Strategic Hedging and Investment Efficiency *

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Abstract

This paper links real investment policy to corporate risk management, endogenizing the costs of external financing. Previous literature finds investment efficiency linked to full hedging. In this model, a firm with proprietary information when deciding its investment in a valuable project, may choose not to fully hedge in equilibrium, and this can improve investment efficiency. The size of the project, the hedged value of cash-earnings, and the timing of information revelation, influence the risk management equilibrium. Results are derived under a constraint of no-distress at the time of the investment decision. When this constraint is relaxed, it results in a bargaining situation between shareholders, the counterparty on the contract utilized to manage risk, and new investors. Depending on the bargaining solution, the firm may still prefer to "not hedge", facing distress in some states. When this occurs investment efficiency is enhanced. The sources of external finance are debt and equity. The issue of disclosure regarding risk exposure plays a crucial role.

JEL Classification: D82, D84, G31, G32

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1 Introduction

Understanding corporate risk behavior is certainly important, as the presence of corporations in derivatives markets is evident. The literature has offered several rationales for corporate risk management, but the discussion is still very much open. The media ususally focus on "stories" of the downside of corporate use of derivatives: situations in which, apparently unjustifiably, firms have large losses in derivatives and sometimes find themselves close to bankruptcy. Maybe we are only looking at half of the story. It is also true that a considerable number of such firms end up reorganizing and undertaking with good new projects. Understanding the linkage between risk management and other aspects of corporate startegy is still on the agenda.

In this paper I endogenously establish a link between corporate risk management and investment efficiency, in a different way from what previous literature considered. In an asymmetric information setting I introduce risk management in the investment problem first identified by Myers and Majluf (1984). I find that a firm with superior information than the market about its prospects, may prefer not to fully hedge in equilibrium because of investment concerns.

Other explanations for corporate risk management have been offered, ranging from shareholder risk aversion (e.g., DeMarzo and Duffie (1991)) to managerial motives (e.g., Smith and Stulz (1985), Campbell and Kracaw (1987), DeMarzo and Duffie (1995) or Breeden and Viswanathan (1996)), avoidance of financial distress (e.g., Smith (1984)), or tax-saving purposes (e.g., Smith and Stulz (1985)). The arguments underlying these theories deserve some thought. Risk aversion of corporate owners as a motive for corporate hedging will most probably not apply to well-diversified, large firms, that are expected to behave as a risk neutral aggregate[1]. Furthermore, it is not clear why hedging should occur at firm level, rather than being implemented by shareholders outside the firm. In an agency framework a managerial motive for hedging is clear under risk aversion; but again one may question why managers do not hedge on their own accounts (even if in some cases this is forbidden by law), rather than at corporate level. Other maybe less obvious managerial reasons for risk management rely on the informational consequences of managerial choices of risk policy, under

[1] Even if shareholders, individually, are risk averse.

asymmetric information. But still there is no precise link established between the use of derivatives by firms and economic efficiency — these are managerial motivations for risk management, rather than an improvement in the value of the firms' fundamentals.

In this sense the clearest exception in the literature is Froot, Scharfstein and Stein (1993)[2], who link corporate risk management to investment efficiency, under risk neutrality and asymmetric information. To put it simply, they consider an exogenous convex function representing the cost of external financing to the firm, which leads to the result that firms would rather hedge their internal liquid funds to reduce the expected cost of financing new investments with external funds. One possible framework to motivate Froot, Scharfstein and Stein (1993)'s cost of external financing set-up is the Myers and Majluf (1984) capital budgeting problem, which I explore in this paper.

When I endogenize the cost of external financing I show that their results do not directly apply to this model. It is possible that a firm prefers not to fully hedge in equilibrium because of investment concerns. A new relationship between investment efficiency and corporate risk management is obtained in a risk neutral world in which firms may need to raise external capital to finance a valuable investment project. I introduce the possibility of hedging or not, at an initial date, some of the firm's assets which will be realized cash earnings when the firm has to decide whether to invest in a new valuable project. I show that firms may benefit from not fully hedging their internally generated cash earnings[3], as opposed to what one might infer from Froot, Scharfstein and Stein (1993).

In this framework, a manager acts on behalf of the original shareholders of a firm who hold their shares until a final date when the firm is liquidated. At the time of taking the investment decision, the manager has superior information about the value of the firm's future assets in place. Underinvestment arises when the new project is marginally profitable, internal funds are not sufficient to cover the investment, and the assets in place are sufficiently more profitable than what is typical — the manager knows that in such circumstances either the issue price of new shares will be less than its

[2]It is recognized that other rationales such as the avoidance of costs of financial distress, or taking advantage of convex tax schedules are also attainable by risk management at corporate level only.

[3]At least as long as the cash flows of the new investment project are uncorrelated with the assets in place or the risky variable underlying the derivative instrument used for risk management.

intrinsic value, or the risk premium inherent to the face value of new debt will be too large. This effect may induce the manager to reject a valuable investment, when he possesses favorable information. This is the inefficiency examined here, adding risk management to the setting.

In this model the cost of external financing is fully endogenized — in fact it can be interpreted as the mispricing due to information asymmetry, or as the underinvestment itself. The main result found is that risk management affects investment efficiency, and that firms will not choose full hedging in a Nash sequential equilibrium, if this implied that underinvestment occured. However what some may view as "speculation" can take place with the sole purpose of mitigating investment inefficiency. The term "speculation" is used in the same sense as "not hedging" or "strategic hedging": a firm may not necessarily take positions in risky financial instruments; it can simply be the case that it does not fully hedge its natural exposure.

For tractability results are derived first under a constraint of no financial distress at the intermediate date when the firm decides whether to invest. By "no distress" it is meant that the firm is able to raise riskless debt to cover any losses in the financial instrument used for risk management in which it invested one period before (i.e., the financial instrument carries no risk premium). It is found that: (1) if when future cash earnings are hedged at the initial date, the firm does not invest when it has positive information one period ahead, then in equilibrium the firm does not hedge. In such case future real investment efficiency increases — this is the value of "not hedging" or "healthy speculation"; (2) if under full hedging of cash earnings the firm invests efficiently in the project independently of its private information, then this efficiency is preserved.

The literature on corporate risk management has largely ignored so far the problem of default on the hedging contract. This serious fault is overcome by endogenizing a default premium on the pricing of the hedging instrument. Financial distress can occur when the firm decides its investment policy, if it is unable to fully cover its losses in the hedging instrument. In the presence of financial distress, a bargaining situation arises between original shareholders ("the firm"), the counterparty on the risky financial instrument used to manage risk, and potential investors interested in the new project. According to the assumptions made about a possible bargaining solution, speculation can still be chosen in equilibrium, and this is "healthy" in this context, as it is associated with enhanced investment efficiency. I find that if the firm has no private information at the time of choosing its risk exposure, then it does not hedge in equilibrium, if otherwise meant that underinvestment would occur. This result induces improved investment efficiency. It may be controversial that investment efficiency is positively related here to financial distress provoked by 'speculation' in a financial instrument at an earlier date. This is due to the bargaining solution considered, which opens way for future research.

Another interpretation for this link between risk management and investment under financial distress comes from a security design perspective: because the firm is constrained not to raise or spend cash to start with, the use of a risk management device that produces randomness at the investment date may lead the firm to overcome the inefficiency caused by its superior information. Because there is randomness in cash earnings, for some realizations the firm can invest efficiently issuing a fairly priced seccurity; whereas for low realizations of cash earnings the bargaining process associated with distress is "as if" the firm had sold its assets-in-place (which are the source of asymmetry) when they are of no value to current shareholders. This may also be viewed as a change in ownership and control to a party that has no superior information and will therefore take efficient investment decisions.

Underlying the derivation of the results is the still currently debated issue of accounting rules for derivatives. A transparent mechanism of disclosure regarding risk exposure is considered. This issue is discussed. The timing of revelation of the value of future assets in place, which is the source of the information asymmetry in the model, is also of interest, and discussed as an extension. If the firm already possesses superior information when choosing its risk exposure then we are faced with multiple equilibria, and it is no longer true that the equilibrium risk exposure is necessarily the level that leads to maximum (constrained) investment efficiency. Trying to motivate the prevalence of any of these equilibria is a matter of further research, as theoretical refinements are of no great help under these circumstances. However, in an ex ante efficiency analysis we still find that an equilibrium with speculation can be more efficient than one with full hedging. It is interesting to note that risk management is not used as a signaling device in this model, as the choice of risk exposure carries no cost. A "lower" type will always pool in the choice of risk level with a "better" type, as this choice is revealed at the time the market reviews its beliefs and prices the new security issued by the firm.

Results are derived assuming debt is the source of external financing, but an extension shows how new equity is also accounted for. The direction of results is not qualitatively affected. For completeness I extend the model to allow for correlations between the cash flows of the new project, the value of assets in place, and the (hedgeable) risk underlying cash earnings. The existence of more than two types of firms, different assumptions about the revelation of the asymmetry, and the possibility of production functions that result in investment decisions that are not simply (0,1) are also discussed.

The structure of the paper is as follows. Section 2 presents a brief description of the model. In section 3 I examine the problem and its solution when the external finance source available is debt. Section 4 discusses several extensions of the model: (i) when both debt and new equity are potential sources of financing; (ii) a different timing for the realization of the information asymmetry; (iii) correlations between variables. Section 5 discusses the possibility of 'speculation' resulting in financial distress at the intermediate investment date; Concluding remarks are presented in Section 6. Examples are in Appendix A, proofs in Appendix B, and figures in Appendix C.

