that the results generated from models such as that of Cox (1984) are robust to some small deviations from the specifications. Indeed, allowing s to be large and yet finite yields strategic incentives that are arbitrarily close to those of a statistically independent framework if s is chosen sufficiently large. It follows that extant models do succeed in modelling strategic voting when a voter's knowledge of the constituency support levels of the candidates is very precise indeed.

6. From Decision Theory to Game Theory

The modelling framework provided here is designed to provide applied researchers with an explicit and tractable model of strategic voting incentives. It is also casts some doubt on the strict version of Duverger's Law. Indeed, when a strategic voter is uncertain of the constituency-wide support of candidates, then the incentive to vote strategically is limited, and hence multi-candidate support becomes possible.

My analysis here, however, is entirely decision-theoretic in nature. By this I mean that I consider the optimal behaviour of a voter conditional on her beliefs. Taking a game-theoretic perspective may restore complete strategic voting and a two-candidate outcome as equilibrium behaviour in a voting game.

Why is this? The standard logic is that strategic voting is self-reinforcing. Suppose that all strategic voters share the same beliefs over the constituency support of candidates, so that, for instance, $\pi_1 < \pi_2 < \pi_3$. The strategic incentive, although finite, remains positive and hence at least some supporters of candidate 1 will switch their vote to candidate 2. Informally, this serves to reduce π_1 and increase π_2. Using Lemma 7, it is clear that this results in an increase in the strategic incentive, and hence a further loss in support for candidate 1. This is a tale of positive feedback, yielding the "bandwagon effect" of Simon (1954). Indeed, it may result in the only stable equilibrium outcomes being one in which all support for a trailing candidate is eventually lost. This story mirrors that of Fey (1997), only with use of the new framework.

As I show in my companion paper Myatt (1999), however, this logic may well be flawed. First, note that the positive-feedback argument assumes that all voters share the same beliefs over the constituency support of candidates. This would follow from a scenario in which all voters commonly observe a public signal or public opinion poll of candidate support levels. In particular, this implies that all candidates know exactly which way a strategic swing is moving. Indeed, such a scenario does not allow for the possibility that individual voters may differ in their opinions of constituency support levels.

In the companion paper, I allow for this possibility. Voters privately observe signals of constituency support levels — equivalently, a private opinion poll of friends and colleagues from the constituency.[4] The strategic incentive varies across voters, and hence multi-directional strategic swings are possible. Perhaps surprisingly, the analysis shows that strategic voting is a self-attenuating phenomenon — it exhibits negative feedback. If a voter anticipates strategic switching by others, this *reduces* rather than increases her incentive to vote strategically. Why is this? Informally, when voters are privately informed and a strategic voter anticipates switching by others, she is uncertain of which way the swing is going. Although the formal analysis is involved, the intuition is that a voter is increasingly wary of switching her vote, due to the worry that she may switch in the wrong direction. The analysis in Myatt (1999) shows that, when voters are privately rather than publicly informed, strategic voting in a game-theoretic equilibrium is substantially less than that observed when strategic voters assume that others will vote sincerely.[5] Moreover, the incentive to vote strategically increases with the precision of information at rate $\sqrt{s}$. This contrasts with the present paper in which it increases at rate s. Importantly, however, when voters are publicly informed, the standard positive-feedback logic continues to operate.

Combining the insights of both this work and the companion paper, I offer the following predictions. When any information on the support of candidates is publicly observed by all voters, and this information is reasonably precise, then I expect extensive strategic switching and a near-Duvergerian outcome. When information is largely private in nature, perhaps generate for individuals' own observations from the constituency, then I expect only limited strategic voting. Moreover, I expect the likelihood of a strategic vote to move with the strategic incentive variable generated by the analysis of this paper. It remains for empirical workers to test these predictions.

[4]Myatt (1999) differs from the present paper in three respects. First, I take a game-theoretic viewpoint and solve for Bayesian Nash equilibria of the appropriate voting game. Second, I simplify the voting game somewhat by considering strategic switching between two candidates and fixing the behaviour of voters who support the third candidate. Finally, I enrich the information structure by allowing voters to sample the preferences of other constituents, rather than their voting intentions. A tractable model with explicit closed-form results obtains.

