Quantitative Research

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Financial Markets

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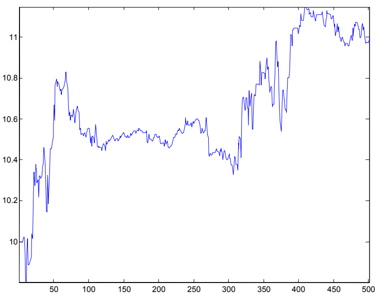

Figure 1: The Price time series generated by the GCMG.

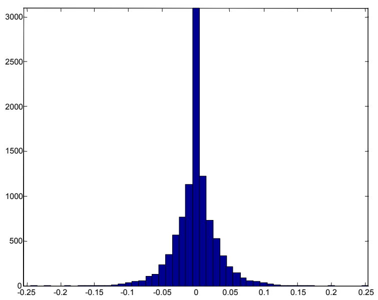

Figure 2: Histogram of price changes for a price time series generated by the GCMG.

These two graphs illustrate the output using a computed based simulation of the Grand Canonical Minority Game. Although the model is a deterministic model, the output superficially behaves in a way that is very similar to that of a stochastic process.

Linked Agent Models

A Microstructure Model of Equity Markets

In this paper, we employ an agent based game model to directly simulate the microstructure of the financial markets. We illustrate how one is able to capture some essential elements of trading behaviour with the simplest of such models.

Extending the model to incorporate resource-linked interactions, agent-linked interactions or news signals, we present the results of simulation establishing a stable method of coupling two market models where the volatility of each market is determined by the market participants' external observations.

Comparing the outputs with the historical data for the DAX index, we illustrate how well the extended model matches the empirical behaviour of the market.

Microscopic Models

It is common practice in financial markets to model an equity price as a log-normal Brownian motion process. By doing this, it is assumed that price changes are a series of samples taken from independent and identical probability distributions. By making these assumptions and disregarding evidence that might point to the contrary, we stand to lose information about the characteristics of equity prices. This may be of concern when developing process trading strategies.

The stochastic approach is of course successful for the study and implementation of option pricing models but here we will be modelling the microscopic behaviour of the underlying equity price using an agent model.

Prototype Agent Market Model

An elementary agent based model describing trader activity in a financial market must incorporate the following features, which describe the trading process and market price formation:

- A trader has to make a decision about the price of a stock, as a first approximation one can simplify this to a binary choice of buy or sell.
- Traders have available to them global information regarding the historical price of the stock that they are trading.
- Traders employ strategies to aid their buy/sell decision based on what they observe in the market.
- The resultant action of the group of traders determines the new price of the stock.
- Traders are rewarded for success and penalised for failure.
- Traders have the ability to learn from their successes and failures.

A variation of the Minority Game (MG) described by Challet and Zhang (1997) can incorporate all of the above features. In the Minority Game, agents compete to be in the minority group by making a decision about which of two groups they will join. One should view this choice as analogous to a buy/sell trading decision. The population of attendance of the minority group on each turn of the game can be used as a measure of demand for a stock and this pressure forces the quoted price to change on the next tick.

This game can be further extended to include additional trader characteristics such as a 'finite memory length' and a 'confidence to trade' parameter. Such a game is known

Resource Linkage

In this simulation of a parameter-linked game, one enforces a dependence on the resource level of Market 2 via the price changes of Market 1

$$\begin{aligned} &\text{if } |\Delta P_1| < 0 \quad && L_2(t+1) = L_2(t) + 1 \\ &\text{if } \Delta P_1 > 0 \quad && L_2(t+1) = L_2(t) - 1 \\ &\text{if } \Delta P_1 = 0 \quad && L_2(t+1) = L_2(t) \end{aligned}$$

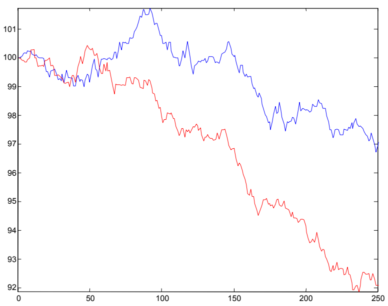

Figure 3: Prices for two markets linked by global resource level. (market 1 in blue, market 2 in red)

Confidence Linkage

An agent decides to play the game only if they are in possession of a strategy that has a virtual score greater than r, known as the confidence level.

