Myopic Traders, Efficiency and Taxation

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Abstract

This paper explores the welfare implications of a securities transaction tax when informed traders act under short-term objectives. The model presented features speculators who can trade on information of differing time horizons, trade by fully rational uninformed agents, endogenous asset prices and profit maximising firms that can use information contained in stock prices to improve their investment decision. The only value enhancing investment available to firms requires a long-term investment. Therefore investment efficiency can only be improved if stock prices contain long-term information. It is shown that when informed traders act under short-term objectives, a *subsidy* on short-term trade can improve welfare. This is because trade by short-term informed speculators exerts a positive externality over the profitability of long-term informed trade. A subsidy on short-term trade thus increases the amount of trade on long-term information in equilibrium. As a result stock prices contain more long-term information, which improves investment efficiency. The model takes full account of the effect of a tax on market liquidity and welfare for all market participants.

Keywords: investment efficiency, short-termism, securities transaction tax, liquidity, welfare.

Journal of Economic Literature Classification Numbers: G14, G18, D60, D82

1. Introduction

This paper addresses the question of how economic welfare is affected by short horizon trading objectives of potentially informed speculators, and how a securities transaction tax may mitigate possible inefficiencies arising from such short objectives. There seems to be a widespread belief that institutional traders, such as pension and mutual fund managers, operate according to short-term objectives due to performance pressure by investors (see Marsh, 1990, and Demirag, 1995, for an overview of the debate on short-termism). Such short-term objectives are typically seen as detrimental to economic welfare, as they may be responsible for the mispricing of long-term assets, leading to inefficient investment decisions by firm managers (Shleifer and Vishny, 1990).

Little work has been done that attempts to formalise the interdependence of speculators' trading horizons, investment efficiency and economic welfare (except Subrahmanyam, 1998, discussed below). Existing work on trading horizons typically focuses on the impact of short-term trading objectives on the informational efficiency of prices in a financial market (see Froot, Scharfstein, and Stein, 1992, Dow and Gorton, 1994, Vives 1995). However, these treatments do not model financial markets as being socially beneficial, which makes it impossible to assess the welfare implications of speculators' exogenous short horizons. Moreover, it remains an open question what the policy implications are, if it is indeed the case that traders' short-term objectives reduce investment efficiency. Can economic welfare be improved by taxing traders? If so, which trades should be taxed and how?

One argument that has been put forward in favour of a securities transaction tax (STT) goes back to Stiglitz (1989) who reasons that traders spend a socially wasteful amount of resources on research in financial markets. Since the information production activity by one speculator exerts a negative externality over the profitability of other speculators' trades, too much effort is spent on producing information compared to the social optimum. He suggests that a STT may correct for this externality, bringing actual effort on information production in line with the social optimum.

Another efficiency argument put forward by Stiglitz (1989) and Summers and Summers (1989) is concerned with managerial short-termism. As mentioned above, short-term objectives by fund managers are alleged to induce short-termist behaviour and inefficient investment decisions by firm managers. Since a STT affects speculators who trade frequently more than those who hold their positions for longer periods, the introduction of a tax may increase holding periods and thus mitigate firm managers' bias for inefficient short-term investment.

However, as Summers and Summers (1989) point out, the relation between short horizons (and thus short holding periods) by fund managers and trading on short-term information (which is what ultimately leads to short-termism by firm managers) is not straightforward. As Summers and Summers (1989) put it:

> "Even if it is granted that portfolio managers care only about returns over a short horizon, they nonetheless necessarily must care about the price at which they can unload the stock. This will depend on tomorrow's demand for that stock, which will in turn depend upon tomorrow's expectations about corporate performance thereafter. It should be clear that a holder of corporate stock today who anticipates quickly selling to a sequence of future short-term holders should nonetheless be concerned about his company's profitability over the long term..."

In this paper we consider an economy of overlapping generations of (potentially) informed speculators who have short-term objectives, but who can produce information regarding the long-term prospects of a company. In this setting myopic speculators may be willing to produce long-term information only, if there are future generations of short-term traders that move the price of the stock towards its fundamental value. Only if this is the case, are speculators who trade on long-term information able to unwind their positions profitably after a short period. There is thus a positive externality of a short-term speculator's trade over the profitability of the long-term speculator's trade.[1]

Interestingly, Stiglitz' argument about excessive information gathering activity by speculators is turned on its head when speculators have short term objectives: information production by one trader is now no longer always detrimental to another speculator's trading profits. On the contrary, production of and trade on short-term information is necessary to enable some myopic speculators to trade profitably on long-term information. We thus show that when trade on long-term information can improve investment efficiency, it may be socially beneficial to subsidise trade on short-term information.

Regarding the social desirability of informed trade, note that the informational efficiency of stock prices tends to increase with an increase in informed trade. There are various reasons as to why improved informational efficiency of stock prices may be socially useful. Diamond (1967) argues that if markets are incomplete, stock prices can convey information to firms regarding agents' preferences, which in turn allows firms to take welfare enhancing investment decisions. Hirshleifer (1971) argues that costly activities related to improving 'foreknowledge' of future states of the world may be socially useful, again because it allows improved investment decisions. Holmstrom and Tirole (1993) explore the social benefit of informative stock prices in a setting where managerial incentives can be improved by making their contracts contingent on stock price.[2] Dow and Gorton (1997) assign a dual role to the stock market: a prospective one (stock prices guide future investment decisions) and a retrospective one (stock prices allow improved monitoring of past managerial performance). They only find a tenuous link between informational efficiency of stock prices and economic efficiency.

In this paper an increase in informational efficiency of stock prices increases firms' investment efficiency, because firms use the information contained in prices to learn about their optimal investment level. This corresponds to the role of stock prices as in Hirshleifer (1971) and Dow and Rahi (1997). In order to capture the idea that short-term information is socially less useful than long-term information, we consider a setting in which firms need to take a 'long-term' investment decision. Therefore, only information that is reflected in prices a sufficiently long time before an investment pays off can be useful in guiding a firm's investment decision. This captures the idea that excessive trade on short-term information is detrimental to firms' investment efficiency.

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[1] Dow and Gorton (1994) show in a related context that long-term arbitrage chains may be impossible when speculators have short horizons. Their model exhibits a similar positive externality of trade by one generation of speculators over the profitability of the preceding generation's trades.

[2] Fishman and Hagerty (1989) consider a setting in which improved price efficiency increases firm managers' incentives to invest, which increases firm value. In this context they explore a firm's incentives to disclose information voluntarily, when firms compete for traders to produce costly information about their own firm.

A number of other arguments have been put forward in favour of a securities transaction tax. Summers and Summers (1989) argue that a STT would be a convenient source of revenue to the tax authorities, yielding an estimated US$10 billion for a 0.5% tax on transactions carried out on US financial markets. Campbell and Froot (1993) maintain that revenue figures are typically overstated, as estimates fail to take into account the behavioural changes that a STT would cause.[3] Summers and Summers (1989) also argue that a tax may be an effective way to curb excessive speculation and noise trade, responsible for excess volatility in financial markets.

This argument, however, is based on the presumption, that there is a significant amount of trade originating from irrational market participants, and that as a result financial markets actually do exhibit excess volatility. Despite extensive research regarding the efficiency of financial markets, there does not seem to be conclusive evidence concerning excess volatility (see e.g. Fama, 1991 and 1998). Moreover, it is theoretically unclear whether a STT would reduce volatility, since not only destabilising noise traders would be driven out of the market, but also price stabilising informed traders (Schwert and Seguin, 1993). Umlauf (1993) presents evidence from Sweden, where a 1% round trip tax was introduced in 1984, which was increased to 2% in 1986. His findings suggest that the introduction of a tax not only reduced trading volume sharply, but also increased volatility.

When discussing the effect of a STT it is not only important to understand the impact such a tax has on the trading behaviour of different types of market participants. As Ross (1989) points out, it is also crucial to consider the welfare impact of the tax on the different types of agents. Thus, if a tax were to reduce trading volume by irrational 'noise' traders, excess volatility may be reduced, but it remains unclear what the welfare implications of such a policy are. In this paper we consider the welfare implications of a tax on all types of market participants.

We do not address the issue of excess volatility in financial markets. Instead, it is assumed that all market participants are fully rational, i.e. there are no noise traders present. There are some risk averse traders that participate in the market for insurance reasons. They are thus (rationally) willing to pay a premium in the form of trading losses to the informed speculators, in exchange for insuring uncertain wage income

[3] Behavioural changes include migration of trade to overseas markets, reduction in trading volume,

that is correlated with the value of a stock. Endogenising liquidity trade in this way allows us to capture the feature that a STT affects both the amount of informed and of liquidity trade. Thus, market liquidity is endogenous in our model and depends on the magnitude of the security transaction tax. Moreover, endogenising liquidity trade is necessary in order to make a complete assessment of the welfare impact of a securities transaction tax.

