

# Optimal exchange-rates: a market-microstructure approach

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## **Abstract**

We analyze exchange-rate management by the central bank when it makes the FX market for the sake of social-welfare objectives. It is assumed that markets are incomplete, so that agents are exposed to exchange-rate volatility against which they cannot fully hedge. It follows that the central bank may provide insurance by smoothing the exchange rate. However, smoothing the exchange rate also creates arbitrage opportunities for speculators. We show that the central bank cannot smooth the exchange rate and deter speculation at the same time. A Tobin tax may provide a way out.

## **1 Introduction**

Recent research on the market microstructure of the foreign exchange (FX) market has made some important contributions to our understanding of this area.

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Evidence has been discovered for the presence of privately-informed traders, to the predictive power of the order flow and to the effect of the trading mechanism on the evolution of prices; see Lyons (2001) and Frankel *et. al.* (1996). In this paper we argue that the market-microstructure approach can provide a fresh insight into a problem that, for a long time, has evaded a satisfactory solution: the welfare analysis of various exchange rate regimes.<sup>1</sup> We investigate under what conditions, and in what manner, the central bank should interfere in the management of the exchange rate. Our theory is not only a normative one. It can explain important stylized facts like the widespread ‘fear of floating’, especially among emerging markets; see Calvo and Reinhart (2000). We also explain how fixed exchange rates are related to financial repression and the Tobin tax.

The market-microstructure approach starts by questioning the analytical value of economics’ most famous metaphor: the Walrasian auctioneer. Rather, the approach views the FX market more as an intermediary, where most of the trade is executed via specialized *dealers*, or *market makers*. Indeed, a 1998 survey by the Bank of International Settlement (BIS) shows that in the highly ‘competitive’ USD-DEM market, where prices are virtually free to float, 66% of the turnover is inter-dealer and 23% is executed against financial institutions; only the remaining 11% is executed against non-financial customers. The numbers for the USD-JPY market are 59%, 17% and 24%, respectively.<sup>2</sup> Even more importantly, the dealers’ role is not limited just to managing the market — quoting prices and executing orders — but also to providing liquidity by clearing a certain amount of the *order flow* against their own stocks.

From that point view, exchange-rate management should not be considered as

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<sup>1</sup>See Obstfeld and Rogoff (1996), p. 632: “...most of the literature on fixed versus flexible exchange rates ... is based on models that lack the microfoundations needed for meaningful welfare analysis.”

<sup>2</sup>See Table E-2 In the BIS’ Triennial survey of foreign exchange markets, available at [www.bis.org](http://www.bis.org).

interfering with some ideal spontaneously-ordered atomistic market. Rather, the central bank takes over from the private intermediaries, and makes the FX market on its own.<sup>3</sup> It is interesting to note that in other aspects of their operations, central banks may be viewed as modifiers of ordinary commercial practices. For example, Gorton (1985) shows how the FED's role as a prudential regulator and a lender of last resort has been shaped after the New-York City Clearing House System.<sup>4</sup> The reason for the intervention was the feeling that banking panics have wide-spread macro-economic implications, and should thus be managed to the benefit of the whole economy, not just the banking industry. Goodhart (1988) develops a general theory of the evolution of central banks along these lines. The crucial step occurred after commercial banks had started to deposit their reserves with some leading bank, often a private institution. As the leading bank 'centralized control over the metallic (gold) reserves' the commercial banks came to 'rely on it to provide extra liquidity when in difficulties'. The commercial nature of the relationship was so dominant that 'the full ramifications of [the central bank's] role as bankers' bank were only dimly perceived at the time of their founding' (p. 5).

If one is to substantiate an argument for exchange rate management with rigorous welfare analysis, an underlying market failure has to be clearly identified. We follow the market-microstructure theoretic literature assuming that some traders operate under an insurance motive. They hedge against fluctuations in export prices by taking certain positions in the FX market. Crucially, this insurance-based trade is affected by market incompleteness as it is executed *after* some *speculators* have already acquired private information about future

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<sup>3</sup>In many cases, the central-bank's role in managing the exchange rate is indistinguishable from its role in making the market. For example, in Israel, the central bank runs the (daily) batch-auction in which the exchange rate is determined, and absorbs the slack left after all orders are served (at the price it has announced); see Djivre (1993).

<sup>4</sup>"... it is almost literally true that the Federeal Reserve System, as originally conceived, was simply a nationalization of the private clearinghouse system" (p. 277).

export prices. (For evidence in favor of private information in the FX market see the next section.) Then, the orders of the *hedgers* and the speculators mingle into the same order flow. As a result, market-prices adjust to the order flow as the market maker suspects that some orders are driven by private information. This well-known “Hirshleifer effect”, by which early revelation of information destroys insurance opportunities (see Hirshleifer, 1971), explains one of our key results: A FX market operated by a competitive market-making industry, where the exchange rate is allowed to float freely, will be generically inefficient.

One might think that fixing, or at least smoothing the exchange rate, would solve the problem: if agents are exposed to exchange-rate volatility against which they cannot hedge, then decreasing that volatility may recover some insurance opportunities. We show that such a conclusion would be too simple-minded. Indeed, a major part of our paper deals with the severe constraints speculators impose on public policy. As we shall see, a competitive market sets prices in such a way that ‘no money is left on the table’. Once the central bank deviates from the market’s pricing rule it will generate new trading opportunities. Particularly, speculators will profit by trading on private information with a smaller movement of prices against them. Obviously, the speculators’ trading profits have their counterpart in the central bank’s trading losses. These trading losses will have to be covered by taxation. As it happens, the tax will fall on the same agents for whom the volatility was smoothed in the first place. The analysis thus boils down to a welfare accounting, where the insurance benefits of smoother exchange rate are traded off against the direct cost of the smoothing policy.

There are three results that highlight the special features of the approach we use. Firstly, we show that a policy of fixing the exchange rate may not be feasible at all. The reason is that when the speculators rationally foresee the central-bank’s policy, they adjust their trading strategy accordingly. In the limit, as the central bank adopts a very smooth policy function, the opportunities of profiting from private information tend to infinity. Secondly, if the bank wants

to curb the speculators' trading profits, it has to *increase* the responsiveness of the exchange rate to the order flow, and thus increase the volatility of the exchange rate. Indeed, we show that it is easy to derive examples where the welfare gain from a smoother exchange rate is dominated by the welfare loss from the bank's trading profits. As a result, under the optimally-managed exchange-rate, volatility is *higher* than under a free-float. Maybe this is the main result of our paper: it is common wisdom among central bankers that policy should decrease exchange-rate volatility and deter speculation. We show that these objectives, while perfectly feasible each on their own, may be mutually incompatible. The third result worthy of emphasis is that a Tobin tax may provide a way of achieving fixed exchange rates, and containing the cost of smoothing the exchange rate. Unfortunately, the practical enforceability of a Tobin tax is under debate.

The direct cost of exchange-rate policy plays a key role in our analysis. Although often ignored in academic discussions of exchange-rate policy, their magnitude seem to increase over time (as international capital markets get deeper), making direct costs harder to ignore. Britain's ejection from the European exchange-rate mechanism (ERM) in September 1992 drew much attention to this problem. According to *The Economist*, 'total intervention could have been a hefty \$30 billion'. A major part of that amount was spent during a single day ('Black Wednesday' on September 16th) when the 'Bundesbank is thought to have lent the Bank of England over \$23 billion'. Of course, one should not confuse the expenditure on FX operations with the social cost of the policy. Back-of-the-envelope calculations with some 'gross simplifications... imply a national capital loss of \$3billion', (about 1.3% of the government's yearly consumption expenditure). Maybe, says *The Economist*, it 'has been the right thing to do, but at a time when public expenditure restraint is supposed to be at the top of the government's agenda, it is worth asking what was the cost to the taxpayer'.<sup>5</sup>

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<sup>5</sup>See, *The Economist*, October 10, 1992, p. 71.

