Stock Based Compensation: Firm-specific risk, Efficiency and Incentives

Vicky Henderson

University of Oxford

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Nomura Centre for Quantitative Finance,

24-29 St Giles',

Oxford. OX1 3LB. UK.

Tel: +44 (0)1865 270502

Email: henders1@maths.ox.ac.uk

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We propose a continuous time utility maximization model to value stock and option compensation from the executive's perspective. We allow the executive to invest non-option wealth in the market and riskless asset but not in the company stock itself. This enables executives to adjust exposure to market risk, but they are subject to firm-specific risk for incentive purposes. Since the executive is risk averse, this unhedgeable firm risk leads them to place less value on the options than their cost to the company, given by their market or Black Scholes value.

By distinguishing between these two types of risks, we are able to examine the effect of stock volatility, firm-specific risk, and market risk on the value to the executive. Executives do not necessarily want to increase stock volatility, as their risk aversion can outweigh the option's convexity effect. Firm-specific risk generally reduces option value, although if market risk is held fixed and the options are out-of-the-money it is possible for the reverse to hold. Generally, market risk has a positive effect on option value.

An implication of the model is that the Black Scholes formula exaggerates the incentives for the executive to increase the company stock price. We examine the relationship between risk and optimal incentives, and find firm-specific risk decreases optimal incentives (regardless of which parameters are fixed) whilst market risk may decrease optimal incentives depending on other parameters.

The research has implications for future design of compensation plans, in light of the recent trend towards expensing all options. The model supports the use of stock if the company can adjust cash pay when granting stock or options, and does not support the use of indexed options.

JEL G13, G30, G32, J33, M12

Key words: executive stock options, unhedgeable risks, incentives, utility maximization, firm-specific risk, equity based compensation plans, stock compensation.

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1 Introduction

Stock based compensation now represents a significant proportion of executive remuneration. In the US, executive stock options represented $893bn worth of shares in June 2000 [1] and Hall and Murphy (2002) cite that the grant date value of stock options accounted for 40% of total pay for S&P500 CEO's in 1998. Stock compensation is also important for rank-and-file workers : 45% of US companies awarded options to exempt salaried employees in 1998, 12% and 10% to non-exempt and hourly employees.

During the bull market of the late 1990's, many executives experienced large gains in the value of their options, and those who could exercise at the right time converted this into profits. However, many options will currently be underwater (out-of-the-money) because of stock market falls since early 2000. Despite this, stock based compensation continues to be used and has recently attracted much media attention. The recent corporate accounting scandals have brought under scrutiny the accounting treatment of stock options. The Financial Accounting Standards Board (FASB) require options which are not granted at-the-money to appear as expenses in the company statements. At-the-money grants only appear in a footnote to the accounts. Not surprisingly, most options granted are done so at-the-money to obtain this preferential accounting treatment. However, the International Accounting Standards Board[2] (IASB) required all options to be expensed in mid-July 2002, putting pressure on the US to follow. Many US companies have voluntarily agreed to expense all options including those granted at-the-money[3] and Standard and Poors will report earnings for companies after options are included in expenses. Since the IASB's announcement, there has been much speculation as to whether the FASB will also change it's rules. It was agreed at a joint meeting in September 2002 that the two bodies would begin eliminating differences between their rules. The Washington Post reported "the FASB has said it will reconsider requiring companies to treat stock options as expenses following a similar movement by the international board", see Spinner (2002).

This is important, since until now, companies have designed compensation to some degree around at-the-money options to take advantage of the accounting rules. Now this should no longer be the case, how should companies compensate their executives? This is one of the main themes of this paper, which concludes that companies should use more stock rather than options. The paper

[1] see Nohel and Todd (2001)

[2] The ISAB covers the EU whilst the US follows FASB.

[3] On August 20th 2002 about 1% of public companies in the US had agreed to expense options, according to Bloomberg News.

also examines the issues of the value of the compensation to the executive, its incentive benefits, the effects of risks on option value and incentives and whether companies should use indexed options.

There are strong theoretical arguments in favor of stock based compensation as the link between executive wealth and stock price provides incentives, and serves to align interests and mitigate agency problems between shareholders and management, see Jensen and Meckling (1976). An additional reason equity based compensation is used is to retain employees, as the options are usually long dated.

