Risk Averse Banks and Uncertain Correlation Values: A Theory of Rational Bank Panics

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Abstract

We present a model for financial fragility in which banks are risk-averse portfolio managers and there is uncertainty over risk management parameters. There is a danger of heightened risk aversion and projects in small economies are assumed to be riskier than those in large economies. In this situation there is a danger that a rise in project correlations will lead to a rational but unnecessary credit crunch. We conclude firstly that greater transparency in the dissemination of correlation parameters is desirable and secondly that regulators should respond to heightened financial fragility by relaxing capital adequacy requirements.

KEY WORDS: Banking, systemic risk, financial fragility, panics, capital adequacy, bank regulation, risk management, Value at Risk.

JEL CLASSIFICATION: C72, G21, G28.

What causes credit crunches? In other words, why do banks reduce lending volumes and call in outstanding loans when doing so diminishes the quality of their remaining assets and may lead to a recession? These questions have been a subject for economic investigation at least since Irving Fisher's (1933) introduction of debt deflation as a possible explanation for the 1929 stock market crash. In this paper, we provide an explanation for credit crunches in terms of uncertainty over risk management parameters when banks are rational but risk averse portfolio managers.

In our model, all projects have the same variance of returns and the same pairwise correlation ρ between returns. Banks are face an increasing cost of external funds and expect to have positive NPV projects in the future: as Froot, Scharfstein and Stein (1993) and Froot and Stein (1998) demonstrate, this will render them risk-averse and their risk aversion will be decreasing in their capital endowment. The monitoring activities of banks are subject to an (unmodelled) network externality. In consequence, an increased level of bank intermediated debt diminishes the volatility of returns of *every* bank monitored project. A consequence of these assumptions is that there is a range of ρ values within which with perfect information there are two possible rational bank investment levels. In this range the economy is *fragile*: for higher investment levels it is *healthy* and for lower levels it is *recessionary*. A move from the healthy to the recessionary economy occurs as a direct consequence of a reduction in bank lending and we therefore term such a movement a *credit crunch*.

In contrast with previous papers we consider a model of repeated one period bank contracts with no savings decision so that liquidity-related explanations for disinvestment cannot be employed. Instead, we allow a temporary increase in bank risk aversion: this leads to rational disinvestment and to a credit crunch. Disinvestment may also occur in response to an increase in ρ and this need not lead to a recession. At the end of each investment period, uninformed investors can observe total investment levels but cannot distinguish between increased risk aversion and increased correlation. We demonstrate that when the ex ante fear of increased risk aversion is sufficiently high credit crunches will occur in the wake of a change in correlation.

Our assumption that risk aversion can increase is unusual and requires further examination. As we observe above, Froot and Stein (1998) demonstrate that a bank's risk aversion is decreasing in its capital base and is increasing in its cost of refinancing after a negative shock to its capital levels. An increase in risk aversion could therefore occur for two reasons: it might arise after a significant loss, or it might be a consequence of an increased cost of recapitalising. The cost of recapitalisation could be affected by standard factors such as a ratings downgrade. It might also be raised as a consequence of regulatory stipulations such as an increase in the level of capital which a bank is required to hold against its assets.

The desire of risk averse banks to achieve portfolio diversification is the motivation for the Value at Risk (VaR) approach to risk management which was instituted by J.P. Morgan (1996) for management of bank trading risks. This approach has been adopted and enforced by the regulatory authorities. More recently, it has been discussed as a possible approach for the management of traditional portfolios of banking assets[1]. By assuming bank risk aversion we are able to investigate the systemic implications of these innovations. Banks are requried to set aside capital equal to a fixed multiple of their VaR figure. The multiplier is increased in response to poor trading performance. This inevitably results in a higher cost of capital and hence increases the effective risk aversion of the bank.

Credit crunches which arise in the wake of an upward revision of project correlation ρ need not occur and they reduce productive activity within the economy: we therefore style them *panics*. A voluminous literature examines financial panics. Allen and Gale (2000*a*) extend Fisher's work, demonstrating how shocks to the credit system can lead to asset price bubbles and to their subsequent bursting when there are agency effects between the owners of capital and those who deploy it. Other authors have explained panics in terms of liquidity problems[2], portfolio linkages[3] and liquidity problems[4]. Credit crunches have been specifically discussed by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), who examine the consequencies of second-best investment decisions which occur as a consequence of non-verifiable project returns. In these models collateral mitigates the contracting problems. A shock to output in one period affects collateral in the following periods and may therefore have a persistent effect.

The panics in our model occur as a consequence of utility maximising actions by agents with rational expectations. Rational expectations models have previously been adopted by Chari and Jagannathan (1988) and Chen (1999) in the context of panics and they have also been employed in the microstructure and corporate control literatures[5]. In models of this type it is possible for real economic effects to lead to disinvestment and hence to recession. Investors receive poor quality signals of the fundamentals and may in certain circumstances elect rationally to withdraw funds in response to a signal which was not generated by a downturn in fundamental conditions.

As in our model, recessions arise in Chari and Jagannathan's and Chen's models as a consequence of confusion over the motivation for the withdrawal of investor funds. In contrast to us, they rely upon two-period Diamond and Dybvig (1983)-style models of risk-neutral banks which manage portfolios of illiquid assets. Chari and Jagannathan show how liquidity shocked investors can be mistaken for informed investors with advance knowledge of poor second period returns. In Chen's model bank prospects vary and will be revealed only after a proportion of investors become perfectly informed. We consider a succession of single period models in which liquidity shocks cannot arise and confusion arises between increases in correlation and in banker risk aversion.

Our model abstracts away entirely from the details of the risk-sharing mechanisms available to banks and hence avoids institutional features. It explicitly excludes agency effects and contracting problems. All agents in our model act rationally in response to received stimuli and do not cluster upon uncertain parameters. In contrast to herding models of the Banerjee (1992) type the actions of our agents change their world. Our results follow from portfolio effects which are of relevance only because we assume banks to be risk averse.

Our model demonstrates that in the presence of uncertainty over risk management parameters, bank risk aversion can lead to panics. We make two policy prescriptions. Firstly, regulatory sanctions for poor performance should be relaxed in times of heightened fragility. We argue that this will have a similar effect to a reduction in risk aversion so that it will reduce the impact of panics and will also diminish their likelihood. Secondly, we suggest that regulators should require general disclosure by banks of the parameters which they use in their risk management systems. The consequential reduction in uncertainty would also serve to reduce the likelihood of panics.

The paper is organised as follows. Section 1 describes a simple economy which is populated by risk averse bankers and entrepreneurs and shows how financial fragility can arise for a range of correlation parameters. Section 2 sets out and solves the game which agents play to select their investments when there is uncertainty over risk aversion and the correlation between project returns. Section 3 contains concluding remarks. The proofs are contained in the appendix.

1 A Simple Model of the Economy

We consider a stylized model of a simple economy which is populated by N bankers and by m entrepreneurs[6], each of whom manages a project. Activity in the economy unfolds in discrete time with bankers taking fresh deposits at the start of each period and dispersing the proceeds of their investments at the end. We model this by assuming that at the start of each period all bankers have a deposit endowment of \$1. At the end of the period the slate is wiped clean and bankers start again with \$1. For the reasons discussed in the introduction, bankers exhibit risk aversion: every banker has an identical CARA utility function

$$u(z) = -e^{-az}$$

for end of period consumption z. At the start of each period every banker invests a fraction α of his deposit endowment in entrepreneur-managed projects so as to maximise his expected end-of-period utility. Note that our assumption of fund disbursement and of single-period utility maximisation precludes intertemporal considerations in the investment decision.

We assume that efficient risk-sharing can be accomplished between banks so that if a total of V is invested in the economy, $\frac{V}{m}$ will be allocated to each project and each banker will receive a share of each project's returns in proportion to his initial investment. All projects are identical and have random end-of-period return $\tilde{R}$. We assume that $\tilde{R}$ has the normal distribution ϕ:

$$\tilde{R} \sim \phi\left(r, \sigma^2\right),$$

where r is a constant.