These concerns must have been exacerbated by the fact that the huge profits were distributed among such a small group of professional speculators. During 1992, George Soros' Quantum Fund yielded a 68% return, net of 1% management fee and 15% incentive fees.<sup>6</sup> The direct cost of exchange-rate policy have been a concern for other countries, such as Brazil, or Israel.<sup>7</sup>

## 1.1 Relation to the literature

The literature on optimal exchange-rate regimes has two main strands. The first, by far the more voluminous, is based on the assumption that commodity prices are 'sticky' so that adjustment to shocks takes time. The optimal exchange rate should facilitate the adjustment and prevent some of the distortions that might arise from a too-lengthy adjustment process. Most of this literature is based on *ad hoc* IS-LM modeling, see Marston (1985). Recently, important progress has been made in modeling this effect within an 'optimization framework' where the welfare accounting can be properly executed; see Devereux and Engel (1998). In contrast to the above papers, our analysis assumes no price stickiness whatsoever; the friction that we explore has little to do with the mechanism of spot trading, but rather with the instruments through which risk is allocated and hedged.

In that respect we are much closer to the incomplete markets approach; see Helpman and Razin (1982) or Neumeyer (1998). The former, while making a strong case for the absence of neutrality between various exchange-rate regimes when markets are incomplete, is somewhat inconclusive on which regime welfare-dominates the other. The later argues that the basic incomplete-markets logic tends to favor flexible exchange rates on strictly fixed ones because more markets will span more insurance opportunities. Our setting differs from the incomplete-

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<sup>6</sup>See *Financial World*, May 11, 1993.

<sup>7</sup>See Jeffrey Sachs, *Financial Times* (January 21, 1999), or Djivre and Tsiddon (2001), respectively.

markets setting in two respects: we add private information which gives rise to speculation, and we allow for a market maker who provides liquidity even under a flexible exchange rate.<sup>8</sup> Another important difference is that our formalization is simpler. This allows us to extend the analysis beyond the extreme cases of fixed and flexible exchange rates, and to relate the generically optimal managed float to the structural coefficients of the model: risk aversion, the quality of information, and the magnitude of the various external shocks.

Our framework is also sufficiently simple to allow the analysis of a Tobin tax in affecting exchange rate policy. Such a tax was originally proposed by Tobin (1978) and has been the subject of much debate ever since. Subrahmanyam (1998) and Dow and Rahi (2000) examine the effect of a transactions tax on informed trading and welfare in a Kyle (1985) setting. Such a transactions tax works to throw sand in the wheels of financial markets, just as a Tobin tax does. Dow and Rahi (2000) show that a small tax may be Pareto improving even if the tax revenue goes to waste.

Our setting shares a number of features with the above paper, notably the microstructure approach to modelling the effect of a tax. However, we focus on the role of a Tobin tax in complementing exchange rate policy, rather than treating a tax as the sole policy instrument available. We show that a Tobin tax may support a policy of smoothing the exchange rate. Speculators trade less aggressively when a Tobin tax is levied, which may reduce the cost of smoothing the exchange rate to the central bank. While a Tobin tax also imposes a cost of currency trading on the local population, we show that a small tax is desirable under a wide range of parameters.

The remainder of the paper is organized as follows. Section 2 outlines the model, followed by a discussion of equilibrium without central bank intervention

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<sup>8</sup>Needless to say, the idea that private information should play an important role in the analysis of monetary policy is an old one; see Lucas (1973). Moreover, Weiss (1980) pointed out that in such a setting an active monetary policy may welfare-dominate a passive policy.

in Section 3. Section 4 derives the main results on optimal exchange rate policies. A Tobin tax is examined in Section 5. Section 6 contains a discussion of the robustness of our findings, and Section 7 relates our model to empirical evidence on exchange rate stability and financial repression.

## 2 The model

Consider a small-open economy that operates for three periods:  $t = 0, 1, 2$ . The economy has a local currency, which we shall dub ‘*the peso*’. International trade is executed in the world currency, which we shall dub ‘*the dollar*’. We denote the inverse of the exchange rate by  $Q$  (\$/peso-unit), so that a high  $Q$  signifies a ‘strong peso’. The market for foreign currency operates at  $t = 1$ . Production is completely specialized in a single staple commodity, which we shall dub ‘*copper*’. The market for copper is opened at  $t = 2$ , when trade is executed at a spot price  $P$  (\$/copper-unit). Due to the small size of the economy,  $P$  is determined exogenously by world-market conditions. Yet, it is a random variable with a distribution,

$$P = \mu_P + p, \quad p \sim N(0, \sigma_p^2).$$

$\mu_P$  is the unconditional mean of the world’s price, and  $p$  is a normally-distributed deviation from that mean. Consumption goods are imported from abroad, their dollar-price normalized to unity. Hence, we shall use the words ‘consumption good’ and ‘dollars’ interchangeably, and use it as a *numeraire*.

There are four types of players in the economy: ‘noise’ traders and speculators whom we shall think of as foreigners. In addition, there are producers of copper and the market-maker. They are locals and the focus of our welfare analysis. The central bank is the agency of public policy in our model. We turn, next, to the more detailed description of these players.

*The central bank*, henceforth ‘the bank’ has two roles. The first is to issue



local currency. This will be the bank's only role in a *flexible* exchange rate regime. In a *managed* exchange rate regime, the bank may be called to intervene in the determination of the exchange rate. As to its first role, we assume that the bank issues peso notes against a portfolio of local assets. For simplicity, suppose that these assets are riskless bonds of local producers, denominated in copper. Hence, the bank's balance-sheet is simply

$$M_0 = \text{copper-denominated bonds}, \quad (1)$$

where  $M_0$  is the money base. At the end the second period, the bank unwinds its position and pays one unit of copper for every peso. It is worth-while making a few comments on issues that occupy much attention in traditional exchange-rate theory.

(i) Local currency is, essentially, a future-contract on copper. As a result, money is priced just like any other short-maturity future contract. This 'commodity money' approach is obviously crude, and abstracts from important issues concerning the pricing of 'fiat money' in an infinite-horizon economy. Nevertheless, the approach captures one aspect, that is both realistic and plays a crucial role in our theory: the peso will be 'strong' when the world price of locally-produced commodities is expected to be high.

(ii) We assume that the bank can commit not to inflate the economy by issuing more peso notes against an existing amount of real bonds. Needless to say, the commitment assumption is made for analytical purposes rather than for realism: we can then focus the analysis on the welfare-improving policies that a perfect and benevolent 'planner' can undertake. It is noteworthy that in the absence of commitment, monetary expansion would be analyzed as a simple dilution of financial claims. In other words, in spite of the economy's imperfect markets, the quantity-theory of money holds. Unlike in most of the traditional analysis, non-neutrality of money is not a necessary condition for a meaningful policy analysis.

(iii) The bank can change the real quantity of money, changing both the money base and its bond holdings by the same amount without causing any inflation. This policy is also neutral, this time by Ricardian-equivalence considerations. All that will happen is that some players will substitute private contracts for pesos in their portfolios. In other words, the substitution of ‘inside money’ for ‘outside money’ is frictionless.

(iv) Some countries have opted for ‘dollarization’ — linking the value of the national currency to the dollar — by holding reserves in the form of foreign currency. However, a policy that ‘stabilizes’ the value of the local currency in terms of foreign currency, does not stabilize the dollar prices of local goods services or assets, as countries that went through dollarization have discovered, to their cost. In the terminology of our model, the central bank could hold dollar reserves against the peso, turning the currency from a future contract on copper to a claim on the dollar. But that would not have any effect on the dollar prices of privately-issued contracts, which are the topic of our investigation. It follows that dollarization just changes the labels on our problem: from ‘should the central bank conduct exchange rate policy’ to ‘should the central bank conduct monetary policy (stabilizing the value of the currency in terms of local assets)’. In substance, however, it is exactly the same problem.

That completes our discussion of the currency issuing function of the central bank. We defer, for the while, the bank’s possible role in making the FX market.

There is a measure 1 continuum of *copper producers* with constant absolute risk aversion (CARA) preferences,

$$u_h(c) = -e^{-\rho_h c}.$$

Their (aggregate) copper production is  $Y$ , deterministically. As noted above, no insurance markets are opened at  $t = 0$ . Yet, copper producers can hedge their consumption by taking FX positions at  $t = 1$ . Their  $t = 2$  consumption is thus

$$c_h = YP + \pi_h, \quad \pi_h = m_h(P - Q), \quad (2)$$

where  $m_h$  is the hedgers' peso position and  $\pi_h$  their trading profits (henceforth, we shall use the words 'hedgers' and 'copper producers' interchangeably). We shall assume that the hedgers, like any other player in our model, are free from short-sales constraints. Equation (2) points at the (deceptively) simple logic in favor of fixing the exchange rate. Suppose the bank fixes the exchange rate at a level of  $\mu_P$ . Then, by holding  $m_h = -Y$ , copper producers can perfectly hedge themselves against price fluctuations on the world market. Since  $\mu_P$  is a 'fair price', and since the hedgers are risk averse, they will indeed buy full insurance. As we shall see below, the market maker is risk neutral and is thus willing to bear this risk at a zero premium. So fixing the exchange rate seems to be a Pareto improvement. Alas, this 'intuition' does not work, and the reason has to do with the next type of player.