Although the cost to the company of stock based compensation can be reasonably approximated by it's market value[4], this is not a good estimate of the value that is transferred to the executive receiving the stock or options. However, early models to value executive stock options from the executive's perspective used the Black Scholes model, see Johnson and Tian (2000a). In fact, much of the empirical research in this area uses Black Scholes values as standard. It has been recognized that the cost of the compensation to the company (its market value) overstates the value that executives place on it, due to restrictions faced by executives in trading their own company's stock. This has been highlighted in a number of papers, including Lambert et al (1991), Kulatilaka and Marcus (1994), Huddart (1994), Detemple and Sundaresan (1999) and Hall and Murphy (2002). These papers argue that there is an asymmetry between the cost to shareholders and the value to the executive caused by trading restrictions. Each of these papers defines the value to the executive to be the amount of cash the manager would exchange for the options or stock, a certainty equivalence or utility indifference amount. Short sales are restricted, and private wealth is invested in the riskless asset. Each finds that under a power utility framework, the value to a manager can be significantly less than the cost perceived by shareholders.[5]

However the drawback is that these models cannot examine the risks faced by the executive (when he receives an option grant) separately, they can only consider total volatility. In this paper, by allowing the executive to trade in the market portfolio, but not in the stock of the company, we may distinguish the effect of market (or systematic) and firm-specific (or nonsystematic) risks on value. The executive is exposed to total risk but can only hedge market risk. Firm-specific risk cannot be hedged, although the executive must be exposed to it for incentive purposes. It is

[4]The market value is taken to be the Black Scholes value in the case of options.

[5]All these papers are structural models which maximize expected utility subject to restrictions. It is worth noting that another strand of literature models early exercise as an exogenous stopping time, see Carpenter (1998) and Carr and Linetsky (2000). Carpenter (1998) examines two models where non-option wealth is invested in the Merton no-option portfolio. The first has an exogenous stopping state where the executive must optimally forfeit or exercise. The second model is a utility model where the executive is offered a reward for leaving the firm each instant.

this exposure to unhedgeable risk, combined with the risk aversion of the executive which leads to the drop in value compared with the market value. The difference in the cost to the company and the value to the executive is the loss due to non-tradeability. We adopt the terminology of 'deadweight cost' to represent this loss, as introduced by Meulbroek (2001a). By allowing trading in the market, the results and implications will differ from the earlier work cited with only one asset. Jenter (2001) remarks that if managers can only invest in the riskless asset, the compensation and incentives appear more effective than if they can invest in the market.

This paper proposes a continuous time utility model to value stock and option compensation, under the assumption that executives cannot trade in the stock of the company.[6] Trading in the market ensures that executives may diversify away market risk, but for incentive purposes remain exposed to firm-specific risk. The general framework allows us to value compensation from the executive's perspective, examine the effect of total, market and firm-specific risks on this value and on the incentives produced, to question the form of compensation which is optimal for the company and analyze if companies should use indexed options. As such, our results span various strands of the literature. This paper is the first to address all of these issues in a single, continuous time utility model.

With the exception of Meulbroek (2001a), Jenter (2001) and Jin (2002), most research treats systematic and non-systematic risks together. Our work is most closely aligned to that of Meulbroek (2001a). She concentrates on the valuation issue and treats stock compensation, rather than options. The model uses the Sharpe ratio of the market as a benchmark for the rate of return a manager requires to compensate for the total volatility of the firm, not simply beta risk. For stock compensation, this gives a cost which is the difference in this required return and the actual expected return on the stock, where expected return is found using CAPM. Then the private value of the stock is simply the market value discounted by the difference in required and actual returns. Her analysis is consistent with assuming a mean variance investor. Merton (2001) extends this to the option case and obtains a pde for the private value of the option.