To summarise, the main features of our model are the following: (i) asset prices are endogenously determined on a semi-strong efficient financial market (in the spirit of Kyle, 1985), (ii) firms take an investment decision, based on observed stock prices so as to maximise the firm value, (iii) speculators decide whether or not they wish to incur a cost of information production and choose a trading strategy, (iv) speculators are differentially informed either about the long-term or the short-term prospects of a firm, (v) risk averse agents participate in the stock market for insurance reasons and choose a trading strategy so as to maximise their expected utility.

To our knowledge the only paper dealing with the issue of information horizon of trades, investment efficiency and taxation is Subrahmanyam (1998). In contrast to our paper, he explores the impact of a tax on trade size, where traders are risk averse and have unlimited horizons. He shows that such a tax can reduce speculators' incentives to engage in a socially wasteful race to discover information shortly before other traders do. Discovering information shortly before other traders only carries a private benefit in Subrahmanyam's setting, because the time difference of information discovery is too short to allow firms to act on this information. The scenario we have in mind is one in which traders can discover information a long time before others, which renders this information socially useful, because it can improve firms' investment decisions. Subrahmanyam also considers a setting where traders can acquire long-term versus short-term information where the latter is socially less useful. He shows that a tax on trade size can increase the proportion of agents trading on long-term information, thus increasing firm value. Again, the result is derived in a setting that features risk averse traders with infinite planning horizons.

The remainder of the paper proceeds as follows. In Section 2 the model is presented. Equilibrium order flows by all types of traders are derived in Section 3,

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and trade in assets that are close substitutes but taxed at a lower rate.

while Section 4 states basic results concerning the equilibrium price setting rule by the market maker and investment rules by firms. Section 5 describes the equilibrium with myopic speculators and general tax rates. In Section 6 it is shown under which conditions a subsidy on short-term speculators may be desirable. Section 7 considers the benchmark case of infinitely lived speculators. Section 8 concludes.

2. The model

We consider an economy with an infinite sequence of dates t=-∞,...,∞. At each date t there is exactly one firm F_t that is liquidated. Firms are all equity firms whose shares are traded on a stock market at all dates preceding liquidation. At the date of liquidation, shareholders get paid their claim on the liquidation value of the firm. The liquidation value v_t of firm F_t depends on an investment that has to be taken two periods before liquidation.[4] The investment decision consists of determining a level of investment k_t, where the value maximising investment depends on a future state of the world, unknown to the firm manager at the date when the decision needs to be taken. In particular it is assumed that firm value takes the following functional form

$$\tilde{v}_t = \tilde{y}_t k_t - \tfrac{c}{2} k_t^2 + \tilde{e}_t + \tilde{h}_t \ , \tag{1}$$

where y_t denotes a random variable that determines the optimal investment level. One could think of y_t as for example the future demand for a firm's output or as its productivity level (the latter interpretation being given by Dow and Rahi, 1997). It is assumed that y_t can take the values a or b ($a < b$) with equal probability. The random variables e_t and h_t denote independent terms that affect firm value. As will become apparent later, it is analytically convenient (but not necessary) to allow e_t and h_t to take the values -B or B with equal probability, where $B=\dfrac{b^2 - a^2}{4c}$.

Moreover, it is assumed that e_t becomes publicly known at t-1 (i.e. the date preceding liquidation), while h_t is only revealed at the date of liquidation. This assumption is made, because it provides a convenient way of modelling endogenous liquidity trade for a firm's share in both periods before its liquidation.

[4] One could think of the firm as coming into existence two periods before it is liquidated.

There are four types of traders in the economy. Two types of traders are short-lived (myopic) agents that at a cost can acquire information regarding a firm's productivity parameter before it becomes publicly known. These potentially informed speculators live for two periods and are distinguished by their genetically endowed ability to *either* acquire short-term *or* long-term information. For simplicity, the agent that is able to acquire short-term (long-term) information is called the short-term (long-term) speculator. At date zero of her live, a speculator can decide whether or not to incur the cost of information acquisition. During the subsequent period she then receives information with probability q, if the cost of information production as been incurred. At the following date (date 1 of a speculator's life), she can trade on the information received and unwind the position after one period, at the end of her life. Speculators are risk neutral and only care about their wealth at the end of their lives. There are no borrowing or short sales constraints. At each date a new pair of speculators (one of each type) is born.[5]

If at date t a speculator incurs the cost k of information acquisition, she receives a signal by date t+1 with probability q, concerning the productivity parameter y_{t+2} (y_{t+3}) if she is a short-term (long-term) speculator. The signal is perfectly informative for either type of speculator. With probability 1-q no signal is received at all. Whether or not a particular speculator who has incurred the cost of information production receives a signal is independent across speculators.

The remaining two types of traders consist of risk averse agents who participate in the stock market, because they wish to hedge uncertain wage income. The reader may find it convenient to think of a new generation of two period lived hedgers being born at each date, although infinitely lived hedgers with appropriately evolving hedging needs would be analytically equivalent. Hedgers are assumed to be infinitely small and of total mass normalised to one, where a mass m are 'h-hedgers' and the remaining 1-m are 'e-hedgers'.[6] The ε-hedgers born at date t-2, have a wage

[5] The assumption that speculators cannot choose which type of information they wish to acquire avoids the problem of co-ordination between the traders. In equilibrium each type of information acquisition supports only one trader. As a result there would always be two equilibria: one in which speculator 1 acquires short-term and speculator 2 acquires long-term information, and one where their roles are swapped. To avoid the possibility of co-ordination failure in the reduced form game of information acquisition, we assume that information acquisition is type dependent.

[6] For the derivation of the demand by hedgers it is necessary to assume that they are competitive. The assumption plays no role otherwise.

when old that is correlated with e_t, while η-hedgers' wage at old age is correlated with h_t. A hedger's wage income at date t, can thus be written as $\tilde{w}_{e,t} = \tilde{z}_{e,t}\tilde{e}_t$ and $\tilde{w}_{h,t} = \tilde{z}_{h,t}\tilde{h}_t$ for the ε-hedger and η-hedger respectively. Whether wage is positively or negatively correlated with the firm's value depends on the realisation of $z_{e,t}, z_{h,t} \in \{-1,1\}$ with equal probability. The realisations of the direction of the shock are independent of each other and serially uncorrelated. Realisations are observed by the hedgers when they are born, i.e. an ε-hedger (η- hedger) born at date t-2 observes $z_{e,t}(z_{h,t})$ at date t-2.

The risk averse hedgers only care about consumption at the end of their lives and have a piece wise linear utility function $U(x) = \min\{x, ax\}$, where $0 \leq \alpha < 1$.

At each date t there are thus (potentially) informed speculators and risk averse hedgers that submit an order for assets in either the firm that is liquidated after the following period (F_{t+1}) or two periods from t (F_{t+2}). Orders are submitted to a market maker, who sets the price of each asset and matches demand out of his inventory. This approach follows Kyle (1985) with modified assumptions concerning the distribution of random variables and allowing for endogenous noise traders (as for example in Spiegel and Subrahmanyam, 1992). Following Kyle, market makers are unable to discern the originator of an order as they observe only total order flow for an asset at a particular date. Moreover, it is also assumed that a market maker faces Bertrand competition and thus sets the price for an asset equal to its expected value, given his information set.

Denote by $Q_{t,t+1}$ ($Q_{t,t+2}$) total order flow at date t for firm F_{t+1}'s (F_{t+2}) stock. Denote by $q_{s,t,t+i}$, $q_{l,t,t+i}$, $q_{e,t,t+i}$, $q_{h,t,t+i}$, the individual orders at date t from the short-term informed speculator, the long-term informed speculator, an individual ε-hedger and an individual η-hedger, respectively for the stock of firm F_{t+i} . Moreover, denote by $p_{t,t+2}$ ($p_{t+1,t+2}$) firm F_{t+2}'s share price at date t (t+1).

In the spirit of Hirshleifer (1971) and Dow and Rahi (1997), it is assumed that firm managers do not know the productivity parameter of their own firm. However, they can observe at which price shares of their firm trade on the stock market, which allows them to draw inferences concerning the true productivity of their firm.

Since a firm has to invest two periods before liquidation, the information contained in prices one period before liquidation, cannot be used in guiding the investment decision. Hence, short-term traders have no impact on the value of the firm. Long-term traders on the other hand may have an impact on the real value of the firm, since they trade in the stock two periods before liquidation and thus their private information concerning the firm's productivity may get reflected in the stock price. This in turn allows a firm to learn its own productivity before investing, with an endogenously determined probability. The resulting investment decision is based on this information, which can improve investment efficiency and thus firm value.

An equilibrium consists of

(i) A price schedule set by the market maker, such that the price of an asset equals its expected value, given the order flow observed by the market maker.

(ii) An investment rule by the firm that maximises the expected liquidation value conditional on information contained in stock prices.

(iii) A (possibly mixed) strategy of short-term and long-term speculators to incur the cost of information acquisition, and a trading rule that maximises expected trading profits.

(iv) A trading rule by both types of hedgers that maximises their expected utility.

Throughout the paper we focus on stationary equilibria.