One can model the impact of price moves in one market on the other market, by relating the confidence level of one market to the price changes of the other market.

$$\begin{aligned} &\text{if } |\Delta P_2| \le x \quad && r_1(t+1) = r_1(t) \\ &\text{if } \Delta P_2 > x \quad && r_1(t+1) = r_1(t) - \mu \\ &\text{if } \Delta P_2 < -x \quad && r_1(t+1) = r_1(t) + \mu \\ &\quad\text{where} && 3 \le r_1(t) \le 12 \\ &\quad \mu = && \text{confidence reset amount} \\ &\quad x = && \text{confidence trigger level} \end{aligned}$$

as the Grand Canonical Minority Game (GCMG) introduced by Johnson (2000). In this game, each agent has a basket of strategies each of which, together with a finite length string of historical price data, determines a possible buy/sell action for the agent. The agent chooses to follow the strategy which has demonstrated the best past performance and only follows it if this performance is above some threshold level, otherwise he or she does not trade on that time-step of the game.

The price time series generated by a GCMG for a particular set of parameters is shown in Figure 1 and the corresponding histogram of price changes is shown in Figure 2.

Parameter Linked Markets

The structures in the time series generated by the GCMG arise purely because of endogenous effects; agents compete amongst themselves for limited resources without influences from external sources. By modifying the nature of the game, through either its parameters or its players, we can include exogenous influences on the behaviour of our model (Douali 2003).

The MG is in fact a particular implementation of Arthur's El Farol Bar Game (1994). In this game, a parameter specifies the maximum population of the winning group, known as the resource level, L. In the MG this parameter has the value N/2, where N is the number of agents playing the game, hence the wining group, having attendance <N/2, is indeed the minority group. This leads us to a natural way of linking two games or market models; large magnitude price changes observed in Market 1 cause a change in the resource level, L_2, in Market 2 and similarly, large magnitude changes in Market 2 cause a change in the resource level, L_1, in Market 1. In this way, the two markets are dynamically linked as shown in Figure 3. It can be seen that there is a tendency for one market to track the other, similar perhaps to the debt & equity markets.

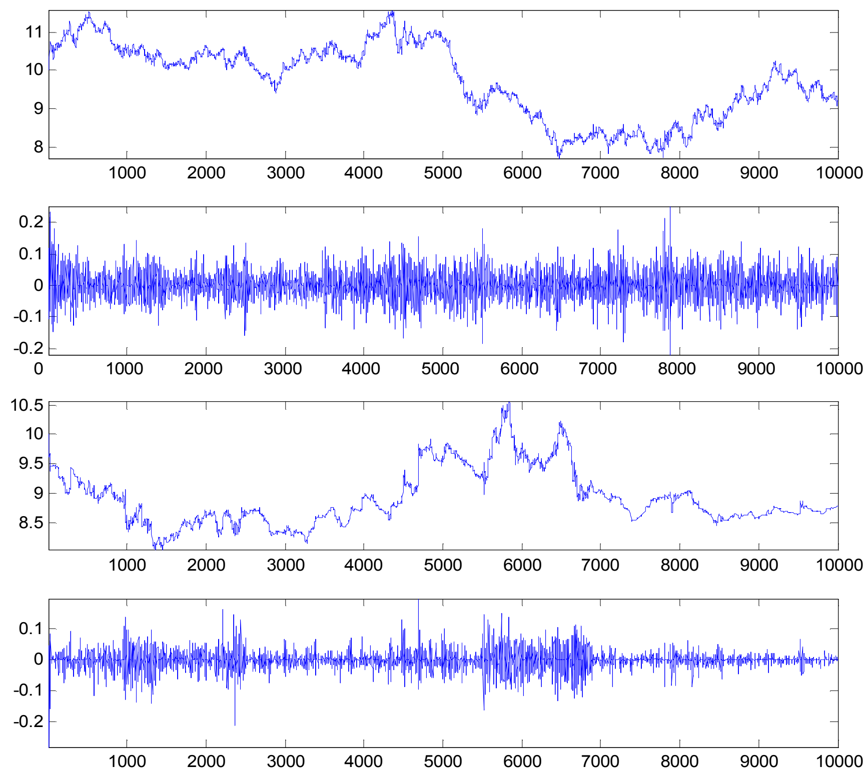

Figure 4: 'r'-linked model. Price and price changes for market 1 and market 2 respectively.