[5]The requirement for a stable equilibrium with multi-candidate support is that private information sources are relatively more important than public information sources.

7. Omitted Proofs

7.1. **Election Outcomes and Constituency Uncertainty.** Omissions from Section 3.

Proof of Lemma 1. Consider the function $\prod_{j=1}^{m} p_j^{\gamma_j}$. This is a strictly quasi-concave function over $\triangle$, with a unique and strict global maximum at $p = \gamma$. For arbitrarily small $\epsilon < \prod_{j=1}^{m} \gamma_j^{\gamma_j}$, define the following subset of $\triangle$:

$$\triangle_\epsilon = \left\{ p \in \triangle \mid \prod_{j=1}^{m} p_j^{\gamma_j} \geq \prod_{j=1}^{m} \gamma_j^{\gamma_j} - \epsilon \right\}$$

This is a compact and convex subset of $\triangle$. The first step is to construct a lower bound to the expression of interest. Begin by noting that:

$$\int_\triangle f(p) \left[\prod_{j=1}^{m} p_j^{\gamma_j} \right]^n dp \geq \underline{f}_\epsilon \int_{\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j} \right]^n dp \quad \text{where} \quad \underline{f}_\epsilon = \min_{p \in \triangle_\epsilon} f(p)$$

The minimum is well defined, following from the compactness of $\triangle_\epsilon$ and the continuity of $f(p)$. It follows that:

$$\begin{aligned} \frac{\int_\triangle f(p) \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp}{\int_\triangle f(\gamma) \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp} &\geq \frac{\underline{f}_\epsilon}{f(\gamma)} \frac{\int_{\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp}{\int_{\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp + \int_{\triangle / \triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp} \\ &= \frac{\underline{f}_\epsilon}{f(\gamma)} \frac{1}{1 + \left[\int_{\triangle/\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp\right] / \left[\int_{\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp\right]} \end{aligned}$$

The next step is to show that the ratio of integrals in the denominator of this expression tends to zero for large n. To see this:

$$\frac{\int_{\triangle/\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp}{\int_{\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp} \leq \frac{\int_{\triangle/\triangle_\epsilon} \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp}{\int_{\triangle_\epsilon} \left[\prod_{j=1}^{m} \gamma_j^{\gamma_j} - \epsilon\right]^n dp} = \frac{1}{\int_{\triangle_\epsilon} dp} \int_{\triangle/\triangle_\epsilon} \left[\frac{\prod_{j=1}^{m} p_j^{\gamma_j}}{\prod_{j=1}^{m} \gamma_j^{\gamma_j} - \epsilon} \right]^n dp$$

Of course, for $p \notin \triangle_\epsilon$ it must be that $\prod_{j=1}^{m} p_j^{\gamma_j} < \prod_{j=1}^{m} \gamma_j^{\gamma_j} - \epsilon$. Hence:

$$\int_{\triangle/\triangle_\epsilon} \left[\frac{\prod_{j=1}^{m} p_j^{\gamma_j}}{\prod_{j=1}^{m} \gamma_j^{\gamma_j} - \epsilon} \right]^n dp \to 0 \quad \text{as} \quad n \to \infty$$

It follows that:

$$\lim_{n\to\infty} \frac{\int_\triangle f(p) \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp}{\int_\triangle f(\gamma) \left[\prod_{j=1}^{m} p_j^{\gamma_j}\right]^n dp} \geq \frac{\underline{f}_\epsilon}{f(\gamma)}$$

Now, it is clear that $\gamma \in \triangle_\epsilon$. For sufficiently small ϵ, $\triangle_\epsilon$ is an arbitrarily small neighbourhood of γ, since γ is the unique and strict global maximiser of the continuous function $\prod_{j=1}^{m} p_j^{\gamma_j}$. Thus, for a sufficiently small choice of ϵ, $\underline{f}_\epsilon$ may be drawn arbitrarily close to $f(\gamma)$. This implies that:

$$\lim_{n\to\infty} \frac{\int_\triangle f(p)\left[\prod_{j=1}^m p_j^{\gamma_j}\right]^n dp}{\int_\triangle f(\gamma)\left[\prod_{j=1}^m p_j^{\gamma_j}\right]^n dp} \geq 1$$