3. Order flows in equilibrium

We now proceed to describing the equilibrium order flows by all types of traders. First, consider the trading strategy by ε-hedgers. At date t a mass of identical ε-hedgers is born, whose wage when old is correlated with e_{t+2}. Since e_{t+2} is revealed at date t+1, it will be fully reflected in the price $p_{t+1,t+2}$ (the 'short-term' price of an asset) and hence the ε-hedger can only hedge his wage risk by trading at date t. Moreover, if the ε-hedger were to hold his position until the liquidation date t+2, he would be exposed to the additional risk originating from h_{t+2}, which is revealed only at date t+2. Since no informed trade occurs on h_{t+2}, the price $p_{t+1,t+2}$ will certainly not reflect h_{t+2}.

Hence, if the ε-hedger wishes to participate in the stock market at all, he will trade in the stock of firm F_{t+2} at date t and unwind the position at date t+1. Hence, we get that $q_{e,t,t+1}$=0, while $q_{e,t,t+2} \neq 0$, if the ε-hedgers decide to participate in the stock market. As a result at each date t there will be a buy or sell order of size $|q_{e,t,t+2}|(1-m)$ for asset F_{t+2}, originating from ε-hedgers. Since we are concerned with a stationary equilibrium and since orders are symmetric around zero for either realisation of $z_{e,t}$, we can safely drop the time index in the ε-hedger's order size. Thus denote by $q_e \equiv |q_{e,t,t+2}|$ the size of an individual ε-hedger's order.

	t-1	t	t+1	t+2
Long-term speculator	Long-term speculator born. Produces information with probability n_l	Signal is received with probability $n_l q$. Trades in F_{t+2}'s stock.	Unwinds the position taken at date t, consumes and dies.	
Short-term speculator		Short-term speculator born. Produces information with probability n_s	Signal is received with probability $n_s q$. Trades in F_{t+2}'s stock.	F_{t+2} is liquidated and short-term trader's claim settled. Speculator consumes and dies.
ε-hedger		Generation of ε-hedgers born. Trade in F_{t+2}'s stock.	Generation of ε-hedgers unwind their position in F_{t+2}'s stock.	
η-hedger		Generation of η-hedgers born. Wage is correlated with h_{t+2}.	η-hedgers trade in F_{t+2}'s stock.	F_{t+2} is liquidated and η-hedgers' claims settled.

Table 1: Illustrates which agents trade at what point in time in firm F_{t+2}'s stock.

Now consider the trading strategy by the η-hedger born at date t. If he were to trade at date t, he would be exposed to the risk due to e_{t+2}, which is not reflected in the price $p_{t,t+2}$ (the 'long-term' price). At date t+1, e_{t+2} gets impounded into the asset price and hence the η-hedger can avoid exposure to this risk by only trading in firm F_{t+2}'s stock at date t+1. Thus, if η-hedgers wish to participate in the stock market at all, they will first trade in firm F_{t+2}'s stock at date t+1. Hence, $q_{h,t,t+2}=0$ and $q_{h,t,t+1}\neq 0$. Therefore, at every date t, there is a mass m of h-hedgers who submit total order size mq_h for firm F_{t+1}'s stock, where $q_h\equiv|q_{h,t,t+1}|$.

Suppose at every date the short-term (long-term) speculator decides to produce information with probability n_s (n_l). This means that a long-term speculator born at date t-1, receives information regarding y_{t+2} at date t with probability $n_l q$. She may then trade an amount $q_{l,t,t+2}$ in firm F_{t+2}'s stock at date t and unwind this position when old (i.e. at date t+1). When submitting her order to the market maker, there will also be an order from a generation of ε-hedgers born at date t (of size $(1-\mu)q_e$).

Note that in any equilibrium in which the speculator does acquire information with a positive probability, the market maker will price the asset such that the speculator's information is reflected in price, whenever the market maker is able to extract the speculator's information from total order flow. Therefore, if the speculator follows a pure strategy as a trading rule (i.e. for each possible signal received, the speculator submits a determinate order) and submits orders that are of a different size from multiples of $(1-m)q_e$, the market maker could always infer the speculator's order from total order flow.[7] In that case the speculator could never make a trading profit.

Suppose now that the speculator submits orders of size $(1-m)q_e$. In the proof of Proposition 1, we show that in equilibrium it is optimal for the speculator to do so. This results in the possible total order flows given below.

$Q_{t,t+2}\in\{-2(1-m)q_e,\ 2(1-m)q_e\}$: The long-term speculator and ε-hedgers either both submitted a buy order, or both submitted a sell order. Either possibility occurs with probability $1/4n_l q$. In either case the market maker can infer the direction of the speculator's trade and concludes in equilibrium that if the speculator submitted a buy (sell) order, she received information indicating that the firm's productivity level is high (low).

[7] Generally an informed trader may use a mixed strategy as a trading rule. However, this cannot be an equilibrium trading rule, unless the trader can commit to actually using the mixed strategy once he is called upon to trade. For a treatment of this issue see Biais and Germain (1998). For a more detailed treatment of the out-of-equilibrium beliefs of a market maker, which support equal order sizes by informed trader and hedger, see the Appendix and for example Dow and Gorton (1994), section VI.

$Q_{t,t+2} \in \{-(1-m)q_e,\ (1-m)q_e\}$: Since ε-hedgers always submit an order when having received information, the market maker knows that only the hedgers submitted an order. Hence, order flow contains no information about the productivity of the firm. Either realisation occurs with probability $1/2(1-n_l q)$.

$Q_{t,t+2} = 0$: Either the speculator submitted a buy order and the hedger a sell order, or vice versa. Since each of the two is *ex ante* equally likely, the market maker cannot infer any information about the productivity parameter of the firm. The probability of zero total order flow is $1/2n_l q$.

Remark: Apart from the equilibrium examined here, there may be another equilibrium in which the speculators submit orders of size $2(1-m)q_e$, resulting in possible total order flows $-3(1-m)q_e$, $-(1-m)q_e$, $(1-m)q_e$, $3(1-m)q_e$. In that case, orders always reveal some information. The basic properties of the model remain unchanged, i.e. a higher probability of information production by the long-term speculator improves price efficiency, which improves investment efficiency. The model becomes considerably less tractable, however, because prices are *partially* revealing. In the equilibrium considered throughout the paper, prices either fully reveal private information or do not reveal it at all. This feature of equilibrium results in quite simple formulae for prices, investment policy, trading profits etc, that would be complicated by having partially revealing prices.

Another reason why we do not focus on the equilibrium in which speculators submit orders of size $q_e(1-m)$, is that it fails to exist when the probability q of receiving a signal is high. To see why this is the case suppose that q=1. In that case the speculator always trades, since he always receives information. Therefore, when total order flow, say $(1-m)q_e$, the market maker knows that the hedgers must have submitted a sell order and the speculator submitted a large buy order. Prices thus *fully* reveal the speculator's information in all states of the world when q=1. He therefore has an incentive to deviate to submitting smaller orders that sometimes result in the non-revealing zero total order flow. The equilibrium we are considering exists for all values of q.

Regarding order flow at date t for asset F_{t+1}, note that there is a long-term speculator and a generation of ε-hedgers who unwind their positions in asset F_{t+1} at date t. Since the order flow due to unwinding old positions is known to the market maker in equilibrium, he can infer from total order flow $Q_{t,t+1}$, how much trade is due to 'new' orders, i.e. orders from short-term speculators and η-hedgers. Define the 'new' order flow as $Q'_{t,t+1} = Q_{t,t+1} + Q_{t-1,t+1}$. Again, suppose that informed short-term speculators submit orders of size mq_h. This yields the possible total order flows $Q'_{t,t+1}$ with associated probabilities.

$Q'_{t,t+1} \in \{-2mq_h, 2mq_h\}$ where each of the two outcomes has probability $1/4n_sq$.

$Q'_{t,t+1} \in \{-mq_h, mq_h\}$, where each outcome has probability $1/2(1-n_sq)$.

$Q'_{t,t+1} = 0$, with probability $1/2n_sq$.

4. Price setting and investment policy

Let us now turn to the market maker's equilibrium price setting mechanism, the firm's investment policy and its resulting liquidation value.

Proposition 1: *Suppose the long-term speculator born at date t-1 acquires information with probability n_l. When the speculator observes $y_{t+2} = b$ ($y_{t+2} = a$) at date t, she submits a buy (sell) order of size $(1-m)q_e$. The market maker applies the following price rule $p_{t,t+2}(Q_{t,t+2})$:*

$$p_{t,t+2}(Q_{t,t+2}) = \begin{cases} \dfrac{b^2}{2c} & \text{if} \quad Q_{t,t+2} = 2(1-m)q_e \\ \dfrac{(a+b)^2}{8c} & \text{if} \quad Q_{t,t+2} \in \{-(1-m)q_e, 0, (1-m)q_e\} \\ \dfrac{a^2}{2c} & \text{if} \quad Q_{t,t+2} = -2(1-m)q_e \end{cases} \tag{2}$$

Moreover, the price dependent investment decision by the firm is given by:

$$k_{t+2} = \begin{cases} \dfrac{b}{c} & \text{if} \quad p_{t,t+2} = \dfrac{b^2}{2c} \\ \dfrac{a+b}{2c} & \text{if} \quad p_{t,t+2} = \dfrac{(a+b)^2}{8c} \\ \dfrac{a}{c} & \text{if} \quad p_{t,t+2} = \dfrac{a^2}{2c} \end{cases} \tag{3}$$

This results in expected firm value of

$$E[v_{t+2}] = \frac{n_l q}{2} \cdot \frac{(b-a)^2}{8c} + \frac{(b+a)^2}{8c}. \tag{4}$$

Proof see Appendix

As can be seen from Proposition 1, the equilibrium pricing and investment rules are particularly simple. If order flow reveals to the market maker the underlying productivity parameter of the firm, this will be reflected in price. The firm can then extract this information from the price and choose a first-best investment level, given its productivity. If the price does not contain any information concerning productivity, the firm makes a medium sized investment, which is optimal given the firm's (lack of) information. The expected firm value, however, is lower when stock prices do not contain and reveal to the firm what its underlying productivity parameter is. Therefore, the expected firm value in equilibrium is an increasing function of the probability n_l with which the long-term speculator produces information.