2 The Model and Main Assumptions

The model has three dates, $t = 0, 1, 2$. An appealing way of interpreting it is by looking at the firm as a set of three projects. There is a long term project that lasts from $t = 0$ to $t = 2$, and which I call "assets in place", $\tilde{a}$. Then, there is one short term project that lasts from $t = 0$ to $t = 1$, and which is what I call cash earnings or financial slack, $\tilde{S}$. This is correlated with a hedgeable risk $\tilde{X}$, to which the firm can choose its level of exposure θ at time $t = 0$. Finally there is a third short lived project, lasting from $t = 1$ to $t = 2$. This project requires investment I, and offers random net present value $\tilde{y}$. Figure C.2 represents the sequence of events according to this interpretation.

Starting with time $t = 1$, the firm has to decide whether to invest in a unique and indivisible risky investment project. This project requires investment $I > 0$, and offers net present value $\tilde{y}$. The random net present value of the project, $\tilde{y}$, has support $[-I, \infty)$. The density function of $\tilde{y}$ is $f(y)$, and the cdf is denoted by $F(y)$. The project is assumed to be valuable as its expected net present value, $b \equiv E_{t=1}(\tilde{y})$, is positive. The realization of $\tilde{y}$ occurs at time $t = 2$. d_1 (d_0) denotes the decision to undertake (reject) the project.

The firm is run by a manager who defends the interests of original shareholders. As in Myers and Majluf (1984) original shareholders do not invest in securities issued at $t = 1$. At the time of making the investment decision ($t = 1$) the manager knows exactly what the future value of its assets in place will be — these are realized at $t = 2$. On the contrary, investors only know at $t = 1$ that the future value of assets in place $\tilde{a}$ can assume one of two possible values $\{a_L, a_H\}$ with equal probability. It is also assumed that $0 < a_L < a_H$. I denote by p (q) the probability with which the firm forgoes the project at $t = 1$ when it learns the state to be H (L). The firm faces limited liability, and is liquidated at $t = 2$.

At time $t = 1$ the firm has a realized amount of "financial slack" or cash earnings $\tilde{S}$, that can be used to finance the project. $\tilde{S}$ is realized at time $t = 1$, being the result of the firm's past activity at $t = 0$. These cash earnings are correlated with a random variable $\tilde{X}$, whose value is realized at time $t = 1$. I assume that there exists a financial instrument whose realization is $\tilde{X}$. Depending on the natural exposure and on the positions the firm undertakes on this instrument $\tilde{X}$ at time $t = 0$, it will end up with a level of exposure denoted by θ: the value of the cash earnings can then be denoted by $\tilde{S} = S + \theta\tilde{X}$, where S is a constant. The choice of θ determines the risk management policy of the firm. Results are derived under the assumption that the support of $\tilde{X}$ is $\{-1, 1\}$. Other supports are discussed in Sections 5 and 6. If a firm fully hedges its liquidity at time $t = 0$ — i.e., $\theta = 0$ — it removes uncertainty regarding its internally generated liquid assets. Otherwise, financial slack remains "not hedged".

All agents in this economy are risk neutral. So the objective function of the original shareholders of the firm is the expected value at $t = 0$ of their claim in the firm. I assume that the firm is initially all-equity financed. The market of outside investors is assumed to be competitive, so that in equilibrium an investor will expect to break even. The same competitiveness takes place in the market where the financial instrument $\tilde{X}$ is traded — traders expect to break even. For simplicity the discount rate is normalized to zero. I also ignore the existence of margins requirements for the instrument that is used for risk management of $\tilde{S}$.

The results obtained are interpreted under the assumption that the manager does not know the realization of $\tilde{a}$ — which is the source of information asymmetry — at time $t = 0$. Nature reveals to him the state H or L at $t = 1$. A different timing is considered in section 4. Figure C.1 represents the time line of events of the game.

It is also assumed that firms can disclose their investment in the financial instrument $\tilde{X}$, thereby revealing their risk exposure. This is in line with the existing accounting requirements for derivatives and the new proposals that actually tend to require more disclosure. Given the structure of the model this amounts to saying that cash earnings at $t = 1$ are observable.

As a benchmark I start by stating the first best efficient outcome under symmetric information at time $t = 1$.

Lemma 1 *(First Best) Under symmetry of information at time $t = 1$, the firm undertakes the project for both state H and L. The expected value of this outcome to original shareholders in state H is $V_H^{FB} = a_H + S + b$, and in state L is $V_L^{FB} = a_L + S + b$. Moreover, the corporate risk management policy undertaken, and the type of financing requested, are irrelevant.*

3 Equilibrium Risk Management and Investment Policies

The game is solved recursively. I start by examining the equilibrium investment decision that occurs at time $t = 1$, when the firm is already aware of the state being H or L, conditional on the realization of the level of cash earnings $\tilde{S}$. I then step back in time to $t = 0$ and determine the optimal risk management policy θ^*, relating it to investment efficiency. Results are derived under the constraint of no financial distress at time $t = 1$. This means that when the realization of cash earnings $\tilde{S}$ is negative, the firm is still able to raise riskless debt in order to cover these losses in the financial instrument $\tilde{X}$ (i.e., $-(S - \theta) \leq a_L$, for $\theta > 0$). This constraint is relaxed in section 5.

3.1 The Investment Decision at $t = 1$

If the level of cash earnings realized at $t = 1$ were $\bar{S} \geq I - a_L$, independently of the sate being H or L, the firm would always go ahead with the new project (i.e., $p = q = 0$). Therefore in equilibrium there would be no investment inefficiency. The intuition for this result is that, independently of the state being H or L, the firm is able to raise $\max(0, I - \bar{S})$ risklessly. Hence, when the firm learns that future assets in place are a_H, it does not need to pool with the worst type when raising new funds in the market. Either internally generated financing covers the investment in the new valuable project, or external funds involve no risk premium.

On the other hand when the realization of cash earnings, say $\bar{S}$, is not sufficiently high to fully cover risklessly the cost of the new project $(\bar{S} < I - a_L)$, then the Nash sequential equilibrium of the game involves both the investment decision by the firm, given state H or L (unobserved by the market), and the break even of investors in the new security the firm may issue. Equilibrium can be defined as follows.

Definition 1 *The equilibrium of the subgame starting at $t = 1$, given that the realization of cash earnings is $\bar{S} < I - a_L$, is such that:*

1. *The probabilities are updated according to Bayes' Theorem, yielding posteriors*[4]*:*

$$\begin{cases} \Pr(H \mid d_1) &= \frac{1-p}{2-p-q} \\ \Pr(L \mid d_1) &= \frac{1-q}{2-p-q} \\ \Pr(H \mid d_0) &= \frac{p}{p+q} \\ \Pr(L \mid d_0) &= \frac{q}{p+q} \end{cases}$$

2. *The pricing rule in a competitive market with rational expectations, when the firm issues debt with face value D is such that:*

$$\begin{aligned} I - \bar{S} &= \frac{1-p}{2-p-q}\int_{-I}^{\infty} \min\{D, a_H + I + y\} f(y)dy + \\ &+ \frac{1-q}{2-p-q}\int_{-I}^{\infty} \min\{D, a_L + I + y\} f(y)dy \end{aligned}$$

3. *When the state revealed to the firm is H, it chooses p^* so that:*

$$\begin{aligned} p^* = \arg\max_p\{(1-p)\int_{-I}^{\infty} \max\{0, a_H + I + y - D\} f(y)dy + \\ +p(a_H + \bar{S})\} \quad s.t. \\ p \in [0,1] \\ q \in [0,1] \end{aligned}$$

4. *When the state revealed to the firm is L, it chooses q^* so that:*

$$\begin{aligned} q^* = \arg\max_q\{(1-q)\int_{-I}^{\infty} \max\{0, a_L + I + y - D\} f(y)dy + \\ +q(a_L + \bar{S})\} \quad s.t. \\ p \in [0,1] \\ q \in [0,1] \end{aligned}$$

[4] For completion, in case $(p = q = 0)$ or $(p = q = 1)$ in equilibrium, I examine out-of-equilibrium beliefs in the proofs.

The following two lemmas state the equilibrium investment decisions taken by the firm, conditional on some level of realized cash earnings $\bar{S}$ at time $t = 0$. Lemma 2 describes the conditions under which full investment efficiency is reached[5], whereas Lemma 3 identifies the underinvestment problem.

Lemma 2 *Suppose that the realization of cash earnings at $t = 1$ is $\bar{S}$. If $\bar{S} < I - a_L$ and the following condition (1) is not satisfied, or if $\bar{S} \geq I - a_L$, then in equilibrium the firm invests with probability 1, independently of the state being H or L.*

Condition (1) is necessary and sufficient for the firm to prefer not to invest, when its private information is that the state is H:

$$\begin{aligned} E_{t=1}[V_H(d_0) \mid \bar{S}] &> E_{t=1}[V_H(d_1) \mid \bar{S}] \\ a_H + \bar{S} &> \int_{-I}^{\infty} \max\{0, a_H + I + y - D^P\} f(y) dy \end{aligned} \tag{1}$$

where V_H denotes the value of the firm in state H, whereas D^P is the face value of debt in equilibrium if the firm invests in both states H and L when cash earnings are $\bar{S}$. It is defined implicitly as:

$$\begin{aligned} I - \bar{S} &= \frac{1}{2} \int_{-I}^{\infty} \min\{D^P, a_H + I + y\} f(y) dy + \\ &+ \frac{1}{2} \int_{-I}^{\infty} \min\{D^P, a_L + I + y\} f(y) dy \end{aligned} \tag{2}$$

It is intuitive that when the firm learns it is in an unfavorable state (L) it will behave in the same way as if state H had been privately revealed, as long as this means that in both states H and L the firm will issue the new security to invest in the new project. By doing so the firm expects to be priced according to "average" quality future prospects, which is better than what it already knows. Only if the firm rejects the new project when aware of the state being H, will it still prefer to invest in the new project when it faces state L.

