An investigation of clean surplus value-added pricing models using time series methods for the UK 1983-1996

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Abstract: in this paper a family of clean-surplus models are developed from standard accounting and financial identities. The models rely on the use of non-traditional performance measures of clean surplus in relation to value-added, and growth in value-added, in order to establish market value to value-added ratios. These measures are relevant both to business strategy and to industrial organisation. They provide an explicit and robust means to link strategy formulation to industrial context and valuation, avoiding problematic aspects of traditional economic-value-added (EVA) measures. The time-series behaviour of the ratio of residual surpluses to value-added is modelled as simple ARIMA (1, 0, 0), (0, 0, 1), (0, 1, 1) and (1, 1 0) processes resulting in four families of valuation model. Using data on publicly quoted British companies available from Datastream to test the models, evidence is provided to support the value-relevance of the performance measures. The models suffer from problems of negative value predictions and excess sensitivity. Adjustment of the empirical data to mitigate these effects yields statistically significant results for three of the four specific models developed, suggesting that further testing of the models on other data sets is warranted.

1. Introduction

The objective of the research[1] which lies behind this paper is to find a series of robust performance measures generally applicable to business which are well captured by accounting systems, that are relevant to strategy formulation and which result in valuation models for companies which are empirically well grounded. The origins of this work lie in the consensus shared by senior managers, consultants and financial officers that existing cash-flow valuation methods are flawed and open to manipulation.[2] Instead of using the long time horizons and somewhat arbitrary discount rates that characterise existing cash-flow methods, it is possible to imagine valuing a business or a strategy by assessing the performance of the associated business over a credible management horizon using performance measures which have a solid foundation in microeconomics and competitive strategy, and for which we have empirically derived valuation scales. Such an approach, as will be shown in this paper, may be made entirely consistent with traditional financial theory. The merit of such an approach would be that in its application it would be less susceptible to assumptions of rate and horizon.

The second section of the paper develops certain value-added performance measures and relates them to strategy and industrial economics. In Section 3, explicit formulations of four families of value-added pricing models are developed, each family corresponding to a simple auto-regressive or moving-average process on either levels or differences for residual surpluses scaled by value-added. The data set used for testing the models is described in Section 4. Companies for which data are available are allocated to one of the four model types using time-series analysis of residual scaled surpluses performed on SPSS. Regression results relating to the relevance of the chosen performance measures to valuation and the performance of the specific models are reviewed and discussed. Section 5 summarises the research to date and suggest avenues for further development.

2. Choice of Performance Measures

Value-added, defined as firm revenues minus the cost of raw materials and purchases[3], is an important factor in the determination of competitive success. The structure of an industry and how it evolves can be well captured by the analysis of the distribution of value-added between different industry participants and how it shifts over time. Similarly, within what strategy

[1] I am grateful to Professor Colin Mayer for his support and guidance in carrying out this research, and to the Master and Fellows of Balliol College for funding it.

[2] For an optimistic modern account of cash-flow methods see Copeland, Koller and Murrin (1995).

[3] Clearly there are questions of the value-added boundary of a firm. For instance, should factory electricity costs be included in value-added? The author believes these questions might settled based upon considerations of "returnability". Where input factors might immediately be returned without price erosion, the factors may be held to be a purchase. Those inputs which are not returnable, or which suffer price erosion, entail a degree of specificity to the firm in question which warrants their inclusion in the value-added structure. Under normal circumstances, imported factory electricity cannot normally be re-exported and hence would be included in value-added.

consultants call a *strategic segment* of an industry[4], or what academics call *mobility groups*, much competitor activity can be considered to be a struggle to control and safeguard profitable value-added through strategies based upon relative cost position, superior price realisation through differentiation, or through technological advantage.[5] Within the firm, value-added corresponds to the resource base which managers control and which they use to implement strategies.[6] Two key imperatives for competitive success are to grow the value-added of the firm, and to achieve a satisfactory level of return on that value-added. We represent the growth of value-added for a firm by g_{VA}, and return on value-added by *ROVA*.

Economists will be familiar with the concept of value-added which they often term *net output*. Many studies in industrial organisation have examined the relationship between profitability and net output in different structural contexts.[7] A measure often chosen for this research is the Price-Cost-Margin or *PCM*, which is defined as

$$\frac{\text{Net output - employee compensation}}{\text{Net output}}$$

If employee compensation represents a large majority of value-added costs, then the numerator in the above expression will be approximately equal to profit and

$$ROVA \approx PCM$$

A focus on average levels of *ROVA* within an industry is also consistent with the Structure-Conduct-Performance model elucidated by Bain (1959) and others.

Furthermore, growth and profitability in relation to value-added accommodate two approaches to business strategy which, while complementary, are often considered to represent opposing views: the resource-based view of the firm as developed by Prahalad and Hamel (1990), and portfolio-based strategy popularised by General Electric, McKinsey and the Boston Consulting Group. The performance of a firm depends both upon the structural context in which all competing firms find themselves, and the firm's ability to establish a competitive advantage relative to its competitiors. The context will determine the magnitude of value-added over which firms compete, the growth of that value-added, and typical levels of profit that may be sustained in relation to that value-added. Competitive advantage will determine the profitability and development of value-added for the individual firm relative to its competitors.

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[4] See Grant, R.M., (1992), *Contemporary Strategy Analysis: Concepts, Techniques, Applications*, Oxford: Blackwell.

[5] For instance see Porter (1980,1985)

[6] This is a simplification insofar as we overlook the need to achieve a competitive level of raw material and purchase prices through effective purchasing.

[7] For example: Fairburn J., Geroski P., (1993) The Empirical Analysis of Market Structure and Performance in Kay J., Bishop M., *European Mergers and Merger Policy*, Oxford: OUP.

In other words, the level of *ROVA* achieved by a firm will depend upon how well it uses it resources relative to competitors (the resource-based view), as well as the attractiveness of the segments it operates in (the portfolio-based view), and the structure of the industry surrounding these segments (Porter Five Forces). Similarly, the ability to generate superior returns in a segment, and the attractiveness of segments for new business development will be strongly influenced by the growth of value-added.

ROVA allows interesting and meaningful comparisons between businesses which have markedly different capital requirements: contrast for example a hotel business, a contract catering business and a restaurant business. Traditional measures of performance such as return on capital employed or return on sales do not produce meaningful comparisons. [8]

While the resource-oriented nature of *ROVA* may meet with approval, it will perhaps be objected that we have failed to take account of the need to provide adequate returns to capital. This is traditionally done by deducting a charge for book capital from earnings to yield residual earnings, more popularly known as Economic Value Added (*EVA*):

$$x_t^a = x_t - (R-1) \; y_{t-1}$$

where x_t^a are residual earnings in period *t*, x_t are earnings in period *t* , *R* is 1 plus the cost of capital *r*, and y_{t-1} is the closing book value of the previous period. It can then be shown (Peasnell 1982) that

$$P_t = y_t + \sum_{\tau=1}^{\infty} E_t\left[x_{t+\tau}^a \; R^{-\tau}\right]$$

where P_t is the market value of a company at time *t*. $E_t[Q]$ represents the expected value at time *t* of variable Q. (In general going forward $E_t[\;]$ will not be specified for the sake of clarity, unless expressly required). This expression states that the value of the company is equal to its book value plus the sum of discounted future residual earnings. The second sum on the right-hand side may also be considered to be unrecorded goodwill.

O'Hanlon (1996) has modified this approach by developing valuation models that incorporate residual returns scaled by the book capital in the company. He

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8 *EVA* fares no better, because (i) the hotel business is in fact a combination of an investment business and an accommodation renting business, (the former warrants a capital charge derived from the market value of the hotel property, whereas the latter does not); (ii) the capital employed in contract catering is often negative because payables exceed the combined value of debtors, stocks and fixed assets. As we shall see later, separation of the operational and funding aspects of a business, allows us to develop a comparative measure which reflects the efficiency with which businesses use resources, capturing in a much sounder way the opportunity costs associated with the consumption of resources by particular businesses.

introduces the Rate of Residual Income (*RRI*), denoted by χ^a, which is defined as

$$\chi_t^a = \frac{x_t^a}{y_{t-1}} = \frac{x_t - (R-1)y_{t-1}}{y_{t-1}} = A_t - (R-1)$$

where A_t is the accounting rate of return on capital (*ARR*).