We now consider how one might incorporate the effect of a news signal on our market model. A news signal might be thought of as modulating confidence in the market. In the agent games considered here, this is achieved by making the confidence to trade parameter, r, time dependent. Figures 4, 5 & 6 show the typical results obtained by making r_2 a function of the price changes observed in Market 1 and r_1 a function of the price changes observed in Market 2. In the GCMG the volatility of the time series is highly dependent on r and hence we are making the local volatility in each market a function of the volatility of the other.

We have established a stable method of coupling two market models where the volatility of each market is determined by the market participants' external observations. The models discussed could be improved by investigating different linking functions. In practise, different markets react at different rates to news announcements; this could be related to the rate of change of these functions. Perhaps functions similar to the step functions used here may represent credit markets and smoother functions might represent equity markets.

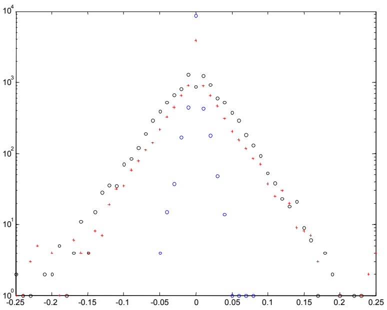

Figure 5: Histogram of price changes for the 'confidence threshold' or 'r' linked-game. (r = 3 in black, r = 12 in blue, r = r(t) in red)

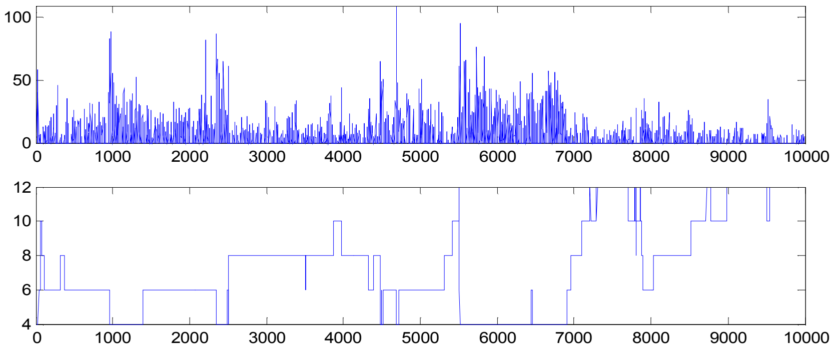

Figure 6: Trading volume and parameter r as a function of time in market 1 of the 'r' linked model

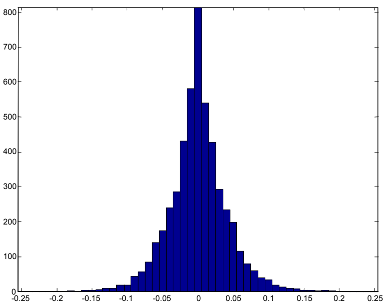

Figure 7: Histogram of price changes for agent linked markets.

Agent Linked Markets

We now introduce an agent linked market model. In this model, agents are active in each of two markets and so have a market impact. Here we attempt to model their cross-market activity.

- The two markets considered are Market 1 and Market 2, and there are three groups of agents, Group A, Group B and Group C.
- Agents in Group A play only in Market 1, Agents in Group C play only in Market 2 but the agents in Group B are free to decide which game they will play on each time step of the game.

Thus, Group A agents can be considered producers for Market 1, and Group C agents producers for Market 2. Group B agents are thus speculators with the choice to play one of two markets. Group B agents hold a single set of strategies but with each strategy are associated two virtual performance scores; one accumulated from performance in Market 1 and the second accumulated from performance in Market 2.