To complete the treatment of the equilibrium pricing rule, we can state the following straightforward result concerning the pricing of a firm's stock, one period before liquidation.

Proposition 2*: In equilibrium the market maker prices the stock for firm F_{t+1} at date t as follows. The price $p_{t,t+1}$ is a function $f(p_{t-1,t+1}, Q'_{t,t+1})$, where*

$$f(p_{t-1,t+1},Q'_{t,t+1}) = \begin{cases} p_{t-1,t+1} + e_{t+1} & \text{for} \quad p_{t-1,t+1} = \dfrac{b^2}{2c} \\ \dfrac{4b^2-(b-a)^2}{8c} + e_{t+1} & \text{for} \quad p_{t-1,t+1} = \dfrac{(a+b)^2}{8c}, Q'_{t,t+1} = 2mq_h \\ p_{t-1,t+1} + e_{t+1} & \text{for} \quad p_{t-1,t+1} = \dfrac{(a+b)^2}{8c}, Q'_{t,t+1} \in \{-mq_h, 0, mq_h\} \\ \dfrac{4a^2-(b-a)^2}{8c} + e_{t+1} & \text{for} \quad p_{t-1,t+1} = \dfrac{(a+b)^2}{8c}, Q'_{t,t+1} = -2mq_h \\ p_{t-1,t+1} + e_{t+1} & \text{for} \quad p_{t-1,t+1} = \dfrac{a^2}{2c} \end{cases}$$

Proof see Appendix.

Note, that the firm cannot use the information that may potentially be revealed by the short-term speculator's trade, since its investment decision must be taken two periods before liquidation. Hence, the probability n_s with which the short-term speculator acquires information, does not affect investment efficiency.

5. Equilibrium for general tax rates

Now, consider the speculators' choice of probabilities n_s, n_l with which to produce information. The long-term speculator dies one period before the firm is liquidated. Therefore, she can only trade profitably, if the price at which she can unwind her position reflects more of the information on which she originally traded. This in turn can only be the case, if there is a short-term speculator in the period before liquidation who trades on the same (or a correlated) piece of information as the long-term speculator. From this it becomes clear that the short-term speculator exerts a positive externality over the profitability of the long-term speculator's trade.

On the other hand, the short-term speculator can only trade profitably, if the long-term speculator's trade has not yet revealed his private information to the market maker. Hence, the higher the probability that the long-term speculator produces information, the lower the probability that the short-term speculator can trade profitably. From this it is clear that informed trade by the long-term speculator exerts a negative externality over the profitability of the short-term speculator's trade. When a social planner chooses a tax policy, he thus has to take into account the two types of externalities that each type of speculator exerts on the other speculator.

Concerning the taxation of trade we consider the following set-up. It is assumed that a social planner can tax 'long-term' and 'short-term' trade separately. This assumption is less motivated by the author's belief that this is practically feasible than by the nature of this enquiry. The purpose of the paper is to improve our theoretical understanding of the impact of a securities transaction tax on the trading behaviour of differentially informed speculators. By allowing to tax traders depending on the point in time at which they trade, we explore the theoretically more general case of a tax. Our main result derives some of its interest from the fact that only short-term speculators should be subsidised, although only long-term traders improve investment efficiency. In order to work out this result in its clearest form, it is necessary to allow for differential tax treatment of the two types of traders.

We assume that the following types of taxes/subsidies may be levied. Trade two periods (one period) before the liquidation of an asset can be taxed at rate t_l (t_s), where the tax is imposed linearly on the traded volume. Hence trading a number q of shares incurs total transaction cost (on one round trip) of qt.[8]

Since we allow the tax rates to be negative, it is necessary to make the following provision on the payability of a subsidy $t_s<0$. It is assumed that the tax authority only pays a subsidy on 'short-term' trade, if the share price in the preceding

[8] Subrahmanyam (1998) and Dow and Rahi (1999) have to assume that the tax is a quadratic function of traded volume, as their models cannot otherwise be solved. This has the conceptual disadvantage that it gives traders an incentive to split their orders into many very small orders, rather than submitting one large order. To avoid this problem, they need to assume that traders simply cannot do this.

period has not yet been revealing. Otherwise, the speculator (or any risk neutral agent for that matter), would immediately wish to trade infinitely large quantities of the stock, because at the time of trading the stock already is fairly priced, and orders no longer move the price (see Proposition 2). In reality such problems may be mitigated by virtue of the fact that (i) even speculators are typically risk averse, (ii) trading large quantities is likely to move the price, even if trade contains no information.[9]

In what follows we focus on the case where α=0. For the purpose of the following proposition, we suppose that speculators trade if and only if they have received information. As was shown in Section 3, in the absence of a tax/subsidy a speculator does not trade unless she received information. However, when a subsidy on trade is paid, speculators may wish to trade even without having received information. Although they may incur a loss due to trade on noise, a sufficiently high subsidy on trade could compensate for that loss. Lemma 1 states conditions on possible subsidies, such that speculators only trade if they have received information..

Proposition 3*: Suppose tax rates are such that speculators only trade if they receive information. Then, the equilibrium features long-term (short-term) speculators producing information with probability n_l (n_s), where*

$$n_l = \begin{cases} 0 & \text{if} \quad L < 0 \\ L & \text{if} \quad 0 \leq L \leq 1 \\ 1 & \text{if} \quad L > 1 \end{cases}$$

with

$$L = \frac{q q_h m\left(\frac{B}{2} - t_s\right) - k}{\frac{q^2}{2} q_h m\left(\frac{B}{2} - t_s\right)} \,. \tag{5}$$

Moreover,

[9] This may be the case, because in real world settings, there is not usually just one piece of information that is either fully revealed or not revealed at all. Hence, real world market participants constantly face the possibility of trading with someone who has still superior information.

$$n_s = \begin{cases} 0 & \text{if} \quad S < 0 \\ S & \text{if} \quad 0 \leq S \leq 1 \\ 1 & \text{if} \quad S > 1 \end{cases}$$

with

$$S = 4\frac{qq_e(1-m)t_l + k}{q^2 Bq_e(1-m)} \quad . \tag{6}$$

Furthermore, for $t_s \in \left(-B\frac{2-n_s q}{2+n_s q}, B\frac{2-n_s q}{2}\right)$, *the h-hedgers' order size is given by*

$$q_h = \frac{B}{2B + t_s} \,. \tag{7}$$

and for $t_l \in \left(-B, B\frac{1-n_s q + \frac{n_s n_l q^2}{4}}{1+\frac{n_s n_l q^2}{4}}\right)$, *the e-hedgers' order size is given by*

$$q_e = \frac{B}{B + |t_l|} \,. \tag{8}$$

Proof see Appendix.

Suppose the underlying parameter values of the model are such that we are indeed in a mixed strategy equilibrium in the absence of taxation. From equations (5)-(8) it is straightforward to work out for which set of parameters this is the case. Consider then the effects of an introduction of a tax on the trading behaviour of the various traders. It is straightforward to see from (5) that the probability with which the long-term speculator is willing to incur the cost of information production, is a decreasing function of t_s.

To understand the relation between the equilibrium choice of n_l and tax rate t_s, consider the following. The mixed strategy equilibrium of the reduced form game of the choice of probabilities n_s, and n_l is given by the conditions, that

(i) given the short-term speculator's choice of n_s, the long-term speculator is indifferent between acquiring and not acquiring information,

(ii) given the long-term speculator's choice of n_l, the short-term speculator is indifferent between acquiring and not acquiring information.

A reduction in t_s has two effects. Firstly, it directly increases the short-term speculator's expected trading profits by reducing the speculator's tax burden (or by increasing the subsidy she receives). Secondly, it increases the h-hedgers' order sizes, which in turn allows the short-term speculator to submit larger orders. Effectively, this increase in liquidity also increases the short-term speculator's expected trading profits.

Since an increase in n_l affects the short-term speculator's profits negatively, she only remains indifferent between acquiring and not acquiring information, if an increase in n_l is compensated for by a reduction in t_s. If n_l were to be increased without the corresponding reduction in t_s, the short-term speculator would cease to ever wish to incur the cost of information acquisition. In this case, however, the long-term speculator would no longer find it worthwhile to produce information either. In the absence of short-term informed trade a myopic long-term speculator cannot make any trading profits and therefore would not engage in information production.