This proof constructed a lower bound to the required expression. A symmetric procedure yields an upper bound with the same properties, and the result obtains. □

Proof of Corollary 1. From the compactness of $\triangle$, uniform convergence means that:

$$\int_\triangle f(p)\left[\prod_{j=1}^m p_j^{\gamma_j}\right]^n dp \underset{n\to\infty}{\approx} \int_\triangle f(\gamma)\left[\prod_{j=1}^m p_j^{\gamma_j}\right]^n dp$$

Defining $\gamma_j = x_j/n$, the result follows. □

Proof of Proposition 1. From an application of Corollary 1 to the expression for $\Pr[x]$:

$$\Pr[x] = \frac{\Gamma(n+1)}{\prod_{j=1}^m \Gamma(x_j+1)} \int_\triangle f(p) \prod_{j=1}^m p_j^{x_j}\, dp \underset{n\to\infty}{\approx} \frac{\Gamma(n+1)}{\prod_{j=1}^m \Gamma(x_j+1)} \int_\triangle f\left(\frac{x}{n}\right) \prod_{j=1}^m p_j^{x_j}\, dp$$

Multiplying and dividing by $\Gamma(n+m)$ obtain:

$$\Pr[x] \underset{n\to\infty}{=\approx} f\left(\frac{x}{n}\right) \frac{\Gamma(n+1)}{\Gamma(n+m)} \int_\triangle \frac{\Gamma(n+m)}{\prod_{j=1}^m \Gamma(x_j+1)} \prod_{j=1}^m p_j^{x_j}\, dp = f\left(\frac{x}{n}\right) \frac{\Gamma(n+1)}{\Gamma(n+m)}$$

The last step follows from recognition of the last integrand as a Dirichlet density. Both n and m are integers. Recalling that $\Gamma(n+1) = n!$ and $\Gamma(n+m) = (n+m-1)!$:

$$\frac{\Gamma(n+1)}{\Gamma(n+m)} = \frac{1}{\prod_{k=1}^{m-1}(n+k)} \underset{n\to\infty}{\approx} \frac{1}{n^{m-1}}$$

From this the result follows. □

7.2. **Voting Behaviour.** Omissions from Section 4.

Proof of Lemma 3. Focusing on the case of a two-way tie between candidates 1 and 2:

$$\Pr[x_1 = x_2 > x_3] = \sum_{\frac{n}{3} < i \leq \frac{n}{2}} \Pr[i, i, n-2i] \quad \text{where} \quad \Pr[i, i, n-2i] = \Pr\begin{bmatrix} x_1 = i \\ x_2 = i \\ x_3 = n-2i \end{bmatrix}$$

Multiply through by n^{m-2} to obtain:

$$n^{m-2}\Pr[x_1 = x_2 > x_3] = \frac{1}{n} \sum_{\frac{n}{3} < i \leq \frac{n}{2}} n^{m-1}\Pr[i, i, n-2i] \underset{n\to\infty}{\approx} \frac{1}{n} \sum_{\frac{n}{3} < i \leq \frac{n}{2}} f\left(\frac{i}{n}, \frac{i}{n}, \frac{n-2i}{n}\right)$$

Examining the sum on the right hand side, this has approximately $n/6$ elements. Multiplying and dividing by 6, this summation defines a Riemann integral over the set $[1/3, 1/2]$. Indeed:

$$n^{m-2}\Pr[x_1 = x_2 > x_3] \underset{n\to\infty}{\approx} \frac{1}{6}\frac{1}{n/6} \sum_{\frac{n}{3} < i \leq \frac{n}{2}} f\left(\frac{i}{n}, \frac{i}{n}, \frac{n-2i}{n}\right) \to \frac{1}{6}\int_{1/3}^{1/2} f(z, z, 1-2z)\, dz$$

which completes the proof. □

7.3. **Modelling Constituency Uncertainty.** Omissions from Section 5. It is useful to begin by considering the basic properties of the function $g(z, \pi)$ from Equation 6 of Lemma 5.