We impose an exogenous assumption that a reduction in the size of the banking sector will result in more second-best entrepreneurial behaviour and hence will increase the riskiness of project returns. As we discussed in the introduction, this is motivated by an assumption that there is a positive externality associated with the (unmodelled) monitoring activities of bankers. The assumption is sufficient to ensure the fragility of the real economy for certain parameter values. Specifically, we make σ a deterministic function of the total volume V of funds which is invested in the economy:

$$\sigma(V) = \begin{cases} \sigma_H & \text{if } V \geq C \\ \sigma_R & \text{if } V < C \end{cases}, \tag{1}$$

where $\sigma_H < \sigma_R$ and σ_H, σ_R, C are all common knowledge: in the following, volatility will be σ_H in healthy economies and will be σ_R in recessionary ones.

Finally, we assume that the correlation between any two projects is ρ. ρ need not be common knowledge.

Bankers in this economy cannot directly observe each others' investment decisions but at the end of each investment period they learn the total volume V which was invested in the economy – this is an imperfect indicator of investment activity and will condition their beliefs in the subsequent investment period.

1.1 Financial Fragility

With m projects and a total investment of V the return on bank-intermediated debt will be

$$\begin{aligned}\tilde{P} = \sum_{i=1}^{m} \frac{1}{m} \tilde{R}_i &\sim \phi\left(r, \frac{\sigma^2(V)}{m} + m(m-1)\frac{\sigma^2(V)}{m^2}\rho\right) \\ &= \phi\left(r, \sigma^2(V)\left(\frac{1+(m-1)\rho}{m}\right)\right).\end{aligned}$$

In each investment period, all bankers will invest a proportion $\alpha \in [0,1]$ of their start of period endowment of \$1 and will hoard the remaining $\$(1-\alpha)$ until the end of the period. When σ, ρ are respectively the volatility of and the correlation between project returns this will generate an end of period income of

$$\tilde{W} = (1-\alpha).1 + \alpha.\tilde{P} \sim \phi\left(1+\alpha(r-1), \alpha^2\sigma^2\left(\frac{1+(m-1)\rho}{m}\right)\right).$$

Each banker therefore selects α to maximise his expected end of period utility:

$$\begin{aligned}E\left[-e^{-a\tilde{W}}\right] &= U\left(\rho, \sigma^2, \alpha\right) \\ &\equiv -\exp\left\{-a\left(1+\alpha(r-1) - \frac{a}{2}\alpha^2\sigma^2\left(\frac{1+(m-1)\rho}{m}\right)\right)\right\}. \quad (2)\end{aligned}$$

The first order condition from this equation yields the following value for α:

$$\alpha_a(\rho, \sigma) = \frac{(r-1)m}{a\sigma^2(1+(m-1)\rho)}. \quad (3)$$

Define $\alpha_R(\rho) \equiv \alpha_a(\rho, \sigma_R)$ and $\alpha_H(\rho) \equiv \alpha_a(\rho, \sigma_H)$. Note that $\frac{\partial\alpha_a}{\partial\rho} < 0$ and $\frac{\partial\alpha_a}{\partial\sigma^2} < 0$ so that investment levels are falling in ρ and σ^2.

When the correlation ρ between projects is common knowledge define a volatility σ^* to be *rational* when

$$\sigma\left(N\alpha_a\left(\rho,\sigma^*\right)\right)=\sigma^*.$$

When σ^* is rational a universal assumption that $\sigma=\sigma^*$ will be self-fulfilling: in this case we say that $\alpha_a\left(\rho,\sigma^*\right)$ is a rational level of investment.

Proposition 1 *Suppose that the correlation ρ between projects is common knowledge and define*

$$\rho\left(\sigma\right)\equiv\frac{N\left(r-1\right)m-a\sigma^2C}{a\sigma^2C\left(m-1\right)}.$$

1. *σ_R is a rational volatility precisely when $\rho>\rho_H\equiv\rho\left(\sigma_R\right)$;*

2. *σ_H is a rational volatility precisely when $\rho\leq\rho_R\equiv\rho\left(\sigma_H\right)$.*

Proof: σ_R is rational precisely when $N\alpha_R\left(\rho\right)<C$; rearranging this expression yields part 1. Part 2 follows similarly from the observation that σ_H is rational precisely when $N\alpha_H\left(\rho\right)\geq C$. Q.E.D.

We say that a *healthy* level of investment obtains when $\alpha=\alpha_H\left(\rho\right)$ and that a *recessionary* level of investment obtains when $\alpha=\alpha_R\left(\rho\right)<\alpha_H\left(\rho\right)$. Since $\rho'\left(\sigma\right)=-\frac{N(r-1)m}{a\sigma^2(m-1)}<0$ we must have $\rho_H<\rho_R$. It follows that there is a range of correlation parameters within which both healthy and recessionary investment levels are rational: in this range we say that the economy is *fragile*.

Proposition 2 *When ρ is common knowledge agents prefer healthy investment levels to recessionary levels.*

[Figure 1]

Figure 1 shows rational investment levels plotted against ρ when ρ is common knowledge. It suggests that we could employ ρ as a barometer of systemic risk. With high ρ levels only recessionary investment levels are possible. We are interested in movements between fragile levels of investment. To this end, we assume that

$$\rho \in \{\rho_S, \rho_C\}, \text{ where } \rho_H < \rho_S < \rho_C \leq \rho_R. \tag{A1}$$

2 Trading Game

2.1 Game Specification

We describe a game which is played when it is common knowledge that in the previous investment period, $\rho = \rho_S$ and $\sigma = \sigma_H$. At the start of the game a perturbation to the model parameters occurs. This is imperfectly communicated to the bankers and some uncertainty arises concerning the state of the world. This is resolved during subsequent investment periods as investment volumes are revealed.

In common with previous informational models of systemic failure (Chen, 1999; Chari and Jagannathan, 1988), we assume that some parameter changes will lead inevitably to disinvestment. A panic occurs when unnecessary disinvestment occurs in the absence of such a change. We will show that panics can arise as a rational response to changes in the correlation parameter.

Specification of informational models of panics requires a parameter modification which leads inevitably to disinvestment. Chen uses an exogenous possibility of poor project returns in a model of two period deposit contracts of the Diamond and Dybvig (1983) type; Chari and Jagannathan allow an exogenously-imposed liquidity squeeze to arise, also in a two-period deposit contract. Our model of repeated single period investments will not admit such a possibility. Instead, we assume a possibility of temporarily increased risk aversion. As discussed in the introduction, this may arise as a consequence of trading losses within the bank or as a result of regulator-imposed policy changes.

Changes in risk aversion last for two investment periods. In the first period the risk aversion of a proportion ι_W of banks increases to $a\left(\frac{\sigma_R}{\sigma_H}\right)^2$: the consequence of this change is that the investment level selected in healthy economies by withdrawers is the same as the one that other agents would select in a recessionary economy. In the second a fixed proportion γ of the remaining banks also increase their risk aversion parameter; γ is an exogenous contagion parameter which reflects concern about illiquidity amongst these banks.

[Figure 2]

The timing of the trading game is pictured in figure 2. Nature makes the first move, choosing to leave all parameters unchanged or to perturb either risk aversion parameters or project correlations. With probability $\delta < 1$ she does not perturb investment parameters. With probability $(1-\delta)\,\pi$ she raises risk aversion for a proportion ι_W of banks as detailed above. With probability $(1-\delta)\,(1-\pi)$ she changes the correlation between project returns to ρ_C. Only a fraction ι_L of banks will receive a signal of this change.

After nature's move, all players simultaneously select an investment level, as shown in figure 2. After all players have invested the total volume invested is revealed: this occurs at the point identified in figure 2 by dashed lines. Players use this information to update their belief system and they then re-invest. The game ends when ρ is once again common knowledge. We will show that in equilibrium this will take at most two investment stages, as in the figure.

When nature makes her move she creates three types of bankers. We call bankers with no signal *followers*, those with increased risk aversion *withdrawers* and those with knowledge of changed correlation *leaders*. While withdrawers and leaders are perfectly informed about nature's initial move followers are not.