There are  $N$  risk neutral *speculators*. Their (aggregate) trading profits are

$$\pi_s = m_s(P - Q). \quad (3)$$

Each speculator receives, at  $t = 1$ , a noisy signal,  $s$  about the  $t = 2$  spot price of copper, such that

$$s = p + \varepsilon, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2).$$

(Note that the signal  $s$  is already defined as a deviation from the mean spot-price).  $\varepsilon$  is un-correlated with any of the other random variables in the model. By standard results,

$$E(p \mid s) = \theta s, \quad \theta = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_\varepsilon^2}, \quad (4)$$

where  $\theta$  is the precision of the signal.

Unlike in equity markets, the assumption of private information in the FX market is not that natural. It might be argued that for assets whose 'fundamental value' is a function of public policy and macro-economic conditions, and whose market is among the deepest in the world, private information is scarce,

and quickly revealed in those rare cases that it is generated. Nevertheless, recent research has assembled an impressive body of evidence to support the claim that FX market is affected by private information. We cite just a small number of results.<sup>9</sup> Ito *et. al.* (1998) have demonstrated how opening the Tokyo market for lunchtime trade has changed the pattern of JPY/USD volatility along a trading day: lunch-time volatility has increased and the typical intra-day U-shape had been flattened. Obviously, these effects are inconsistent with the idea that fundamentals alone determine prices. The results are consistent with microstructure effects, where private information plays a critical role.

Cheung and Wong (2000) conduct a questionnaire survey of opinions among FX dealers in Asian financial centers (Hong Kong, Tokyo and Singapore). They report that “in all the three foreign exchange markets, ‘large customer base’ and ‘better information’ are selected as the two main sources of large players’ competitive advantage” (p. 410). Naranjo and Nimalendran (2000) measure the amount of private information in the FX market through bid-ask spreads. Interestingly, they find that the market responds to the policy executed by the central bank, as happens in the theoretical analysis below.

Noise traders are assumed to take positions in order to hedge an endowment shock, as in Spiegel and Subrahmanyam (1992).<sup>10</sup> Within this model these traders only serve the function of adding noise to order flow. Hence, although they are referred to as noise traders they are actually rational expected utility maximizing agents. Noise traders are competitive and in total there is a continuum of measure one of them. They resemble domestic hedgers in that they also trade for hedging reasons. However, domestic hedgers have a constant exposure known to everyone, while noise traders are subject to a quantity shock  $n \sim N(0, \sigma_n^2)$ , the realization

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<sup>9</sup>See Lyons (2001) for an exhaustive survey of the results.

<sup>10</sup>An alternative way of introducing rational ‘noise traders’ is due to Bhattacharya and Nicodano (2001). They replace noise traders by utility maximizing agents who are subject to liquidity shocks.

of which is known only to themselves. The shock is realized at  $t = 0$ . Noise traders have CARA preferences with coefficient of risk aversion  $\rho_n$  :

$$u_n(c) = -e^{-\rho_n c},$$

where

$$c_n = nP + \pi_n. \quad (5)$$

Obviously, their trading profits are

$$\pi_n = m_n(P - Q). \quad (6)$$

The *market maker* is risk neutral. Traders may submit market orders only. The market maker observes the order flow and quotes the exchange rate accordingly, executing all orders at the quoted price. Under any exchange rate, his policy function is of the form

$$Q = \mu_P + \lambda \cdot (m - \mu_m), \quad m = m_s + m_h + m_n. \quad (7)$$

$m$  is the aggregate order flow for pesos, to be supplied in exchange for dollars;  $\mu_m$  is the unconditional mean of  $m$ . Note that the market maker has information about the aggregate order flow  $m$ , but not about individual orders, let alone the type of player who placed them.  $\lambda$  is the market maker's response coefficient, to be determined endogenously below.

Under a flexible exchange rate,  $\lambda$  is determined by commercial considerations. We follow the usual formalization by which the market maker prices a security according to its expected value, conditional upon the information contained in the order flow (see Kyle, 1985). One may interpret this assumption as a reduced form of a competitive market-making industry where market forces push prices down to the zero-profit point.  $\lambda > 0$  means that when the market maker observes a strong demand for pesos he suspects that the speculators might have received information indicating a high price of copper at  $t = 2$ ; he responds by making the

peso stronger. The linear form of equation (7) is a commonplace in this literature; we shall comment on linearity in the next section.

As noted, the central bank may want to ‘nationalize’ the market-making industry and manage the exchange rate on its own. We model the scheme as follows: the central bank will select a new  $\lambda$ , which will be the solution of a social welfare problem. The market maker will simply execute the policy dictated by the central bank. However, a policy of that sort may expose the market maker to some trading losses. We assume that the bank raises a lump-sum tax from the local producers so as to compensate the market maker and keep him at his original welfare level, namely zero-profit. We assume that all this takes place *ex ante*, at  $t = 0$ , so that the tax and the transfer payment equal the market-maker’s *expected* trading loss. This makes sense from a social-welfare point of view as the market maker bears the profit uncertainty with no welfare loss. However, the copper producers, for whom the whole scheme is provided, will have to pay for his trading losses. That makes the social-welfare problem non-trivial. We defer the details of the Tobin tax to Section 5 below.

There are two more points worth noting. Firstly, traders, particularly the hedgers, are exposed to ‘execution risk’: while making their decisions about market orders, they have no observation of the execution price (although they know its probabilistic properties).

Secondly, we have assumed, implicitly, that all players in the model are free from insolvency constraints. Their pockets are deep enough to bear any trading losses that might occur in some states of nature. (Note, however, that trading losses are generated while executing trading strategies that are derived optimally, taking into consideration *expected* trading profits and losses across all states of nature). Needless to say, this assumption is made more for tractability than for realism. As for the government, its freedom from wealth constraints is a function of the accessibility of an unlimited tax base, and its ability to extract revenue out of this base without any dead-weight losses.

Evidently, we take a very different view from other authors who have assumed that even small shortages of assets over liabilities may generate a bank-run; see Krugman (1979), Morris and Shin (1998) and many others. It seems that both assumptions are equally extreme and call for modification - a task left for future research.

### 3 Flexible exchange rate equilibrium

Let us first consider the case where the central bank does not interfere in the currency market, i.e. the exchange rate is flexible. This will help to illustrate some of the basic effects at work in this model. It will also allow us to show an important result, namely that there is scope for welfare improvement from central bank intervention.

Solving the model entails finding a Nash equilibrium of the following game. After observing a private signal, each speculator submits an order so as to maximize expected profits, given a price setting strategy of the market maker and trading strategies of all other traders. Similarly, hedgers submit orders to maximize expected utility, taking as given a price setting strategy of the market maker. The market maker sets prices after observing total order flow so as to break even in expectation, again taking as given all other traders' strategies. This corresponds to a zero profit constraint, which has to be satisfied when the exchange rate is determined in a competitive market.

In line with the existing literature working with variants of the Kyle (1985) model, we restrict attention to linear equilibria. In a linear equilibrium the market maker sets the exchange rate as a linear function of order flow (equation 7), speculators submit orders as a linear function of their signal and noise traders submit orders as a linear function of their endowment shock. Note that in this equilibrium there is no restriction on the choice of strategies, i.e. all agents choose linear strategies, because it is optimal to do so, given other agents' choice

of strategies. Hence, we restrict attention to an equilibrium that may not be the unique equilibrium of the game, but we do not restrict the strategy space.