[5] Actually there is also a separating equilibrium in which the firm invests only in state L. Daniel and Titman (1995) have pointed out that the separating equilibrium would not be part of a Perfect Sequential Equilibrium. In their experiments Cadsby, Franks and Maksimovic (1990) also find that markets tend to converge to the pooling rather than the separating equilibrium in these cases.

Lemma 3 *Suppose that the realization of cash earnings at time* $t = 1$ *is* $\bar{S}$*. If* $\bar{S} < I - a_L$ *and condition (1) is satisfied, then in equilibrium the firm will undertake the project if the state is* L *(*$q^* = 0$*), but will invest with probability zero if the realization of* $\tilde{a}$ *is state* H *(*$p^* = 1$*). Payoffs expected at* $t = 1$ *by original shareholders of the firm in states* H *and* L *are, respectively,* $V_H^h = a_H + \bar{S}$ *and* $V_L^h = a_L + \bar{S} + b$*.*

The underinvestment problem has been identified. If the amount of external funds required by the new project is sufficiently large, then a firm aware that its quality is greater than average prefers not to undertake a valuable project simply to avoid having to pool and pay too large a risk premium on external funds, due to the lower quality expected by the market.

3.2 The Risk Management Policy at $t = 0$

Given the equilibrium investment decisions derived in lemmas 2 and 3, it is possible now to motivate the optimal risk management policy under various scenarios. Proposition 1 characterizes the conditions under which full hedging is an equilibrium strategy. To do so define first $\tilde{\theta}(S)$: it is the largest level of exposure (in absolute value) for which the firm invests both in states L and H, independently of the realization of $\tilde{X}$. A rigorous definition is in Appendix C.4.

Proposition 1 *If* $S < I - a_L$ *and condition (1) is not satisfied, or if* $S \geq I - a_L$*, then full hedging (*$\theta = 0$*) is an equilibrium strategy. The optimal level of exposure* θ^* *to risk* $\tilde{X}$ *is any element of the interval* $[-\tilde{\theta}(S), +\tilde{\theta}(S)]$*. Moreover at time* $t = 1$ *the firm undertakes the new project with probability 1 (*$p^* = q^* = 0$*), independently of the state being* H *or* L*, and of the realization of* $\tilde{X}$*. The expected value of the firm at time* $t = 0$ *is* $E_{t=0}[V] = \frac{a_H + a_L}{2} + S + b$*.*

Figure C.4(b) provides the intuition for the result. It was established before that when the state revealed to the manager at time $t = 1$ is L, the firm always undertakes the new project. Under the conditions of the proposition, if the firm is fully hedged it invests in the new project, even if in state H. If the firm chose a value of θ beyond the mentioned interval $[-\tilde{\theta}(S), +\tilde{\theta}(S)]$, then for at least one realization of $\tilde{X}$, the firm would prefer to reject the new project, had it been in state H. It is this property that defines $\tilde{\theta}(S)$. The hedging policy is such that first best investment efficiency is preserved. The firm has no incentive to deviate from fully hedging its short lived project that last from $t = 0$ to $t = 1$.

Proposition 2 states the conditions under which the firm does take some risk ($\theta^* \neq 0$) in equilibrium. What some may view as 'speculation' in the financial instrument $\tilde{X}$, results in a level of investment efficiency that is superior to what the firm would have reached had it fully hedged at $t = 0$. Hence this speculation in $\tilde{X}$ is healthy in the sense that it improves investment efficiency, and further induces a reduction in the variance of investment decisions (investment decisions are hedged), across states H and L. First define $\hat{\theta}$: it is the smallest level of risk exposure (in absolute value) for which, in state H, the firm invests in the project for one realization of $\tilde{X}$. A rigorous definition is in appendix C.4(a).

Proposition 2 *If $S < I - a_L$ and condition (1) is satisfied, then in equilibrium the firm will choose some $\theta^* \neq 0$, belonging to the set: $[-(a_L + S), -\hat{\theta}(S)] \cup [\hat{\theta}(S), (a_L + S)]$, when the set is not empty. If this set is empty the firm is indifferent between any level of exposure θ belonging to $[-(a_L + S), a_L + S]$.*

Again Figure C.4(a) gives the intuition for the result. If the firm fully hedges at $t = 0$ the cash earnings it will have at $t = 1$, underinvestment occurs for all realizations of $\tilde{X}$ when the manager learns that the state of future assets is H — the firm prefers to reject a valuable project, to pooling with a worse type at the eyes of the market when raising new financing (Lemma 3). Therefore in equilibrium the firm chooses non-zero exposure to risk $\tilde{X}$. In such case, if $\theta^* > 0$ and in the conditions stated, even if state H is revealed to the manager the firm will undertake the new project when $x = 1$[6]. This fact occurs either because the firm is able to finance the project solely with internally generated cash earnings or riskless debt, or because raising a smaller amount of external funds is not so costly — the firm ends up being pooled in the market when its relative superiority to the value it would be worth in state L is not so important. Hence it is found that some risk-taking is chosen in equilibrium, and this induces superior investment efficiency compared to what it would be if the firm had fully hedged at $t = 0$. However first best efficiency is not attained.
In more detail, given the equilibrium choice of θ^*, three possible cases can occur. I describe them for $\theta^* > 0$, and the analysis symmetrically applies to $\theta^* < 0$:
Case 1: $S + \theta^* \geq I - a_L$. The firm invests both when the state is H or L with probability 1 when $x = 1$; when $x = -1$, the firm still invests

[6] Symmetrically for $\theta^* < 0$ and $x = -1$.

with probability 1 in state L, and in state H it invests with probability 0. So the expected pay-offs to original shareholders at time $t = 1$ are: $V_H^u = a_H + S + \frac{b}{2} > V_H^h$ if the state is H, and $V_L^u = a_L + S + b = V_L^h$ if the state is L. Original shareholders at time $t = 0$ have expected payoff $E_{t=0}[V] = \frac{a_H + a_L}{2} + S + \frac{3}{2}b$.

Case 2: $S + \theta^* < I - a_L$ and condition (3)[7] is satisfied:

$$a_H + S \leq \int_{-I}^{\infty} \max\{0, a_H + I + y - D^\theta\} f(y)dy - \theta \tag{3}$$

where D^θ is defined implicitly as:

$$\begin{aligned} I - S - \theta &= \frac{1}{2}\int_{-I}^{\infty} \min\{D^\theta, a_H + I + y\} f(y)dy + \\ &+ \frac{1}{2}\int_{-I}^{\infty} \min\{D^\theta, a_L + I + y\} f(y)dy \end{aligned} \tag{4}$$

In this case the firm chooses to invest with probability 1 when $x = 1$, even when the state revealed to the manager is H. The expected pay-offs at $t = 1$ to original shareholders are such that: $V_H^u > a_H + S = V_H^h$ in state H, and $V_L^u > a_L + S + b = V_L^h$ in state L. The payoff expected by original shareholders at time $t = 0$ is $E_{t=0}[V] = \frac{a_H + a_L}{2} + S + \frac{3}{2}b$.

Case 3: $S + \theta^* < I - a_L$ and condition (3) is not satisfied. The equilibrium θ can be any value[8]. In such case the firm will always invest with probability 0 when the manager knows that the realization of $\tilde{a}$ is H. The expected pay-offs at $t = 1$ to original shareholders are $V_H^u = a_H + S = V_H^h$ if the state is H, or $V_L^u = a_L + S + b = V_L^h$ if the state is L.

4 What if...

The existence of equity as another source of external finance is also considered, though not altering the direction of results regarding optimal risk management policies, and their relationship with real investment decisions. A different timing for the information asymmetry to arise is also discussed. The optimal risk management policy is briefly examined for the case in which the manager learns the realization of $\tilde{a}$ (H or L) immediately at $t = 0$, before the choice of θ is made. The way results are affected by the existence of correlations between variables is briefly discussed as well. A discussion of

[7]Shown to be less restrictive than the converse of (1) in the proof of Proposition 2.

[8]Satisfying the constraint of no financial distress at the intermediate date $t = 1$.

the possibility of a different production technology and of a different support for $\tilde{a}$ and $\tilde{X}$ is left for the last section of the paper.

4.1 ... New Equity Can be Issued

When the model is generalized to account for access to both debt and equity, results in terms of investment efficiency and risk management policies are still in line with what was found before. Results were presented in more detail with debt as the sole finance source in order to focus on the risk management equilibrium, rather than preference for issuance of one security over the other. The co-existence of both securities does not qualitatively alter the direction of the previous results.

Instead of choosing a probability p of not investing as opposed to raising debt, in state H the firm has now a larger set of possible decisions: with some probability p_E it can invest and raise equity, with probability p_D it can invest and raise debt, and with probability $(1-p_E-p_D)$ it does not invest. Analogously it is possible to define q_E and q_D in state L. For example, in equilibrium the market perceives that if a firm issues equity the probability that it is type H is $\frac{p_E}{p_E+q_E}$, and similarly for debt issues[9].