When followers are unable to distinguish between volume levels in the wake of some withdrawal signals ι_W and of some leader signals ι_L a panic is possible.

For panics to be possible we require total post-signal volume to be the same in the wake of withdrawal and leader signals with positive probability. We will accomplish this by selecting deterministic values for ι_W and ι_L which ensure that post-shock volume is independent of the type of shock. A more involved approach with (say) uniformly distributed values for ι_L and ι_W would yield qualitatively the same results but at the cost of far greater complexity.

Write $N \equiv \{U, W, L\}$ for nature's set of moves (Unchanged, Withdrawal, Leader) and $\Sigma = \{\sigma_H, \sigma_R\}$ for the set of possible project return variances. Define the correlation ρ_T associated with a type $T \in N$ as follows:

$$\rho_T \equiv \begin{cases} \rho_S, & T = W \\ \rho_C, & T = L \end{cases}$$

The *beliefs* of followers are given by a probability measure μ on N. Write M for the set of beliefs. The initial follower beliefs μ_1 reflect their prior information as follows:

$$\left.\begin{aligned} \mu_1(U) &= \delta \\ \mu_1(W) &= (1-\delta)\pi \\ \mu_1(L) &= (1-\delta)(1-\pi) \end{aligned}\right\} \tag{4}$$

The equilibrium investment decisions of bankers will be informed by the beliefs of followers and also by their knowledge of the previous period's investment volume: our solution to the game will employ an assumption of sticky volatility expectations. The investment decisions of followers, withdrawers and leaders will therefore be given by respective functions $\lambda_F : M \times \Sigma \to [0,1]$, $\lambda_W : M \times \Sigma \to [0,1]$, $\lambda_L : M \times \Sigma \to [0,1]$.

A *strategy* is a triple $\lambda = (\lambda_F, \lambda_W, \lambda_L)$ of investment functions. Suppose that the follower beliefs are μ. If the previous period's volatility of returns was σ then investment according to strategy λ will lead after a move by nature of type T to the following total investment volume:

$$
\begin{aligned}
&v(\lambda,\mu,T,\sigma)\\
&\equiv\left\{\begin{array}{ll}
N\left[\lambda_F(\mu,\sigma)(1-\iota_W)+\lambda_W(\mu,\sigma)\,\iota_W\right], & T=W \text{ and } \mu(U)\neq 0\\
N\left[\lambda_F(\mu,\sigma)(1-\iota_W)(1-\gamma)\right. & \\
\qquad\left.+\lambda_W(\mu,\sigma)(\iota_w+\gamma(1-\iota_w))\right], & T=W \text{ and } \mu(U)=0\\
N\left[\lambda_F(\mu,\sigma)(1-\iota_L)+\lambda_L(\mu,\sigma)\,\iota_L\right], & T=L
\end{array}\right.
\end{aligned}
$$

The dependence upon $\mu(U)$ when $T=W$ arises because of the effect of the contagion parameter. In the first investment stage no information has been revealed and the followers place a non-zero probability upon nature leaving the investment parameters unchanged. If nature triggers a withdrawal at this stage it will be amongst a proportion ι_W of banks. If the followers are aware that a perturbation has occurred then if it is a withdrawal it will be in its second stage and the proportion of withdrawers will be increased by the effects of contagion to $\iota_W+\gamma(1-\iota_W)$.

We employ a rational expectations equilibrium (Radner, 1979 and Grossman and Stiglitz, 1980):

Definition 3 *A* rational expectations equilibrium *for the investment game is a strategy* $\lambda=(\lambda_F,\lambda_W,\lambda_L)$ *and a series* $(\mu_i)_{i\in\mathbb{N}}$ *of follower beliefs for* $i=1,2,\ldots,n$ *such that:*

1. μ_1 *satisfies equation 4;*

2. *Let* V_i *be the revealed second period volume and write* $\sigma(V_i)=\sigma_i$ *.* μ_{i+1} *is derived from* μ_i *by Bayesian updating in accordance with* λ *and* V_i*:*

$$
\mu_{i+1}(T)=\left\{\begin{array}{ll}
0, & v(\lambda,\mu_i,T,\sigma_i)\neq V_i\\
\frac{\mu_i(T)}{\mu_i\{T'\in N\mid v(\lambda,\mu_i,T',\sigma_i)=V_i\}}, & \textit{otherwise}
\end{array}\right.
$$

3. *λ depends upon the previous period's volatility σ_{i-1} and current beliefs μ_i as follows:*

$$\begin{aligned}
\lambda_F(\mu_i, \sigma_{i-1}) &= \arg\max_{\alpha} \sum_{T \in N} \mu_i(T)\, U\left(\rho_T, \sigma^2\left(v\left(\lambda, \mu_i, T, \sigma_{i-1}\right)\right), \alpha\right); \\
\lambda_W(\mu_i, \sigma_{i-1}) &= \alpha_{a\left(\frac{\sigma_R}{\sigma_H}\right)^2}\left(\rho_S, \sigma\left(v\left(\lambda, \mu_i, W, \sigma_{i-1}\right)\right)\right); \\
\lambda_L(\mu_i, \sigma_{i-1}) &= \alpha_a\left(\rho_C, \sigma\left(v\left(\lambda, \mu_i, L, \sigma_{i-1}\right)\right)\right),
\end{aligned}$$

where $U(.)$ and $\alpha_a(.)$ are defined in equations 2 and 3 respectively.

4. *The support of μ_n has size* 1.

Part 1 of the definition ensures that the initial beliefs of the followers are consistent with their prior information. Part 2 ensures that beliefs are correctly updated in response to revealed investment volumes. Part 3 states that each agent will select his investment so as to maximise his expected utility and that the total realised investment volume will be consistent with the volatility assumptions used to derive investment volumes. Part 4 states that the game ends when all uncertainty has been resolved.

Note that at the end of the game, part 3 of the definition implies that the economy has a volatility σ^* which is rational in the sense of section 1.1. A *panic* occurs when $\iota_W = 0$ and $\sigma^* = \sigma_R$.

2.2 Game Solution

There are several equilibria for the game described in section 2.1 which depend upon the total investment levels which players select. For example, a general assumption that the volatility of projects will alternate between σ_H and σ_R whenever possible will be self-fulfilling and will lead to an equilibrium, although it does not seem a sensible one. We provide a solution in which players assume that volatility parameters change as little as possible. The motivation for this approach is the difficulty of coordinating a significant modification of investment policy. Since disinvestment occurs when volatility increases our assumption leads to a world in which the propensity for disinvestment and for recessions is minimised.

Accordingly, we define an investment strategy λ^* so that each agent will maximise his expected utility in the next period conditional upon the smallest possible rationally assumed change in the input parameter σ. The formal definition is as follows.

Let σ_P be the volatility of asset returns during the previous investment period and let μ be an arbitrary belief system. For $n = 1, 2, \ldots$ we define investment mappings $\zeta_F^n : M \rightarrow [0,1]$, $\zeta_W^n : M \rightarrow [0,1]$, $\zeta_L^n : M \rightarrow [0,1]$ and volatility functions $\sigma_n : N \rightarrow [0,1]$ by simultaneous induction as follows:

$$\left.\begin{array}{r}\sigma_0\left(T\right) = \sigma_P \\ \sigma_n\left(T\right) = \sigma\left(v\left(\zeta^n, \mu, T, \sigma_{n-1}\left(T\right)\right)\right)\end{array}\right\} \tag{5}$$

$$\left.\begin{array}{l}\zeta_F^n\left(\mu\right) = \arg\max_a \sum_{T\in N} \mu\left(T\right) U\left(\rho_T, \sigma_{n-1}^2\left(T\right), \alpha\right) \\ \zeta_W^n\left(\mu\right) = \alpha_{a\left(\frac{\sigma_R}{\sigma_H}\right)^2}\left(\rho_S, \sigma_{n-1}\left(W\right)\right) \\ \zeta_L^n\left(\mu\right) = \alpha_a\left(\rho_C, \sigma_{n-1}\left(L\right)\right)\end{array}\right\} \tag{6}$$

Let k be the lowest integer such that

$$\forall T \in N . \mu\left(T\right) \neq 0 \rightarrow \sigma_{k+1}\left(T\right) = \sigma_k\left(T\right) \tag{7}$$

and define

$$\lambda^*\left(\mu, \sigma_P\right) = \zeta^k\left(\mu\right).$$

Equations 5 and 6 define ζ^1 to be the investment strategy obtained by assuming an unchanged volatility parameter. If this volatility assumption is not self-fulfilling the agents take the volatilities which it generates as their new assumption and they recompute their investments. The induction stops when the volatility assumptions are unchanged by the optimising process. We require $\mu\left(T\right) \neq 0$

in equation 7 so as to ensure that the condition is only applied to conceivable economies: in other words, to those whose existence is in accordance with publicly available information.