Lemma 10. *For $1/3 < z < 1/2$, the function $g(z, \pi)$ is strictly decreasing in π and satisfies $0 < g(z, \pi) < 1$. At the endpoints of the interval $g(1/3, \pi) = 1/3$ and $g(1/2, \pi) = 0$.*

Proof. To show that $g(z, \pi)$ is decreasing in π, write:

$$g(z,\pi) = z\left[\frac{1-2z}{z}\right]^{\pi} \quad \Rightarrow \quad \frac{\partial g(z,\pi)}{\partial \pi} = g(z,\pi)\log\left[\frac{1-2z}{z}\right] < 0 \quad \text{for} \quad \frac{1}{3} < z < \frac{1}{2}$$

The remaining properties hold by inspection. □

Lemma 11. *The expression* $\log\left[\int_{1/3}^{1/2} g(z,\pi)^s\, dz\right]$ *is convex in π.*

Proof. Examining the expression:

$$\frac{\partial \log\left[\int_{1/3}^{1/2} g(z,,\pi)^s\, dz\right]}{\partial \pi} = \frac{1}{\int_{1/3}^{1/2} g(z,\pi)^s\, dz}\int_{1/3}^{1/2} sg(y,\pi)^{s-1}\frac{\partial g(y,\pi)}{\partial \pi}\, dy$$

where I have used to different variables of integration for each separate integral in order to avoid condufusion. Turning to the function $g(y, \pi)$:

$$g(y,\pi) = y^{1-\pi}(1-2y)^{\pi} \quad \Rightarrow \quad \frac{\partial g(y,\pi)}{\partial \pi} = g(y,\pi)\log\left[\frac{1-2y}{y}\right]$$

which on substitution and multiplication through by -1 yields:

$$-\frac{\partial \log\left[\int_{1/3}^{1/2} g(z,\pi)^s\, dz\right]}{\partial \pi} = s\int_{1/3}^{1/2} \log\left[\frac{y}{1-2y}\right]\left[\frac{g(y,\pi)^s}{\int_{1/3}^{1/2} g(z,\pi)^s\, dz}\right] dy \tag{7}$$

The multiplication by -1 ensures that we are dealing with a positive expression, since $y > 1-2y$ on the required interval. I wish to show that that this expression is decreasing in π. Now, observe that $\log[y/1-2y]$ is strictly increasing in y. Also observe that:

$$\frac{g(y,\pi)^s}{\int_{1/3}^{1/2} g(z,\pi)^s\, dz} \tag{8}$$

is a well defined density function — it is positive valued, and integrates to 1 over the interval $[1/3, 1/2]$. It follows that the expression in Equation 7 is the expectation of an increasing function. It is thus sufficient to show that a reduction in π induces a first order stochastically dominant shift upwards in the density described by Equation 8. To demonstrate this, I must show that, for any y where $1/3 < y < 1/2$:

$$\pi_j < \pi_k \quad \Rightarrow \quad \frac{\int_{1/3}^{y} g(w,\pi_j)^s\, dw}{\int_{1/3}^{1/2} g(z,\pi_j)^s\, dz} \leq \frac{\int_{1/3}^{y} g(w,\pi_k)^s\, dw}{\int_{1/3}^{1/2} g(z,\pi_k)^s\, dz} \quad \Leftrightarrow \quad \frac{\int_{1/3}^{y} g(w,\pi_j)^s\, dw}{\int_{1/3}^{y} g(w,\pi_k)^s\, dw} \leq \frac{\int_{1/3}^{1/2} g(z,\pi_j)^s\, dz}{\int_{1/3}^{1/2} g(z,\pi_k)^s\, dz}$$

To demonstrate that this inequality holds it is sufficient to show that the left hand side term is maximised at $y = 1/2$. To show this, it is sufficient to show that its derivative is positive on the interval $1/3 < y < 1/2$. To show this requires:

$$\begin{aligned}\frac{\partial}{\partial y}\log\left[\frac{\int_{1/3}^{y} g(w,\pi_j)^s\,dw}{\int_{1/3}^{y} g(w,\pi_k)^s\,dw}\right] \geq 0 \qquad &\Leftrightarrow \quad \frac{g(y,\pi_j)^s}{\int_{1/3}^{y} g(w,\pi_j)^s\,dw} \geq \frac{g(y,\pi_k)^s}{\int_{1/3}^{y} g(w,\pi_k)^s\,dw} \\ &\Leftrightarrow \quad \frac{\int_{1/3}^{y} g(w,\pi_j)^s\,dw}{g(y,\pi_j)^s} \leq \frac{\int_{1/3}^{y} g(w,\pi_k)^s\,dw}{g(y,\pi_k)^s}\end{aligned}$$