The difficulty with this approach is that it conflates questions of economic efficiency with questions of funding. Criticism along these lines has been voiced by Kwong, Munro and Peasnell (1994). Part of the capital y_t is required to fund working capital because of the operating cycle of the business. This need for capital has nothing to do with the efficiency of the use of resources by the firm in competitive markets.[9]

Consider the case of a business which is newly established and where invoices are settled instantaneously, where all equipment is efficiently rented at a cost equal to the economic rate of depreciation of the assets involved, and where all profits are immediately paid over to the owners. In these circumstances, the question of the economic use of resources by the business still arises, but the book value of the company is zero. Economic value-added and residual earnings are equal to earnings, but it is not possible to assess whether the use of resources by the business amounts to an opportunity (utility) gain or loss.

A similar situation arises if we contemplate an extremely lengthy accounting period for a business where what are customarily the unexpired costs of capital assets are treated as period expenses. Again, starting and ending book values would be zero. *EVA* assessments of performance would appear to be subject to accounting conventions with regard to periods, and strongly influenced by the operating cycles of the business.

As an alternative, we may separate conceptually the operational funding of a business from the contribution to the value of the firm that arises from the efficient or inefficient use of resources. This separation is similar to the separation of tax and financing effects from an all-equity valuation that arises in Adjusted Present Value approaches to discounted cash flow.[10] Let us set aside the question of the funding of the operational cycle of the business: any capital which exists and is recorded on the balance sheet should be regarded, under this approach, as equivalent to cash or marketable investments, which do not feature in the valuation of the business as a going economic concern.[11]

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[9]A specific difficulty for *EVA* as traditionally formulated, is that a capital charge is made against fixed operating capital and inventories, with no account taken of the other elements of working capital.
[10] Brearley and Myers (1981) or Luehrman (1997)
[11] Assets other than cash should be valued by discounting their associated flows at a risk-adjusted rate *r*. If the rate of depreciation is equal to the economic rate of depreciation no bargain or loss will occur in relation to book asset values.

In other words, we may consider the value of the firm to be comprised of two elements: an investment component and an operational component. The investment component not only includes cash and marketable securities, but also working capital viewed as a largely involuntary or passive investment in the company. It would also include any holding gain expected to arise from the retention of physical assets in excess of the purchase price of the asset. A risk-adjusted rate of return would be required on the investment component.

The operational component of valuation would be determined by the level and development of returns on value-added, and would be independent of the book values of physical assets employed, once account had been taken of any expected holding gains. The value of the physical assets deployed would be entirely captured in the future value-added returns in the business. No charge for book capital would be made in the evaluation of the operational component, but the question would remain as to how to gauge whether the returns quantified in the operational component are adequate to satisfy investors. This will be considered in due course. Note that in drawing a distinction between investment and operational components of value, the intention is not to diminish the practical importance of tight control of inventories and working capital, but to focus upon the microeconomic linkages that support the value-added performance measures which have been introduced.

One immediate corollary of this approach is that, if assets are depreciated according to their true economic returns and these returns are capitalised in the balance sheet i.e. the assets are efficiently priced, the value of the firm will be independent of starting book value, putting the realities of working capital and the payments cycle aside. The value of the firm would be entirely captured by the flows which occur as a result of its operations, and would not include stock variables.

Consider the case of a new firm operating on an instantaneous payment cycle which has purchased or leased assets for its business on an efficient basis. Assume instantaneous full pay-out of dividends Since there was no prior period operation, the value of the P_t is given by:

$$
\begin{aligned}
P_t &= \sum_{\tau=1}^{\infty} x_{t+\tau} R^{-\tau} \\
&= \sum_{\tau=1}^{\infty} ROVA_{t+\tau}.\upsilon_{t+\tau} R^{-\tau}
\end{aligned}
$$

where $ROVA_{t+\tau}$ and $\upsilon_{t+\tau}$ are respectively the return on value-added and value-added in period $t+\tau$. If value-added grows at a compound rate g, P_t is equal to:

$$P_t = \sum_{\tau=1}^{\infty} ROVA_{t+\tau}.\upsilon_{t+1} R^{-\tau} (1+g)^{\tau-1}$$

$$= \sum_{\tau=1}^{\infty} (ROVA_{t+\tau} - r).\upsilon_{t+1} R^{-\tau} (1+g)^{\tau-1} + \sum_{\tau=1}^{\infty} r.\upsilon_{t+1} R^{-\tau} (1+g)^{\tau-1}$$

$$= \upsilon_{t+1} \sum_{\tau=1}^{\infty} \gamma^{-\tau} (ROVA_{t+\tau} - r) + \frac{r\,\upsilon_{t+1}}{r-g}$$

where $\gamma^{-\tau} = \frac{(1+g)^{\tau-1}}{R^{\tau}}$.

The second term on the right-hand side is equal to the capitalised normal returns expected on the value-added flows of the company. In an ideal accounting system these flows, which result from contracts with employees, customers and suppliers, would be recorded as assets and liabilities in the balance sheet, and their sum would represent the book value of the firm and equal the replacement cost of the firm's resources. These assets and liabilities are distinct from the investments historically made to fund the company which have already been discussed. The assets and liabilities recorded are the yet-to-be-incurred costs and yet-to-be-recovered revenues of the firm which together give rise to the stream of normal profits arising on the value-added $\upsilon_{\tau+1}$ which grows at rate *g*. If we divide the left-hand side of the above expression by the book value of the firm, we obtain:

$$Q = 1 + \frac{r-g}{r} \sum_{\tau=1}^{\infty} \gamma^{-\tau} (ROVA_{\tau} - r)$$

In the case of $g = 0$, this simplifies to:

$$Q = 1 + \sum_{\tau=1}^{\infty} R^{-\tau} (ROVA_{\tau} - r)$$

This equation states that for the idealised firm, the ratio of the market to book value of the firm is given by one plus the sum of the discounted marginal revenue products of the firm i.e. Tobin's Q. The magnitude of Q is determined by $ROVA_{\tau} - r$ and g making explicit the importance of excess returns on value-added and the growth in value-added in the creation of shareholder wealth through competitive advantage.

It may be objected that the importance of residual returns on value-added $ROVA_{\tau} - r$ has been overstated since we may create pseudo-residual returns for other measures of profitability, which are not of strategic relevance to valuation. Consider for instance returns on sales, ROS_τ, where σ_τ represents the corresponding level of sales and g' is the compound growth in sales:[12]

$$
\begin{aligned}
P_t &= \sum_{\tau=1}^{\infty} x_{t+\tau} R^{-\tau} \\
&= \sum_{\tau=1}^{\infty} ROS_{t+\tau}.\sigma_{t+\tau} R^{-\tau} \\
&= \sum_{\tau=1}^{\infty} ROS_{t+\tau}.\sigma_{t+1} R^{-\tau} (1+g')^{\tau-1} \\
&= \sum_{\tau=1}^{\infty} (ROS_{t+\tau} - r).\sigma_{t+1} R^{-\tau} (1+g')^{\tau-1} + \sum_{\tau=1}^{\infty} r.\sigma_{t+1} R^{-\tau} (1+g')^{\tau-1} \\
&= \sigma_{t+1} \sum_{\tau=1}^{\infty} \gamma^{-\tau} (ROS_{t+\tau} - r) + \frac{r\,\sigma_{t+1}}{r-g'}
\end{aligned}
$$

Thus the value of the firm is equal to the discounted sum of residual returns on sales plus a capitalised normal return on sales.

This result does not undermine the significance of the corresponding equations for $ROVA$. The key difference is that the value-added in each period equals the net cash flow of the firm upon which the required rate of return *r* must be earned.[13] This is not the case with the ROS returns, which do not equate to the stream of cash flows of the firm. One may visualise this situation by considering the history of the firm to comprise a series of one period share offerings and liquidations. The amount of investment required in each period is equal to the value-added of that period, assuming materials are billed direct to customers; shareholders require a return of *r* during the period.[14] Any returns on the resources used up by the firm in the period beyond this rate create unexpected additional wealth for shareholders.