This implies that a social planner can improve investment efficiency by reducing the tax on short-term speculators, or even giving them a subsidy. Whether or not the increase in investment efficiency outweighs the monetary cost of the subsidy for the social planner depends on the underlying parameters of the model.

As mentioned above, the payment of a high subsidy can give speculators an incentive to trade at random, rather than only trading when they received information. The following Lemma states the highest possible subsidy payments such that speculators only trade after having received information.

Lemma 1*: The long-term and short-term speculators do not trade when they have not received information when tax rates are*

$$t_l \geq -\frac{n_s qb(b-a)}{8c} \tag{9}$$

and

$$t_s \geq -\frac{B}{2} \tag{10}$$

Proof see Appendix.

Equation (9) ensures that the subsidy to the long-term speculator is not so large as to induce trade without information (churning). If the subsidy exceeded the level given in equation (9), the long-term speculator would be better off always trading and never acquiring any information. A similar bound has to be imposed on the subsidy for short-term trade, which leads to condition (10).

6. Improving investment efficiency

From equation (5) it becomes clear that t_l has no impact on n_l. This is due to the way in which the mixed strategy equilibrium is determined (see above). It follows that a tax/ subsidy on the long-term speculator and ε-hedger only redistributes welfare between traders and the tax authority, without affecting investment efficiency. Since we are interested in the way in which investment efficiency can be improved through a securities transaction tax, we can safely set t_l=0.

From Proposition 3 it is clear that if the exogenous parameters of the model are such that a mixed strategy equilibrium results, the only possible way to improve investment efficiency is by subsidising short-term traders.

Note that the η-hedgers are clearly made better off by a subsidy. Moreover, it is straightforward to show that for a=0, ε-hedgers are equally well off, regardless of the tax on short-term speculators. Generally, ε-hedgers may be affected by the tax rate t_s, since it affects the probability n_l with which a long-term speculator acquires information and trades on it. As a result the probability with which the long-term price of a stock is revealing depends on this tax rate. For the special case α=0 this does not matter, and the tax rate t_s does not affect ε-hedgers' welfare.

Hence, a very weak criterion for the desirability of a subsidy on short-term speculators is, whether the expected increase in firm value more than outweighs the cost of paying a subsidy to some of the traders.[10] Regarding the exact rate of a subsidy, we have to make sure that it does not exceed the value in condition (10) and the restriction in Proposition 3. A subsidy that satisfies these conditions is referred to as a 'small subsidy'. The following Proposition 4 states for which parameter values a small subsidy may be socially desirable.

Proposition 4*: The cost of paying a small subsidy to short-term traders is less than the monetary value of the efficiency gain in firm investment, if*

$$4km < Bq(1-m)\left(\frac{5(b-a)}{2(b+a)}-m\right). \qquad (11)$$

No trader is made worse off by the payment of such a subsidy.

Proof see Appendix.

From inequality (11) it follows that an increase in *b-a* (keeping *b*+*a* constant) increases the set of parameters *k*, *m*, *q*, *c*, for which a subsidy on short-term speculators is desirable. This is the case, because as the difference between the two possible productivity levels increases, the firm's liquidation value becomes more sensitive to taking the 'right' investment decision. Essentially, the difference in firm value from knowing the productivity parameter and investing accordingly, and from not knowing the productivity parameter and taking a 'middle-of-the-road' investment decision increases with *b-a*.

Moreover, an increase in the cost of information production *k* and an increase in the mass *m* of *h*-hedgers, makes it less beneficial to grant short-term speculators a subsidy. An increase in *m* essentially makes a subsidy payment more expensive for the tax authority, because total quantities traded are larger, and therefore a higher total subsidy needs to be paid out, which is undesirable. On the other hand, an increase in *k* means that the short-term speculator is more inclined not to acquire information. In order for her to accept acquiring information with a positive probability, for a given

[10] In a more general setting where $\alpha>0$ is possible, one could include the liquidity traders' expected utilities and trade off the gain of a η-hedger from receiving a subsidy with the loss of an ε-hedger from having more informed trade in his trading period. A similar result could be derived for that case.

level of n_l, therefore requires a higher subsidy. Again this is undesirable for the tax authority.

7. Equilibrium with infinitely lived speculators

Let us consider as a benchmark the case where the speculators are not myopic.[11] The aim of this section is to illustrate that the need for a subsidy for short-term speculators arises because of speculators' myopia. We will therefore show that long-term speculators are willing to produce information and trade on it for a larger set of parameter values when they are infinitely lived. Hence, there is a range of parameter values for which long-term speculators produce information if their planning horizon is infinite, while they do not produce information when their horizon is short.

Proposition 5*: An infinitely lived long-term speculator incurs the cost k of information production if*

$$k \pounds k^{*} \circ max\left\{B \tfrac{q}{4}\left(q_e(1-m)(1+q)+q_h m(1-q)\right), B\tfrac{q}{2} q_e(1-m)\right\} \qquad (12)$$

Proof see Appendix.

Proposition 5 gives a sufficient condition for the cost of information production such that the infinitely lived long-term speculator produces information and trades on it. When speculators are myopic, we know from Proposition 3 that a long-term speculator incurs the cost of information acquisition only if

$$\kappa \leq B\tfrac{q^2}{4} q_e(1-m). \qquad (13)$$

Note that condition (12) constitutes a sufficient condition for information production for the infinitely lived speculator, while inequality (13) constitutes a necessary condition for the short-lived speculator to incur the cost of information production. Since $k^{*} > B\tfrac{q^2}{4} q_e(1-m).$, it follows that there is a set of parameter values for which the cost of information acquisition is such that an infinitely lived long-term speculator

[11] Throughout this section we continue to assume that hedgers are short lived. As mentioned above, this is just a convenience, rather than a necessity. With appropriately evolving hedging needs, hedgers could be thought of as infinitely lived.

would acquire information, while a short-lived long-term speculator would not acquire information.

This confirms the frequently put forward conjecture (see Introduction) that short horizon objectives by speculators may reduce the ‘amount’ of trade on long-term information.

Another point that deserves mentioning here is the following. As was shown in the previous section, the amount of trade on long-term information can be increased by subsidising short-term traders. This ceases to be the case when speculators are infinitely lived. To see why this is the case consider the following. When the long-term speculator is infinitely lived, she can always hold her position until the date of the firm’s liquidation. If she chooses to do so, her profits are independent of short-term speculators’ decisions regarding information acquisition and trade. In that case short-term speculators are irrelevant and taxing/subsidising them does not change the long-term speculator’s participation in the market.

On the other hand, it may be optimal for the long-term speculator not to hold a position until the date of liquidation. Instead, expected trading profits may be higher if she unwinds her position and trades on the same information again (for details on how this works see the proof of Proposition 5). In that case trading profits are lower when there is also a short-term informed speculator in the market. Hence, such a speculator exerts a negative externality over the long-term speculator’s trades. We are thus in the case described by Stiglitz (1989), where each informed trader exerts a negative externality over all other informed traders. In that case it may be socially desirable to tax the short-term speculators out of the market, as this can result in increased market participation by long-term speculators. It cannot be emphasised enough, however, that this conclusion is only valid when all speculators have infinite planning horizons.

8. Conclusion

In the preceding study, we explore the role of a securities transaction tax in improving the investment efficiency of firms whose shares are traded by potentially informed, myopic speculators. We consider a setting in which stock prices can be used by firms to guide their investment decisions. Firms can only make long-term investments and therefore only speculators who trade on long-term information can directly improve investment efficiency by rendering stock prices more efficient. We show that when all speculators have short-run objectives, it may be desirable for a social planner to subsidise speculators who trade on short-term information. This is because trade on short-term information exerts a positive externality over the profitability of trading on long-term information. Myopic speculators who trade on long-term information can only trade profitably when they can unwind their positions after a short period at prices that reflect more of the information on which they originally traded. The presence of speculators who trade on short-term information is therefore necessary in order to provide an incentive to other speculators to trade on long-term information.

We consider a setting in which liquidity trade originates from fully rational agents who participate in the stock market for insurance reasons. This allows us to endogenise the effect of a tax on liquidity. Moreover, when assessing the desirability of a tax, a complete appraisal of all agents' welfare is possible. We use a very weak criterion for the desirability of a tax/subsidy. We first establish that a subsidy on short-term speculation leaves all traders no worse off than no subsidy. We then check under which conditions the increase in firm value due to improved investment efficiency more than outweighs the cost to the tax authority of actually paying a subsidy. We give a precise condition under which a subsidy for trade on short-term information is desirable according to this criterion.

Appendix

Proof of Proposition 1:

We start the proof by characterising the information available to the market maker in equilibrium and the resulting price setting behaviour and investment policy. We do so supposing that the speculator chooses order size $(1\text{-}m)q_e$ and submits a buy (sell) order only after having received good (bad) news. We complete the proof by characterising the market maker's out-of-equilibrium beliefs that support this order size and trading rule as an equilibrium choice by the speculator.