We demonstrate below that k is at most 3 so that λ^* is well defined.

By construction, $\sigma\left(v\left(\lambda^*, \mu, T, \sigma_k\left(T\right)\right)\right) = \sigma_k\left(T\right)$ so it is a trivial consequence of equation 6 that λ^* satisfies part 3 of definition 3 and hence that when $\left(\mu_i\right)_{i\in\mathbb{N}}$ is defined by parts 1 and 2 of the same definition, $\left(\lambda^*, \left(\mu_i\right)_{i\in\mathbb{N}}\right)$ is a rational expectations equilibrium for the investment game. We now examine the properties of the equilibrium.

We require some additional assumptions to derive our results. Firstly, recall from our earlier remarks that we require total first period investment volume to be the same after either a withdrawal or a correlation signal. We show shortly that the following assumption guarantees this:

$$\frac{\iota_W}{\iota_L} = \frac{\alpha_H\left(\rho_S\right) - \zeta_F^1\left(\mu\right)}{\alpha_R\left(\rho_S\right) - \zeta_F^1\left(\mu\right)}. \tag{A2}$$

We require the first wave of disinvestment after a risk aversion increase to be insufficient to trigger universal second period disinvestment. We demonstrate below that the following suffices:

$$0 \leq \iota_W \leq I \equiv \frac{\sigma_R^2}{\sigma_R^2 - \sigma_H^2} - aC\frac{\sigma_H^2\sigma_R^2\left(1 + \left(m-1\right)\rho_S\right)}{N\left(r-1\right)m\left(\sigma_R^2 - \sigma_H^2\right)}. \tag{A3}$$

Proposition 4 *When condition A3 is satisfied the volatility of assets in the first period of the equilibrium $\left(\lambda^*, \left(\mu_i\right)_{i\in\mathbb{N}}\right)$ is always σ_H. Moreover, when assumption A2 holds moves W and L by nature lead to the same the first period investment volume.*

Proposition 4 states that no signal from nature can cause an immediate descent into recession. Moreover, it tells us that although there is no uncertainty for withdrawers and leaders, followers will be unable after revelation of first period investment volumes to distinguish between withdrawals and correlation changes.

Proposition 5 demonstrates that it will take at most three periods for all uncertainty in the economy to be resolved.

Proposition 5 *Write $\|\mu_i\|$ for the length of the series $(\mu_i)_{i\in\mathbb{N}}$. In the equilibrium $\left(\lambda^*, (\mu_i)_{i\in\mathbb{N}}\right)$:*

1. *$\|\mu_i\| = 2$ when nature does not perturb the economy;*
2. *If nature does perturb the economy, $\|\mu_i\| = 3$ and μ_2 and μ_3 are independent of π. All uncertainty is resolved at the end of the second investment stage.*

[Figure 3]

Withdrawers and leaders will always know precisely what nature's move was. The evolution of the information structure for followers is illustrated in figure 3: the times in the figure refer to labels in figure 2. The horizontal line of dots at each stage represents the set of possible signals. Given that a signal has occurred, the set of possible signals which the followers believe to be possible is indicated by the box which contains that signal; the probabilities which the followers assign to each signal are shown in each non-trivial partition. Note that at time t_2, every signal is contained in a singleton box, so that there is no ambiguity and the game has ended.

We now characterise the economy which will obtain at the end of the game when the equilibrium (λ^*, μ_i) is selected. To do so we require two further assumptions.

Firstly, we require there to be sufficient bankers in the economy for each to be unconcerned about the consequences of his own actions upon the total volume invested. The following assumption will suffice :

$$N > aC\sigma_R^2 \frac{(1 + (m-1)\rho_S)}{m(r-1)}. \tag{A4}$$

We also require increases in the herding parameter γ to lead unambiguously to more systemic risk. The following assumption will accomplish this for us:

$$\sigma_H^2 \rho_C < \sigma_R^2 \rho_S. \tag{A5}$$

Think of $\sigma^2 \rho$ as a measure of the riskiness of an economy which has pairwise correlation ρ between projects when the volatility of project returns is σ. Assumption A5 states that the low volatility economy is always less risky than the high volatility economy.

With these assumptions the risk of recession is related to the size of the contagion parameter γ and also to the ex ante probability π which players place upon an increase in risk aversion. Panics can occur when both γ and π are high enough:

Proposition 6 *Suppose that nature starts the game by perturbing the economy.*

1. *If* $\gamma \leq \frac{I-\iota_W}{1-\iota_W}$ *then the economy will not enter a recession;*
2. *If* $\gamma > \frac{I-\iota_W}{1-\iota_W}$ *then withdrawals will end in recession. Moreover, there is* ι_L^* *such that:*
   (a) $\iota_L^* = \frac{C-N\alpha_R(\rho_S)}{N(\alpha_H(\rho_C)-\alpha_R(\rho_S))}$, $\iota_L^* \in (0,1)$ *and* ι_L^* *is increasing in* ρ_S *and in* ρ_C*;*
   (b) *If* $\iota_L \geq \iota_L^*$*, a change in correlation will not lead to a recession;*
   (c) *If* $\iota_L < \iota_L^*$ *there is* $\pi_L \in (0,1)$ *such that correlation changes will lead to recessions precisely when* $\pi > \pi_L$*.* π_L *is increasing in* ι_L*.*

Proposition 6 is the key result of this paper. Firstly, it states that there is a threshold level for the contagion parameter γ below which recessions will not occur. In view of the acknowledged significance of contagion in financial downturns this is an unsurprising result. Secondly, when the contagion parameter is sufficiently high, *all* increases in risk aversion result in recessions. In this case, if the ex ante probability which uninformed traders place upon rises in risk aversion is sufficiently high then *any* perturbation of the economy by nature will result in recession. We provide the intuition for this result below.

Given a high contagion parameter γ and a high withdrawal probability, suppose that uninformed investors observe a disinvestment caused by a correlation change. They do not know the reason for the disinvestment. Increased risk aversion will result in a recession and followers will reduce their investment to reflect the ex ante probability which they place upon this. When the effect of the followers' disinvestment is sufficiently large it will precipitate an unnecessary recession so that it will become rational for the leaders to disinvest. In this case $\iota_W = 0$ and the game terminates with the volatility of investment projects equal to σ_R: in other words, a panic occurs.

[Figure 4]

The case where the contagion level γ is sufficiently high for all withdrawals to result in recession is illustrated in figure 4. This illustrates all possible combinations (ι_L, π) for economies which result from a perturbation by nature of the project correlation: if a recession then occurs, it does so as a consequence of panic. For combinations (ι_L, π) which lie towards the top left of the box there are a low number of uninformed followers who ascribe a very low probability π to a withdrawal: with these parameters, panics cannot occur. Combinations which lie towards the bottom right of the box have a high number of uninformed investors who attach a high probability to a panic movement: these parameter values will cause panics to occur.

The dividing line between the recessionary and non-recessionary regions is shown as a bold line. For parameter values south east of this line correlation increases will result in panics. For $\iota_L \geq \iota_L^*$, there are so few uninformed investors that panics cannot happen. For lower values of ι_L the bold line is given by $\pi = \pi_L$. The increasing width of the recessionary region as ι_L decreases reflects the increasing relevance of the opinion of the large follower group.