In equilibrium, the maximizing players rationally foresee that the market-maker will set the exchange rate according to the function,

$$E(p \mid m) = \lambda_F (m - \mu_m) \quad (8)$$

(remember  $p$  is the deviation of the spot-price from its mean). Note that hedging demand  $m_h$  is non-random: hedgers make decisions based on non-random fundamentals and with no additional information. As will become obvious below,  $E(m_s) = 0$ . Hence,

$$m - \mu_m = m_s + m_n. \quad (9)$$

Denote by  $m_i$  the market order of the  $i$ -th individual speculator, such that  $m_s = \sum_i m_i$ . Denote by  $m_{-i}$  the aggregate market order of all speculators beside the  $i$ 'th speculator. Then, using equation (3) the maximization problem of the  $i$ -th speculator is

$$\max_{m_i} m_i \cdot E(p - q \mid s), \quad (10)$$

where  $q = Q - \mu_P$  and  $\mu_P$  cancels out. Using equations (4), (7) and (9), noting that  $E(m_n) = 0$ , equation (10) boils down to

$$\max_{m_i} m_i \cdot [\theta s - \lambda (m_i + m_{-i})].$$

Assuming that all  $N$  speculators are symmetric, the solution to this optimization problem is given by

$$m_i = \frac{\theta}{\lambda(N+1)} s, \quad m_s = \delta s \quad (11)$$

and

$$\delta = \frac{\theta N}{\lambda(N+1)} \quad (12)$$



Moreover, from (5), we can calculate the currency demand by noise traders. Due to CARA preferences and the normality assumption, the noise traders' problem can be written using the certainty equivalence of expected utility:

$$\max_{m_n} E(c_n) - \frac{1}{2} \rho_n \cdot \text{Var}(c_n).$$

This can be written as

$$\max_{m_n} -\lambda m_n \bar{m}_n - \frac{1}{2} \rho_l \cdot ((n + m_n(1 - \lambda\delta))^2 \sigma_p^2 + (m_n \lambda\delta)^2 \sigma_\varepsilon^2),$$

where  $\bar{m}_n$  denotes the demand by all other liquidity traders, which each individual trader takes as given. Solving the first-order condition with respect to  $m_n$  and setting  $m_n = \bar{m}_n$  yields the following demand by liquidity traders:

$$m_n = -n \frac{\rho_n (1 - \lambda\delta) \sigma_p^2}{\lambda + \rho_n ((1 - \lambda\delta)^2 \sigma_p^2 + (\lambda\delta)^2 \sigma_\varepsilon^2)} \quad (13)$$

It is then straightforward to prove the following result.

**Proposition 1** *Within the class of linear equilibria, there exists a unique flexible-exchange rate equilibrium, if and only if*

$$K > 1, \quad K = \frac{\rho_n^2 \sigma_p^2 \sigma_n^2 (1 - \theta \frac{N}{N+1})^2}{\theta \frac{N}{(N+1)^2}}.$$

*In this equilibrium the market-maker responds to the order flow according to the coefficient  $\lambda_F$  given by*

$$\lambda_F = \rho_n B \frac{\sqrt{K} + 1}{K - 1}, \quad (14)$$

*with  $B = (1 - \theta \frac{N}{N+1})^2 \sigma_p^2 + (\theta \frac{N}{N+1})^2 \sigma_\varepsilon^2$ .*

**Proof.** The market maker will set the exchange rate so as to break even in expectation, given the information contained in total order flow.

$$q = E[p|m_s + m_n]$$

From standard inference rules, we know that

$$E[p|m_s + m_n] = \lambda(m_s + m_n), \quad \lambda = \frac{Cov(p, m_s + m_n)}{Var(m_s + m_n)}.$$

Using the result in (11) yields

$$\lambda = \frac{\delta \sigma_p^2}{\delta^2 (\sigma_p^2 + \sigma_\varepsilon^2) + Var(m_n)}.$$

Note that  $\lambda > 0$ . Hence, one may substitute in the  $\delta$  as computed in equation (12) and the variance of liquidity trader demand from (13). Solving for  $\lambda_F$  yields a quadratic equation which has one positive real root if and only if  $K > 1$ . This root is given by (14). ■

It is noteworthy that a competitive equilibrium may not exist. This corresponds to the finding of Spiegel and Subrahmanyam (1992) that hedgers are only willing to trade if their demand for insurance is sufficiently high (high values of  $\rho_n, \sigma_p^2, \sigma_n^2$ ) to justify the trading losses against the informed speculators.

Regarding speculators' trading behavior, three points are worth noting. Firstly, the speculators trade more aggressively the more precise their private information is (namely the higher is  $\theta$ ). This can be seen from equation (11). Secondly, the speculators, as a group, trade more aggressively the more numerous they are. This is because of the Cournot nature of this equilibrium: the fewer speculators there are, the more they utilize their monopoly power by revealing less of their private information. One way to measure the informativeness of the exchange rate would be to look at the sensitivity with which it reacts to private information of the speculators. This sensitivity is given by

$$\frac{\partial q}{\partial s} = \lambda \delta = \theta \frac{N}{N+1},$$

which is increasing in the number  $N$  of speculators. Lastly, the speculators trade more aggressively the lower  $\lambda$  is. This constitutes the main obstacle to a policy of a managed exchange rate. The speculators will respond to a policy that smooths

the exchange rate by increasing the intensity of their trading. Moreover, they do so in a way that leaves the sensitivity with which the exchange rate reacts to private information unchanged. We will return to this last point in the next section. But before doing so, it is interesting to note that

**Corollary 1** *A competitive market maker smooths the exchange-rate in the sense that*

$$Var(q | \lambda_F) = \theta \frac{N}{N+1} \sigma_p^2 < \sigma_p^2. \quad (15)$$

This result is relevant, because it turns out that the hedgers' welfare is directly related to the variance of the exchange rate. To understand this consider the hedger's maximization problem. Again, it is convenient to use the hedger's certainty equivalence as maximand.

$$\max_{m_h} E(c_h) - \frac{1}{2} \rho_h \cdot Var(c_h). \quad (16)$$

We can treat the maximization problem (16) as a standard mean-variance portfolio problem with two risky assets. Using equation (2) we can rewrite equation (16) as

$$\max_{m_h} Y \mu_P - \frac{1}{2} \rho_h [(Y + m_h)^2 \sigma_p^2 - 2(Y + m_h) m_h \sigma_{pq} + m_h^2 \sigma_q^2]. \quad (17)$$

To find the covariance between  $p$  and  $q$  note that under a flexible exchange rate, the market maker sets the exchange rate so that it will reflect all the available information about  $p$ . Since  $q$  is an efficient predictor of  $p$ , the regression coefficient of  $p$  on  $q$  equals 1. Hence,

$$\frac{Cov(q, p | \lambda_F)}{Var(q | \lambda_F)} = 1. \quad (18)$$

This enables us to determine the optimal demand for pesos by domestic hedgers.

**Corollary 2** *Solving the problem (17), and using equation (15), it is easy to see that under a flexible exchange rate, the hedger holds all his wealth in dollars, namely*

$$m_h | \lambda_F = -Y. \quad (19)$$

This seems like a somewhat surprising result. The hedger's problem is a standard mean-variance portfolio problem with two risky assets: pesos and dollars. Pesos, being futures on copper, bear a standard maturity risk: it is uncertain what the value of copper in terms of consumption goods will be. Although dollars are futures on consumption goods, they are not safe either. This is because the hedger short-sells pesos for dollars not knowing what the exchange rate will be. (Remember that the market maker announces the price *after* observing the order-flow, and that only market orders are allowed; hence the hedgers face 'execution risk'). Note that both assets have the same mean return, but the variance on dollars is smaller (see Corollary 1). Still, there might be some motivation for holding dollars for diversification. Obviously, this motivation diminishes as the covariance between  $p$  and  $q$  increases. Evidently, it vanishes completely under the covariance generated by a flexible exchange rate regime (i.e. the market-maker's response coefficient  $\lambda_F$ ).