If the market maker observes an order flow $Q_{t,t+2} = 2(1\text{-}m)q_e$, he knows that the informed trader must have submitted a buy order. Given the informed trader's strategy, it is clear that she must have observed a high productivity level ($y_{t+2}=b$). Since the market maker thus knows y_{t+2}, this information will get fully reflected in the equilibrium price. This in turn allows the firm to deduce in equilibrium its own productivity level from observing the price.

From (1) we can write down the first-order condition for firm value maximisation: $y_{t+2}\text{-}ck_{t+2} = 0 \quad \Rightarrow \quad k_{t+2}{}^* = y_{t+2}/c.$

Hence, if the price is fully revealing and $y_{t+2}=b$, the optimal investment level is $k_t{}^*=b/c$, resulting in a liquidation value $v^*{}_{t+2}=b^2/2c$. Since in equilibrium the market maker anticipates this investment decision, he sets $p_{t,t+2} = b^2/2c$. Similarly, equilibrium prices and investment levels can be derived for $Q_{t,t+2} = -\ 2(1\text{-}m)q_e$.

Now consider the case $Q_{t,t+2} \in \{-\ (1\text{-}m)q_e,\ 0,\ (1\text{-}m)q_e\}$. As described in Section 3, these order flows reveal no information to the market maker and therefore, the equilibrium price will not reflect any information concerning y_{t+2}.

If the firm does not know its own productivity, and maximises expected liquidation value, the first-order condition becomes

$$1/2(a\text{-}ck_{t+2})+1/2(b\text{-}ck_{t+2}) = 0 \quad \Rightarrow \quad k_{t+2}{}^*=(a+b)/2c.$$

This results in expected firm value of

$$E[v^*_{t+2}] = \frac{(a+b)^2}{8c}. \tag{14}$$

Again, setting price equal to the expected liquidation value of the firm, results in the price as stated in the proposition.

The formula for expected firm value follows straightforwardly, when one considers that with probability $n_I q/2$ the state of the world is such that the firm learns its productivity parameter, while it does not learn anything with the complementary probability.

Now consider the optimality of the speculator's trading rule. Firstly, note that a speculator has no incentive to trade without information. This is the case, because by submitting an order she is likely to move the price, which results in trading losses, because the direction of the price move does not correspond to the underlying value of the asset more often than it does. Secondly, consider the market maker's out-of-equilibrium beliefs that support order size $(1\text{-}m)q_e$. Suppose that the speculator receives good news (the case of trading on bad news can be dealt with analogously), and submits a buy order of size $(1\text{-}m)q_e + d$, where $d \in (0, (1\text{-}m)q_e)$. Then total order flow can take the values d or $2(1\text{-}m)q_e + d$. Now suppose that the market maker's out-of-equilibrium pricing rule is as follows:

$$p_{t,t+2} = \begin{cases} \dfrac{(a+b)^2}{8c} & \text{for} \quad 0 \le Q_{t,t+2} \le (1-m)q_e \\ \dfrac{b^2}{2c} & \text{for} \quad (1-m)q_e < Q_{t,t+2} \le 2(1-m)q_e \\ \dfrac{b^2}{2c} + B & \text{for} \quad Q_{t,t+2} > 2(1-m)q_e \end{cases} \tag{15}$$

Note, that for $Q_{t,t+2} > 2(1\text{-}m)q_e$, the price set by the market maker is supported by his belief that if total order flow exceeds $2(1\text{-}m)q_e$, this must be the case, because there is another informed trader who trades on information concerning e_{t+2}. The out-of-equilibrium belief then leads the market maker to adjust price, taking into account that the ε-informed trader knows that $e_{t+2} = B$. It is then straightforward to show that for $d \in (0, (1\text{-}m)q_e]$, expected profits from trading (given that a signal has been received) are zero. For d=0, expected profits from trading are $Bq(1\text{-}m)q_e/4 > 0$. Hence, the long-term speculator does not wish to deviate from the proposed equilibrium order size.

q.e.d.

Proof of Proposition 2: In contrast to the price $p_{t\text{-}1,t+1}$ which only depends on current order flow, the 'short-term' price $p_{t,t+1}$ depends on current order flow *and* the previous price. If, at date t-1, order flow already revealed the speculator's private information, this information will also get reflected in the following price. Hence, whenever $p_{t\text{-}1,t+1}$ reveals the speculator's information, the price $p_{t,t+1}$ will only change by the amount e_{t+1}, which becomes publicly known at date t.

If the 'long-term' price does not reveal private information, we know from Proposition 1 that the firm will take the 'middle-of-the-road' investment decision $k_{t+1}=(a+b)/2c$. Whenever the order flow $Q_{t,t+1}$ reveals the short-term speculators private information, the market maker can calculate the expected firm value, by simply substituting the investment level k_{t+1} and the revealed y_{t+1} into (formula for firm value), which yields the prices given in the proposition. Straightforward calculation of expected firm value, when the investment level is (above) and the productivity level is not known yields the remaining price.

q.e.d.

Proof of Proposition 3: Start by considering the short-term and long-term speculators' expected payoffs, depending on each of the two speculator's actions (decision to produce or not to produce information).

The formula for long-term trading profits is derived for a strategy of trading at date t and unwinding the long-term position at date t+1. Straightforward calculation yields

$$E[p_l | e_s = 1] = \frac{q^2}{4} Bq_e (1-m) - qq_e (1-m) t_l - k \qquad (16)$$

	Short-term speculator	*YES*	*NO*
Long-term speculator	Probabilities	ν_s	$1-\nu_s$
YES	ν_l	$E[\pi_l\|e_s=1]$, $E[\pi_s\|e_l=1]$	$E[\pi_l\|e_s=0]$, 0
NO	$1-\nu_l$	0, $E[\pi_s\|e_l=0]$	0, 0

Table 2: Shows the payoff matrix for each type of speculator depending on the speculator's decision to incur the cost of information production.

When there is no short-term speculator, next period's prices will certainly not move towards the fundamental value, and therefore expected profits consist of tax payments (receipts) minus the cost of information production. Hence,

$$E[p_l|e_s=0] = -qq_e(1-m)\,t_l - k. \tag{17}$$

When there is no long-term speculator trading, 'long-term' prices will certainly not reflect the asset's fundamental value and therefore the short-term speculator always makes a trading profit when she receives information and her own order remains unrevealed. This happens with probability $q/2$. Hence,

$$E[p_s|e_l = 0] = \tfrac{q}{2}Bq_h m - qq_h m t_s - k \tag{18}$$

If, on the other hand, a long-term speculator produces information, the 'long-term' price does not reveal information with probability $1-q/2$. Hence,

$$E[p_s|e_l = 1] = \left(1-\tfrac{q}{2}\right)\left(\tfrac{q}{2}Bq_h m - qq_h m t_s\right) - k \tag{19}$$

For the case that $E[p_l|e_s=1] > 0$ and $E[p_s|e_l=0] > 0$, and $E[p_s|e_l=1] < 0$ and $E[p_l|e_s=0] < 0$, we get a mixed strategy equilibrium of the reduced form game, in which each of the two types of speculator chooses a probability n_l, n_s with which they produce information. Note that $E[p_l|e_s=0] < 0$ is always the case, unless the long-term speculator receives a substantial subsidy. However, if the long-term speculator were to receive such a subsidy, he would not have an incentive to actually acquire the information, and would instead be better off just churning, hence not increasing the informational efficiency of 'long-term' prices.

If the above conditions are indeed satisfied a mixed strategy equilibrium is given by

$$n_s E[p_l|e_s=1] + (1-n_s)\, E[p_l|e_s=0] = 0, \qquad (20)$$

and

$$n_l E[p_s|e_l=1] + (1-n_l)\, E[p_s|e_l = 0] = 0. \qquad (21)$$

Substituting (16) and (17) into (20) and solving for n_s yields (6). Similarly, substituting (18) and (19) into (21) and solving for n_l yields (5).

For completeness, consider the maximum tax rates that can be levied without resulting in a degenerate equilibrium in which neither type of speculator acquires information. The long-term speculator would never incur the cost of information production if $E[p_l|e_s=1]<0$. Substituting (8) in (16) and solving for t_l yields

$$t_l \leq B \frac{\frac{q}{4} Bq(1-m) - k}{Bq(1-m) - k} \qquad (22)$$

Similarly, the short-term speculator would never wish to produce information, if $E[p_s|e_l=0]<0$. From (7) and (18) we can solve for the maximum permissible tax rate:

$$t_s \leq 2B \frac{\frac{q}{4} Bm - k}{Bqm + k} \qquad (23)$$

Now consider the trading strategy by ε-hedgers. As was shown in the main text, an ε-hedger born at date t will trade at date t and unwind his position at date t+1 (if he trades at all). Hence, we only need to derive the order size of an ε-hedger. Consider the following event tree for the case that at date t, $z_{e,t}$ = -1, i.e. the agent has a positive hedging demand ($q_{e,t} = q_e$).