Finally, we provide some observations about ι_L^*. As $\sigma_R \rightarrow \sigma_H$ the size of the fragile region diminishes, the post crash investment level $\alpha_R(\rho_C)$ increases towards $\frac{C}{N}$, the healthy investment level $\alpha_H(\rho_C)$ decreases towards $\frac{C}{N}$ and $\iota_L^* \rightarrow 0$. In other words, increasing the severity of recession by increasing $\sigma_H - \sigma_R$ will increase the economy's susceptibility to financial fragility and will also increase the likelihood of financial panics.

3 Conclusion

This paper considers an economy in which banks are risk averse, project riskiness is a decreasing function of the size of the banking sector and there is a possibility that banks may experience a short-term increase in their risk aversion. In this situation the economy will exhibit fragility in the sense that with perfect information both high and low investment levels are possible. Increases in risk aversion lead unavoidably to recessions but increases in the correlation between project returns need not. If bankers receive imperfect signals of parameter changes then a credit crunch may occur as a rational response to an increase in correlation. This will result in an unnecessary recession and we therefore call this phenomenon a panic.

Two regulatory suggestions follow. Firstly, we have drawn upon the results of Froot and Stein (1998) to argue that regulatory sanctions for poor performance may act to increase the effective risk aversion of banks[7]. In our model heightened risk aversion leads to panics and also increases their severity. We therefore suggest that relaxing capital requirements in times of heightened financial fragility may reduce the danger of panics.

Our second observation is that better dissemination of risk management parameters would remove the confusion which this model identifies and hence would diminish the risk of financial panics. As bankers typically regard this data as commercially sensitive it may be necessary for regulators to compel its disclosure.

Proofs

Proof of Proposition 2

For fixed ρ, regard $\alpha(\rho,\sigma)$ as a function of σ^2 so that $\frac{\partial\alpha}{\partial\sigma^2} = -\frac{\alpha}{\sigma^2}$ and the expected utility $U(.)$ defined in equation 2 is a function of σ^2 only. Then

$$\begin{aligned}\frac{\partial}{\partial\sigma^2}U\left(\sigma^2\right) &= -aU\left(\sigma^2\right)\frac{\partial}{\partial\sigma^2}\left(1+\alpha(r-1)-\frac{a}{2}\alpha^2\sigma^2\left[\frac{1+(m-1)\rho}{m}\right]\right)\\ &= aU\left(\sigma^2\right)\frac{(r-1)^2}{2a\sigma^4\left(1+(m-1)\rho\right)} < 0,\end{aligned}$$

which establishes the result.

The following lemma describes how an agent will choose his investment level when his beliefs have a two element support. We will require it to derive the behaviour of the followers under strategy λ^*.

Lemma 7 *Suppose previous period volatility was σ_{-1} and that a follower has beliefs μ with a two element support T_1 and T_2. Fix assumptions $\sigma\left(v\left(\lambda,\mu,T_i,\sigma_{-1}\right)\right) = \sigma_i$ for $i = 1,2$ and let α_1 and α_2 be the assignments which the follower would choose given that he was certain of T_1 and T_2 respectively. Write $m\left(\alpha_1,\alpha_2,\mu\left(\alpha_1\right)\right)$ for the assignment which the follower chooses given beliefs μ. Then $m\left(\alpha_1,\alpha_2,1\right) = \alpha_1$, $m\left(\alpha_1,\alpha_2,0\right) = \alpha_2$ and $m(.)$ is monotonic in $\mu\left(\alpha_1\right)$.*

Proof: Write ζ for $\mu\left(\alpha_1\right)$ and let ρ_1, ρ_2 be the respective correlation parameters after moves T_1 and T_2. The agent will select α to maximize expected utility $W(\alpha)$:

$$W(\alpha) = \zeta U\left(\rho_1,\sigma_1^2,\alpha\right) + (1-\zeta)U\left(\rho_2,\sigma_2^2,\alpha\right), \tag{8}$$

where $U\left(\rho,\sigma^2,\alpha\right)$ is defined by equation 2. The agent solves the following first order condition:

$$\begin{aligned}&\zeta U\left(\rho_1,\sigma_1^2,\alpha\right)\left(r-1-a\alpha\sigma_1^2\left(\frac{1+(m-1)\rho_1}{m}\right)\right)+\\ &\qquad(1-\zeta)U\left(\rho_2,\sigma_2^2,\alpha\right)\left(r-1-a\alpha\sigma_2^2\left(\frac{1+(m-1)\rho_2}{m}\right)\right) = 0.\end{aligned}$$

Denote by $F(\alpha)$ and $S(\alpha)$ respectively the first and second terms in the definition of $W(\alpha)$ and without loss of generality, assume $\alpha_1 < \alpha_2$. Then $F'(\alpha_1) = 0$ and $S'(\alpha_1) > 0$ so $W'(\alpha_1) > 0$. Similarly, $W'(\alpha_2) < 0$ so by the intermediate value theorem, $\alpha_1 < a(\alpha_1, \alpha_2, \zeta) < \alpha_2$. By definition, $m(\alpha_1, \alpha_2, 1) = \alpha_1$ and $m(\alpha_1, \alpha_2, 0) = \alpha_2$. As ζ increases, $|S'(\alpha_2)|$ decreases for a given α_2 so $W'(\alpha_1)$ and $W'(\alpha_2)$ decrease and by concavity of W, the solution to $W'(\alpha) = 0$ moves closer to α_1, as required. Q.E.D.

Proof of Proposition 4

Following our definition for λ^*, we firstly derive the investment functions ζ_F^1, ζ_W^1 and ζ_L^1 conditional upon an assumption that volatility remains unchanged at σ_H. With this assumption we get

$$\zeta_W^1(\mu_1) = \frac{(r-1)\,m}{a\left(\frac{\sigma_R}{\sigma_H}\right)^2 \sigma_H^2 \left(1 + (m-1)\,\rho_S\right)} = \alpha_R(\rho_S)$$

and $\zeta_L^1(\mu_1) = \alpha_H(\rho_C)$, where $\alpha_R(.)$ and $\alpha_H(.)$ are defined in section 1.1. With probability $\delta + (1-\delta)\,\pi$, followers should select $\alpha_H(\rho_S)$ and with probability $(1-\delta)(1-\pi)$ they should select $\alpha_H(\rho_C)$. From lemma 7, they will select $\zeta_F^1(\mu_F^1) = m(\alpha_H(\rho_S), \alpha_H(\rho_C), \delta + (1-\delta)\,\pi)$. ζ^1 will be adopted provided for every $T \in N$, $v(\zeta^1, \mu_1, T, \sigma_H) = \sigma_H$. This is trivially the case for any economy for which $T \neq W$. When $T = W$,

$$\begin{aligned} v\left(\zeta^1, \mu_1, T, \sigma_H\right) &= N\left[\iota_W \alpha_R(\rho_S) + (1-\iota_W)\,\alpha_H(\rho_S)\right] \\ &= \frac{N(r-1)\,m}{a\left(1+(m-1)\,\rho_S\right)}\left(\frac{1-\iota_W}{\sigma_H^2} + \frac{\iota_W}{\sigma_R^2}\right) \\ &\geq \frac{N(r-1)\,m}{a\left(1+(m-1)\,\rho_S\right)}\left(\frac{1-I}{\sigma_H^2} + \frac{I}{\sigma_R^2}\right) = C, \end{aligned}$$

where the inequality is a consequence of assumption A3. We have therefore demonstrated that $\lambda^*(\mu_1, \sigma_H) = \lambda^1(\mu)$ and that first period volatility will be σ_H for every $T \in N$.

For first period investment volume to be the same in the wake of W and L moves by nature, we require

$$N\left[\zeta_F^1(\mu_1)(1-\iota_W)+\alpha_R(\rho_S)\iota_W\right]=N\left[\zeta_F^1(\mu)(1-\iota_L)+\alpha_H(\rho_S)\iota_L\right].$$

This is an immediate consequence of assumption A2.