As noted above, we focus our welfare analysis on local citizens. Since the market maker is risk neutral, makes zero profits, and is not affected by price volatility, the welfare criterion of our model boils down to the hedger's value function. Substituting the hedger's demand (19) into his objective function (17) one finds that welfare depends directly on exchange rate volatility:

$$W_F = Y\mu_P - \frac{1}{2}\rho_h \cdot Y^2 \cdot \text{Var}(q \mid \lambda_F). \quad (20)$$

Using (15), we can rewrite (20) as

$$W_F = Y\mu_P - \frac{1}{2}\rho_h \cdot Y^2 \cdot \theta \frac{N}{N+1} \sigma_p^2. \quad (21)$$

The following points are worth emphasizing. As one would have expected, welfare falls with the hedgers' degree of risk aversion and the model's fundamental risk,  $\sigma_p^2$ . Secondly, welfare *falls* as competition (the number  $N$  of speculators) increases. This is because hedgers are affected by speculators via the Hirshleifer effect only. To be more precise, when competition increases, the speculators'

trading profits fall, to the benefit of the noise traders (who are not part of our welfare accounting). Since the hedgers make zero trading profits, they are not affected by the decreasing profits of the speculators.<sup>11</sup> However, as noted in Corollary 1, the market provides the hedgers with some degree of insurance as the volatility of the exchange rate is smaller than the volatility of the spot-price. As speculators compete more strongly, they reveal more of their private information, which destroys more insurance opportunities. Moreover, for the same reason, welfare falls when the quality of the speculators' information increases.

Interestingly, welfare is *not* affected by the amount of noise in the model. As we have shown in Proposition 1, the market maker absorbs the order flow with less of a price change when he knows that there is more noise (higher variance  $\sigma_n^2$ ). As equation (12) shows, the speculators will trade more aggressively in response to this more accommodating behavior by the market. In the competitive equilibrium (flexible exchange rate) the reduction in  $\lambda$  exactly off-sets the price impact of an increase in  $\sigma_n^2$  and speculators' trading strategies adapt in such a way as to leave the impact of information on prices unchanged. The overall effect of a change in the variance of noise trade on hedgers' welfare is therefore nil.

## 4 The optimal exchange rate

The flexible exchange rate regime is characterized by a value of  $\lambda$  such that the market maker breaks even in expectation, i.e. the speculators' profits are the noise traders losses. Once the central bank intervenes by setting  $\lambda$  at a level other than its 'competitive' level  $\lambda_F$ , this is no longer true. In particular, net profits or losses will be born by the market maker, depending on the value of  $\lambda$ .

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<sup>11</sup>Spiegel and Subrahmanyam (1992) show that hedgers' welfare may fall with the number of informed traders, even when the hedgers incur trading losses to the informed traders. In their setting the insurance effect dominates the effect of reduced trading losses when the number of speculators increases.

The central bank will have to compensate the market for any trading losses, and finance the transfer via a lump-sum tax  $T$ , such that

$$T = E(\pi_n) + E(\pi_s). \quad (22)$$

Note that the hedgers make zero trading profits under any  $\lambda$ . This is the case, because their demand is deterministic and therefore does not affect prices. Using equation (6) it is easy to see that the noise-trader's profit is

$$E(\pi_n) = -\lambda \delta_n^2 \sigma_n^2, \quad (23)$$

where  $\delta_n = \frac{m}{n}$ . The speculators' profits are given by

$$E(\pi_s) = \frac{\theta}{\lambda(N+1)^2} \sigma_p^2. \quad (24)$$

This allows us to derive an important result:

**Proposition 2** *A fixed exchange rate ( $\lambda = 0$ ) is not feasible.*

**Proof.** Using equations (13), (23) and (24), it is easy to verify that

$$\lim_{\lambda \rightarrow 0} T = \infty.$$

■

Intuitively, a fixed exchange rate regime will diminish the noise trader's trading losses (equation 23). But it will also provide the speculators with an incentive to trade more and more aggressively, with increasing profits. This is the case, because when the bank deviates from the informationally efficient pricing policy as defined in equation (14), it will leave some 'money on the table', which the speculators will pick up. At a fixed exchange rate arbitrage by the risk neutral speculators grows without bound, which renders this policy infeasible.

Let us now turn to the policy maker's welfare optimization problem. Suppose that only locals can be taxed, i.e. the tax falls on the copper-producers. It follows that the economy's social-welfare function is given by:

$$W = Y \mu_P - T - \frac{1}{2} \rho_h \cdot Var(c_h). \quad (25)$$

Hence, the welfare optimization problem is to maximize  $W$  given by (25), subject to compensating the market maker for expected losses (equation 22), and the trading behavior induced by a particular value of  $\lambda$  (equations 12, 13 and solving 17).

We can now prove a key result of this paper.

**Proposition 3** *Suppose that  $K > 1$ , i.e. a competitive equilibrium exists. Then (i) the social welfare optimization problem has an interior solution  $0 < \lambda^* < \infty$ , and (ii) a flexible exchange rate is generically inefficient ( $\lambda^* \neq \lambda_F$ ).*

**Proof.** Firstly, we prove (i) by showing that  $\frac{\partial W}{\partial \lambda} < 0$  for sufficiently large  $\lambda$ , i.e. the optimum  $\lambda^* < \infty$ . The previous proposition establishes that  $\lambda^* \neq 0$ .

We can calculate the currency demand by domestic hedgers by solving (17). Straightforward calculation yields

$$m_h = -Y \frac{Var(p) - Cov(p, q)}{Var(p - q)}. \quad (26)$$

Substituting this back into (17), we find that

$$Var(c_h) = Y^2 \sigma_p^2 \frac{\sigma_q^2 (1 - \phi^2)}{\sigma_p^2 - 2\phi\sigma_p\sigma_q + \sigma_q^2}, \quad (27)$$

where  $\phi = \frac{\sigma_{pq}}{\sigma_p\sigma_q}$  is the correlation coefficient between  $q$  and  $p$ . Moreover, we can calculate

$$Cov(p, q) = \lambda\delta\sigma_p^2 = \theta \frac{N}{(N+1)} \sigma_p^2, \quad (28)$$

and

$$Var(q) = (\lambda\delta)^2 (\sigma_p^2 + \sigma_\varepsilon^2) + (\lambda\delta_n)^2 \sigma_n^2. \quad (29)$$

Using this and (13), it is straightforward to show that

$$\frac{\partial Var(c_h)}{\partial \lambda} > 0. \quad (30)$$

Next we can calculate

$$-\frac{\partial T}{\partial \lambda} = \theta \frac{N}{\lambda^2 (N+1)^2} \sigma_p^2 + \frac{\rho_n B - \lambda}{(\lambda + \rho_n B)^3} \rho_n^2 \sigma_p^4 \sigma_n^2 \left(1 - \theta \frac{N}{N+1}\right)^2,$$

Hence,  $-\frac{\partial T}{\partial \lambda} < 0$  whenever

$$K \frac{\lambda^3}{(\lambda + \rho_n B)^3} > 1 + \frac{\lambda^2 \rho_n B (N+1)^2}{(\lambda + \rho_n B)^3 \theta N}. \quad (31)$$

As  $\lambda \rightarrow \infty$ , the inequality (31) is satisfied whenever  $K > 1$ . Hence, at high  $\lambda$ , tax income to domestic hedgers increases by reducing  $\lambda$ , which also reduces the variance of consumption. It follows that for sufficiently high  $\lambda$ ,  $\frac{\partial W}{\partial \lambda} = -\frac{\partial T}{\partial \lambda} - \frac{\rho_h}{2} \frac{\partial \text{Var}(c_h)}{\partial \lambda} < 0$ . Therefore, the optimal  $\lambda$  must be finite.

We now proceed to the proof of (ii). We differentiate the tax function by  $\lambda$  and evaluate the derivative at the point of the flexible exchange-rate:

$$\frac{\partial T}{\partial \lambda} \big|_{\lambda=\lambda_F} = -2\delta_n \sigma_n^2 \left( \lambda \frac{\partial \delta_n}{\partial \lambda} \big|_{\lambda=\lambda_F} + \delta_n \right). \quad (32)$$

Next, using equation (17) and the envelope theorem we compute

$$\frac{\partial \text{Var}(c_h)}{\partial \lambda} \big|_{\lambda=\lambda_F} = Y^2 \cdot \frac{\partial \text{Var}(q)}{\partial \lambda} \big|_{\lambda=\lambda_F}. \quad (33)$$

Using equation (12) for  $\delta$ , we can rewrite (29) as

$$\text{Var}(q) = \theta \frac{N^2}{(N+1)^2} \sigma_p^2 + (\lambda \delta_n)^2 \sigma_n^2. \quad (34)$$

Hence, one can evaluate the derivative at the point of the flexible exchange rate:

$$\frac{\partial \text{Var}(q)}{\partial \lambda} \big|_{\lambda=\lambda_F} = 2\lambda_F \delta_n \sigma_n^2 \left( \lambda \frac{\partial \delta_n}{\partial \lambda} \big|_{\lambda=\lambda_F} + \delta_n \right). \quad (35)$$