Define the following variables: $q_l \equiv n_l q$, $q_s \equiv n_s q$, $n_l \equiv q_e(1-m)$, $n_s \equiv q_h m$.

From Table 3 and the event tree in Figure 1, we can conclude that the following outcomes with associated probabilities result in equilibrium if the ε-hedger chooses an order size q_e.

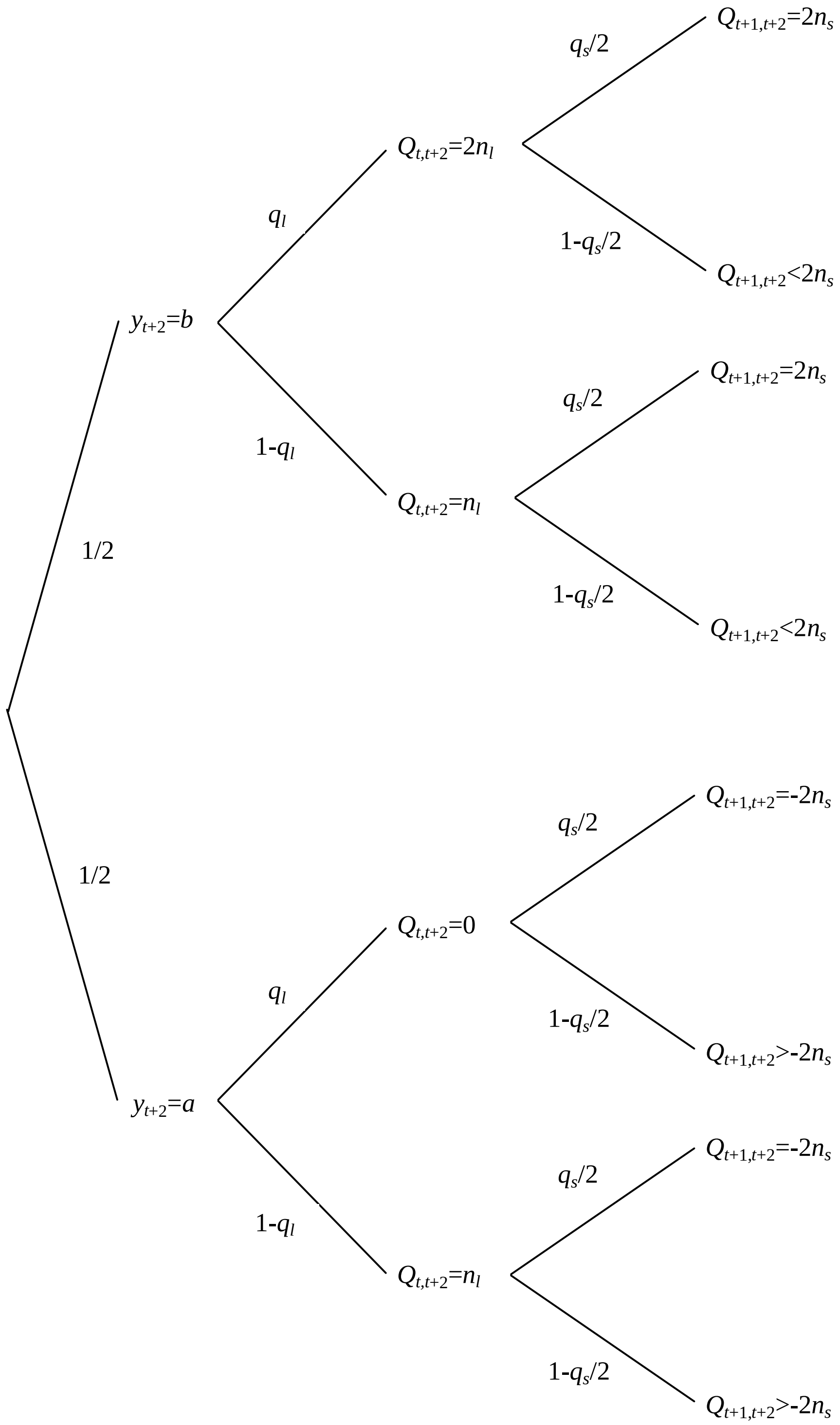

Figure 1: Illustrates the event tree relevant to the ε-hedgers' choice of order size.

y_{t+2}	$Q_{t,t+2}$	$Q_{t+1,t+2}$	$p_{t,t+2}$	$p_{t+1,t+2}$
b	$2n_l$	$2n_s$	$\frac{b^2}{2c}$	$\frac{b^2}{2c} + e_{t+2}$
b	$2n_l$	$<2n_s$	$\frac{b^2}{2c}$	$\frac{b^2}{2c} + e_{t+2}$
b	n_l	$2n_s$	$\frac{(a+b)^2}{8c}$	$\frac{4b^2-(a-b)^2}{8c} + e_{t+2}$
b	n_l	$<2n_s$	$\frac{(a+b)^2}{8c}$	$\frac{(a+b)^2}{8c} + e_{t+2}$
a	0	$-2n_s$	$\frac{(a+b)^2}{8c}$	$\frac{4a^2-(a-b)^2}{8c} + e_{t+2}$
a	0	$>-2n_s$	$\frac{(a+b)^2}{8c}$	$\frac{(a+b)^2}{8c} + e_{t+2}$
a	n_l	$-2n_s$	$\frac{(a+b)^2}{8c}$	$\frac{4a^2-(a-b)^2}{8c} + e_{t+2}$
a	n_l	$>-2n_s$	$\frac{(a+b)^2}{8c}$	$\frac{(a+b)^2}{8c} + e_{t+2}$

Table 3: Illustrates possible states of the world with associated probabilities that are payoff relevant to the ε-hedgers.

With probability

$1-q_s(1-q_s q_l/2)/2$ $\quad x=(q_e-1)e_{t+2}- q_e t_l$

$q_s(1-q_l)/4$ $\quad x=q_eB + (q_e-1)e_{t+2}-q_e t_l$

$q_s/4$ $\quad x= -q_eB + (q_e-1)e_{t+2}-q_e t_l$,

where e_{t+2} takes the values $-B$, B with equal probability. This yields six possible outcomes for the ε-hedger, which we can number and call $x_1, x_2,...,x_6$. For each of the possible outcomes we can determine whether they are increasing or decreasing in q_e (assuming that t_l is sufficiently close to zero). We can moreover determine for which q_e each of the x_i are equal to zero. This supplies us with the necessary case distinctions in order to determine expected utility, which is piece wise linear with a kink at zero. For each of the resulting cases, we can determine whether or not expected utility on that interval of q_e is increasing or decreasing in q_e. This exercise has to be carried out separately for $t_l>0$ and for $t_l<0$. As a result we find that when

$t_l \geq 0$, then for $q_e < B/(B+t_l)$, expected utility is increasing in q_e, iff

$$t_l < t_u = B\frac{1-n_s q + \frac{n_s n_l q^2}{4}}{1+\frac{n_s n_l q^2}{4}}.$$

If $t_l > t_u$, the hedger would not want to participate in the market.

Moreover, for $q_e > B/(B+t_l)$, expected utility is decreasing in q_e for any positive tax rate t_l. Hence, for a positive tax rate the optimal order size is $q_e = B/(B+t_l)$ (or zero if the tax rate is too high).

Similarly, for the case $t_l < 0$, it is possible to show that when $q_e < B/(B-t_l)$, then for any negative tax rate, the hedger wishes to increase order size. If, on the other hand, $q_e > B/(B-t_l)$, then the expected utility is decreasing in order size, as long as $t_l>-B$, which has to be the case for the long-term speculator to produce information, rather than to simply trade without information (see Lemma 1).

Hence, the optimal order size for an *e*-hedger is given by $B/(B+|t_l|)$ for an interval of t_l as specified in the proposition.

Now consider the trading strategy by an η-hedger. We need to distinguish two cases. Firstly, the long-term price $p_{t,t+2}$ may already have revealed the speculator's private information. In that case the tax rate is simply set to zero. Moreover, the short-term price $p_{t+1,t+2}$ is then simply set equal to the actual asset value plus the shock e_{t+2} (see Proposition 2). Hence, the income, including the traded position, of a η-hedger is simply: $x_{h,t+2} = q_{h,t+1,t+2}\eta_{t+2} + z_{h,t+2}\eta_{t+2}$. From this it follows straightforwardly that full hedging, i.e. $q_{h,t+1,t+2} = -z_{h,t+2}$ is optimal.

If, on the other hand, the long-term price does not reveal the information, the η-hedger faces a residual risk exposure to y_{t+2}. Suppose the hedger faces a positive hedging need ($z_{h,t}$= -1) and chooses a positive order $q_{h,t}$=q_h. Then, with probability $q_s/2$ the productivity level is high, *and* revealed to be so by short-term prices. In this case the risk associated with y_{t+2} is cancelled. With complementary probability the short-term price does not reflect the underlying productivity parameter, which can be either high or low. Altogether, this yields the following outcomes for the η-hedger:

With probability	$q_s/2$	$x_{h,t+2} = (q_h-1)\eta_{t+2} - q_h t_s$
	$(1-q_s)/2$	$x_{h,t+2} = q_h B+(q_h-1)\eta_{t+2} - q_h t_s$
	$1/2$	$x_{h,t+2} = -q_h B+(q_h-1)\eta_{t+2} - q_h t_s$.