Finally, to show this last inequality it is sufficient to show that, for all w and y satisfying $1/3 < w \leq y < 1/2$:

$$\frac{g(w,\pi_j)}{g(y,\pi_j)} \leq \frac{g(w,\pi_k)}{g(y,\pi_k)} \quad \Leftrightarrow \quad \left[\frac{y(1-2w)}{w(1-2y)}\right]^{\pi_j} \leq \left[\frac{y(1-2w)}{w(1-2y)}\right]^{\pi_k}$$

This last inequality holds, since $\pi_j < \pi_k$ and $y(1-2w) \geq w(1-2y)$ for $1/3 < w \leq y < 1/2$. Assembling calculations so far, I have demonstrated that the right hand side of Equation 7 is decreasing in π. It follows that:

$$\frac{\partial^2 \log\left[\int_{1/3}^{1/2} g(z,\pi)^s\,dz\right]}{\partial \pi^2} \geq 0$$

which is the required result. □

Lemma 12. *Define $g^*(\pi)$ as the maximum of $g(z,\pi)$ on $1/3 \leq \pi \leq 1/2$.*

$$\lim_{s\to\infty}\left\{\frac{\log\left[\int_{1/3}^{1/2} g_j(z)^s\,dz\right]}{s\log g^*(\pi)}\right\} = 1$$

Proof. Both numerator and denominator tend to $-\infty$. Apply l'Hôpital's rule to obtain:

$$\lim_{s\to\infty}\left\{\frac{\log\left[\int_{1/3}^{1/2} g_j(z)^s\,dz\right]}{s\log g^*(\pi)}\right\} = \frac{1}{\log g^*(\pi)} \times \lim_{s\to\infty}\left\{\int_{1/3}^{1/2}\log g(z,\pi)\left[\frac{g(z,\pi)^s}{\int_{1/3}^{1/2} g(y,\pi)^s\,dy}\right]dz\right\}$$

The second term in the integrand defines a density function. As $s \to \infty$, it focuses increasing weight around the maximum of $g(z,\pi)$, and hence the integral tends to $g^*(z)$. A full formal proof of this follows an identical structure to the proof of Lemma 1. □

Proof of Lemma 5. Recall that the asymptotic strategic incentive λ_∞ satisfies:

$$\lambda_\infty = \log\frac{2p_{12}+p_{23}}{2p_{12}+p_{13}}$$

From Proposition 3 and Equation 5:

$$p_{12} = \int_{1/3}^{1/2} f(z, z, 1-2z)\, dz \propto \int_{1/3}^{1/2} [z^{\pi_1} z^{\pi_2} (1-2z)^{\pi_3}]^s\, dz$$
$$= \int_{1/3}^{1/2} [z^{1-\pi_3}(1-2z)^{\pi_3}]^s\, dz = \int_{1/3}^{1/2} g(z, \pi_3)^s\, dz$$

Similar operations for p_{13} and p_{23} yield the first part of Lemma 5. To obtain the explicit expression, for $j \in \{1, 2, 3\}$, make the change of variable $w = 1 - 2z$ to obtain:

$$\begin{aligned}\int_{1/3}^{1/2} [z^{1-\pi_j}(1-2z)^{\pi_j}]^s\, dz &= \frac{1}{2^{(1-\pi_j)s+1}} \int_0^{1/3} [w^{\pi_j}(1-w)^{1-\pi_j}]^s\, dw \\ &= \frac{2^{\pi_j s} B_{1/3}(\pi_j s + 1, (1-\pi_j)s + 1)}{2^{s+1}}\end{aligned}$$

where the last equality follows from recognising the integral expression as the incomplete Beta function evaluated at $1/3$ with the specified parameters. Form the asymptotic strategic incentive λ_∞, and cancel appropriate terms to obtain:

$$\begin{aligned}\lambda_\infty &= \log \frac{2^{1+\pi_3 s} B_{1/3}(\pi_3 s + 1, (1-\pi_3)s + 1) + 2^{\pi_1 s} B_{1/3}(\pi_1 s + 1, (1-\pi_1)s + 1)}{2^{1+\pi_3 s} B_{1/3}(\pi_3 s + 1, (1-\pi_3)s + 1) + 2^{\pi_2 s} B_{1/3}(\pi_2 s + 1, (1-\pi_2)s + 1)} \\ &= \log \frac{B_{1/3}(\pi_3 s + 1, (1-\pi_3)s + 1) + 2^{(\pi_1-\pi_3)s-1} B_{1/3}(\pi_1 s + 1, (1-\pi_1)s + 1)}{B_{1/3}(\pi_3 s + 1, (1-\pi_3)s + 1) + 2^{(\pi_2-\pi_3)s-1} B_{1/3}(\pi_2 s + 1, (1-\pi_2)s + 1)}\end{aligned}$$

which is the desired expression. This completes the proof. □

Proof of Lemma 6. From the definition of λ_∞, it is clear that:

$$\lambda_\infty > 0 \quad \Leftrightarrow \quad \frac{\int_{1/3}^{1/2} [2g(z,\pi_3)^s + g(z,\pi_1)^s]\, dz}{\int_{1/3}^{1/2} [2g(z,\pi_3)^s + g(z,\pi_2)^s]\, dz} > 1 \quad \Leftrightarrow \quad \int_{1/3}^{1/2} g(z,\pi_1)^s\, dz > \int_{1/3}^{1/2} g(z,\pi_2)^s\, dz$$

From Lemma 10, if $\pi_2 > \pi_1$ then $g(z, \pi_1) > g(z, \pi_2)$ for z within the range of integration. Hence this condition holds. The reverse argument holds for $\pi_1 < \pi_2$, and $\lambda_\infty = 0$ when $\pi_1 = \pi_2$. □

Proof of Lemma 8. Consider first the case where $\pi_3 > \pi_2 > \pi_1$, so that $c > 0$. In terms of c and d, π satisfies:

$$\pi = \frac{1}{3} \begin{bmatrix} 1 - 2d - c \\ 1 + d - c \\ 1 + d + 2c \end{bmatrix} \quad \Rightarrow \quad \frac{\partial \pi_1}{\partial c} = -\frac{1}{3} \quad \frac{\partial \pi_2}{\partial c} = -\frac{1}{3} \quad \frac{\partial \pi_3}{\partial c} = \frac{2}{3} \tag{9}$$

Next consider the asymptotic strategic incentive λ_∞. Using Proposition 3:

$$\lambda_\infty = \log \frac{2p_{12} + p_{23}}{2p_{12} + p_{13}} = \log \frac{2(p_{12}/p_{13}) + (p_{23}/p_{13})}{2(p_{12}/p_{13}) + 1}$$

Since $\pi_2 > \pi_1$, the strategic incentive is positive. Equivalently, $p_{23} > p_{13}$. It follows that λ_∞ is decreasing in (p_{12}/p_{13}). Using the results of Lemma 5:

$$\frac{p_{12}}{p_{13}} = \frac{\int_{1/3}^{1/2} g(z,\pi_3)^s \, dz}{\int_{1/3}^{1/2} g(z,\pi_2)^s \, dz}$$

From Equation 9 above, an increase in c reduces π_2 and increases π_3. Using the results of Lemma 10, this increases $g(z,\pi_2)$ and reduces $g(z,\pi_3)$. Combining, it follows that (p_{12}/p_{13}) is decreasing in c. This means that, to demonstrate the claim of the Lemma, it is sufficient to show that (p_{23}/p_{13}) is increasing in c. To do this, differentiate to obtain:

$$\begin{aligned}\frac{\partial \log(p_{23}/p_{13})}{\partial c} &= \frac{\partial \log \int_{1/3}^{1/2} g(z,\pi_1)^s \, dz}{\partial \pi_1}\frac{\partial \pi_1}{\partial c} - \frac{\partial \log \int_{1/3}^{1/2} g(z,\pi_2)^s \, dz}{\partial \pi_2}\frac{\partial \pi_2}{\partial c} \\ &= \frac{1}{3}\left\{\frac{\partial \log \int_{1/3}^{1/2} g(z,\pi_2)^s \, dz}{\partial \pi_2} - \frac{\partial \log \int_{1/3}^{1/2} g(z,\pi_1)^s \, dz}{\partial \pi_1}\right\} > 0\end{aligned}$$