The crucial point to retain is that: if the firm undertakes the project at $t=1$ in state H, then in state L it will choose the same financing portfolio. The firm will never risk separating from its state H equilibrium financing decision (when the project is undertaken in state H), when it is in state L. On the contrary, if the firm chooses not to invest at all (either with debt or equity) in state H, then in state L the firm still invests and is indifferent between any combination of debt and equity for this effect — no mispricing will take place. Hence the decision taking process at time $t=1$ is not at all different because of adding equity to the model. The risk management policy follows the reasoning of section 3. Only if we considered a different support for the state $\tilde{a}$ would we start finding the possibility of different "types" choosing different financing schemes, as discussed in section 6. Given this discussion I state the result:

Corollary 1 *The possibility of issuing new equity does not alter the direction of results of propositions 1 and 2. Conditions analogous to (1) and (2) can be found for equity.*

[9] Out-of-equilibrium beliefs are examined to support equilibria, as before.

4.2 ... the Asymmetry is revealed immediately at $t = 0$

Inefficiency arises in the model because the manager learns the realization of state $\tilde{a}$ before all other participants in the market. It was assumed that this asymmetry arose only after the risk management policy was undertaken — i.e., at $t = 1$. Consider now the sequence of events of Figure C.3(b) in which the manager learns whether it is in state H or L at time $t = 0$. How is the optimal hedging policy different from section 3? It is important to remember that the realization of cash earnings at $t = 1$ is observable. If the firm receives bad news (L) at $t = 0$, then it will always choose the same level of exposure (θ) to $\tilde{X}$ as if it were in state H. The firm in state L will always try to pool with H's equilibrium choices. Therefore we do find multiple equilibria.

Going back to the conditions of proposition 2, if the firm fully hedges (in both states H and L) investment inefficiency takes place as the project is rejected in state H. As in proposition 2, in equilibrium the firm may choose $\theta^* \neq 0$ (for both H and L). The difference here is that the firm benefits more from "not hedging" in state H the larger θ is, in absolute value. This can be understood from the payoffs given in the proposition. And, hence, this is the equilibrium choice of θ that the firm would prefer in state H. However this equilibrium is not unique. On the contrary, the firm in state L would have preferred still an equilibrium with $\theta^* \neq 0$, but just enough to induce H to invest in some states, so that the mispricing is maximized. The same result in terms of investment efficiency would be reached. The equilibria we find are the same as when the choice of risk exposure θ occurs before firms have superior information, but the preference for each equilibrium differs for each "type" — it is not conceivable that the market can make any inference about the state being H or L, on the basis of the choice of θ.

As in proposition 1, if $S \geq I - a_L$ the firm invests independently of the realization of $\tilde{a}$, and has no reason to deviate from full hedging. But if $S < I - a_L$ and condition (1) is not satisfied — i.e., the firm in state H would invest, and full hedging would be an equilibrium choice had the firm not known the realization of $\tilde{a}$ when choosing θ— we find now that if the firm is aware of the state of $\tilde{a}$ before choosing θ, there are equilibria in which the firm chooses $\theta \neq 0$ in state H[10], and this implies diminished investment efficiency in comparison to what would arise under full hedging, and in comparison to what would arise if the asymmetry of information were revealed later. Overall, under the conditions of proposition 1, the set

[10]Which is also chosen in state L in equilibrium.

of possible equilibria is enlarged and there are now equilibria preferred by H that result in less efficient investment outcomes. Therefore:

Corollary 2 *Under the conditions of proposition 2, the equilibrium choice of θ that the firm prefers in state H is the largest possible in absolute value, and the converse holds true in state L. A linkage between investment efficiency and choice of risk exposure can be established as before. However, in the conditions of proposition 1 it is possible that the firm chooses $\theta \neq 0$ and does not always invest when the state is H, even if it would have invested with probability 1 if $\theta = 0$. Thus the link between enhanced investment efficiency and risk management policy is broken.*

One could also think of intermediate situations in which with some probability firms either learn state H or L at time $t = 0$ or at time $t = 1$. Again we do find pooling in the choice of θ for fims in state H and L. Conclusions are not changed in any interesting way.

4.3 ... there are Correlations between variables

If the cash flows on the new project, $\tilde{y}$, were correlated with either assets in place $\tilde{a}$ or with the cash flows of the financial instrument $\tilde{X}$, then results would not be qualitatively modified. Constraints analogous to (1) can be easily found, and it is straightforward to implement the same analysis.

5 Unconstrained Risk Exposure and Financial Distress at the Investment Date

Consider now the possibility of financial distress at the intermediate date $t = 1$. This can occur if the support of $\tilde{X}$ is $(-\infty, +\infty)$ and $\theta \neq 0$, or if θ is sufficiently large (in absolute value) even when the support is not $\Re$. The main difference from the previous analysis stems from the existence of a risk or default premium required by risk neutral competitive traders of the financial instrument $\tilde{X}$. When losses in derivatives are very large the firm is not able to finance them at $t = 1$. But still the firm has the ongoing project $\tilde{a}$, and faces the valuable new investment opportunity. A bargaining situation arises between three parties: (i) traders, who are the counterparty on the financial contract $\tilde{X}$, want to receive what they are due at $t = 1$; (ii) original shareholders want to maximize the value of their residual claim in the firm; (iii) investors in a competitive market are aware of the firm's distress at $t = 1$ and also of its valuable new project that requires external financing in order to go ahead.

I consider the following workout may occur. The firm may still raise debt at time $t = 1$ in order to finance the project, and this new debt will take priority over any other claim on the firm. At time $t = 2$ debtholders, who again are assumed to be risk neutral and competitive, are the first to be paid. From what is left (if anything) the debt (fully or partly) due to losses in trading $\tilde{X}$ is also repaid; finally if anything is left it accrues to original shareholders of the firm. This seems to be a reasonable assumption in this context, and close to "debtor in possession" financing in distressful situations, even if the absolute priority rule is violated as new debtholders are paid at $t = 2$ before traders who should have been repaid at $t = 1$.

The claim that this workout is reasonable, or that it arises naturally, is based on the fact that all parties are better off with it than without financing the new project at all. The total value to split is higher at $t = 2$, and so it pays for the traders to wait, accept priority given to new debtors and end up with a higher payoff in expected terms than otherwise. As the market for debt is competitive, in equilibrium new debtholders must expect to break-even. Therefore it is possible that shareholders themselves, when undertaking the new project, will receive strictly a positive pay-off in some states of the project at $t = 2$. Furthermore what I find in an example in Appendix A is that if debt is not given priority, then it is not issued at all — the new project is not undertaken, and all parties are worse off. If, however, other solutions to the bargaining obtain, then these results should not apply.

Both the face value of debt and the risk premium on $\tilde{X}$ are derived endogenously and simultaneously. This makes a full derivation of equilibria under an interval as a support for $\tilde{X}$ impractical. In order to obtain a closed-form solution to the problem, in which the risk premium on $\tilde{X}$ — conditional on θ — the face value of debt, investment decisions, and risk management equilibria are determined simultaneously, I resort to the example in Appendix A. There I compare the choice between fully hedging cash earnings by choosing $\theta = 0$ at time $t = 0$, and unhedging by choosing some $\theta \neq 0$ that is sufficiently large to cause distress at $t = 1$ — and large enough so that losses in $\tilde{X}$ are greater than a_H.

From the example, had the firm not been aware of the state (H or L) at time $t = 0$ it would actually choose to 'speculate' in $\tilde{X}$ in equilibrium. The intuition behind the result is that in distressful situations the firm will still finance the new project whether the state is H or L, as in state H shareholders have nothing to lose when going ahead with the project. This fact results in a lower risk premium on instrument $\tilde{X}$, and consequently, in increased investment efficiency. First best investment efficiency is restored, and this is where the value of "healthy speculation" comes from. The outcome of this example is perhaps controversial. When 'speculation' in financial instrument $\tilde{X}$ leads to financial distress the workout considered leads to first best investment efficiency[11]. The example serves as proof of the result:

Proposition 3 *As long as underinvestment would occur if $\theta = 0$, it is possible that the firm chooses a large risk exposure so that it faces financial distress in some states. This fact can lead to overall improved investment efficiency.*

Unlike before (proposition 2), it is in state L only that the firm benefits from unhedging. Therefore, if aware at $t = 0$ of the state being H or L, the firm would prefer to fully hedge rather than to pool later when raising debt in distressful situations. And this is so even when balanced with the benefits of investing with riskless debt in the good states of nature. But again we face multiple equilibria, as L and H pool in the choice of θ. The relative benefits of unhedging in state H or L are reversed in the presence of financial distress.

Another interpretation for the outcome of this workout lies on the fact that because the firm is constrained not to raise or spend cash at $t = 0$[12], the use of a risk management device that produces randomness at the investment date may lead the firm to overcome the inefficiency caused by its superior information. Because there is randomness in cash earnings, for some realizations the firm can invest efficiently issuing a fairly priced security; whereas for low realizations of cash earnings the bargaining process associated with distress is "as if" the firm has sold its assets-in-place (which are the source of asymmetry) when they are of no value to current shareholders. This may also be viewed as a change in ownership and control to a party that has no superior information and will therefore take efficient investment decisions.

If exogenous costs of financial distress were to be explicitly considered, these results should be qualified, but the arguments given above still hold. A final remark on the source of financing. In the presence of distress and

[11] Of course for a different support of $\tilde{X}$, efficiency could still be improved, but first best would not necessarily be reached.

[12] Otherwise the Myers and Majluf problem would not exist.

for the workout described giving priority to new claimants, debt is clearly advantageous.