Proof of Proposition 5

1. We know from proposition 4 that

   $$\lambda_F^*(\mu_1)=m(\alpha_H(\rho_S),a_H(\rho_C),\delta+(1-\delta)\pi)\equiv\alpha_{F1},$$

   $\lambda_W^*(\mu_1)=\alpha_R(\rho_S)$ and $\lambda_L^*(\mu_1)=\alpha_H(\rho_C)$.

   The reported end of period volumes will therefore be $V_{1U}=N\alpha_{F1}$ after no perturbation, $V_{1W}=Nv(\lambda^*,\mu_1,W,\sigma_H)$ after a rise in risk aversion and $V_{1L}=Nv(\lambda^*,\mu,L,\sigma_H)$ after a change in correlation. By assumption A2 $V_{1W}=V_{1L}$: moveover, $V_{1W}<V_{1U}$. It follows that when no perturbation occurs all players will know immediately after the first period volume figure is announced that nature's move was U and the game will end.

2. If $V_1=V_{1W}=V_{1L}$ is announced at the end of the first stage of a game the followers will update their beliefs in accordance with part 2 of definition 3 and will assign probability π to W and probability $(1-\pi)$ to L. Let α_{F2}, α_{W2}, α_{L2} be the respective assignments of followers, withdrawers and leaders: we determine these in the proof of proposition 6. V_2 will be

   $$V_{2W}\equiv N\left(\left[\iota_W+\gamma(1-\iota_W)\right]\alpha_{P2}+(1-\gamma)(1-\iota_W)\alpha_{F2}\right)\qquad(9)$$

   after move W by nature and will be

   $$V_{2L}\equiv N\left(\iota_L\alpha_{L2}+(1-\iota_L)\alpha_{F2}\right)\qquad(10)$$

   after move L. $V_{2W}\neq V_{2L}$ and so all uncertainty will be resolved when V_2 is revealed.

Proof of Proposition 6

Recall from proposition 4 that the first period realised volatility is σ_H. We compute $\lambda^*(\mu_2)$. Firstly, let $\gamma \leq \frac{I-\iota_W}{1-\iota_W}$. We determine optimal allocations ζ^1 assuming an unchanged second period volatility of σ_H. In this case the optimal investment for leaders is $\zeta_L^1(\mu_2) = \alpha_H(\rho_C)$, for withdrawers is $\zeta_W^1(\mu_2) = \alpha_R(\rho_S)$ and, using lemma 7, for followers is $\zeta_F^1(\mu_2) = m(\alpha_H(\rho_S), \alpha_H(\rho_S), \pi)$. If $v(\zeta^1, \mu_2, T, \sigma_H) \geq C$ for $T = W$ and L then $\lambda^*(\mu_2) = \zeta^1(\mu_2)$ and recessions will not occur. This condition holds trivially for $T = L$ as $\zeta_F^1(\mu_2) > \alpha_H(\rho_C)$. It will be true for $T = W$ provided

$$[\iota_W + \gamma(1-\iota_W)]\,\alpha_R(\rho_S) + (1-\gamma)(1-\iota_W)\,\zeta_F^1(\mu_2) \geq \frac{C}{N}; \tag{11}$$

this will be true whatever the value of π provided

$$[\iota_W + \gamma(1-\iota_W)]\,\alpha_R(\rho_S) + (1-\gamma)(1-\iota_W)\,\alpha_H(\rho_C) \geq \frac{C}{N}. \tag{12}$$

This requirement is equivalent to

$$\frac{1}{\sigma_H^2(1+(m-1)\rho_C)} - [\iota_W + \gamma(1-\iota_W)]\frac{\sigma_R^2 - \sigma_H^2 + (m-1)(\sigma_R^2\rho_S - \sigma_H^2\rho_C)}{\sigma_R^2\sigma_H^2(1+(m-1)\rho_S)(1+(m-1)\rho_C)} \geq \frac{aC}{(r-1)mN}.$$

It is a consequence of assumption A5 that the left hand side of this expression is decreasing in γ. Inserting the assumed maximum value $\frac{I-\iota_W}{1-\iota_W}$ of γ into the left hand side, we obtain

$$\frac{(m-1)(\rho_C-\rho_S)}{(\sigma_R^2-\sigma_H^2)(1+(m-1)\rho_S)(1+(m-1)\rho_C)} - \frac{aC\sigma_R^2(m-1)(\rho_C-\rho_S)}{Nm(r-1)(\sigma_R^2-\sigma_H^2)(1+(m-1)\rho_C)} + \frac{aC}{Nm(r-1)}.$$

Inserting the maximum value for N from assumption A4 into this expression, we obtain its minimum value $\frac{aC}{Nm(r-1)}$ as required. It follows that $\sigma(v(\zeta^1, \mu_2, T, \sigma_H)) = \sigma_H$ for $T \in \{W, L\}$ and $\lambda^*(\mu_2) = \zeta^1(\mu_2)$ so that when V_L is revealed and all uncertainty is resolved the economy has a healthy level of investment.

Now suppose that $\gamma > \frac{I-\iota_W}{1-\iota_W}$. Assuming again an unchanged second period volatility of σ_H the optimal allocation will again be ζ^1 as in the first part of the proof. As before, $v\left(\zeta^1, \mu_2, L, \sigma_H\right) > C$. However,

$$\begin{aligned} v\left(\zeta^1, \mu_2, W, \sigma_H\right) &= N\left(\left[\iota_W + \gamma\left(1-\iota_W\right)\right]\alpha_R\left(\rho_S\right) + \left(1-\gamma\right)\left(1-\iota_W\right)\alpha_{F1}\right) \\ &< N\left(\left[\iota_W + \gamma\left(1-\iota_W\right)\right]\alpha_R\left(\rho_S\right) + \left(1-\gamma\right)\left(1-\iota_W\right)\alpha_H\left(\rho_S\right)\right) \\ &< C, \text{ because } \gamma > \frac{I-\iota_W}{1-\iota_W}. \end{aligned}$$

It follows that $\lambda^*\left(\mu_2\right) \neq \zeta^1\left(\mu_2\right)$. Following the definition of λ^*, we compute optimal allocations with assumed volatilities $\sigma_1\left(L\right) = \sigma_H, \sigma_1\left(W\right) = \sigma_R$. Conditional upon these assumptions, optimal allocations are given by

$$\begin{aligned} \zeta_F^2 &= m\left(\alpha_R\left(\rho_S\right), \alpha_H\left(\rho_C\right), \pi\right), \\ \zeta_L^2 &= \zeta_L^1, \\ \zeta_W^2 &= \frac{N\left(r-1\right)\sigma_H^2}{a\sigma_R^4\left(1+\left(m-1\right)\rho_S\right)}. \end{aligned}$$

Then $v\left(\zeta^2, \mu_2, W, \sigma_R\right) < C$ so $\lambda^*\left(\mu_2\right) = \zeta^2\left(\mu_2\right)$ if $v\left(\zeta^2, \mu_2, L, \sigma_H\right) > C$. This inequality condition is true provided

$$\iota_L\alpha_H\left(\rho_C\right) + \left(1-\iota_L\right)m\left(\alpha_R\left(\rho_S\right), \alpha_H\left(\rho_C\right), \pi\right) \geq \frac{C}{N}. \tag{13}$$

Condition 13 will be satisfied for any π provided $\iota_L\alpha_H\left(\rho_C\right) + \left(1-\iota_L\right)\alpha_R\left(\rho_S\right) \geq \frac{C}{N}$ - in other words, if

$$\iota_L \geq \iota_L^* \equiv \frac{C - N\alpha_R\left(\rho_S\right)}{N\left(\alpha_H\left(\rho_C\right) - \alpha_R\left(\rho_S\right)\right)}. \tag{14}$$

Since $C \geq N\alpha_R\left(\rho_S\right)$ and $\alpha_H\left(\rho_C\right) > \alpha_H\left(\rho_S\right)$, $\iota_L^* > 0$. Moreover, since $N\alpha_H\left(\rho_C\right) > C$, $\iota_L^* < \frac{C-N\alpha_R(\rho_S)}{C-N\alpha_R(\rho_S)} = 1$. The sign of $\frac{\partial\iota_L^*}{\partial\rho_S}$ is the same as the sign of

$$\begin{aligned} &\left(N\left(\alpha_H\left(\rho_C\right) - \alpha_R\left(\rho_S\right)\right)\left(-N\alpha'\left(\rho_S\right)\right)\right) - \left(C - N\alpha_R\left(\rho_S\right)\right)\left(-N\alpha'\left(\rho_S\right)\right) \\ &\qquad = \left(-N\alpha'\left(\rho_S\right)\right)\left(N\alpha_H\left(\rho_C\right) - C\right) > 0, \end{aligned}$$

since $\alpha_H(.)$ is decreasing in ρ. The sign of $\frac{\partial \iota_L^*}{\partial \rho_C}$ is the same as the sign of

$$(C - N\alpha_R(\rho_S))(-N\alpha'_H(\rho_C)) > 0.$$

We have therefore shown that ι_L^* has the required properties.