Clearly, it is optimal for the bank to intervene in the FX market whenever

$$-\frac{\partial T}{\partial \lambda} \big|_{\lambda=\lambda_F} - \frac{1}{2} \rho_h Y^2 \cdot \frac{\partial \text{Var}(q)}{\partial \lambda} \big|_{\lambda=\lambda_F} \neq 0 \Leftrightarrow 1 - \frac{1}{2} \rho_h Y^2 \lambda_F \neq 0. \quad (36)$$

■

Note that although a managed float is generically optimal, a smoothing policy (decreasing  $\lambda$ ) is not necessarily optimal. Indeed, Proposition 3 identifies two



motives for intervention. There is the *revenue motive* identified by equation (32) and there is the *insurance motive* identified by equation (33). It is easy to see that the revenue motive always pushes the central bank towards a more responsive policy ( $\lambda > \lambda_F$ ), and thus *increasing* the exchange-rate volatility. The motive is simple: by definition, the economy is a monopolist on its national currency. It should thus try to extract as much rent as possible from foreigners coming to trade in its money market. That means forcing them to trade against a steeper pricing function, increasing prices more vigorously against the order flow. On the other hand, it is easy to see from (30) that the insurance motive always pushes the central bank towards a lower  $\lambda$  and thus a *lower* exchange-rate volatility.

Let us consider the effect of a change in  $\lambda$  on the variance of consumption in somewhat more detail. It is straightforward to show that  $Var(c_h)$  is increasing both in the correlation  $\phi$  between  $p$  and  $q$ , and in the variance of the exchange rate  $\sigma_q^2$ . As the exchange rate  $q$  is more strongly correlated with the spot price  $p$ , hedging becomes less effective, because of the Hirshleifer effect: more of the uncertainty regarding the future value of  $p$  is reflected in the current exchange rate. Insurance against the uncertainty thus revealed is impossible. Interestingly, the covariance between  $p$  and  $q$  is independent of  $\lambda$  (equation 28). Therefore, somewhat surprisingly, smoothing the exchange rate leaves the degree of correlation between  $p$  and  $q$  unchanged. This is because speculators respond by trading more aggressively, so as to exactly offset any effect of a smoother exchange rate.

A reduction in  $\lambda$ , however, does have an effect on  $\sigma_q^2$  and therefore reduces  $Var(c_h)$ . A smaller  $\lambda$  reduces the amount of noise that gets impounded into the exchange rate by the noise traders. Hence, hedging becomes more effective, because the currency becomes a more precise insurance instrument. To summarize, smoothing the exchange rate (reducing  $\lambda$ ) improves insurance for domestic producers, because it reduces the amount of noise reflected in the exchange rate, but *not* because it affects the sensitivity with which the exchange rate reacts to speculators' information.

A flexible exchange rate regime is generically inefficient, because there is no reason for the optimal trade-off point between revenue and insurance to coincide with the flexible exchange rate. Regarding the optimal trade-off between the revenue generated by higher  $\lambda$  and reduced insurance, it is straightforward to show the following result.

**Corollary 3** *It is optimal to increase  $\lambda$  above  $\lambda_F$  if*

$$\frac{\rho_h}{2} Y^2 \lambda_F < 1.$$

This follows straightforwardly from equation (36). The result is very intuitive. If hedgers are not too risk averse (small  $\rho_h$ ), or if exposure to fundamental risk is not too high (small  $Y$ ), then the revenue motive dominates the insurance motive and the central bank should increase  $\lambda$  above the fully flexible level.

## 5 The Tobin tax

In addition to direct interference in the currency market, the central bank may be able to affect exchange rate fluctuations via a tax on capital flows. This section analyzes how the ability to set a transactions tax on trade in the currency (which we shall call a Tobin tax) may complement a policy of direct interference discussed in the previous sections. A Tobin tax may serve two important functions in our setting: (i) it may generate tax revenue, and (ii) it may discourage speculation and thus allow the central bank to interfere in the currency market at lower cost. However, a Tobin tax also renders hedging for domestic agents more costly and may therefore be undesirable. In this section we show that introducing a Tobin tax as a policy instrument, renders a fixed exchange rate feasible. Moreover, a small Tobin tax is shown to be desirable for a wide range of model parameters.

Suppose the central bank can levy a tax, which we assume to be a quadratic function in the order flow of each trader:  $\frac{1}{2}\tau m^2$ . The assumption of a quadratic

tax is necessary to preserve differentiability of the optimization problem, and corresponds to previous work in this area (see Subrahmanyam, 1998, Dow and Rahi, 2000). The quadratic tax function is not very elegant, because it faces problems of implementability. Traders have an incentive to split their orders into infinitely small orders in order to evade tax. We therefore have to assume that traders cannot split their orders in such a way as to avoid tax. However, it should be noted that the results do not hinge on the quadratic form of the tax. All that is required is a wedge between the ‘buying’ and ‘selling’ price of the currency and as such a linear tax would be sufficient.

The total amount of lump sum tax  $T$  that needs to be raised from domestic hedgers in order to support a given exchange rate regime thus is reduced by the amount of Tobin tax raised

$$T = E(\pi_n) + E(\pi_s) - \frac{1}{2}\tau E(Nm_i^2 + \overline{m}_h^2 + \overline{m}_n^2), \quad (37)$$

where that  $\overline{m}_h$  and  $\overline{m}_n$  are equilibrium demands and hence are not a choice variable in the hedgers’ and noise traders’ optimization problems. Apart from yielding direct tax revenue, the Tobin tax affects hedging and speculative demand for currency. Let us first consider the impact of a tax on speculation. A Tobin tax reduces the aggressiveness with which speculators pursue exchange rate arbitrage. This in turn affects speculative profits and hence the required ex ante transfer from hedgers to market makers. A Tobin tax may thus serve as a complementary policy instrument to support a smooth exchange rate. In particular, it is straightforward to show the following.

**Proposition 4** *A Tobin tax  $\tau > 0$  renders a fixed exchange rate regime feasible.*

**Proof.** In the presence of a Tobin tax, the speculator’s optimization problem can be rewritten as

$$\max_{m_i} m_i \cdot [\theta s - \lambda(m_i + m_{-i})] - \frac{1}{2}\tau m_i^2.$$

Solving for the equilibrium  $m_i$  yields

$$m_i = \frac{\theta}{\lambda(N+1) + \tau} s, \quad m_s = \delta s, \quad \delta = \frac{\theta N}{\lambda(N+1) + \tau}. \quad (38)$$

For  $\tau > 0$ ,  $\delta$  is therefore bounded above for  $\lambda = 0$ . Hence,  $E(\pi_s)$  is also bounded. Moreover, it is straightforward to show that all other terms in equation (37) are finite at  $\lambda = 0$ , and hence  $T$  is finite also. ■

Hence, a Tobin tax enables the central bank to set a fixed exchange rate, which we know is impossible in the absence of such a tax. In itself this does not mean, of course, that it is actually desirable to levy a Tobin tax. Such a tax renders hedging more costly, and therefore its desirability cannot be taken for granted. Moreover, if noise traders also scale back their order sizes under a Tobin tax, it is unclear how exchange rate volatility will be affected. We will investigate these issues in the following paragraphs.

In the presence of a Tobin tax, hedgers determine their demand for pesos, taking into account the tax cost of trading out of pesos. Their optimization problem can be written as:

$$\max_{m_h} Y\mu_P - T - \frac{1}{2}\tau m_h^2 - \frac{1}{2}\rho_h \text{Var}(c_h). \quad (39)$$

This yields a reduced demand for dollars, because it is costly to hedge. Straightforward calculation yields

$$m_h = -Y \frac{\rho_h (1 - \lambda\delta) \sigma_p^2}{\tau + \rho_h [(1 - \lambda\delta)^2 \sigma_p^2 + (\lambda\delta)^2 \sigma_\epsilon^2 + (\lambda\delta_n)^2 \sigma_n^2]}. \quad (40)$$

Interestingly, it can be shown that a small Tobin tax reduces the variance of consumption in spite of the reduction in hedging demand. This is because speculators scale back their trades and in that way destroy less insurance compared to the case without a tax. This is summarized in the following Lemma.