Use of residual $ROVA$ returns is not inconsistent with the fundamental notions that support EVA, and may be held to be a logical improvement. In EVA, a charge is made against the book capital of the business, and we obtain the familiar:

$$P_t = y_t + \sum_{\tau=1}^{\infty} E_t \left[\left(x_{t+\tau} - (R-1)\, y_{t+\tau-1} \right) R^{-\tau} \right]$$

[12] We are not assuming here that materials and purchases are billed direct to the customer i.e. sales are not equal to value-added.
[13] Assuming economic depreciation of assets.
[14] Any holding gains or timing differences from using depreciation schedules different from economic depreciation are assumed to be captured in the investment component of valuation discussed above.

It is later shown that if this equation is modified to accommodate *ROVA* measures of profitability, we obtain:

$$P_t \quad = \quad \upsilon_{t+1} \sum_{\tau=1}^{\infty} \gamma^{-\tau} \left(ROVA_{t+\tau} - \quad r \right) \quad + \quad \frac{r\,\upsilon_{t+1}}{r-g} \quad - \quad \frac{g_B}{r-g_B} y_t$$

where g_B is the growth in book value y_t which reflects the historic funding of the firm, not the replacement cost of resources contracted by the firm. This equation is similar to the equation derived for the value of a firm which does not require capital, and where assets are efficiently priced, but an extra term is introduced which represents the capitalised stream of additional investment absorbed by the business to fund assets and working capital. Hence

Value	=	abnormal returns on resources i.e. value-added	+	normal returns on resources	-	additional capital to support application of resources

Note, in particular, that if there is no growth in book value, the value of the firm is independent of the value of starting capital y_t as conjectured. The associated Q ratio is given by:

$$Q' \quad = \quad 1 \quad + \quad \frac{r-g}{r} \quad \left(\sum_{\tau=1}^{\infty} \gamma^{-\tau} \left(ROVA_{\tau} - r \right) \quad - \quad \frac{g_B}{r-g_B} \alpha \right)$$

where α is the ratio of book value y_t to initial value-added υ_1.

In order to complete the account of the relationship between *EVA* and *ROVA*, a reconciliation of these two measures for an idealised company is required. If the replacement value of the resources to be consumed by the company is efficiently priced and recorded on the balance sheet, then the opening asset value y_t (equal to the initial book value of the company) will be $r\upsilon_1 / (r-g)$ as detailed above. It remains to show that for this company $Q' = Q$, i.e. that $g_B = 0$. Let us assume initially that the company pays out dividends equal to ry_t, consistent with *EVA* methods. Let us consider the development of the book value of the firm B_{t+1}, which we know was initially equal to the replacement value of the resources contracted i.e. $B_t = y_t$. From accounting identities we know

$$B_{t+1} \quad = \quad B_t \quad + \quad \upsilon_1 ROVA_1 \quad - \quad \left(y_t - y_{t+1} \right) \quad - \quad rB_t$$

If we substitute known identities and assume assets follow economic depreciation, we obtain:

$$B_{t+1} \quad = \quad \frac{r\upsilon_1}{r-g} + \quad \upsilon_1 ROVA_1 \quad - \quad \left(\frac{r\upsilon_1}{r-g} - \frac{r\upsilon_2}{r-g} \right) \quad - \quad r.\frac{r\upsilon_1}{r-g}$$

Substituting $\upsilon_2 = (1+g)\,\upsilon_1$ yields

$$B_{t+1} \quad = \quad \frac{r\upsilon_1}{r-g} \quad + \quad \upsilon_1 ROVA_1 \quad + \quad g.\frac{r\upsilon_1}{r-g} \quad - \quad r.\frac{r\upsilon_1}{r-g}$$

$$= \quad \frac{r\upsilon_1}{r-g} \quad + \quad \upsilon_1 (ROVA_1 - r)$$

$$= \quad B_t \quad + \quad \upsilon_1 (ROVA_1 - r)$$

Thus book value will increase by the level of abnormal returns in the period if these abnormal returns are retained. The assumption of retention is not consistent, however, with the assumptions used to determine the replacement value of the resources contracted. In attributing a replacement value of $r\upsilon_1 / (r-g)$, it was assumed that the replacement value was equal to the discounted marginal products of the growing stream of value-added of the firm. If abnormal earnings are retained, additional return will be earned on the retentions and the replacement values for future periods ($r\upsilon_2 / (r-g)$etc.) will require adjustment.[15] The replacement values adopted reflect full pay-out of abnormal returns, in which case $B_{t+1} = B_t$ and $g_B = 0$.

For an idealised firm, the book value of equity will be constant if dividends are paid equal to a normal return upon the replacement value of resources contracted to the firm plus any abnormal returns earned on those resources. Under these conditions $Q' = Q$ and the use of *ROVA* as a value-creating measure of performance is entirely reconciled with the customary *EVA* approach.

3. Time-series ARIMA Processes

Having provided a theoretical grounding for the use of *ROVA* measures, we may turn to the modelling of residual *ROVA* returns using time series analysis to investigate the response of these returns to transitory and permanent shocks.

Define *residual earnings on value-added,* f_t^a, *residual returns on value-added,* ϕ_t^a, and *returns on value-added,* $ROVA_t$, as follows:

$$f_t^a \quad = \quad x_t \quad - \quad (R-1)\,\upsilon_t$$

[15] This is discussed in Ohlson (1995) p.673.

$$\phi_t^a \quad = \quad \frac{f_t^a}{\upsilon_t} \quad = \quad \frac{x_t}{\upsilon_t} - \quad (R-1) \quad = \quad ROVA_t - r$$

$$ROVA_t \quad = \quad \frac{x_t}{\upsilon_t}$$

Then we may substitute residual earnings on value-added for customary residual earnings[16]

$$x_t^a \quad = \quad x_t \quad - \quad (R-1)\, y_{t-1}$$

$$= \quad f_t^a + \quad (R-1)(\upsilon_t - y_{t-1})$$

and apply this expression to the standard formula for firm value to obtain:

$$P_t \quad = \quad y_t \quad + \quad \sum_{\tau=1}^{\infty} f_{t+\tau}^a R^{-\tau} \quad + \quad (R-1)\sum_{\tau=1}^{\infty} \upsilon_{t+\tau} R^{-\tau} \quad - \quad (R-1)\sum_{\tau=1}^{\infty} y_{t+\tau-1} R^{-\tau}$$

If value-added grows at a compound rate of g and book value at g_B, then

$$P_t \quad = \quad y_t \quad + \quad \sum_{\tau=1}^{\infty} f_{t+\tau}^a R^{-\tau} + (R-1)\left[\frac{\upsilon_{t+1}}{R} + \frac{(1+g)\upsilon_{t+1}}{R^2} + \frac{(1+g)^2\upsilon_{t+1}}{R^3} + \dots\right]$$
$$- \quad (R-1)\left[\frac{y_t}{R} + \frac{(1+g_B)y_t}{R^2} + \frac{(1+g_B)^2 y_t}{R^3} + \dots\right]$$

Summing the series yields:

$$P_t \quad = \quad y_t \quad + \quad \sum_{\tau=1}^{\infty} f_{t+\tau}^a R^{-\tau} + \frac{(R-1)\,\upsilon_{t+1}}{(r-g)} - \frac{(R-1)\,y_t}{(r-g_B)}$$

Substituting ϕ_t^a for f_t^a yields

$$P_t \quad = \quad y_t\left(\frac{-g_B}{r-g_B}\right) \quad + \quad \upsilon_{t+1}\left(\frac{r}{r-g}\right) \quad + \quad \sum_{\tau=1}^{\infty} \upsilon_{t+\tau}\phi_{t+\tau}^a R^{-\tau}$$

and since $\upsilon_{t+\tau} = \upsilon_{t+1}(1+g)^{\tau-1}$

$$P_t \quad = \quad y_t\left(\frac{-g_B}{r-g_B}\right) \quad + \quad \upsilon_{t+1}\left(\frac{r}{r-g}\right) \quad + \quad \upsilon_{t+1}\sum_{\tau=1}^{\infty} \gamma^{-\tau}\ \phi_{t+\tau}^a$$

[16] Note that in all cases we use clean surplus earnings

$$= \quad y_t\left(\frac{-g_B}{r-g_B}\right) \quad + \quad \upsilon_{t+1}\left(\frac{r}{r-g}\right) \quad + \quad \upsilon_{t+1}\sum_{\tau=1}^{\infty} \gamma^{-\tau}\left(ROVA_{t+\tau}-r\right)$$

which is the result cited in Section 2 above.