Now suppose that condition 14 is not satisfied so that for at least some value of π it is not the case that $\lambda^*(\mu_2) = \zeta^2(\mu_2)$. In this case, condition 13 is false for $\pi = 1$ and true for $\pi = 0$. Since the left hand side of equation 13 is monotonic in π there is a unique $\pi_L \in (0,1)$ such that condition 13 is satisfied precisely when $\pi \leq \pi_L$. Since the extent of the failure in equation 13 is decreasing in ι_L, π_L is an increasing function of ι_L. $\lambda^*(\mu_2) = \zeta^2(\mu_2)$ precisely when $\pi \leq \pi_L$. If $\pi > \pi_L$ we determine optimal allocations given the assumed volatility function $\sigma_2(T) = \sigma_R$ for $T \in \{W, L\}$; they are given by $\zeta_F^3 = m(\alpha_R(\rho_S), \alpha_R(\rho_C), \pi)$; $\zeta_L^3 = \alpha_R(\rho_C)$ and $\zeta_W^3 = \zeta_W^2$. Then $v\left(\zeta^3, \mu_2, T, \sigma_R\right) < C$ for $T \in \{W, L\}$ and $\lambda^*(\mu_2, \sigma_H) = \zeta^3(\mu_2)$.

When $\gamma > \frac{I-\iota_W}{1-\iota_W}$ we have shown that $\lambda^*(\mu_2) = \zeta^2(\mu_2)$ whenever $\iota_L \geq \iota_L^*$ and for lower values for ι_L whenever $\pi \leq \pi_S$. Since $v\left(\zeta^2, \mu_2, W, \sigma_H\right) < C$, in the wake of an increase in risk aversion the game will always end in a recessionary state with $\rho = \rho_S$. For these values of ι_L and π, $v\left(\zeta^2, \mu_2, L, \sigma_H\right) \geq C$ so that after a change of correlation the game will end in a non-recessionary state with $\rho = \rho_C$. If $\iota_L < \iota_L^*$ and $\pi > \pi_L$, $\lambda^*(\mu_2) = \zeta^3(\mu_2)$. In this case increases in risk aversion will again lead to recessionary investment levels and, since $v\left(\zeta^3, \mu_2, Lm\sigma_H\right) < C$, a change of correlation will cause the economy to move into a recessionary state with $\rho = \rho_C$.

Notes

[1]The Basle Committee (1999) has for the time being rejected VaR as an approach for computation of regulatory capital requirements, but this topic remains important to practitioners and to regulators.

[2]Diamond and Dybvig (1983) show how panics can develop when bank depositors force early liquidation of assets absent liquidity needs.

[3]Rochet and Tirole (1996), Lagunoff and Schreft (1998), Allen and Gale (2000*b*), Freixas, Parigi and Rochet (1999), Freixas and Parigi (1998).

[4]Diamond and Dybvig (1983) show how panics can develop when bank depositors force early liquidation of assets absent liquidity needs.

[5]Grossman and Stiglitz (1980), Kyle (1985), Pagano (1989).

[6]We adopt a loose definition of "project." We require different projects to have substantively different properties so that a broad selection of projects automatically provides risk diversification. One could therefore interpret a 'project' as an amalgamation of all of the organisations operating in a particular sector of the economy. It follows that N will typically exceed m.

[7]See also Flannery (1989), who argues that regulators may render banks risk averse in their selection of individual credits, while they continue to seek portfolio volatility.

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Figure captions

Figure 1. Possible aggregate investment levels against project return correlation. When correlation between projects is common knowledge the economy is financially fragile whenever two possible levels of aggregate investment are consistent with utility maximization. The economy is recessionary when only the lower level is possible and is healthy when the higher level is possible. Possible investment levels are plotted here for $r = 1.1$, $m = 10$, $a = 3$, $\sigma_R = 0.8$, $\sigma_H = 0.5$, $N = 10,000$ and $C = 2,000$.

Figure 2. Timing of the investment game. At the start of the game the state of the economy in the previous investment period is common knowledge. Nature introduces uncertainty by deciding whether to perturb an economic parameter. It will take at most two investment periods for the uncertainty to resolve itself.

Figure 3. Evolution of the Followers' Information Structure. Each line of dots represents the possible signals which nature could have selected. The box which contains a signal T contains all of the signals which followers believe to be possible when T actually occurred. Since the U signal appears in a singleton box at time t_1 the game terminates in one stage when parameters are unchanged. For W and L signals, the game terminates in two stages.

Figure 4. Unnecessary recession boundary for correlation changes. If all players knew for sure that nature had changed the correlation between investment projects then a recession would never ensue. When uninformed traders assign a sufficiently high probability to increases in risk aversion they will disinvest. When there are sufficiently many such traders a panic will occur. The range of parameters where panics occur is the "unstable" region in the figure.