6 Concluding Remarks

Some literature relates to the model of this paper. Myers and Majluf (1984) and Froot, Scharfstein and Stein (1993) were already mentioned. Mello and Parsons (1995,1996) also provide an example in which liquidity and cash flow timing problems associated with hedging lower the firm value absolutely. The problem they deal with is not of the same nature as that of this paper. Rather than an asymmetric information setup they have a two stage investment decision where the main source of inefficiency are bankruptcy costs.

Degeorge, Moselle and Zeckhauser (1996) consider an adverse selection problem, in which managers of two different firm types will choose the level of risk of their firms' earnings, which are observed by investors and used to make inferences about the firms' types. Although their framework is a risk neutral economy with risk neutral agents, they assume exogenously a functional relationship between expected earnings and their variance (concave, increasing up to a point and then decreasing), so that a trade off between risk and expected return actually exists. Here I examine an investment decision for which ex ante risk management is relevant; in their model, risk management matters first of all because it *is* the choice of average earnings, and also because firms are aware of their types when choosing risk exposures. They show that if the choice of risk is observable there is pooling behavior, among which better firms would rather have lower variance (more revealing), as opposed to worse firms. The issue of limited liability is disregarded.

Without focusing on the role of risk management, others have attempted to find solutions to the inefficiencies encountered in Myers and Majluf (1984). A fully satisfactory solution to the investment inefficiency problem has not yet been found. In a contracting perspective Dybvig and Zender (1991 argued that the inefficiency lies in the objective function chosen for the manager. However Persons (1994)[13] shows that the contractual solution suggested by Dybvig and Zender (1991) is not robust to renegotiation, whereas that in Myers and Majluf (1984) is dynamically consistent. The focus of most of the literature has been on security design. Examples are

[13]With whom this paper shares the support for the project's cash flows.

Noe (1988)[14], Nachman and Noe (1994), DeMarzo and Duffie (1996),Constantinides and Grundy (1989), Brennan and Kraus (1987), Stein (1992) and Rebello (1995). All these concentrate efforts in determining financing strategies that enhance investment efficiency. The issue of corporate risk management discussed in this paper does not receive their attention. But, as pointed out before, the use of a risk management instrument can be interpreted as part of the security design program of the firm (as any financial decision is, for that matter).

This paper proposes a rationale for corporate risk management in an asymmetric information framework. Using the basic set-up of Myers and Majluf (1984), I introduce the possibility of a firm managing its exposure to some risk underlying the realization of the firm's cash earnings at an intermediate date in which a real investment decision is made. The assumptions of this model are subject to the same criticism that Myers and Majluf (1984) and subsequent literature faces. As consolation one may argue that there is no reason to believe that this set of assumptions is less reasonable than others. In our case even if we considered a different objective function for the manager in which maximizing passive current shareholders' is not the ony ingredient, as long as it is there to some extent, our results may be mitigated, but are present.

The discussion of the results' robustness to some of the assumptions of the model was left to this last section. What happens if, instead of a simple binary variable, the support of the assets in place is something else? For instance suppose that there are three possible realizations of $\tilde{a}$ (a_L, a_M and a_H). If there is no asymmetry at the time of choosing θ, the only possibility of separation or "signaling" is having the intermediate type issuing a different security from what the other two types do. For this line of results we can revert to Noe (1988). This does not alter the conclusions about the value of the choice of risk exposure θ. If the firm is already aware of its type when choosing θ, then it is possible that we find in equilibrium the intermediate type choosing a level of θ that is different from what the others choose. Still we will always find in equilibrium pooling in the choice of θ by the extreme types. This happens if we consider also a continuous support $[\underline{a}, \bar{a}]$for $\tilde{a}$. If anything, the choice of risk management policy becomes even more important and revealing.

Considering a different support for the hedgeable risk $\tilde{X}$ does not qualitatively change conclusions. If we consider a bounded interval as support,

[14] Using the same definition of equilibrium considered here.

we now define for different values of θ what is the critical value of $\tilde{X}$ above (or below) which the firm invests in state H. And the same analysis as before is carried out. The case of unbounded $\tilde{X}$ was mentioned in section 5.

The model considers an investment project of fixed scale, allowing for its associated cash flows to be random. It is obvious that considering a different production function will involve the same type of results in terms of relevance of the choice of θ, as long as there is some sort of discontinuity, or step in this function. It may be tempting to analyse a risk management and investment equilibrium in the presence of a continuous production function. This poses several problems, particularly as we are considering random payoffs and need to impose some structure on the riskiness of the project. For this reason we are not able to derive a general result that applies to "all" production functions — which shows how crucial and detailed this analysis must be when conducted in real life situations. I considered several examples of concave production functions and found the same richness of results in terms of optimal choices of θ and investment policy (when scale allows for separation of different types). For example, for a function $E[f(I)] = -\frac{1}{400}I^2 + 0.54I$ with equiprobable cash flows $(I+2f, -I)$, $a_L = 60$, and $a_H = 200$, I find that if equity is the only source of financing — as in Myers and Majluf (1984) — a firm will be better off if it does not fully hedge its intermediate cashflows when the hedged level is -50; but if the firm knows its type at the time of choosing θ, then in state H the firm would have actually preferred the full hedge equilibrium. With debt, again we find complex results in which the choice of θ is relevant, and speculation can be found in equilibrium[15]. Concerns regarding multiple equilibria will still be present, as expected.

Summarizing results, I start with a constraint of non-financial distress at the intermediate date in which the real investment decision is made. By doing so I find that the main contributions of the paper are the following: (1) Corporate risk management is not indifferent to the firm, and influences investment efficiency; (2) there are equilibria in which the firm chooses to "not hedge" its internally generated funds; (3) this apparently speculative behavior is motivated by investment concerns. "Not hedging" enhances efficiency; (4) the importance of a clear disclosure of risk exposures is reinstated; (5) the cost of external financing is endogenously derived; (6) Results hold for the basic case of debt issues, but I also consider equity as a source of external finance. (7) The direction of these results is not affected by having the

[15] More details can be obtained from the author.

cash flows of the new project correlated with either the source of risk under management or the manager's private information; (8) The timing of the revelation of the manager's superior private information influences the equilibrium exposure to risk — the link between risk exposure and investment efficiency is not so clear when the asymmetry is revealed early.

When financial distress is allowed to occur at the investment date, and according to assumptions made about a possible workout, speculation can still be chosen in equilibrium, and this can be "healthy", as it is associated with enhanced investment efficiency. I find that: (1) If the firm has no proprietary information at the time of choosing its risk exposure, then it chooses to 'speculate', and this induces enhanced investment efficiency. Even first best efficiency can be restored. The result is perhaps controversial, as speculation induces investment efficiency when the firm goes through financial distress. This opens way for future research. Another interpretation for this result is that in distressful situations the firm's assets-in-place become worthless to current shareholders. Their manager's superior information is of no value, and investment decisions will be taken according to efficiency criteria. (2) Perhaps surprisingly, if the firm receives private information earlier before choosing the risk management policy, then it would prefer to hedge when its information is favorable. This happens to avoid sharing financial distress costs with "worse types", as the market always perceives a pooled average quality firm. So hedging could take place in equilibrium. Again the strengths at stake differ from the model when no distress is allowed at the investment date and the financial instrument used to manage risk carries no default premium.

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Appendix A - Example: Financial Distress at $t = 1$

Consider the following setup:

$$\begin{array}{lll} S = 0 & a_H = 150 & I = 200 \\ \tilde{S} = \theta(\tilde{X} - r(\theta)) & a_L = 50 & b = 10 \end{array}$$

where $r(\theta)$ is the risk premium required by the dealer, on $\tilde{X}$(can be traded OTC). Define also $R(\theta) = \theta r$ as the total risk premium given exposure θ. Assume that $\tilde{X}$ can assume values 1 or -1 with probability 1/2 each. The investment project has expected NPV of 10, as it can assume with equal probability two values: 120 or -100. For simplicity I am considering that the natural exposure to $\tilde{X}$ is zero so that θ is fully due to trading in the financial instrument.

(1) If there is full hedging, $\theta = 0$, and cash earnings at $t = 1$ are 0. In order to invest firms must raise $I = 200$. The face value of debt D, given break-even for a competitive bank is such that:

$$\begin{aligned} 200 &= 1/4 \min(D, 470) + 1/4 \min(D, 250) + 1/4 \min(D, 370) + 1/4 \min(D, 150) \\ D &= 216.\bar{6} \end{aligned} \tag{5}$$

In state H the firm would not invest[16] because:

$$V_H(d_1) = 1/2(470 - 216.\bar{6}) + 1/2(250 - 216.\bar{6}) < 150 = V_H(d_0)$$

[16] I checked that the same would happen under equity financing.