Next consider the case of the evolution of ϕ_t^a according to a generalised auto-regressive, integrated, moving-average process ARIMA (*p*,*d*,*q*):

$$\Delta^d\phi_t^a \quad - \quad \overline{\Delta}^d\phi^a \quad = \quad \sum_{c=1}^{c=p}\omega_c\left(\Delta^d\phi_{(t-c)}^a-\overline{\Delta}^d\phi^a\right) \quad - \quad \sum_{j=1}^{j=q}\theta_j e_{(t-j)} \quad + \quad e_t$$

where ω_c is an auto-regressive coefficient of order c, θ_j is a moving-average coefficient of order j, and e_t is a zero mean, randomly distributed error term. *p*, *d*, *q* are the orders of the auto-regressive, differencing and moving-average processes respectively.

Adapting the results of O'Hanlon (1994), it is possible to obtain a generalised expression for the impact of ϕ_t^a on the value of P_t:

$$P_t = \upsilon_t\left[\frac{r(1+g)}{r-g}-\frac{\alpha g_B}{r-g}+\left[\begin{array}{l}\overline{\Delta}^d\phi^a\left(\dfrac{\gamma^d}{(\gamma-1)^{d+1}}\right)+\left(\displaystyle\sum_{z=0}^{z=d-1,d>0}\Delta^z\phi_t^a\left(\frac{\gamma^z}{(\gamma-1)^{z+1}}\right)\right)+ \\ \left(\dfrac{1}{\left(\dfrac{\gamma-1}{\gamma}\right)^d\left(1-\displaystyle\sum_{c=1}^{c=p}\frac{\omega_c}{\gamma^c}\right)}-\left(\dfrac{\gamma}{\gamma-1}\right)^d\right)\left(\Delta^d\phi_t^a-\overline{\Delta}^d\phi^a\right)+ \\ \dfrac{1}{\left(\dfrac{\gamma-1}{\gamma}\right)^d\left(1-\displaystyle\sum_{c=1}^{c=p}\frac{\omega_c}{\gamma^c}\right)}\left(\displaystyle\sum_{c=2}^{c=p}\sum_{s=1}^{s=c-1}\omega_c\left(\frac{\Delta^d\phi_{t-s}^a-\overline{\Delta}^d\phi^a}{\gamma^{c-s}}\right)\right)- \\ \dfrac{1}{\left(\dfrac{\gamma-1}{\gamma}\right)^d\left(1-\displaystyle\sum_{c=1}^{c=p}\frac{\omega_c}{\gamma^c}\right)}\left(\displaystyle\sum_{j=1}^{j=q}\sum_{k=0}^{k=j-1}\frac{\theta_j e_{t-k}}{\gamma^{j-k}}\right)\end{array}\right]\right]$$

where three small changes have been made to simplify the result: $\gamma^\tau = (1+r)^\tau/(1+g)^\tau$, incorporating an extra factor of $(1+g)$ in relation to the expression for gamma used previously; υ_t has replaced υ_{t+1} ; and accordingly $\alpha = y_t/\upsilon_t$.

The general expression may be summed for the simplest ARIMA processes to yield the following specific models.

ARIMA (1, 0, 0)

$$P_t \quad = \quad \upsilon_t \quad \left[\frac{-\alpha g_B}{r - g_B} \quad + \quad \frac{\overline{ROVA}}{(\gamma - 1)} \left(\frac{\gamma\,(1-\omega)}{\gamma - \omega} \right) \quad + \quad \frac{ROVA_t}{\gamma - 1} \left(1 - \left(\frac{\gamma\,(1-\omega)}{\gamma - \omega} \right) \right) \right]$$

This is a weight-average of the current level of ROVA and the mean level of ROVA.

ARIMA (0, 0, 1)

$$\begin{aligned} P_t \quad &= \quad \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B} + \frac{\overline{ROVA}}{\gamma - 1} - \frac{\theta}{\gamma} \left[\phi_t^a - \overline{\phi}^a + \gamma \left[\frac{-\alpha\, g_B}{r - g_B} . \frac{G_V}{G_B} + \frac{\overline{ROVA}}{\gamma - 1} - \frac{P_{t-1}}{\upsilon_{t-1}} \right] \right] \right] \\ &= \quad \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B} \left(1 - \frac{\theta\, G_v}{G_B} \right) + \frac{\overline{ROVA}}{\gamma - 1} (1 - \theta) + \theta \left[M_{t-1} - \frac{\left(ROVA_t - \overline{ROVA} \right)}{\gamma} \right] \right] \end{aligned}$$

where $M_t = P_t / \upsilon_t$. Thus the market value is a weight average of mean returns and a term involving the deviation from average returns and the market to value-added ratio in the previous period.

ARIMA (1, 1, 0)

$$P_t \quad = \quad \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B} + \frac{ROVA_t}{\gamma - 1} + \left(\frac{\omega\gamma}{(\gamma - \omega)(\gamma - 1)} \right) . \,\Delta ROVA_t \right]$$

This expression results from another weight-average expression (see Section 6) and combines current and first difference terms.

ARIMA (0, 1, 1)

$$P_t \quad = \quad \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B} \left(1 - \frac{\theta\, G_v}{G_B} \right) + \left(\frac{ROVA_t}{\gamma - 1} \right) (1 - \theta) + \theta\, M_{t-1} \right]$$

The formula is a weight-average of the current level of ROVA and the prior period ratio of market value to value-added.

The models correspond to (i) a simple auto-regressive process on levels of value-added return; (ii) a moving-average process on levels of return; (iii) a simple auto-regressive process on first differences in levels of value-added return; and (iv) a moving-average process for first differences. The models generally involve a weight average of terms which relate to average and current level value-added returns. They are similar to those in O'Hanlon (1994, 1996), Ohlsen (1995) and Ramakrishnan and Thomas (1992). In Section 6 individual derivations are given for each of these results.

In the following section, these simple models are tested longitudinally against UK corporate and market data extracted from the Datastream Company and Market Databases.

4. Empirical Results

Data were extracted for those companies in the FTSE Allshare index for which the component data necessary to compute value-added were available. This yielded 121 companies for which value-added data were available for some of the years 1977 to 1996. Value-added was defined using Datastream definitions and codes as trading income less raw materials and consumables, less other external charges. A separate attempt to create a data-set for the engineering sector was abandoned because of the paucity of data.

For some of the companies data were only available for a small number of years, insufficient for the accurate determination of the appropriate ARIMA model for the company concerned. The best compromise between sample size and the number of years for which data were available yielded 65 companies for which data were available between 1983 and 1996.

For these companies annual figures were extracted for year-end market and book values, value-added and clean surplus. In addition beta values were calculated on a monthly basis, and then averaged to produce an annual beta, which combined with medium-term gilt rates and a geometric mean market premium of 6.6% yielded annual required rates of return for each company.[17] The majority of the companies had December fiscal year end, but no attempt was made to adjust for those which did not, on the grounds that the longitudinal nature of the study would smooth the impact of this mismatch, and that any adjustment was likely to be precarious. The clean surplus and value-added figures were combined to produce clean ROVA measures, which, when combined with market required rates of return, yielded residual returns on value-added.

Growth rates for value-added and the book value of equity were calculated on a five-year compound basis wherever possible. Growth rates for the first five years were based upon a tapered average. Average returns on value-added were calculated over a five year period.