In contrast to Meulbroek (2001a) we use a utility framework to incorporate risk preferences and allow the executive to receive options or stock. According to Meulbroek "...a more precise estimate of the manager's private value..." is obtained via a utility maximization approach, although of course assumptions concerning the utility function of the manager must be made. Although this is not

[6]Our model is adapted and extended from those developed in Henderson (2002) and Henderson and Hobson (2002a) and the reader is referred to those papers for further technical details. A practitioner summary of these papers may be found in Henderson and Hobson (2002b).

straightforward, utility models using reasonable assumptions can at least provide a guide valuation to an individual. In this paper we assume CARA preferences via a negative exponential utility function. Companies are also realizing that individual valuation is necessary, this is highlighted in a quote from transcripts of the FASB Public Hearings on SFAS 123 (taken from Hall and Murphy (2002)):

"We do not believe any model can consistently or accurately value employee stock options. Because of the restrictive nature of these instruments...their value to an individual will vary significantly with individual circumstances, and their cost (if any) to a company may vary significantly from each individual's value"

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The papers of Jenter (2001) and Jin (2002) construct principal agent models to analyze incentives. Jenter (2001) models firm value as being determined by managers effort choice and a multiplicative random shock. Wealth is allocated between the riskless asset and market portfolio. His model, with power utility preferences, is set in a static framework. Jin (2002) analyzes the tradeoff between incentives and firm-specific risk. The compensation is assumed to be linear with mean variance preferences in a static framework.

Our model framework allows us to calculate an explicit expression for option incentives and we examine the impact of total, firm and market risks on this variable. In contrast to both the models of Jenter (2001) and Jin (2002) our framework is continuous time. In addition, we use a utility model and allow executives to receive option grants, unlike Jin (2002).

The utility framework allows us to find explicitly a utility indifference, or certainty equivalence value the executive places on the stock or option compensation.[7] This value depends primarily on the risk aversion parameter and the correlation between the company stock and the market, but also on the volatilities and expected returns of the stock and the market. As the size of the correlation between the stock and the market approaches one, the value of the stock or options to the executive tends to their market value. This is an important feature, and Meulbroek (2001a) has pointed out that not all utility models recover the Black Scholes option value.

The 'deadweight cost' in our model is significant. Using NYSE average stock and index volatilities, an at-the-money 10 year option is valued at only 78% of its market value with correlation 0.90, or 55% of market value with 0.50 correlation. These numbers could drop to 56% and 30%

[7] We use the terms certainty equivalence and utility indifference interchangeably. To be precise, the utility indifference value is simply a dynamic extension of the certainty equivalence value concept and was introduced in Hodges and Neuberger (1989).

for an executive with a high proportion of wealth in the options. For stock compensation, the corresponding figures are 85% and 67% respectively.

This paper differs from other recent work as our model allows us to examine the effect of firm-specific and market risk on the value to the executive, whilst making it clear which parameters are being held constant. Our results on risk and value are as follows. Executives awarded options do not necessarily want to increase stock volatility in general, when correlation is held constant, since their risk aversion can outweigh the convexity effect of the options. This effect is stronger when executives are given stock as then value decreases in total volatility. Secondly, if firm risk varies as a proportion of (fixed) total risk, firm-specific risk has a negative effect on option value, whilst market risk has a positive effect. However, if instead market risk is held constant, firm-specific risk again generally has a negative effect on option value, but not necessarily for out-of-the-money options. This depends on the importance of convexity. Finally, holding firm-specific risk constant, a generally positive relationship between market risk and option value is demonstrated, although for some parameter values it is possible to conclude the reverse. For stock, market risk does not seem to have much effect on value. All these effects can occur in the one model and indicate that it is difficult to make general statements about "risk" and value without clarifying exactly what variables are being held constant and which vary. See Section 4 for more details.

By maximizing incentives for a given total cost to the company and fixing the post-grant value of total compensation to the executive (at its pre-grant value), we show stock rather than options are optimal in our framework. This conclusion holds when the volatility of the stock price is reasonable and is robust to changes in risk aversion and a variety of assumptions concerning which risks are fixed and varying. For very low volatilities, the model makes the reverse conclusion that (a greater number of) out-of-the-money options maximize incentives. This is partly in common with Jenter (2001) who calculates the cost of inducing the same level of effort incentives in a static model, and Hall and Murphy (2000). These models do not obtain the low volatility effect. With the likely expensing of at-the-money options in the near future, all options will be expensed so companies will not gain an advantage by using at-the-money options. Our model suggests if companies can reduce cash when granting options, and stock volatility is not too low, then stock should be given to executives.