Hence in equilibrium: $p^* = 1$ and $q^* = 0$, with $V_H = 150$ and $V_L = 60$, as the face value of debt is finally:

$$\begin{aligned} 200 &= 1/2 \min(D^L, 370) + 1/2 \min(D^L, 150) \\ D^L &= 250 \end{aligned}$$

(2) Suppose now that $\theta = 160$ and so distress might occur at $t = 1$. In that case it is assumed that firms may still raise debt, which will have priority at $t = 2$ over the loss in derivatives with the trader. With equal probability $\tilde{S}$ at $t = 1$ can be either $160 - R$ or $-160 - R$, where R is derived endogenously, so that the dealer breaks even in expected terms in a competitive market. R must satisfy:

$$\begin{aligned} 0 &= 1/2(-160 + R) + 1/8 \min(160 + R, 470 - D) + 1/8 \min(160 + R, 250 - D) + \\ &+ 1/8 \min(160 + R, 370 - D) + 1/8 \min(160 + R, \max(150 - D, 0)) \\ R &= 58.\bar{6} \qquad (6) \end{aligned}$$

In the "bad" state ($x = -1$) debt to the trader at $t = 1$ is $S = -160 - 58.\bar{6} = -218.\bar{6}$. And $D = 216.\bar{6}$, hence:

$$\begin{aligned} V_L(\theta = 160; x = -1) &= 1/2 \max(0, 370 - 216.\bar{6} - 218.\bar{6}) + \\ &+ 1/2 \max(0, 150 - 216.\bar{6} - 218.\bar{6}) \\ &= 0 \qquad (7) \\ V_H(\theta = 160; x = -1) &= 1/2 \max(0, 470 - 216.\bar{6} - 218.\bar{6}) + \\ &+ 1/2 \max(0, 250 - 216.\bar{6} - 218.\bar{6}) \\ &= 17.\bar{3} \qquad (8) \end{aligned}$$

In the "good" state ($x = 1$), $S = 160 - 58.\bar{6} = 101.\bar{3}$ at $t = 1$. To invest in the project risk-free debt of $98.\bar{6}$ can be issued. In this case:

$$\begin{aligned} V_L(\theta = 160, x = 1) &= 1/2(370 - 98.\bar{6}) + 1/2(150 - 98.\bar{6}) = 161.\bar{3} \qquad (9) \\ V_H(\theta = 160, x = 1) &= 1/2(470 - 98.\bar{6}) + 1/2(250 - 98.\bar{6}) = 261.\bar{3} \qquad (10) \end{aligned}$$

Overall for both types:

$$\begin{aligned} \bar{V}_L(\theta = 160) &= 1/2(161.\bar{3} + 0) = 80.\bar{6} > 60 = V_L(\theta = 0) \qquad (11) \\ \bar{V}_H(\theta = 160) &= 1/2(261.\bar{3} + 17.\bar{3}) = 139.\bar{3} < 150 = V_H(\theta = 0) \qquad (12) \end{aligned}$$

If firms are aware of their types at $t = 0$ type H chooses $\theta = 0$ and type L pools. But if the firm is unaware of its type at $t = 0$, then it prefers to speculate facing possibly distress at $t = 1$, but investing always:

$$\begin{aligned} E_{t=0}(V \mid \theta = 0) &= 1/2(150 + 60) = 105 \qquad (13) \\ E_{t=0}(V \mid \theta = 160) &= 1/2(139.\bar{3} + 80.\bar{6}) = 110 \qquad (14) \end{aligned}$$

This example is in fact more general. For cases in which distress is even more serious, $\theta + R > 470 - D$, it is found that $R = \theta - 110 = \theta - E_{t=0}(V \mid \theta)$. When $x = -1$, the firm gets 0 in both H and L. But when $x = 1$, the realization of $\tilde{S}$ is $\theta - R = 110$. The remaining 90 can be raised risklessly. Hence overall $V_H = (270 + 0)/2$ and $V_L = (170 + 0)/2$. Finally at $t = 0$ the firm expects pay-off $E_{t=0}(V \mid \theta \neq 0) = 110 > 105 = E_{t=0}(V \mid \theta = 0)$.

Appendix B - Proofs

B.1. Proof of Lemma 1:

Investors can identify the state H or L at $t = 1$ as well as the firm. If firm i ($i = H, L$) has internal funds $\bar{S} < I$ and requests external financing of $I - \bar{S}$, investors provide $I - \bar{S}$ knowing that they break even in expected terms because the investment project has a positive expected NPV. Firm type i compares the expected payoff of not investing, $a_i + \bar{S}$, to that of investing, $a_i + \bar{S} + b$.
Risk management of $\tilde{S}$ is irrelevant because the expected value to original shareholders of firm i when hedging is $a_i + S + b$; if choosing $\theta \neq 0$, the expected value of investing is $E_{t=0}[a_i + S + \theta\tilde{X} + b] = a_i + S + \theta E_{t=0}(\tilde{X}) + b = a_i + S + b$.
The type of financing is irrelevant because the firm type is observed, which implies that mispricing never occurs. If firm type i issues equity of $I - \bar{S}$, the market correctly values the firm at $P = a_i + b + \bar{S}$ (net of the external financing, valued at $I - \bar{S}$), and type i clearly chooses to invest. If firm type i issues debt to finance the project, the face value of the debt D^i is defined implicitly as:

$$I - \bar{S} = \int_{-I}^{\infty} \min\{D^i, a_i + I + y\} f(y)dy$$

$$I - \bar{S} = \int_{-I}^{D^i - a_i - I} (a_i + I + y)f(y)dy + \int_{D^i - a_i - I}^{\infty} D^i f(y)dy \qquad (15)$$

and the firm always invests, independently of the state being H or L because:

$$\int_{-I}^{\infty} \max\{0, a_i + I + y - D^i\} f(y)dy > a_i + \bar{S}$$

$$\int_{D^i - a_i - I}^{\infty} (a_i + I + y)f(y)dy - \int_{D^i - a_i - I}^{\infty} D^i f(y)dy > a_i + \bar{S} \qquad (16)$$

Together with the definition of D^i in (15), (16) implies that:

$$\int_{D^i - a_i - I}^{\infty} (a_i + I + y)f(y)dy - I + \bar{S} + \int_{-I}^{D^i - a_i - I} (a_i + I + y)f(y)dy > a_i + \bar{S}$$

simplifying to $b \equiv E_{t=1}(\tilde{y}) > 0$, which holds by definition. If $\bar{S} \geq I$ the firm always invests independently of learning H or L, without the need for external financing. This completes the proof. □

B.2. Proof of Lemma 2: Direct from proof of lemma 3.

B.3. Proof of Lemma 3:

1. In state L the firm chooses q^* in order to maximize:

$$V_L(q \mid p, D) = (1-q)\int_{-I}^{\infty} \max\{0, a_L + I + y - D\} f(y)dy + q(a_L + \bar{S}) \quad (17)$$

subject to

$$I - \bar{S} = \frac{1-p}{2-p-q}\int_{-I}^{\infty} \min\{D, a_H + I + y\} f(y)dy + \\ + \frac{1-q}{2-p-q}\int_{-I}^{\infty} \min\{D, a_L + I + y\} f(y)dy \quad (18)$$

In equilibrium, totally differentiating (17) with respect to q and D:

$$dV_L = [a_L + \bar{S} - \int_{D-a_L-I}^{\infty} (a_L + I + y - D) f(y)dy]dq + \\ -(1-q)[1 - F(D - a_L - I)]dD \quad (19)$$

By also totally differentiating (18) I obtain:

$$\{(1-p)[1 - F(D - a_H - I)] + (1-q)[1 - F(D - a_L - I)]\}dD \\ = [\int_{-I}^{\infty} \min\{D, a_L + I + y\} f(y)dy - (I - \bar{S})]dq \quad (20)$$

Hence:

$$dD = \frac{\int_{-I}^{\infty} \min\{D, a_L + I + y\} f(y)dy - (I - \bar{S})}{(1-p)[1 - F(D - a_H - I)] + (1-q)[1 - F(D - a_L - I)]} dq \quad (21)$$

Therefore, in equilibrium $\frac{\partial D}{\partial q} > 0$.

2. Putting (21) together with (19) results in:

$$dV_L = [a_L + \bar{S} - \int_{D-a_L-I}^{\infty} (a_L + I + y - D) f(y)dy]dq + \\ -(1-q)[1 - F(D - a_L - I)]\frac{\partial D}{\partial q}dq \quad (22)$$

Hence $\frac{\partial V_L}{\partial q} < 0$. Finally: $q^* = 0$, if this is an equilibrium strategy, which I check below.

(i) $q^* = 0$ is an equilibrium strategy in state L, as there is no incentive to deviate to any other strategy $w \in (0, 1]$, given any $p \in [0, 1]$, and that the market has rationally formed beliefs:

$$\int_{-I}^{\infty} \max\{0, a_L + I + y - D^e\} f(y)dy \geq w(a_L + \bar{S}) +$$
$$(1-w) \int_{-I}^{\infty} \max\{0, a_L + I + y - D^e\} f(y)dy \tag{23}$$

with D^e defined implicitly as:

$$I - \bar{S} = \frac{1-p}{2-p} \int_{-I}^{\infty} \min\{D^e, a_H + I + y\} f(y)dy +$$
$$\frac{1}{2-p} \int_{-I}^{\infty} \min\{D^e, a_L + I + y\} f(y)dy \tag{24}$$

(23) can be rewritten as:

$$\int_{-I}^{\infty} \max\{0, a_L + I + y - D^e\} f(y)dy \geq a_L + \bar{S} \tag{25}$$

(ii) Define D^L as the face value of debt in case the market believes that the issuing firm is type L with probability 1:

$$I - \bar{S} = \int_{-I}^{\infty} \min\{D^L, a_L + I + y\} f(y)dy \tag{26}$$

Note that $D^L \geq D^e$, for $p \leq 1$, because $a_H + I + y > a_L + I + y$ $\forall y$; this implies that $\min\{D, a_H + I + y\} \geq \min\{D, a_L + I + y\}$ $\forall D$. Since (26) holds with D^L, it must be true that if $D^e = D^L$, in (24) we get an inequality of the sort $I - \bar{S} \geq \frac{1-p}{2-p} \int_{-I}^{\infty} \min\{D^e, a_L + a + I + y\} f(y)dy + \frac{1}{2-p} \int_{-I}^{\infty} \min\{D^e, a_L + I + y\} f(y)dy$. Hence for (24) to hold it must be true that $D^L \geq D^e$.