Again, h_{t+2} can take the values $-B$, B with equal probability, yielding 6 different outcomes that can be numbered $x_1,...,x_6$. As before, it is possible to make case distinctions as to the sign of x_i depending on the size of q_h. Going through the calculations in a similar vein as for the *e*-hedger it can be shown that the optimal order size is as in (proposition) for the corresponding interval for the tax rate t_s. By repeating the exercise for a negative hedging need $z_{h,t+2}$ =1, it can be shown that the optimal order size is $-q_h$.

q.e.d.

Proof of Lemma 1: Firstly, consider the case where the long-term speculator receives a subsidy $t_l<0$. If the subsidy is high, the speculator may wish to trade without having acquired information. Although trade then results in a loss, this is compensated for to some extent by the subsidy on trade. Note that when the long-term trader deviates from the equilibrium of trading only when having received information, such a deviation remains unnoticed by the market maker and the firm. This means that her trade affects prices and possibly the investment decision by the firm, although no information has been acquired.

When trading without information, it does not matter for the speculator whether she buys or short-sells the security. Consider therefore w.l.o.g. the case where she always buys the security. With probability 1/4 the productivity level is high ($y=b$), and the price is ‘revealing’, in which case the deviant speculator makes zero expected trading profits. With probability $1/8q_s$ productivity is high and the price is not revealing and the short-term speculator moves the price in the subsequent period, resulting in expected trading profits of B. With probability $1/4(1-q_s/2)$ we are in the above case, except that the short-term speculator does not move the price, so zero expected trading profits result.

On the other hand, it may happen that the productivity level is low, but long-term prices are as if the productivity level were high (probability 1/4). In that case the firm takes the wrong investment decision ($k=b/c$ when it should be a/c). In the subsequent period, the true productivity level may be revealed by the short-term trader. Note that in equilibrium a short-term speculator does not trade after the long-term price has been revealing. Suppose the market maker’s out-of-equilibrium belief is that short-term trade after revealing long-term prices means that the short-term speculator trades on information, while the long-term speculator traded on noise. In that case the short-term speculator actually has an incentive to trade on his information even if the long-term price has been 'revealing', and the market maker's belief is correct, that the short-term speculator trades on information while the long-term trader does not.

The market maker then sets the price of the stock equal to $b(2a\text{-}b)/2c+e$. On the other hand, the short-term price may remain unchanged if the short-term speculator does not trade or her order remains hidden, in which case zero expected trading profits result. Moreover, if the productivity level is low and the long-term speculator's order remains hidden, she may incur a trading loss of magnitude B, when the short-term price is 'revealing'.

Overall the trading profits from trading without information are

$$E[\hat{p}_l] = -n_l\left(\frac{q_s b(b-a)}{8c} + t_l\right) \qquad (24)$$

Setting (24) smaller or equal to zero and solving for t_l yields the desired result.

On the other hand, trading in the absence of information results in the following loss for the short-term speculator. Note, that the short-term speculator only trades when the long-term price has not yet been revealing, which is the case with probability $1\text{-}q_l/2$.

$$E[\hat{p}_s] = -n_s\left(1-\frac{q_l}{2}\right)\left(\frac{B}{2} + t_s\right). \qquad (25)$$

The inequality (25) follows straightforwardly.

q.e.d.

Proof of Proposition 4: As a criterion for social desirability of a tax/subsidy on short-term trade, we consider whether or not the gains in expected firm value more than outweigh the cost of the subsidy. Note that this is a rather weak criterion, because the ε-hedger is unaffected by a tax t_s, while the η-hedger profits from it. The benefit of the η-hedger is neglected for simplicity.

To start with, we calculate the gain/loss in investment efficiency as a function of the tax rate t_s. Denote this gain by $E[G(t_s)]$. Moreover, denote by $E[v^{rev}]$ the expected firm value, when the long-term price reveals the speculator's information. We get

$$E[v^{rev}] = \frac{b^2 + a^2}{4c}. \qquad (26)$$

Denote by $E[v^{non}]$ the expected firm value when the long-term price is not revealing. This yields $E[v^{non}] = \frac{(a+b)^2}{8c}$. (27)

From this we can calculate the gain in efficiency, by considering a change in the probability with which the long-term speculator produces information, that then gets incorporated into the long-term price. Denote by $Dv \equiv E[v^{rev}] - E[v^{non}]$. This yields

$$E[G(t_s)] = Dv(n_l(t_s) - n_l(t_s = 0))q/2. \qquad (28)$$

Now consider the cost/revenue of a subsidy/tax on short-term trade. Since the tax/subsidy only accrues when the long-term price has not yet been revealing, there will only be any revenue/expenditure with probability $(1 - n_l q/2)$. In that case, the η-hedger trades for sure, while the short-term trader only trades with probability $n_s q$ (remember that the conditions on t_s ensure that the short-term speculator is not willing to trade unless she has actually received information). Tax revenue (expenditure) $E[T(t_s)]$ can thus be written as

$$E[T(t_s)] = (1 + n_s q)(1 - n_l q/2) n_s t_s. \qquad (29)$$

Defining a 'welfare' function $V(t_s) \equiv E[T(t_s)] + E[G(t_s)]$ yields

$$V(t_s) = \left(n_s q + 1 - \frac{5\Delta v}{Bm}\right) \frac{k t_s}{a\left(\frac{B}{2} - t_s\right)} \qquad (30)$$

From this it is straightforward to see that

$V(t_s)$ is a decreasing function of t_s

$\Leftrightarrow$

$$n_s q + 1 - \frac{5\Delta v}{Bm} < 0. \qquad (31)$$

Substituting n_s from equation (6) and Dv from (26) and (27) into (31) yields the result in the proposition.

q.e.d.

Proof of Proposition 5: Consider the trading strategy of a long-term speculator with an infinite horizon (once she has incurred the cost of information production). Suppose the speculator decides at date t-1 to produce information (concerning firm F_{t+2}'s productivity parameter). One possible trading strategy is to trade at date t (when information is received) and hold the position until the liquidation of the firm. In that case the order flow at date t does not reveal the speculator's order with probability 1/2. Given that a signal is received with probability q, the resulting trading profits are:

$E[p_l(\text{hold})]$

$$= \tfrac{q}{2} q_e (1-m)\left[\tfrac{1}{2}\left(\frac{4b^2-(a-b)^2}{8c}-\frac{(a+b)^2}{8c}\right)-\tfrac{1}{2}\left(\frac{4a^2-(a-b)^2}{8c}-\frac{(a+b)^2}{8c}\right)\right]\text{-}k$$

$$= \tfrac{q}{2} B q_e (1-m) - k \qquad (32)$$

where $B=(b^2-a^2)/4c$.

In this case trading profits for the long-term speculator do not depend on whether or not the short-term speculator participates in the market as well.

Alternatively, the long-term speculator can trade at date t, unwind the position at t+1 and trade again and date t+1 a position that matches in size the η-hedgers total order. She thus makes a trading profit on the position traded between date t and t+1, only if short-term prices are revealing, while she makes a profits on the position traded between dates t+1 and t+2 only if short-term prices are not revealing. In that case it does matter whether or not the short-term speculator acquires information and trades on it. The long-term speculator makes smaller profits when the short-term speculator trades as well. This is the case, because the long-term speculator only chooses to trade twice (rather than hold a position for two periods), when there is more liquidity trade in the period preceding liquidation than two periods before liquidation. She thus makes more than half of her expected profits from trade in the second period. However, when there is also a short-term speculator, short-term prices will be revealing with a higher probability, which means that profits accrue with a higher probability from first period trade, thus reducing expected trading profits.

Consider trading profits when the short-term speculator also trades. It is straightforward to calculate that with probability (1-q)q/4 the long-term speculator receives information and both the short-term and the long-term prices do not reveal information. In that case trading profits are $Bq_h m$. With probability (1+q)q/4, the long-term speculator receives information, and only the long-term price but not the short-term price is non-revealing. Trading profits in that case are $Bq_e(1-m)$. This yields overall trading profits of

$$E[p_l(\text{trade twice})|e_s=1\,] = B\tfrac{q}{4}\Big(q_e(1-m)(1+q)+q_h m(1-q)\Big)-k \qquad (33)$$

Of course, the profits from either strategy are identical when the total order size submitted by ε- and η-hedgers is identical, i.e. when $q_e(1-m) = q_h m$. Otherwise, the speculator will choose the trading strategy that allows her to trade at the point in time when there is more liquidity trade. Hence, the speculator follows a ‘trade-and-hold’ strategy when $q_e(1-m) > q_h m$, and a strategy of trading twice when $q_e(1-m) < q_h m$.

The smallest profits that a long-term speculator can achieve are given by the maximum of (32) and (33). Solving for k thus yields a sufficient condition on k for which the long-term speculator is willing to incur the cost of information acquisition.

q.e.d.

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