By employing a richer model, we are able to extend the results of Jin (2002) on the relationship between risk and optimal incentives to include non-linear compensation. Our first conclusion that optimal incentives decrease with total stock volatility (when other parameters are held constant) is consistent with principal agent theory. We find firm-specific risk decreases optimal incentives, regardless of whether total risk or market risk are kept fixed. This extends Jin's conclusion which was achieved for linear compensation. However, we find optimal incentives may increase or decrease with market risk, depending on whether total or firm risk are kept fixed. These results are in contrast to Jin (2002) who found (for linear compensation) market risk has no effect on optimal incentives.

Finally, we show that by reducing holdings of the market asset, the executive achieves a similar effect to indexing, thus giving a possible explanation for the lack of indexation in the US, in agreement with Meulbroek (2001a) and Jenter (2001). This abstracts from accounting issues which in the US have put indexed options at a disadvantage. Despite recent trends leaning towards expensing all options, the model implies US companies do not need to increase their useage of indexed options.

The paper is structured as follows. Section 2 sets up the model to value executive stock and option compensation. The next section uses a certainty equivalence style approach to value options or stock using the exponential utility model. Features of this valuation are explored and numerical examples given. Section 4 separates market and firm-specific risk, and examines their effect on the value to the executive of the options. Incentive benefits are introduced in Section 5 and maximized in Section 6 for a fixed cost to the company and fixed (pre-grant) value to the executive. The relationship between risk and incentives is also explored. Section 7 examines the implications of the model for relative performance based compensation. Concluding remarks are in the final section, and details of derivations are placed in the Appendix.

2 A Utility Maximization Model for Executive Stock Option Valuation

The executive receives European call options on the company's stock S, with expiry T and strike K. These are traditional executive stock options as defined in Johnson and Tian (2000a). Often in practice these options are issued at-the-money, we allow for this, but retain strike K to examine the effect of moneyness on valuation.[8] We can easily consider stock compensation in the model by setting $K = 0$.

We make the assumption that the executive cannot trade the stock, but is able to take positions

[8]We use the terminology "at-the-money" throughout the paper to refer to options with an exercise price equal to the grant-date stock price. These are sometimes called fair market value options . Out-of and in-the-money options are also defined with respect to the grant-date stock price.

in the market portfolio, denoted by M. The dynamics for the market index M are given by a geometric Brownian motion

$$dM/M = \mu dt + \sigma dB \tag{1}$$

where B is a standard Brownian motion. Assume the company's stock S also follows a lognormal process

$$\frac{dS}{S} = \nu dt + \eta(\rho dB + \sqrt{1-\rho^2}dW) \tag{2}$$

where W is a standard Brownian motion independent of the Brownian motion B driving the market.[9] The total variance of S is η^2 and using the above, this can be split into the *market or systematic risk* $\rho^2\eta^2$ and the remaining *firm-specific or non-systematic risk* $(1-\rho^2)\eta^2$. When $\rho = 0$, the risk is all firm risk, and when $|\rho| = 1$ the risk is solely market risk, which can be diversified away. For $|\rho| < 1$ the presence of a second Brownian motion W and the fact that no trading is allowed on S means that the firm risk cannot be hedged away.

The executive is able to invest in the riskless asset r and the market M. Cash amount θ is invested in the market and the remainder earns the riskless rate, giving dynamics for the wealth X of

$$dX = \theta dM/M + r(X - \theta)dt.$$

The aim of the executive is to maximize expected utility of wealth, where, in addition to funds generated by trading, the executive receives λ units of the call option with payoff $(S_T - K)^+$.[10] The value function of the executive is given by

$$V(t, X_t, S_t; \lambda) = \sup_{(\theta_u)_{u \geq t}} \mathbb{E}_t[U(X_T + \lambda(S_T - K)^+)] \tag{3}$$

and is used in the following section to obtain a certainty equivalence or utility indifference value for the options.