(iii) From the proof of Lemma 1 it is known that $\int_{-I}^{\infty} \max\{0, a_L + I + y - D^L\} f(y)dy > a_L + \bar{S}$. Since $D^L \geq D^e$: $a_L + I + y - D^L \leq a_L + I + y - D^e$, $\forall y$. Thus $\int_{-I}^{\infty} \max\{0, a_L + I + y - D^L\} f(y)dy \leq \int_{-I}^{\infty} \max\{0, a_L + I + y - D^e\} f(y)dy$. Finally, because from Lemma 1 it is known that $\int_{-I}^{\infty} \max\{0, a_L + I + y - D^L\} f(y)dy > a_L + \bar{S}$; this implies that $\int_{-I}^{\infty} \max\{0, a_L + I + y - D^e\} f(y)dy > a_L + \bar{S}$. Therefore $q^* = 0$ is an equilibrium strategy for type L. I show next that it is unique.

(iv) $q \in (0, 1)$ is not an equilibrium strategy when $p \in [0, 1]$, because L has incentive to deviate to $q = 0$:

$$\int_{-I}^{\infty} \max\{0, a_L + I + y - D\} f(y)dy \geq q(a_L + \bar{S}) +$$
$$(1-q) \int_{-I}^{\infty} \max\{0, a_L + I + y - D\} f(y)dy$$

which reduces to:

$$\int_{-I}^{\infty} \max\{0, a_L + I + y - D\} f(y)dy \geq a_L + \bar{S} \tag{27}$$

where D is defined implicitly by:

$$I - \bar{S} = \frac{1-p}{2-p-q} \int_{-I}^{\infty} \min\{D, a_H + I + y\} f(y)dy + \frac{1-q}{2-p-q} \int_{-I}^{\infty} \min\{D, a_L + I + y\} f(y)dy \tag{28}$$

So that both (26) and (28) hold, it must be true that $D \leq D^e$. As before, this implies that $a_L + I + y - D \geq a_L + I + y - D^e$, $\forall y$. And $\max\{0, a_L + I + y - D\} \geq \max\{a_L + I + y - D^e\}$. We already found that $\int_{-I}^{\infty} \max\{0, a_L + I + y - D^e\} f(y)dy > a_L + \bar{S}$. Therefore: $\int_{-I}^{\infty} \max\{0, a_L + I + y - D\} f(y)dy > a_L + \bar{S}$, and $q \in (0, 1)$ cannot hold in equilibrium.

(v) $q = 1$ also can not hold in equilibrium. If $p \in [0, 1)$, L would deviate to $q = 0$ as: $\int_{-I}^{\infty} \max\{0, a_L + I + y - D\} f(y)dy > a_L + \bar{S}$, for D defined implicitly by $I - \bar{S} = \int_{-I}^{\infty} \min\{D, a_H + I + y\} f(y)dy$. The argument is exactly as above. If $p = 1$, out of equilibrium beliefs must be considered in case L deviates, and actually issues debt. If the market believes that the deviating firm is type H with probability 1[17] then again L deviates from $q = 1$ to $q = 0$[18],and it is not an equilibrium. If the market assumes that the firm that deviates is type L with probability 1, then the firm again deviates to $q = 0$, for the same arguments as (i)–(iii).

3. In state H the firm chooses p^*, conditional on $q^* = 0$, according to:

$$p^* = \arg\max_p \{p(a_H + \bar{S}) + (1-p)J(p)\} \qquad s.t. \tag{29}$$

$$I - \bar{S} = \frac{1-p}{2-p} \int_{-I}^{\infty} \min\{D, a_H + I + y\} f(y)dy + + \frac{1}{2-p} \int_{-I}^{\infty} \min\{D, a_L + I + y\} f(y)dy \tag{30}$$

where:

$$J(p) = \int_{-I}^{\infty} \max\{0, a_H + I + y - D\} f(y)dy \tag{31}$$

4. Totally differentiating (31) yields:

$$dJ = -[1 - F(D - a_H - I)]dD \tag{32}$$

[17] The most favorable case.

[18] Exactly like when $p \in [0, 1)$.

5. By totally differentiating (30) I find:

$$\frac{\partial D}{\partial p} = \frac{\int_{-I}^{\infty} \min\{D, a_H + I + y\} f(y)dy - (I - \bar{S})}{(1-p)[1 - F(D - a_H - I)] + [1 - F(D - a_L - I)]} > 0 \tag{33}$$

From (34) I learn that $\frac{\partial J}{\partial p} < 0$:

$$dJ = -[1 - F(D - a_H - I)] \frac{\partial D}{\partial p} dp \tag{34}$$

$p^* = 1$ is an equilibrium strategy because for any $w \in [0, 1)$:

$$a_H + \bar{S} > w(a_H + \bar{S}) + (1 - w)J(1) \tag{35}$$

is implied by condition (1)[19] which is satisfied by assumption in Lemma 3. Also $p \in [0, 1)$ cannot hold as equilibria, since there is an incentive to deviate to $p = 1$:

$$a_H + \bar{S} > p(a_H + \bar{S}) + (1 - p)J(p) \tag{36}$$

It was shown before that $J(0) > J(p)$, for $p \in (0, 1]$. Therefore $a_H + \bar{S} > J(p)$. Finally $q^* = 0$ and $p^* = 1$ is the unique NSE of the game. The proof is symmetrical for the case when condition (1) is not satisfied (Lemma 2). This completes the proof. □

B.4. Proof of Proposition 1: Direct from proof of Lemma 2 and Figure C.4.

B.5. Proof of Proposition 2:

- Given that $q^* = 0$, it was shown before that firm H chooses $p = 1$ if condition (1) is satisfied when its cash earnings are S. When cash earnings are not hedged, they can assume the values $S - \theta$ or $S + \theta$. When cash earnings are $S - \theta$ the same arguments of the previous proof hold and $q^* = 0$ and $p^* = 1$ are the equilibrium strategies. When liquid assets are $S + \theta$, again it must hold that $q^* = 0$; but it is possible that firm H will invest ($p = 0$). Firm H will choose $p = 0$ in equilibrium if the following condition is met, for $w \in (0, 1]$:

$$\int_{-I}^{\infty} \max\{0, a_H + I + y - D^{\theta}\} f(y)dy \geq w(a_H + S + \theta) + (1 - w) \int_{-I}^{\infty} \max\{0, a_H + I + y - D^{\theta}\} f(y)dy$$

which reduces to:

$$J(0) \equiv \int_{-I}^{\infty} \max\{0, a_H + I + y - D^{\theta}\} f(y)dy \geq a_H + S + \theta \tag{37}$$

[19] $a_H + \bar{S} > J(0)$.

where D^θ is defined implicitly as:

$$I - S - \theta = \frac{1}{2}\int_{-I}^{\infty} \min\{D^\theta, a_H + I + y\} f(y)dy + \\ + \frac{1}{2}\int_{-I}^{\infty} \min\{D^\theta, a_L + I + y\} f(y)dy \quad (38)$$

Next I show that as cash earnings increase, the condition that determines whether $p = 0$ can hold in equilibrium (condition (1) when slack is S) becomes less restrictive.

- By totally differentiating this condition I show that:

$$dD^\theta = -\frac{2}{[1 - F(D^\theta - a_H - I)] + [1 - F(D^\theta - a_L - I)]} d\theta \quad (39)$$

Hence $\frac{\partial D^\theta}{\partial \theta} < 0$ and $| \frac{\partial D^\theta}{\partial \theta} | > 1$.

- I now define:

$$B \equiv J(0) - (a_H + S + \theta) \quad (40)$$

and totally differentiate (40), obtaining:

$$dB = -[1 - F(D^\theta - a_H - I)]dD^\theta - d\theta \quad (41)$$

Knowing $\frac{\partial D^\theta}{\partial \theta}$ from (39) and replacing in (41), I get:

$$dB = [\frac{2[1 - F(D^\theta - a_H - I)]}{[1 - F(D^\theta - a_H - I)] + [1 - F(D^\theta - a_L - I)]} - 1]d\theta \quad (42)$$

Finally, because $F(D^\theta - a_L - I) \geq F(D^\theta - a_H - I)$, it is proved that $\frac{\partial B}{\partial \theta} > 0$. Therefore, as θ increases, condition (37) — analogous to (1) — that determines whether firm H will invest, becomes less restrictive. So, if θ is large enough, it is possible that when cash earnings are $S + \theta$ firm H invests — i.e., condition (3) is satisfied. It has been established that $p^* = 0$ is an equilibrium strategy when slack is $S + \theta$ and (3) is satisfied. Other strategies might hold in equilibrium. Suppose $p \in (0, 1]$. It is possible that this holds in equilibrium as long as there is no alternative strategy $w \neq p$, where $w \in [0, 1]$, and $J(p) \geq a_H + S + \theta$. But still $J(0) > J(p)$ for $p \in (0, 1]$. Hence in state H the firm chooses $p^* = 1$.

- Summarizing: (i) if θ is large enough so that $S + \theta \geq I$, case 1 of the proposition obtains. And proof is obvious; (ii) when θ is such that $S + \theta < I$ and condition (3) is satisfied[20], case 2 of the proposition occurs; (iii) when condition (3) cannot be reached, the firm in state H still never invests. This completes the proof.□

[20]Which was just shown to be possible, if (1) is also satisfied.