An initial analysis was undertaken to test the strength of the linkages between market-to-value-added and market-to-book ratios and (i) clean surplus returns on value-added, (ii) average clean surplus returns, (iii) residual returns, (iv) growth in value-added for all years for the companies in the sample.

[17] No attempt was made to unlever the estimates of required rates or adjust for tax given (i) the limited impact of tax shields under the UK imputation system; (ii) difficulties related to the market weighting of debt and equity; and (iii) the general level of uncertainties relating to determination of market risk premia and risk-free rates.

Next, the residual clean surplus returns on value-added for 1983-1996 for each of the 65 companies were analysed using the Trends module of SPSS. In two cases, the series of returns contained problematic outliers that made ARIMA modelling unproductive. These two cases were omitted from the final data set. Each company was tested for fit with each of six basic ARIMA models: (1, 0, 0) with and without constant, (0, 0, 1) with and without constant, (0, 1, 1) and (1, 1, 0). The results of each modelling exercise were evaluated based upon (i) inspection of the autocorrelation and partial autocorrelation functions (ACF and PCF) of the residual clean surplus returns on value-added, (ii) investigation of the Box-Ljung statistics of the error series generated by the ARIMA model applied; (iii) the Akaike Information and Schwartz Bayesian criteria; (iv) ANOVA results and tests of significance; and (v) visual inspection, in accordance with traditional Box-Jenkins methods. This resulted in a partition of the company set into four families, each corresponding to one of the simple ARIMA models (the with-constant and without-constant categories were amalgamated).

For each member company of the four ARIMA families, predicted values of P_t / υ_t were calculated for 1983-1996. For each company, the relevant autoregressive or moving-average coefficient, as determined by the ARIMA modelling of residual clean surplus returns on value-added for the case of best ARIMA fit, were utilised in the valuation model. Actual and average returns on value-added, their first differences and rates of growth were incorporated in the model predictions as dictated by the specific nature of the family of models relevant to that company. The results were then analysed by regression on a family basis to compare actual and predicted values; results were also pooled across families to test for statistically significant relationships.

4.1 General linkages

Table 1

Descriptive Statistics for Value-relevantFactors for Datastream Sample of 65 Companies 1983-1996

	Number	Range	Minimum	Maximm	Mean	Std. Dev.
avROVA	801	182.00	-87.53	94.46	11.68	19.00
5yrGVA	798	126.96	-14.91	112.05	14.47	13.18
M/B	795	128.01	0.32	128.33	2.47	4.99
MV/VA	801	37.88	0.09	37.97	2.27	2.46
ROVA	801	706.95	-430.97	275.98	12.02	36.06
xsROVA	801	711.74	-447.56	264.18	-3.86	36.58

	Number	Variance	Skewness	Std.Error	Kurtosis	Std.Error
avROVA	801	361.00	0.11	0.086	5.65	0.173
5yrGVA	798	173.68	2.13	0.087	10.30	0.173
M/B	795	24.89	20.89	0.087	514.35	0.173
MV/VA	801	6.03	7.45	0.086	79.87	0.173
ROVA	801	1300.48	-0.53	0.086	45.44	0.173
xsROVA	801	1338.29	-0.48	0.086	43.88	0.173

Summary statistics for the variables of interest are presented in Table 1.

Table 2 shows the results of simple regressions of market-to-value-added ratios and key value-added measures. The results strongly support the linkages between value and the profitability measures developed, though due regard is to be paid to problems of auto-correlation as reflected in the Durbin-Watson statistics.

Table 2

Preliminary Regressions of Value-relevant Factors for Datastream Sample of 65 Companies 1983-1996

Dependent/Independent variables	Adjustd. R sqd.	F statistic	t statistic	Significance	Coefficient	Durbin Watson
MVVA/avgROVA	0.052	44.8	6.7	.000	0.0298	0.638
MVVA/avgROVA no outliers	0.189	184.7	13.6	.000	0.0265	0.778
MVVA/ROVA	0.021	18.4	4.3	.000	0.0102	0.669
MVVA/ROVA no outliers	0.062	52.7	7.3	.000	0.0084	0.774
MVVA/xsROVA	0.022	19.0	4.4	.000	0.0102	0.669
MVVA/xsROVA no outliers	0.069	58.9	7.7	.000	0.0087	0.754
MVVA/VAgrowth	0.000	0.9	1.0	.332	0.0064	0.620
MVVA/VAgrowth no outliers	-0.001	0.3	0.5	.606	0.0016	0.597
MVVA/MB	0.007	6.6	2.6	.010	0.412	1.564

Outliers are taken to be values beyond three standard deviations of the mean. Surprisingly, no relationship between growth and the ratio of market value to value-added was discernible. This may either be taken to show that there is no link between growth and value, or, more likely, that the growth measure taken is problematic.

Clearly, period, average and residual ROVA measures are highly correlated. Stepwise regression results shown in Table 3 reveal that average ROVA is the most significant driver of value.

Table 3.1

Stepwise Regression Models of Value-relevant Factors for Datastream Sample of 65 Companies 1983-1996

Model	R	R Squared	Adj. R Squared	Std. Error	Durbin Watson
1	0.436	0.190	0.189	0.995	
2	0.489	0.239	0.237	0.965	
3	0.494	0.244	0.241	0.963	
4	0.498	0.248	0.245	0.960	0.840

Table 3.2

ANOVA Results of Stepwise Regression Models of Value-relevant Factors for Datastream Sample of 65 Companies 1983-1996

Model		Sum of Squares	df	Mean Square	F	Significe.
1	Regression	180.83	1	180.83	182.75	.000
	Residual	768.80	777	0.989		
	Total	949.63	778			
2	Regression	226.86	2	113.43	121.79	.000
	Residual	722.76	776	0.931		
	Total	949.63	778			
3	Regression	231.48	3	77.16	83.27	.000
	Residual	718.15	775	0.927		
	Total	949.63	778			
4	Regression	235.93	4	58.98	63.97	.000
	Residual	713.70	774	0.922		
	Total	949.63	778			

Table 3.3

Coefficients of Stepwise Regression Models of Value-relevant Factors for Datastream Sample of 65 Companies 1983-1996

Model		Unstandard. Coefficients	Standard. Std.Error	Coefficients Beta	t	Significe.
1	Constant	1.674	0.044		38.44	.000
	avROVA	2.80E-02	0.002	0.436	13.52	.000
2	Constant	1.503	0.049		30.82	.000
	avROVA	3.20E-02	0.002	0.499	15.32	.000
	M/B	5.03E-02	0.007	0.229	7.03	.000
3	Constant	1.548	0.053		29.35	.000

	avROVA	2.89E-02	0.003	0.450	11.52	.000
	M/B	5.19E-02	0.007	0.236	7.23	.000
	xsROVA	3.19E-03	0.001	0.086	2.23	.026
4	Constant	2.102	0.257		8.17	.000
	avROVA	2.72E-02	0.003	0.425	10.44	.000
	M/B	5.09E-02	0.007	0.232	7.11	.000
	xsROVA	3.64E-02	0.015	0.987	2.40	.017
	ROVA	-3.45E-02	0.015	-0.889	-2.20	.028

where the predictors in the four models are (i) constant with average ROVA; (ii) constant with average ROVA and market-to-book ratios; (iii) constant with average ROVA, market-to-book ratios and residual ROVA; (iv) constant with average ROVA, market-to-book ratios, residual ROVA and period ROVA. The dependent variable is always the market-to-value-added ratio. The cumulative explanatory power of the models is good, reaching an adjusted r-squared value of 0.245, but again the results need to be treated with a degree of caution given the degree of autocorrelation reflected in the Durbin-Watson statistics.

4.2 Autocorrelation and ARIMA classification

The series of residual returns on value-added is highly auto-correlated. To account for this autocorrelation, individual company returns have been modelled as specific ARIMA processes for the period 1983-1996 as described above. In Table 4 the summary results are provided for the estimated cross-sectional distribution of estimated absolute autocorrelations at lags 1 to 4.