In the remainder of the paper, we will consider utilities with constant absolute risk aversion of the form $U(x) = -\frac{1}{\gamma}e^{-\gamma x}, \gamma > 0$. Choice of γ is discussed in the next section. Some justification for this choice is the research of Bliss and Panigirtzoglou (2002) who estimate empirically the market's or representative agent's degree of risk aversion from option prices. They conclude CARA is more consistent with the data. The lack of dependence on wealth can also be an advantage if empirical work were to be done, since the manager's wealth is unobserved.

[9] We can generalize the dynamics of the stock price S to $dS = \nu(S_t, t)dt + \eta(S_t, t)(\rho dB + \sqrt{1-\rho^2}dW)$ provided $\nu(S, t)$ and $\eta(S, t)$ satisfy suitable boundedness and regularity conditions. Details of this extension are available from the author upon request.

[10] We make the standard assumption that the executive's option grant does not have an effect on the value of the company stock.

3 Valuing Stock and Option Compensation

3.1 Certainty Equivalence Valuation

Using the model outlined in Section 2, we can find the amount of cash the executive would forego to receive the stock options. This certainty equivalence amount represents the value of the options to the executive. More specifically, the idea is to compare the expected utility for an executive who does not receive any options to the expected utility of the executive who receives the calls. The adjustment to the initial wealth which makes these two values equal gives the certainty equivalence (or utility indifference) value. Given an initial wealth of x, the utility indifference amount is the solution to the equation $V(0, x-p, S; \lambda) = V(0, x, S; 0)$ (see Hodges and Neuberger (1989)).

We write the value function of the executive given in (3) as

$$V(t, X_t, S_t; \lambda) = -\frac{1}{\gamma} e^{-\gamma X_t e^{r(T-t)}} g(T-t, \log S_t)$$

where $g(0, \log s) = e^{\lambda\gamma(S-K)^+}$. It follows (see Appendix, Section 9.1) that

$$V(t, X_t, S_t; \lambda) = -\frac{1}{\gamma} e^{-\gamma X_t e^{r(T-t)}} e^{-(\mu-r)^2(T-t)/2\sigma^2} [\mathbb{E}^0 e^{-\lambda\gamma(1-\rho^2)(S_T-K)^+}]^{(1-\rho^2)^{-1}}. \tag{4}$$

Solving $V(t, X_t - p^e, S_t, \lambda) = V(t, X_t, S_t, 0)$ for the value p^e of λ calls gives

$$p^e = -\frac{e^{-r(T-t)}}{\gamma(1-\rho^2)} \log \mathbb{E}^0 [e^{-\lambda\gamma(1-\rho^2)(S_T-K)^+}]. \tag{5}$$

Notice the expectation in (4) and (5) is no longer taken with respect to the original probability measure $\mathbb{P}$. Under this new probability measure denoted $\mathbb{P}^0$, the rate of return on the market is r, but the return on innovations orthogonal to the market are left unchanged.[11] Specifically, the market follows

$$\frac{dM}{M} = rdt + \sigma dB^0$$

where $B^0 = B + (\frac{\mu-r}{\sigma})t$, a $\mathbb{P}^0$ Brownian motion and the stock follows

$$\frac{dS}{S} = \left[r + \eta\left(\frac{\nu-r}{\eta} - \frac{\rho(\mu-r)}{\sigma}\right)\right] dt + \eta\rho dB^0 + \eta\sqrt{1-\rho^2} dW \tag{6}$$

where we define $\delta = r + \eta(\frac{\nu-r}{\eta} - \frac{\rho(\mu-r)}{\sigma}) = \nu - \frac{(\mu-r)\eta\rho}{\sigma}$ to be the drift of the stock under $\mathbb{P}^0$.