The highest virtual performance score determines both the strategy to play and the game in which to play it. So Group B agents have available to them information originating from two markets to aid their decisions while the other groups have information from just their own market.

Agents in Groups A, B and C play a GCMG, each with their own parameter set (the number in the group, the memory length, the number of strategies, the reset timescale and the virtual score), and each of the two markets evolves accordingly.

The results of this model are shown in Figures 7 - 11; there is particularly good agreement between the probability distribution generated by this market model and equity index markets such as the DAX shown in Figure 11.

Volatility Clustering

When simulating an Agent-Linked game we found the resulting price changes displayed volatility clustering, and the histograms of price changes in each game show evidence of the fatter tails seen in real financial time series.

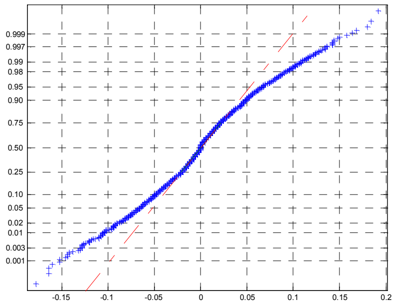

Figure 9: Normal probability plot for agent linked market model.

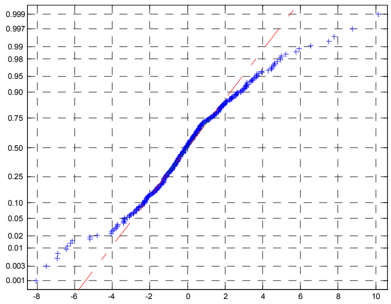

Figure 10: Normal probability plot for DAX.

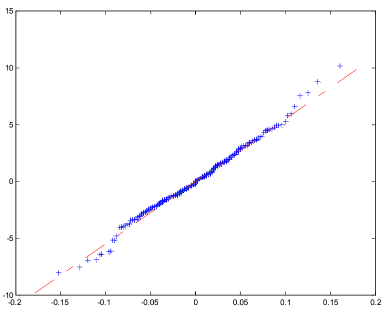

Figure 11: Q-Q plot of DAX against agent linked market model.

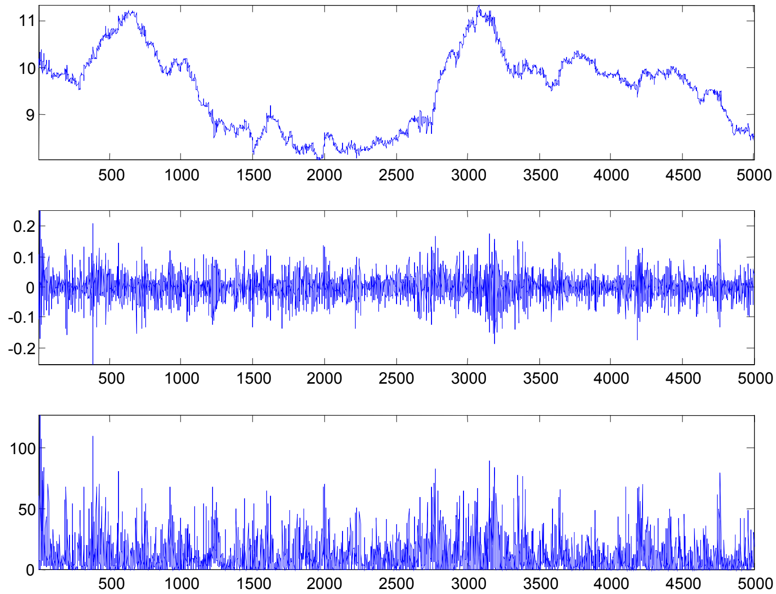

Figure 8: Time series generated by agent-linked market

Conclusion

We have shown that it is possible to take a very simple endogenous agent based game and modify it in such a way so that we can construct realistic market models subject to exogenous news and market influences. As with any microscopic model, the parameters are hidden and their determination may require complex inferences to be made from real market data. One possible area of research is to explore mixed populations of agents with different investment and trading strategies to the heterogeneous group that we have considered here and begin to develop trading models.

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