Table 4

Distribution of Absolute Autocorrelation in Residual Returns on Value-added for Datastream Sample of 63 Companies 1983-1996

Lag	Mean	Median	25^{th} Centile	50^{th} Centile	75thCentile
1	0.200	0.159	0.094	0.159	0.278
2	0.152	0.128	0.059	0.128	0.227
3	0.142	0.118	0.044	0.118	0.218
4	0.167	0.143	0.049	0.143	0.282

The table shows substantial mean absolute correlations for all four lags. Absolute values were taken because the auto-regressive and moving-average coefficients in the ARIMA models of best fit varied in sign. Using SPSS Trends to analyse the best-fitting ARIMA model, companies were allocated to one of the basic families (1, 0, 0), (0, 0, 0), (1, 1, 0) and (0, 1, 1) with the distribution of autoregressive and moving-average coefficients given in Table 5.

Table 5

Distribution of Autoregressive and Moving Average Absolute Coefficients for Datastream Sample of 63 Companies 1983-1996

	(1, 0, 0)	(0, 0, 1)	(1, 1, 0)	(0, 1, 1)
Mean	0.462	0.674	0.604	0.847
Median	0.519	0.656	0.624	0.877
Std.Dev.	0.282	0.298	0.119	0.153
Number	8	23	12	20

Thus, for instance, the average absolute autoregression coefficient for the eight companies, for which the autocorrelated behaviour of residual value-added returns was best described by a first order autoregressive (1, 0, 0) process on levels was 0.462; a typical first order moving-average coefficient on first differences would be 0.877. The actual value of the coefficient for each company was used in the valuation model corresponding to the relevant ARIMA process in order to obtain predicted values of the ratio of firm market value to value-added. The largest fraction (37%) of firms were best described by a simple moving-average process on levels, followed by a moving-average process on first differences (32%). These results may be compared with those of Ramakrishnan and Thomas (1992), who found that the residual earnings of a sample of 511 Compustat firms were best explained by a first order autoregressive ARIMA process on levels in 60% of cases, by a first order moving-average ARIMA process on first differences in 31% of cases, and by a first order autoregressive process on first differences in 9% of cases. They did not investigate first order moving-average processes on levels. In terms of coefficients for the three ARIMA processes they considered, the two studies are broadly consistent, but Ramakrishnan and Thomas do not appear to have encountered significant changes of sign for the coefficients, and hence do not rely upon absolute values.

Table 6

Distribution of Autoregressive and Moving Average Absolute Coefficients for R&T Sample of 511 Compustat Companies

	(1, 0, 0)	(0, 0, 1)	(1, 1, 0)	(0, 1, 1)
Mean	0.67		0.68	0.64
Median	0.68		0.73	0.62
Std.Dev.	0.17		0.27	0.17
Number	308		47	156

O'Hanlon (1996) found in his examination of the time-series behaviour of the ordinary earnings of 140 UK companies, that 51% of companies were best described by ARIMA (1, 0, 0) on levels, followed by (0, 0, 1) and (0, 1, 0) processes at 19% and 18% respectively. For clean surplus earnings, however, he found that 68% of cases were best described by random processes (0, 0, 0), and only 15% by first order autoregressive processes on levels. O'Hanlon attributed the results to the large random effect of extraordinary and other components of clean surplus earnings besides ordinary earnings.

4.3 Value-added Pricing Models

Predicted values were obtained for the ratio of market value to value-added for each of the companies between 1983 and 1996 using the appropriate valuation model developed in Sections 3 and 6. Results for the companies were analysed on both a pooled and a model-specific basis. Two recurrent problems were immediately evident.

First, in a large number of instances the growth rate of value-added, or the growth rate of book value exceeded the market-derived required rate of return. This resulted in negative values for the ratio of market value to value-added. In basic financial theory, firms are assumed to be self-funding, which constrains the rate of growth of book value and value-added. It was not possible to adjust the data to reflect changes in capital structure, or for acquisitions, and an argument may be made to disregard these non-sensical results.

Second, the rates of growth were often similar to the levels of required rates of return. Given that the difference between these two factors frequently appears in the denominator of the valuation models, the results amplify the data uncertainties associated with the estimates of growth factors.

Besides analysing the raw results, two stages of data enhancement were also carried out. First, predictions resulting in growth rates in excess of required rates were excluded, and then as a further step any additional negative values were excluded. One further case, an extreme outlier, was omitted from the (0, 1, 1) results. Table 7 shows the results obtained for the pooled predictions and for the individual families of models.

Table 7

Regression of Predicted against Actual Ratios of Market Value to Value-Added for Pooled and ARIMA Families of Valuation Models 1983-96

Model	Version	Adj. R Squared	Beta t Statistic	df	Durbin Watson
All	All	-0.001	0.124	787	0.617
	r>g	-0.002	-0.590	411	1.441
	r.g,+ve	0.004	1.439	285	0.823
100	All	0.005	0.729	106	2.145
	r>g	0.000	-0.150	54	1.791
	r.g,+ve	0.268	**3.718**	35	0.850
001	All	0.000	-0.220	276	2.473
	r>g	0.022	-2.061	186	1.090
	r.g,+ve	0.554	**12.592**	127	1.789

110	All	0.001	0.292	154	2.028
	r>g	0.008	-0.819	81	2.043
	r.g,+ve	-0.026	0.286	36	2.032
011	All	0.181	7.363	247	1.839
	r>g	0.074	2.804	86	2.495
	r.g,+ve	0.093	**2.901**	83	1.316

Regressions conducted on the full set of predicted values for the pooled and ARIMA family predictions were entirely lacking in significance, with the exception of (0, 1, 1), where the good statistical results are a curiosity. As the data quality improves, however, the statistical significance of the regressions of predicted against actual values increases markedly in three out of the four families, and to a fair measure for the pooled results. Normalising the actual and predicted values produced similar results. The particular strength of the moving-average models is in large part due to the inclusion of the prior period ratio of market value to value-added, but this is not the case with the (1, 0, 0) results which depend upon predictions based on a weighted average of current and averaged returns on value-added.

5. Conclusions

In this paper a theoretical framework has been developed to substantiate the value-relevance of clean surplus returns on value-added. Specific models have been developed to examine the time-series behaviour of residual returns on value-added which are empirically testable.

Investigation of the general linkages between market to value-added ratios shows average and current levels of clean surplus return on value-added to be statistically significant factors. Growth in value-added for the sample of companies analysed was not value-relevant.

Using data derived from the FTSE Allshare set of companies for which value-added information is available from Datastream between 1983 and 1996, predicted values for the ratio of market value to value-added, derived from company specific models, have been regressed against actual values. Although aggregate results are without significance, removal of those values where growth is in excess of required rates of return and any remaining negative values, produces statistically significant results for the value-relevance of three of the four families of value-added pricing models.

The results are sufficiently encouraging to suggest that further empirical investigation is warranted. A larger data set would be advantageous, and work is underway to extend the study to include data from other countries, in particular, the United States. Variations of the models may also be developed which do not suffer from the denominator sensitivities discussed above.