It is important to realize that we have not priced under this measure, it is merely appearing in the expression for executive value, which was derived using utility maximization arguments. To

[11] This is called the minimal martingale measure in the mathematical finance literature, see Föllmer and Schweizer (1990). It is also the same idea used by Hull and White (1987) when they assume the market price of volatility risk is zero in their stochastic volatility model.

reinforce this, we could expand the value in (5) in terms of the number of options, λ, to give

$$p^e = e^{-r(T-t)}\lambda \left(\mathbb{E}^0 (S_T - K)^+ - \frac{\lambda}{2}\gamma(1-\rho^2)Var[(S_T - K)^+] + ... \right). \tag{7}$$

The first term in the expansion is exactly the value of the option under the measure $\mathbb{P}^0$, but there are correction terms to this. We now show that when the market and stock are perfectly correlated, this valuation method gives the correct value for the option, its Black Scholes or market value. Putting $|\rho| = 1$ into the expansion for p^e above eliminates the second order term. Ignoring higher order terms, the value appears to be simply the price under the measure $\mathbb{P}^0$. However, on closer inspection, to avoid arbitrage in the model, the relationship $\nu - r = (\mu - r)\eta\rho/\sigma$ must hold, with either $\rho = 1$ or $\rho = -1$. This implies the drift δ under $\mathbb{P}^0$ is actually just the riskless rate, and hence the value becomes simply the Black Scholes price (as $\mathbb{P}^0$ becomes the risk neutral measure). In this case, there is no firm-specific risk and the executive is fully diversified.

Significantly, this is in contrast to the results in many utility maximization models with one risky asset which do not recover the correct market value when taking the market as their risky stock. Meulbroek (2001a) criticizes the utility method for this reason, see her Footnote 30. Whilst this is a valid criticism of the models with one risky asset, our model does not suffer from the same limitations.

We have established that the limiting case of perfect correlation between the stock and the market gives the Black Scholes value, or company cost. However, for the more realistic case of non-perfect correlation, how does the value differ from the company cost ? Let us first recognize that the value p^e solves a partial differential equation. We write

$$p^e(t,s) = -\frac{e^{-r(T-t)}}{\gamma(1-\rho^2)} \log W(t,s) \tag{8}$$

where $W(t, S_t) = \mathbb{E}^0 e^{-\lambda\gamma(1-\rho^2)(S_T-K)^+}$. Using the pde for W developed in Appendix 9.1, we may derive another pde for the executive value p^e:

$$\dot{p^e} + \frac{1}{2}\eta^2 S^2 (p^e_{ss} + (p^e_s)^2(\gamma(1-\rho^2)e^{r(T-t)})) + \delta S p^e_s = rp^e \tag{9}$$

with $p^e(T,s) = \lambda(s-K)^+$. Again, we can see that taking $|\rho| = 1$ and $\delta = r$ in (9) gives back the Black Scholes pde.

We return to the question of non-perfect correlation between the stock and the market. We can show the value p^e is lower than the company cost and increasing in the size of the correlation. As correlation tends to one, the value to the executive tends to the company cost. We use a comparison argument. Take $(\hat{\rho})^2 < (\tilde{\rho})^2$. Fix δ and define the operator (from (9))

$$\mathcal{L}f = -rf + (\dot{f} + \frac{1}{2}\eta^2 S^2 [f'' + \gamma(1-\tilde{\rho}^2)e^{r(T-t)}(f')^2] + \delta S f').$$

Define h^{ρ} to be the value p^e using correlation ρ. Then $\mathcal{L}h^{\tilde{\rho}} = 0$ and $\mathcal{L}h^{\hat{\rho}} \leq 0$ since $\frac{1}{2}\eta^2 S^2 \gamma e^{r(T-t)}(f')^2$ is positive. Since $h^{\hat{\rho}}$ and $h^{\tilde{\rho}}$ have the same boundary conditions (doesn't depend on ρ) we can use a comparison theorem (see Ishii and Lions (1990)) to conclude $h^{(\hat{\rho})^2} \leq h^{(\tilde{\rho})^2}$ and hence $p^e((\hat{\rho})^2) \leq p^e((\tilde{\rho})^2)$.

We have shown the executive's value is increasing in $|\rho|$, up to the Black Scholes value or company cost. At least qualitatively, this is the same observation made in many recent papers including Meulbroek (2001a) and Jenter (2001).

The risk aversion parameter γ plays an important role. When the executive is awarded options ($\lambda > 0$), higher risk aversion results in him placing a lower value on them. This can be shown using a similar comparison argument to that used for the correlation.