Appendix C — Figures

Figure C.1: Time Line of Events

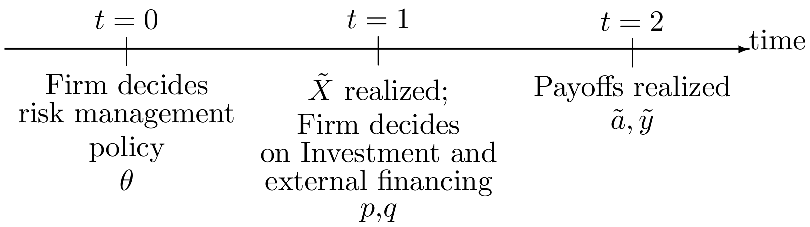


Figure C.2: The Firm as the product of three projects

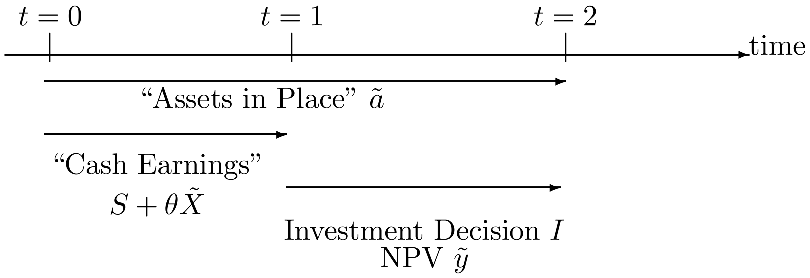
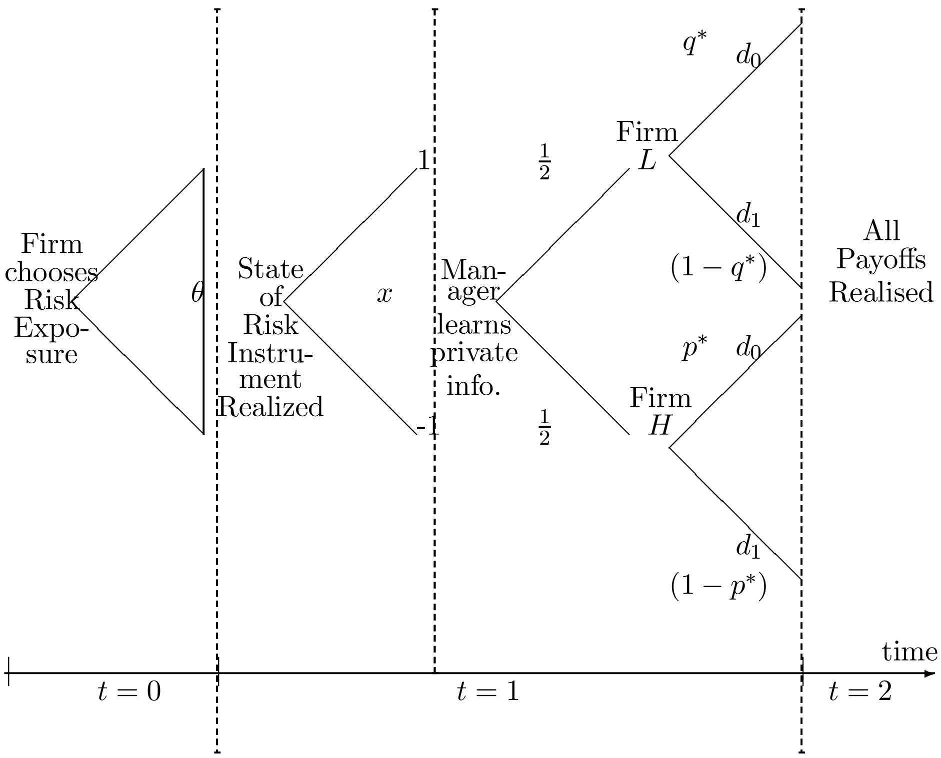


Figure C.3: Sequence of Events of game starting at $t = 0$

(a) Assuming the State H or L is revealed to the manager at $t = 1$



(b) Assuming the State H or L is revealed at $t = 0$

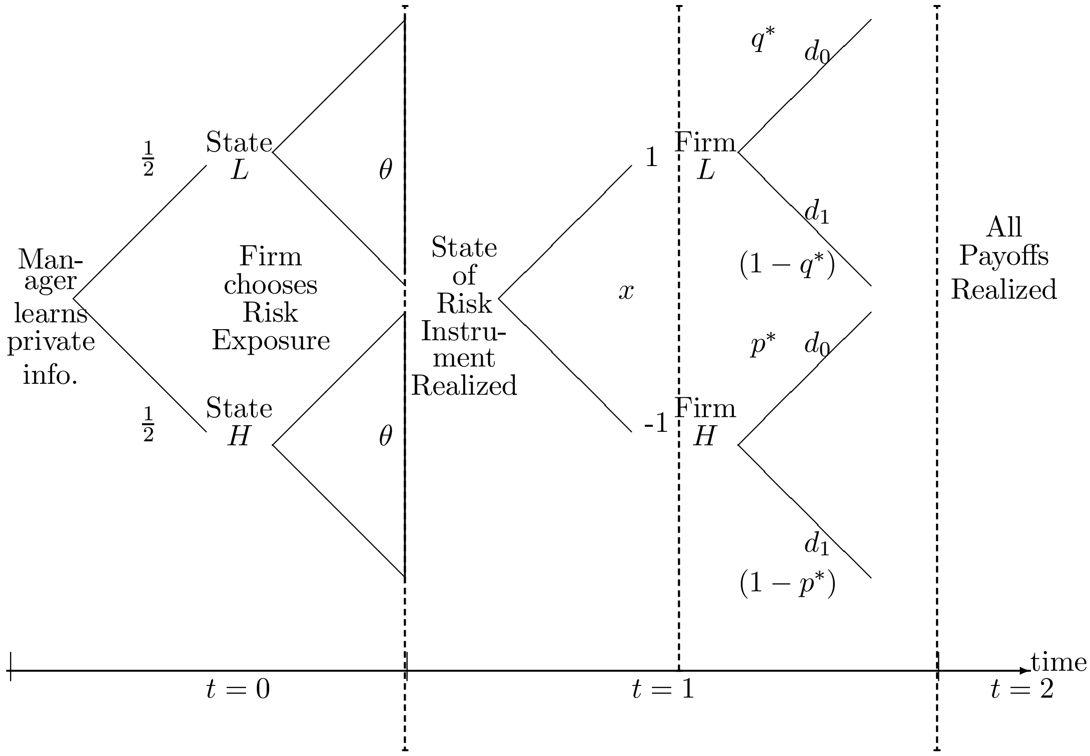
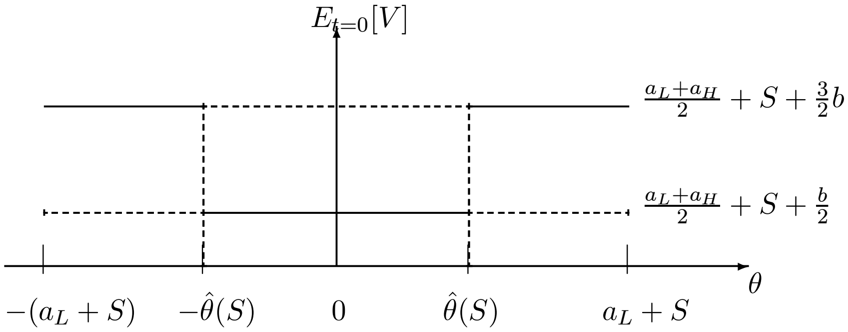


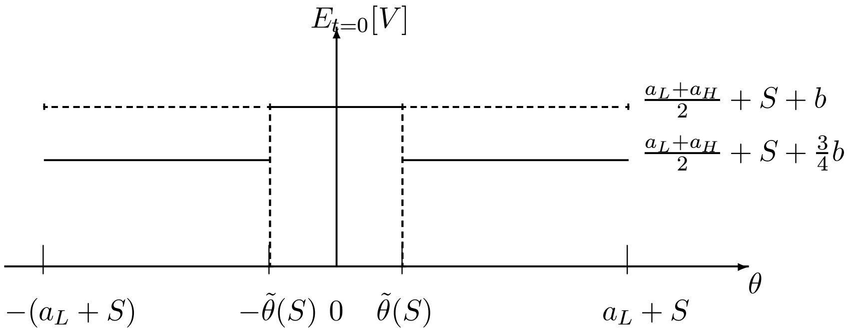
Figure C.4: Expected value of the firm at $t = 0$ depending on risk exposure θ and hedged value of cash earnings S

(a) When $S < I - a_L$ and condition (1) is satisfied



$$
\begin{aligned}
\hat{\theta}(S) \quad &\equiv \quad \inf\{\theta \mid \theta \geq 0; a_H + S \leq \int_{-I}^{\infty} \max(0, a_H + I + y - D^{\theta}) - \theta; \\
&\qquad I - S - \theta = \frac{1}{2} \sum_{i=L,H} (\int_{-I}^{\infty} \min(D^{\theta}, a_i + I + y) f(y) dy)\}
\end{aligned}
$$

(b) When $S \geq I - a_L$; or When $S < I - a_L$ and condition (1) is not satisfied



$$
\begin{aligned}
\tilde{\theta}(S) \quad &\equiv \quad \inf\{\theta \mid \theta \geq 0; a_H + S \geq \int_{-I}^{\infty} \max(0, a_H + I + y - D^{\theta}) + \theta; \\
&\qquad I - S + \theta = \frac{1}{2} \sum_{i=L,H} (\int_{-I}^{\infty} \min(D^{\theta}, a_i + I + y) f(y) dy)\}
\end{aligned}
$$