**Lemma 1** *A small Tobin tax reduces risk exposure:*

$$\frac{\partial \text{Var}(c_h)}{\partial \tau} \Big|_{\tau=0} < 0.$$

**Proof.** Using equation (40) allows us to write

$$\begin{aligned}\frac{\partial Var(c_h)}{\partial \tau}|_{\tau=0} &= 2\frac{\partial \delta}{\partial \tau}|_{\tau=0} \cdot (-m_h \lambda \sigma_p^2 (Y + m_h (1 - \lambda \delta)) + m_h^2 \lambda^2 \sigma_\epsilon^2 \delta) \\ &\quad + 2\frac{\partial \delta_n}{\partial \tau}|_{\tau=0} m_h^2 \lambda^2 \sigma_n^2 \delta_n.\end{aligned}$$

Using (38) allows the further simplification

$$\begin{aligned}\frac{\partial Var(c_h)}{\partial \tau}|_{\tau=0} &= 2m_h \theta \frac{N}{\lambda (N+1)^2} \cdot \left( \sigma_p^2 (Y + m_h (1 - \lambda \delta)) - m_h \theta \frac{N}{N+1} \sigma_\epsilon^2 \right) \\ &\quad - \frac{2m_h^2 \lambda^2 \sigma_n^2 \delta_n^2}{\lambda + \rho_n B}.\end{aligned}$$

Substituting in the optimal hedging demand from (40) enables us to show that

$$\sigma_p^2 (Y + m_h (1 - \lambda \delta)) - m_h \theta \frac{N}{N+1} \sigma_\epsilon^2 > 0.$$

Given that  $m_h < 0$ , it follows that  $\frac{\partial Var(c_h)}{\partial \tau}|_{\tau=0} < 0$ . ■

While it is now clear that a small Tobin tax improves insurance, it remains to be shown whether it can also constitute an overall improvement for domestic agents. This imposes the additional constraint that the market maker has to break even. Once we take into account that a Tobin tax affects the size of the lump sum transfer from hedgers to the market maker (the revenue motive), we can state the following result.

**Proposition 5** *A small Tobin tax may be Pareto improving for domestic agents. A sufficient but not necessary condition for Pareto improvement is  $N < 4$  and  $\lambda < \frac{\rho_n}{3} B$ .*

**Proof.** In the presence of a Tobin tax, the welfare function can be written as

$$W = Y\mu_P - T - \frac{1}{2}\tau m_h^2 - \frac{1}{2}\rho_h \cdot Var(c_h).$$

Taking the first-order condition with respect to  $\tau$  and applying the envelope theorem yields

$$\frac{\partial W}{\partial \tau}|_{\tau=0} = -\frac{\partial T}{\partial \tau}|_{\tau=0} - \frac{1}{2}m_h^2 - \rho_h \frac{\partial Var(c_h)}{\partial \tau}|_{\tau=0}.$$

Given equations (37) and (38), we can evaluate the derivative at  $\tau = 0$ .

$$\begin{aligned} \frac{\partial W}{\partial \tau} \Big|_{\tau=0} = & - \left( -\theta \frac{3N}{2\lambda^2 (N+1)^2} + 2\theta \frac{N^2}{\lambda^2 (N+1)^3} \right) \sigma_p^2 \\ & + \frac{1}{2} \delta_n^2 \sigma_n^2 \left( 1 - 4 \frac{\lambda}{\lambda + \rho_n B} \right) - \rho_h \frac{\partial Var(c_h)}{\partial \tau} \Big|_{\tau=0}. \end{aligned}$$

Since  $\frac{\partial Var(c_h)}{\partial \tau} \Big|_{\tau=0} < 0$ , a sufficient condition for Pareto improvement is that revenue is non decreasing in  $\tau$ . Revenue  $(-T)$  increases with a small Tobin tax when

$$- \left( -\theta \frac{3N}{2\lambda^2 (N+1)^2} + 2\theta \frac{N^2}{\lambda^2 (N+1)^3} \right) \sigma_p^2 + \frac{1}{2} \delta_n^2 \sigma_n^2 \frac{\rho_n B - 3\lambda}{\lambda + \rho_n B} > 0.$$

A sufficient condition for this to be true is  $N < 4$  and  $\lambda < \frac{\rho_n}{3} B$ . ■

The condition for Pareto improvement is weak in that we require the lump sum tax  $T$ , which hedgers pay to the market maker, to fall when a small Tobin tax is levied. If this is the case a Tobin tax is unequivocally desirable, because we know from Lemma 1 that it improves insurance. The condition in Proposition 5 requires that the revenue from the Tobin tax more than outweighs the potentially increased aggregate trading profits that result when such a tax is levied.

Under a Tobin tax noise traders scale back their trades, which reduces their trading losses. For  $\lambda$  not too large ( $\lambda < \frac{\rho_n}{3} B$ ), the reduction in noise traders' losses is more than outweighed by their direct contribution from paying the Tobin tax. Hence, net revenue from noise traders to domestic agents increases when this condition is met.

Direct trading profits of speculators increase when a Tobin tax is levied. Due to competition, aggregate trading profits for  $N > 1$  speculators are smaller than the monopolistic profits. Under a Tobin tax, speculators scale back their trades, which softens competition and therefore increases profits. This explains why a small number ( $N < 4$ ) of speculators are sufficient to ensure that a small Tobin tax is Pareto improving: softening competition is particularly relevant when speculators compete fiercely (high  $N$ ). Hence, only for small  $N$  is the

direct income from the Tobin tax sufficient to make up for the increased trading profits of speculators.

A similar effect has been identified by Dow and Rahi (2000), who show that a transaction tax can be Pareto improving, because it leaves speculators better off at the same time as improving insurance for hedgers. They find a Pareto improvement to result under somewhat different conditions from the ones identified in this paper. This is due to the welfare functions considered. Dow and Rahi are concerned with the welfare of speculators and hedgers. By contrast, we are interested in the welfare of hedgers and implicitly the market maker, but not the speculators. As result, when the speculators make higher trading profits our welfare measure may fall, because the market maker is made worse off and his loss needs to be covered by hedgers.

## 6 Comments about robustness

It was pointed out, above, that models with incomplete markets and private information are sensitive to the exact specification of these imperfections. It would thus be desirable to explain why we have made certain modeling assumptions, and how the results would be affected by alternative specifications. On the whole, our assumptions were made so as to favor managing and smoothing the exchange rate. We assumed that local trade takes place between a risk-averse producer and a risk neutral market maker, both of which are uninformed. We also assumed that ‘fundamental’ uncertainty is about commodity *prices* only. Under such assumptions, if only the foreigners could be excluded from trade, an efficient risk-sharing arrangement would be to fix interim prices completely. It is easy to think of specifications where that would not be the case. For example, suppose fundamental uncertainty was about *quantities*, i.e. agents suffer from supply shocks. Then, optimal risk-sharing would imply fluctuating prices, in particular lower prices in good times and higher prices in bad times (with unit elasticity).

Our assumptions are favorable to intervention. There are lump-sum taxes and prices play no allocative role. Hence, manipulating them so as to improve risk-sharing carries no dead-weight loss. Also, speculators have nothing to contribute to the economy: they add no risk-bearing capacity, they relax no shortage of funds and the information they have is not operational and thus has a zero social value. Hence, a policy against trade in international funds is welfare-improving to the locals. In spite of all these strong assumptions, the model still could not yield unambiguous results in favor of smoothing the exchange rate, and did yield strong results against fixing the exchange rate. We thus interpret our results as favorable to flexible exchange rates. Notably, our strongest results are in favor of a Tobin tax, but that ignores issues of implementability.

It is worth noting, however, that in one respect our assumptions are biased against fixed exchange rates. Suppose information was (privately) costly to produce, and socially valuable. What sort of policy would encourage foreigners to ‘express more interest’ in the local economy and produce more information? The answer is: a policy of smoothing the exchange rate, because it increases speculative profits. Hence, exchange rate management may be a way to encourage and subsidize foreign direct investment.

## **7 Some positive aspects of our theory**

We believe that our theory does quite well against the stylized facts; see Calvo and Reinhart (2000). Firstly, regardless of what countries say about themselves, most adopt a policy of a managed float. Probably, this widespread ‘fear of floating’ reflects some intuitive understanding that markets are incomplete, that exchange-rate volatility expose exporting industries to risks that cannot be fully hedged, and that decreasing exchange-rate volatility is a way by which the central bank can provide these industries with insurance. Secondly, and more interestingly, less developed economies have a greater fear of floating.