6 Derivations

6.1 ARIMA (1, 0, 0)

As a first order autoregressive process it follows that

$$\begin{aligned}
\phi_t^a - \bar{\phi}^a &= \omega \left(\phi_{t-1}^a - \bar{\phi}^a\right) \\
\phi_{t+1}^a - \bar{\phi}^a &= \omega \left(\phi_t^a - \bar{\phi}^a\right) \\
\phi_{t+2}^a - \bar{\phi}^a &= \omega \left(\phi_{t+1}^a - \bar{\phi}^a\right) = \omega^2 \left(\phi_t^a - \bar{\phi}^a\right) \quad etc.
\end{aligned}$$

If we then sum the discounted series $\phi_{t+\tau}^a$ we obtain

$$\begin{aligned}
\sum \phi_{t+\tau}^a \gamma^{-\tau} &= \frac{\omega\left(\phi_t^a - \bar{\phi}^a\right) + \bar{\phi}^a}{\gamma} + \frac{\omega^2\left(\phi_t^a - \bar{\phi}^a\right) + \bar{\phi}^a}{\gamma^2} + \ldots \\
&= \left(\phi_t^a - \bar{\phi}^a\right)\left[\frac{\omega}{\gamma} + \frac{\omega^2}{\gamma^2} + \frac{\omega^3}{\gamma^3} + \ldots\right] + \bar{\phi}^a\left[\frac{1}{\gamma} + \frac{1}{\gamma^2} + \frac{1}{\gamma^3} + \ldots\right] \\
&= \left(\phi_t^a - \bar{\phi}^a\right)\left(\frac{\omega}{\gamma - \omega}\right) + \frac{\bar{\phi}^a}{\gamma - 1}
\end{aligned}$$

If we substitute this formula into the expression for P_t we obtain

$$\begin{aligned}
P_t &= \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{(R-1)G}{(R-G)} + \frac{\bar{\phi}^a}{\gamma - 1} + \frac{\omega}{\gamma - \omega}\left(\phi_t^a - \bar{\phi}^a\right)\right] \\
&= \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{(R-1)}{(\gamma-1)} + \frac{\bar{\phi}^a}{\gamma - 1} + \frac{\omega}{\gamma - \omega}\left(\phi_t^a - \bar{\phi}^a\right)\right] \\
&= \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{\overline{R}\overline{O}\overline{VA}}{(\gamma-1)} + \frac{\omega}{\gamma - \omega}\left(ROVA_t - \overline{R}\overline{O}\overline{VA}\right)\right] \\
&= \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{\overline{R}\overline{O}\overline{VA}}{(\gamma-1)}\left(\frac{\gamma(1-\omega)}{\gamma - \omega}\right) + \frac{ROVA_t}{\gamma - 1}\left(1 - \left(\frac{\gamma(1-\omega)}{\gamma - \omega}\right)\right)\right]
\end{aligned}$$

where $G = (1 + g_{VA})$ and $R = (1 + r)$. This is a weight-average of the current level of ROVA and the mean level of ROVA.

6.2 ARIMA (0, 0, 1)

For this process we know

$$\begin{aligned}
\phi_t^a - \bar{\phi}^a &= -\theta\, e_{t-1} + e_t \\
\phi_{t+1}^a - \bar{\phi}^a &= -\theta\, e_t + e_{t+1} \\
\phi_{t+2}^a - \bar{\phi}^a &= -\theta\, e_{t+1} + e_{t+2} \quad etc.
\end{aligned}$$

Since the error terms e_{t+1} and onwards are randomly distributed about a zero mean, their expected value is zero. If we sum the discounted series of residual returns we obtain

$$\begin{aligned}\sum \phi^a_{t+\tau}\gamma^{-\tau} &= -\frac{\theta\, e_t}{\gamma} + \frac{\bar{\phi}^a}{\gamma} + \frac{\bar{\phi}^a}{\gamma^2} + \frac{\bar{\phi}^a}{\gamma^3} + \dots \\ &= -\frac{\theta\, e_t}{\gamma} + \frac{\bar{\phi}^a}{\gamma - 1}\end{aligned}$$

Substituting into the expression for P_t

$$\begin{aligned}P_t &= \upsilon_t \left[\left[\frac{-\alpha g_B}{r - g_B}\right] + \frac{(R-1)G}{(R-G)} + \left[\frac{\bar{\phi}^a}{\gamma-1} - \frac{\theta\, e_t}{\gamma}\right] \right] \\ &= \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{(R-1)}{(\gamma-1)} + \frac{\bar{\phi}^a}{\gamma-1} - \frac{\theta\, e_t}{\gamma}\right] \\ &= \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{\overline{R}\,\overline{O}\,\overline{VA}}{(\gamma-1)} - \frac{\theta\, e_t}{\gamma}\right]\end{aligned}$$

Rearranging

$$e_t = \frac{\gamma}{\theta}\left[\frac{-\alpha g_B}{r - g_B} + \frac{\overline{R}\,\overline{O}\,\overline{VA}}{\gamma-1} - \frac{P_t}{\upsilon_t}\right]$$

Hence

$$e_{t-1} = \frac{\gamma}{\theta}\left[\frac{-\alpha g_B}{r - g_B}.\frac{G_{VA}}{G_B} + \frac{\overline{R}\,\overline{O}\,\overline{VA}}{\gamma-1} - \frac{P_{t-1}}{\upsilon_{t-1}}\right]$$

Since

$$e_t = \phi^a_t - \bar{\phi}^a + \theta\, e_{t-1}$$

we obtain

$$\begin{aligned}P_t &= \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B} + \frac{\overline{R}\,\overline{O}\,\overline{VA}}{\gamma-1} - \frac{\theta}{\gamma}\left[\phi^a_t - \bar{\phi}^a + \gamma\left[\frac{-\alpha\, g_B}{r - g_B}.\frac{G_V}{G_B} + \frac{\overline{R}\,\overline{O}\,\overline{VA}}{\gamma-1} - \frac{P_{t-1}}{\upsilon_{t-1}}\right]\right]\right] \\ &= \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B}\left(1 - \frac{\theta\, G_v}{G_B}\right) + \frac{\overline{R}\,\overline{O}\,\overline{VA}}{\gamma-1}(1-\theta) + \theta\left[M_{t-1} - \frac{(ROVA_t - \overline{R}\,\overline{O}\,\overline{VA})}{\gamma}\right]\right]\end{aligned}$$

where $M_t = P_t / \upsilon_t$. Thus the market value is a weight average of mean returns and a term involving the deviation from average returns and the market to value-added ratio in the previous period.

6.3 ARIMA (1, 1, 0)

For this process

$$
\begin{aligned}
\left(\phi_{t+1}^a - \phi_t^a\right) - \overline{\Delta}\phi^a &= \omega\left[\left(\phi_t^a - \phi_{t-1}^a\right) - \overline{\Delta}\phi^a\right] \\
\left(\phi_{t+n-1}^a - \phi_{t+n-2}^a\right) - \overline{\Delta}\phi^a &= \omega\left[\left(\phi_{t+n-2}^a - \phi_{t+n-3}^a\right) - \overline{\Delta}\phi^a\right] \\
\left(\phi_{t+n}^a - \phi_{t+n-1}^a\right) - \overline{\Delta}\phi^a &= \omega^n\left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right) \\
\phi_{t+n}^a - \phi_t^a - n\,\overline{\Delta}\phi^a &= \left[\omega^n + \omega^{n-1} + \omega^{n-2} + \ldots + \omega\right]\left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right) \\
\phi_{t+n}^a &= \left[\omega^n + \omega^{n-1} + \omega^{n-2} + \ldots + \omega\right]\left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right) - \phi_t^a - n\,\overline{\Delta}\phi^a
\end{aligned}
$$

The last term on the right-hand side generates a series of perpetuities which when discounted as a series S gives

$$
\begin{aligned}
S &= \frac{\overline{\Delta}\phi^a}{\gamma - 1}\left[1 + \frac{1}{\gamma} + \frac{1}{\gamma^2} + \frac{1}{\gamma^3} + \ldots\right] \\
&= \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma - 1)^2}
\end{aligned}
$$

Using similar methods one can show the polynomial in ω sums to S' where

$$S' = \frac{\omega\left(\omega^n - 1\right)}{\omega - 1}$$

Thus if we discount the series of residual returns we obtain

$$
\begin{aligned}
\sum \phi_{t+\tau}^a\,\gamma^{-\tau} &= \sum\left[\frac{\omega\left(\omega^\tau - 1\right)}{\omega - 1}.\left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right) + \phi_t^a\right]\gamma^{-\tau} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma - 1)^2} \\
&= \frac{\omega}{\omega - 1}.\left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right)\sum\frac{\omega^\tau - 1}{\gamma^\tau} + \frac{\phi_t^a}{\gamma - 1} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma - 1)^2} \\
&= \frac{\omega}{\omega - 1}.\left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right)\left(\frac{\omega}{\gamma - \omega} - \frac{1}{\gamma - 1}\right) + \frac{\phi_t^a}{\gamma - 1} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma - 1)^2} \\
&= \frac{\omega\,\gamma}{(\omega - 1)(\gamma - 1)}.\left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right) + \frac{\phi_t^a}{\gamma - 1} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma - 1)^2}
\end{aligned}
$$