Figure 1

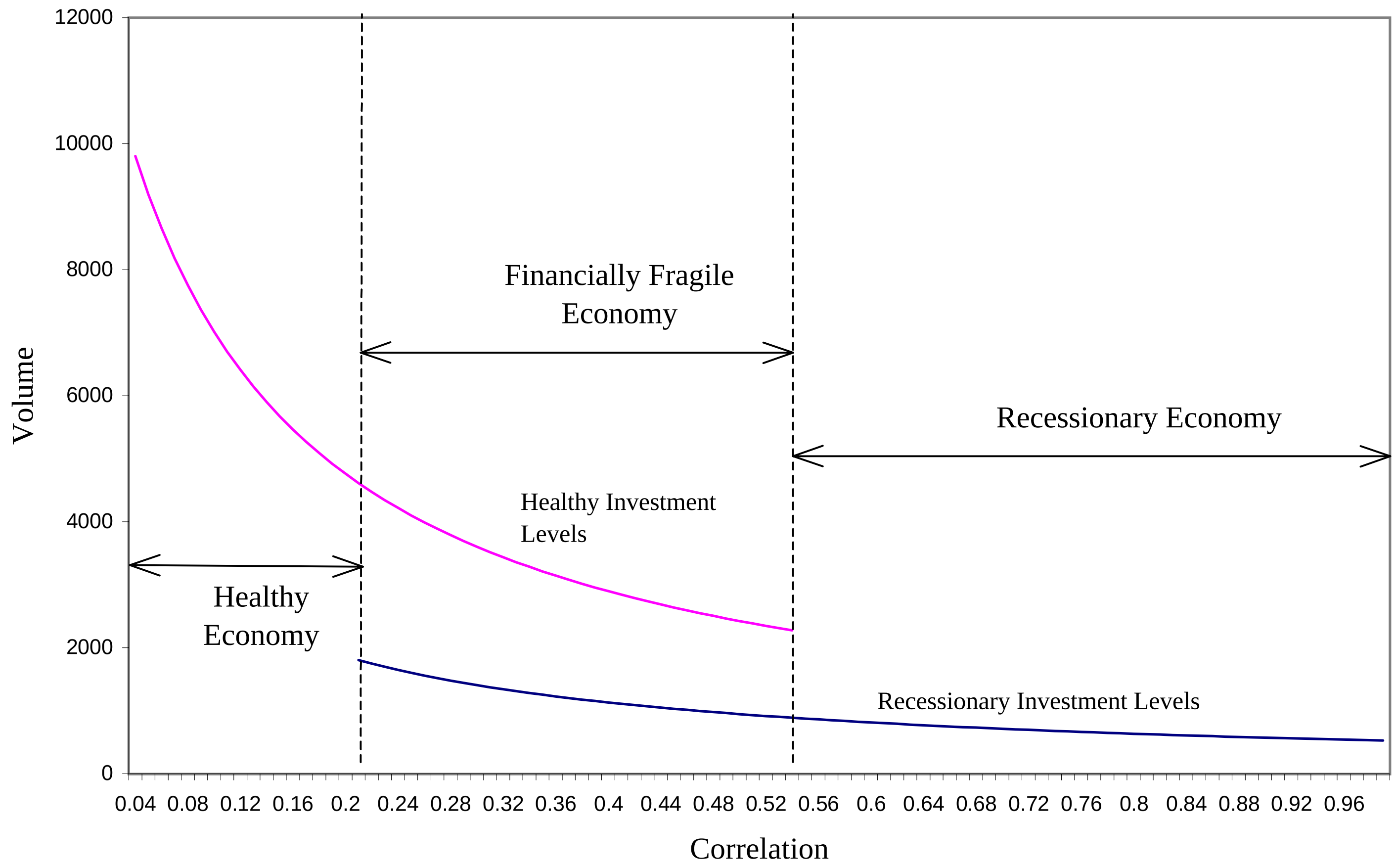
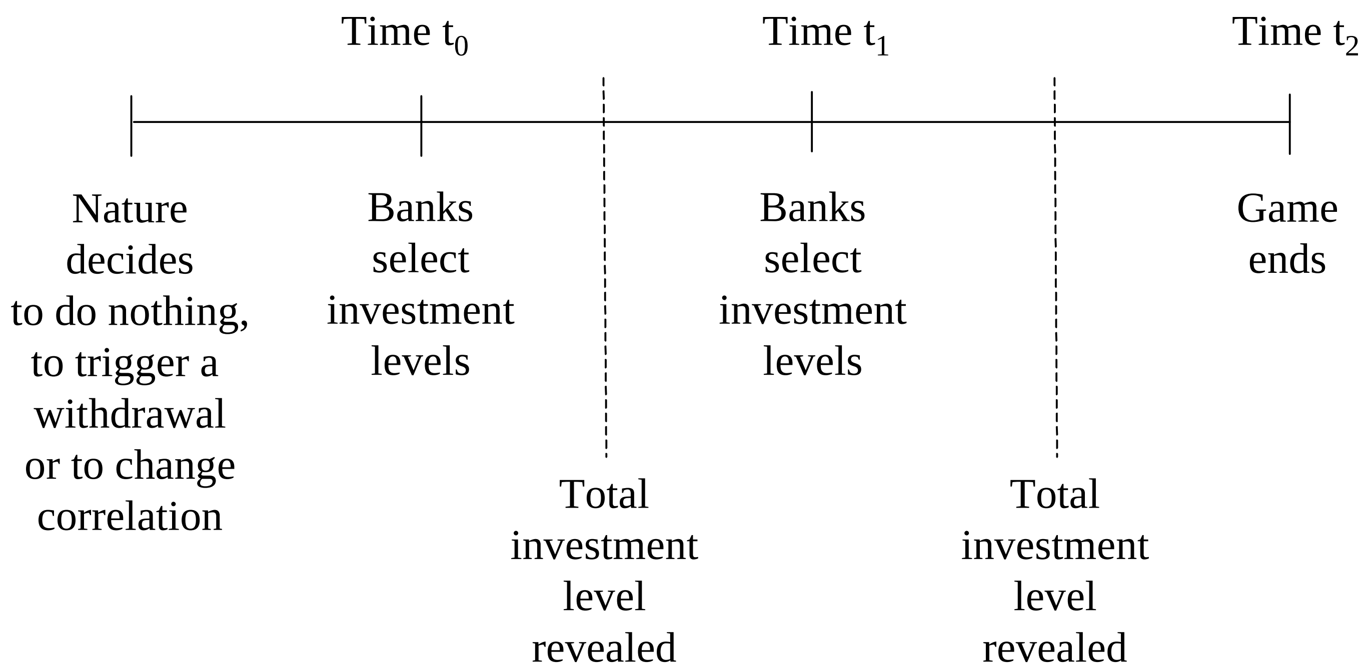

Volume
12000
10000
8000
6000
4000
2000
0
Financially Fragile Economy
Recessionary Economy
Healthy Investment Levels
Healthy Economy
Recessionary Investment Levels
0.04 0.08 0.12 0.16 0.2 0.24 0.28 0.32 0.36 0.4 0.44 0.48 0.52 0.56 0.6 0.64 0.68 0.72 0.76 0.8 0.84 0.88 0.92 0.96
Correlation

Figure 2



Time t_0
Time t_1
Time t_2
Nature decides to do nothing, to trigger a withdrawal or to change correlation
Banks select investment levels
Total investment level revealed
Banks select investment levels
Total investment level revealed
Game ends

Figure 3

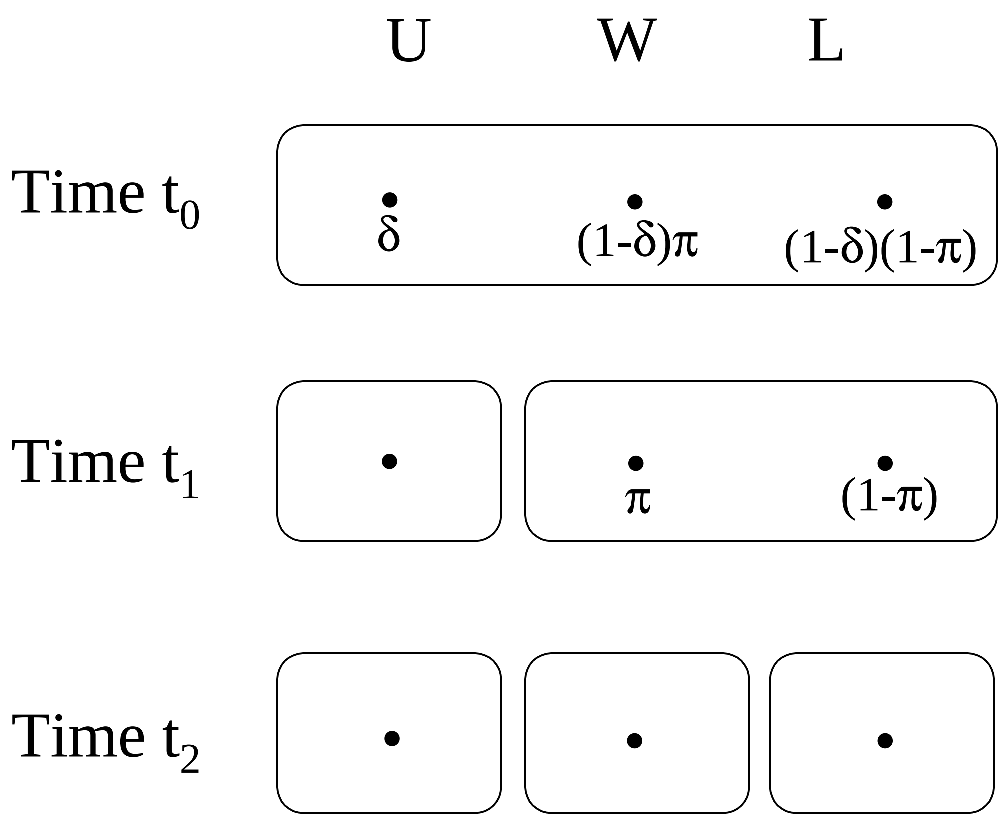


Figure 4