Our theory suggests two explanations for that observation. The first is that more developed capital markets provide market participants with more sophisticated instruments to insure themselves. As a result, exchange-rate volatility is not that closely associated with risk exposure. The pressure on the government and the central bank to ‘stabilize’ the exchange rate will be weaker. The second explanation is that financial repression acts like a Tobin tax. Most emerging markets need not put ‘sand in the wheels’ of trade so as to facilitate the management of the exchange rate; they already have sand aplenty.

To illustrate this idea, we add to the Calvo-Reinhart measure of exchange rate stability, a standard measure of financial repression: interest rate spreads. The results are presented in Table 1 and Figure 1.<sup>12</sup> It is easy to see that all the observations fall above the diagonal, or close to it. Yet, the observations are too scattered to create a clear pattern. Indeed, an estimation of a regression line failed to yield a significant result. On closer inspection, however, the outliers may be explained. Argentina and Canada are dollarized economies. As explained above, the stability of these exchange rates do not reflect any stability in the dollar price of local assets. The Asian tigers (Thailand, Korea, Malaysia and Indonesia) all show a high degree of exchange-rate stability. That is obviously misleading, and a result of averaging pre and post crisis values.

On the other hand, in some countries, financial repression operates more via quantitative constraints than through price channels; such countries may report a low spread. That may explain Egypt. Bolivia is another extreme value, but the results are not sensitive to its inclusion, or exclusion. After these omissions, we find that the regression coefficient (over 25 observations) has a t-statistic of 1.99.

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<sup>12</sup>Peru and Uruguay do not appear in the figure as they report extreme interest-rate spreads, above 40% (net of inflation, see Table 1).

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Calvo-Reinhart measures of exchange-rate stability						Interest-rate spread		
Country	Abbrevia tion	Starting	Ending	Prob +/-1%	Prob +/-2.5%	Starting	Ending	Spread
ARGENTINA	AR	1991	1999	97.9	100	1994	1999	3
AUSTRALIA	AU	1984	1999	28	70.3	1985	1999	3.8
BOLIVIA	BV	1983	1999	73.9	79.9	1984	1999	22.7
BRAZIL	BR	1994	1999	83.1	94.3			n.a.
BULGARIA	BG	1997	1999	65.5	93.1	1998	1999	9.6
CANADA	CA	1970	1999	68.2	93.6	1971	1999	1.3
CHILE	CH	1982	1999	45.5	83.8	1983	1999	5.6
COLOMBIA	CL	1979	1999	15.6	86.8	1986	1999	7.6
COTE D'IVOIR	CI	1970	1999	97.7	99.4			n.a.
EGYPT	EG	1991	1999	95.7	98.9	1992	1998	4.8
ESTONIA	ES	1992	1999	85.4	100	1994	1999	8.2
FRANCE	FR	1979	1999	86.7	97.5	1980	1999	4.4
GREECE	GR	1977	1999	57.5	85	1978	1999	5.8
INDIA	IN	1979	1999	62.3	87.2	1980	1999	5.2
INDONESIA	DS	1978	1999	88.6	91.5	1986	1999	2.5
ISRAEL	IS	1991	1999	45.5	90.9	1992	1999	5.4
JAPAN	JP	1973	1999	33.8	61.2	1974	1999	3
KENYA	KN	1970	1999	55.5	83	1971	1999	5.6
KOREA	KR	1980	1999	74.3	91.4	1981	1999	0.7
LITHUANIA	LT	1994	1999	100	100	1995	1999	6.4
MALAYSIA	ML	1986	1998	68.7	90.1	1987	1998	2.2
MEXICO	MX	1989	1999	51.1	81.4	1993	1999	10.5
NEW ZEALAND	NZ	1985	1999	39.1	72.2	1988	1999	3.9
NIGERIA	NG	1986	1999	66.5	86.2	1987	1999	5.6
NORWAY	NR	1978	1999	58.3	88.4	1979	1999	4.7
PAKISTAN	PK	1982	1999	77.8	92.8			n.a.
PERU	PR	1990	1999	45.2	71.4	1991	1999	45.3
PHILIPPINES	PH	1988	1999	60.7	74.9	1989	1999	4.4
SINGAPORE	SG	1983	1999	62.9	91.3	1984	1999	2.9
SOUTH AFRICA	SA	1983	1999	32.8	66.2	1984	1999	3.5
SPAIN	SP	1984	1999	63.8	92.9	1985	1999	3.2
SWEDEN	SW	1985	1999	47.4	84.3	1986	1999	4.9
THAILAND	TH	1970	1999	88.6	94.7	1977	1999	2.8
TURKEY	TR	1980	1999	12.6	36.8	1987	1999	1.5
UGANDA	UG	1992	1999	52.9	77.9	1995	1999	9.8
UNITED STATES	US	1973	1999	26.8	58.7	1976	1999	2
URUGUAY	UR	1993	1999	22.7	92	1994	1999	41.8
VENEZUELA	VZ	1994	1999	73.1	94.3	1995	1999	9.8

Table 1: Exchange rate stability is the probability that the monthly change of the exchange-rate will be within the range of (+/-1% or 2.5 %). Source: Calvo and Reinhart (2000). Spreads:  $(1+\text{lending rate})/(1+\text{deposit rate})-1$ . Note that under such a definition inflationary expectations cancel out. Source: International Monetary Fund.

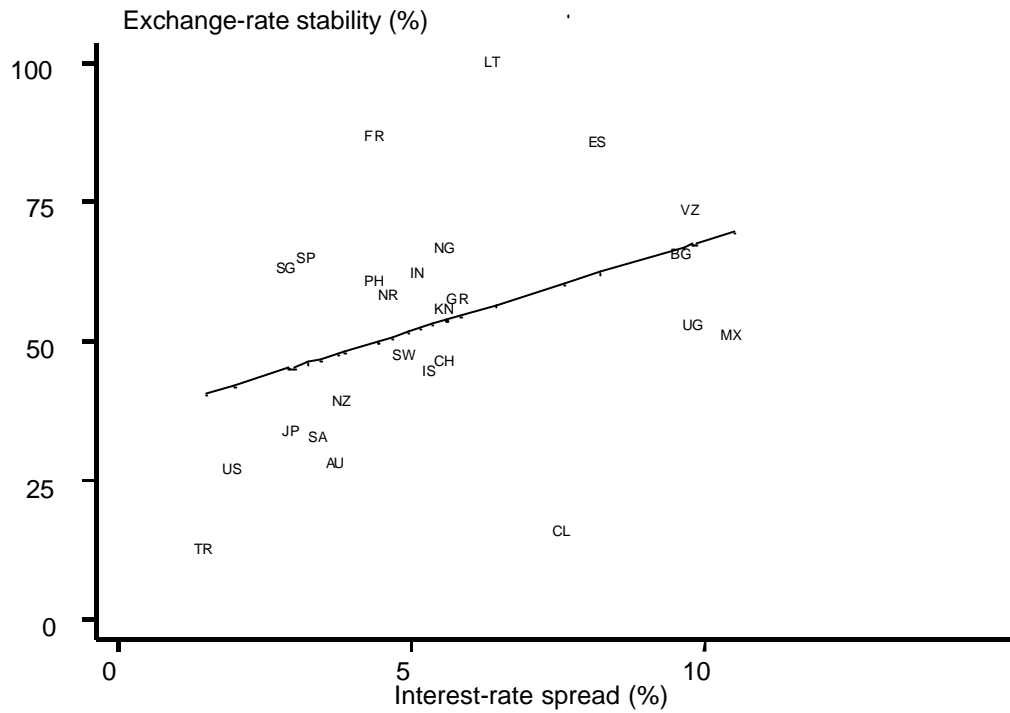


Figure 1: The measure of exchange rate stability is the probability that the monthly change of the exchange-rate will be within the range of  $(+/-1\%)$ . Source (including the country abbreviation used in the figure): Table 1. The figure excludes the Asian Tigers (Indonesia, Malaysia, Thailand and Korea), the dollarized economies (Argentina and Canada), the very regulated (Egypt) and the extreme values (Peru, Uruguay and Bolivia). The t-statistic of the slope: 1.99; number of observations: 25.