The corresponding market price is given by

$$
\begin{aligned}
P_t \;&=\; \upsilon_t \left[\frac{-\alpha\, g_B}{r-g_B} + \frac{(R-1)}{\gamma-1} + \frac{\omega\,\gamma}{(\omega-1)(\gamma-1)} . \left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right) \;+\; \frac{\phi_t^a}{\gamma-1} \;+\; \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma-1)^2} \right] \\
&=\; \upsilon_t \left[\frac{-\alpha\, g_B}{r-g_B} + \frac{(R-1)}{\gamma-1} + \frac{\phi_t^a}{\gamma-1} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma-1)^2} \;+\; \frac{\omega\,\gamma}{(\omega-1)(\gamma-1)} . \left(\phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a\right) \right] \\
&=\; \upsilon_t \left[\frac{-\alpha\, g_B}{r-g_B} + \frac{ROVA_t}{\gamma-1} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma-1)^2} \left[\frac{1}{\gamma-1} - \frac{\omega}{\gamma-\omega} \right] + \frac{\omega\,\gamma}{(\omega-1)(\gamma-1)} . \Delta ROVA_t \right] \\
&=\; \upsilon_t \left[\frac{-\alpha\, g_B}{r-g_B} + \frac{ROVA_t}{\gamma-1} + \frac{\omega\,\gamma}{(\omega-1)(\gamma-1)} . \Delta ROVA_t + \frac{\overline{\Delta} ROVA\,\gamma}{(\gamma-1)^2} \left[\frac{\gamma\,(1-\omega)}{\gamma-\omega} \right] \right] \\
&=\; \upsilon_t \left[\frac{-\alpha\, g_B}{r-g_B} + \frac{ROVA_t}{\gamma-1} + \left[\left(1 - \left[\frac{\gamma\,(1-\omega)}{\gamma-\omega} \right] \right) \Delta ROVA_t + \overline{\Delta} ROVA \left[\frac{\gamma\,(1-\omega)}{\gamma-\omega} \right] \right] \frac{\gamma}{(\gamma-1)^2} \right]
\end{aligned}
$$

This is a weight-average of the current first difference and average first difference plus current level of returns. In the current case the average of the first difference is zero, so we obtain

$$
P_t \;=\; \upsilon_t \left[\frac{-\alpha\, g_B}{r-g_B} + \frac{ROVA_t}{\gamma-1} + \left(\frac{\omega\gamma}{(\gamma-\omega)(\gamma-1)} \right) . \Delta ROVA_t \right]
$$

6.4 ARIMA (0, 1, 1)

For this process on an expected value basis we may derive

$$
\begin{aligned}
\left(\phi_{t+1}^a - \phi_t^a\right) - \overline{\Delta}\phi^a \;&=\; -\theta\, e_{t-1} + e_t \\
\left(\phi_{t+n-1}^a - \phi_{t+n-2}^a\right) - \overline{\Delta}\phi^a \;&=\; 0 \\
\left(\phi_{t+n}^a - \phi_{t+n-1}^a\right) - \overline{\Delta}\phi^a \;&=\; 0 \\
\phi_{t+n}^a - \phi_t^a - n\,\overline{\Delta}\phi^a \;&=\; -\theta\, e_{t-1} \\
\phi_{t+n}^a \;&=\; -\theta\, e_{t-1} - \phi_t^a - n\,\overline{\Delta}\phi^a
\end{aligned}
$$

If we discount the residual returns we obtain

$$
\begin{aligned}
\sum \phi_{t+\tau}^a\, \gamma^{-\tau} \;&=\; -\theta\, e_t \left[\frac{1}{\gamma} + \frac{1}{\gamma^2} + \frac{1}{\gamma^3} + \ldots \right] + \frac{\phi_t^a}{\gamma-1} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma-1)^2} \\
&=\; \frac{-\theta\, e_t}{\gamma-1} + \frac{\phi_t^a}{\gamma-1} + \frac{\overline{\Delta}\phi^a\,\gamma}{(\gamma-1)^2}
\end{aligned}
$$

This gives rise to the following price equation:

$$P_t = \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{(R-1)G}{(R-G)} + \frac{-\theta\, e_t}{\gamma - 1} + \frac{\phi_t^a}{\gamma - 1} + \frac{\overline{\Delta}\phi^a\, \gamma}{(\gamma-1)^2} \right]$$

$$= \upsilon_t \left[\frac{-\alpha g_B}{r - g_B} + \frac{ROVA_t}{(\gamma-1)} + \frac{\overline{\Delta}ROVA\,\gamma}{(\gamma-1)^2} - \frac{\theta\, e_t}{\gamma - 1} \right]$$

Rearranging

$$e_t = \frac{\gamma - 1}{\theta} \left[\frac{-\alpha g_B}{r - g_B} + \frac{ROVA_t}{\gamma - 1} + \frac{\overline{\Delta}ROVA\,\gamma}{(\gamma-1)^2} - \frac{P_t}{\upsilon_t} \right]$$

Hence

$$e_{t-1} = \frac{\gamma - 1}{\theta} \left[\frac{-\alpha g_B}{r - g_B} . \frac{G_{VA}}{G_B} + \frac{ROVA_{t-1}}{\gamma - 1} + \frac{\overline{\Delta}ROVA\,\gamma}{(\gamma-1)^2} - \frac{P_{t-1}}{\upsilon_{t-1}} \right]$$

Since

$$e_t = \phi_t^a - \phi_{t-1}^a - \overline{\Delta}\phi^a + \theta\, e_{t-1}$$

$$= \Delta ROVA_t - \overline{\Delta}ROVA + \theta\, e_{t-1}$$

we obtain

$$P_t = \upsilon_t \left[\begin{array}{l} \dfrac{-\alpha\, g_B}{r - g_B} + \dfrac{ROVA_t}{\gamma - 1} + \dfrac{\overline{\Delta}ROVA\,\gamma}{(\gamma-1)^2} \\ -\dfrac{\theta}{\gamma-1} \left[\Delta ROVA_t - \overline{\Delta}ROVA + (\gamma-1) \left[\dfrac{-\alpha\, g_B}{r - g_B} . \dfrac{G_V}{G_B} + \dfrac{ROVA_{t-1}}{\gamma - 1} + \dfrac{\overline{\Delta}ROVA\,\gamma}{(\gamma-1)^2} - \dfrac{P_{t-1}}{\upsilon_{t-1}} \right] \right] \end{array} \right]$$

$$= \upsilon_t \left[\begin{array}{l} \dfrac{-\alpha\, g_B}{r - g_B} \left(1 - \dfrac{\theta\, G_v}{G_B} \right) + \dfrac{ROVA_t}{\gamma - 1} + \dfrac{\overline{\Delta}ROVA\,\gamma}{(\gamma-1)^2} (1 - \theta) \\ -\dfrac{\theta}{\gamma-1} \left[ROVA_t - \overline{\Delta}ROVA - (\gamma-1) \dfrac{P_{t-1}}{\upsilon_{t-1}} \right] \end{array} \right]$$

$$= \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B} \left(1 - \frac{\theta\, G_v}{G_B} \right) + \left(\frac{ROVA_t}{\gamma - 1} + \frac{\overline{\Delta}ROVA\,\gamma}{(\gamma-1)^2} \right) (1 - \theta) + \theta \left[M_{t-1} + \frac{\overline{\Delta}ROVA}{\gamma - 1} \right] \right]$$

In the current case the average of the first difference is zero, so we obtain

$$P_t = \upsilon_t \left[\frac{-\alpha\, g_B}{r - g_B} \left(1 - \frac{\theta\, G_v}{G_B} \right) + \left(\frac{ROVA_t}{\gamma - 1} \right) (1 - \theta) + \theta\, M_{t-1} \right]$$

The formula is a weight-average of the current level of ROVA and the prior period ratio of market value to value-added.

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