The last inequality follows from Lemma 11 and the fact that $\pi_2 > \pi_1$ by assumption. Combining, it follows that λ_∞ is increasing in c for $c > 0$. Turning to the case of $c < 0$, a similar proof yields the desired result. □

Proof of Lemma 9: Write λ_∞ as:

$$\lambda_\infty = \log \frac{2(p_{12}/p_{23}) + 1}{2(p_{12}/p_{23}) + (p_{13}/p_{23})} \tag{10}$$

Take, for example the ratio (p_{13}/p_{23}). Apply Lemma 12 to obtain:

$$\log \frac{p_{13}}{p_{23}} = \log \frac{\int_{1/3}^{1/2} g(z,\pi_2)^s \, dz}{\int_{1/3}^{1/2} g(z,\pi_1)^s \, dz} \longrightarrow s\left[\log g^*(\pi_2) - \log g^*(\pi_1)\right] \quad \text{as} \quad s \to \infty$$

When $\pi_1 < \min\{\pi_2, \pi_3\}$ it is straightforward to observe that $\pi_1 < 1/3$ and show that $g^*(\pi_1) > \max\{g^*(\pi_2), g^*(\pi_3)\}$. It follows that $(p_{13}/p_{23}) \to 0$ as $s \to \infty$. Similar operations on the remaining elements of Equation 10 demonstrate that $\lambda_\infty \to \infty$. A similar proof applies to the case of $\pi_3 < \pi_1 < \pi_2$. □

8. Numerical Calculations

In this (optional) appendix, I give a brief description of the issues involved in implementing the strategic incentive variable. Using the Dirichlet distribution and applying Lemma 5 the strategic incentive solves:

$$\lambda_\infty = \log \frac{B_{1/3}(\pi_3 s + 1, (1-\pi_3)s + 1) + 2^{(\pi_1 - \pi_3)s - 1} B_{1/3}(\pi_1 s + 1, (1-\pi_1)s + 1)}{B_{1/3}(\pi_3 s + 1, (1-\pi_3)s + 1) + 2^{(\pi_2 - \pi_3)s - 1} B_{1/3}(\pi_2 s + 1, (1-\pi_2)s + 1)}$$

where $B_{1/3}(a,b)$ is the incomplete Beta function evaluated at $1/3$, satisfying:

$$\begin{aligned} B_{1/3}(a,b) &= \int_0^{1/3} z^{a-1}(1-z)^{b-1}\,dz \\ &= \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \times \underbrace{\int_0^{1/3} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} z^{a-1}(1-z)^{b-1}\,dz}_{\text{Regularised Incomplete Beta Function}} \end{aligned}$$

Of course, the regularised incomplete Beta function is the cumulative distribution function of the Beta distribution. Excel implements this using the `BETADIST` worksheet function. Furthermore, the Gamma function is implemented via the `GAMMALN` function, satisfying $\texttt{GAMMALN}(\alpha) = \log\Gamma(\alpha)$. Hence, using Excel notation:

$$B_{1/3}(a,b) = \texttt{EXP}\left(\texttt{GAMMALN}(a) + \texttt{GAMMALN}(b) - \texttt{GAMMALN}(a+b)\right) * \texttt{BETADIST}(1/3, a, b)$$

Manipulating the Γ components assists with numerical accuracy when using Excel. An alternative to Excel is a full matrix language such as MATLAB. MATLAB once again implements the regularised incomplete function using the function `BETAINC`. In addition, MATLAB also defines the Beta function $\texttt{BETA}(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$. Hence:

$$B_{1/3}(a,b) = \texttt{BETA}(1/3, a, b). * \texttt{BETAINC}(1/3, a, b)$$

in MATLAB notation.

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