3.2 Features of the Executive Valuation and Numerical Examples

We now illustrate some of the features of our analytical formula for executive valuation further by varying the parameters in (5). Dividing (5) by the number of units λ, gives a per-unit value $\frac{p^e}{\lambda}$. We can observe that the parameters λ and γ only appear together in the per-unit value, and we set $q = \lambda\gamma$. We motivate our numerical choice of this parameter by equating local absolute risk aversion with the more popular power utility with constant relative risk aversion. This is useful since many authors use values of R around 3-4 and we can adopt similar values in our numerical examples.[12] The equating of local risk gives the relation $\gamma = \frac{R}{x}$, where R is the coefficient of relative risk aversion in the power utility and x is private wealth.

Say the initial value of the stock is 100 cents. Now consider a number of scenarios concerning the ratio of options or stock to private wealth of the executive. Suppose the executive receives \$25, 000 worth of shares so $\lambda = 25,000$. If this represents a very small fraction of his wealth, say $\frac{1}{40}$, then $x = \$1,000,000$ or $x = 10^8$ cents and a suitable risk aversion parameter is $\gamma = 4/10^8$ and $\lambda\gamma = 4 \times 25,000/10^8 = 1/1000$. This scenario might describe a wealthy CEO. At the other extreme, if the shares represent $\frac{1}{4}$ of the executive's wealth, then $x = \$100,000$, $\gamma = 4/10^7$ and $\lambda\gamma = 4 \times 25,000/10^7 = 1/100$. This would be more suitable for firms such as Internet firms where many employees have a large part of their wealth in options or stock of the company. Other values between these two values of γ that we use later are $\gamma = 4/2500$ and $\gamma = 4/1500$. Of course, these numbers can be scaled. The executive who receives \$ 2, 500, 000 worth of shares has $\lambda = 2,500,000$

[12]Many papers, including Jenter (2001), Nohel and Todd (2001), Lambert et al (1991) and Hall and Murphy (2002) use R about 3-4. Although there is a debate as to whether R should be much larger, (see the discussion of the equity premium puzzle in Cochrane (2001)) this would only magnify the effects observed in our model.

and if this represents $\frac{1}{40}$ of his wealth, x also gets multiplied by 100. This results in the same value for $q = \lambda\gamma$. This provides some justification for our choices of risk aversion γ and shows how implicitly, this choice may reflect private wealth.

In each of the graphs (unless specified otherwise) we use parameters $T = 10$ years, $\rho = 0.8$, $\mu = 0.10, \eta = 0.45, \sigma = 0.35, r = 0.05$. We set the drift of S, δ under $\mathbb{P}^0$ to be r and let ν vary according to the relationship given earlier. Figure 1 is a plot of the 'Executive Value lines' for option compensation, expressed as value per unit option. These are the executive value plotted as a function of stock price, as used in Hall and Murphy (2002). The Black Scholes or company cost represented by the highest line on the graph is significantly higher than either of the executive values, the two broken lines. The executive values use two values of q. This represents two executives with the same number of options λ and different risk aversion parameters, but can also be thought of as representing two executives with the same risk aversion parameters but holding different numbers of options.

This explains why executives often demand large premiums to accept options in place of cash payments. This difference is the amount the company effectively loses by granting the options, as it is the value that is not passed on to the executive. As predicted from the valuation formula, as risk aversion increases (moving from the higher to lower of the two broken lines), the value placed on the options falls. The difference in the company cost and value is greater when the option is far in-the-money.

The executive's valuation of stock compensation is given in Figure 2. The company cost is again much greater than the values calculated from the expression (5).

We can examine the effect of changing the option's expiry, correlation between the market and stock and q, representing risk aversion and number of options, in a table. Table 1 considers an at-the-money option and reports the ratio of the executive value to the company cost for 3, 5 and 10 year options. We fix σ and η and vary ρ, which also changes the beta of the stock.

As predicted, when correlation tends to one, the ratio approaches one, as the executive value tends to the company cost. In this case, the executive is fully diversified and will not require a return premium. Again, we can think of different q as representing executives with the same number of options but different risk aversions, or the same risk aversions with different numbers of options. Taking the former view, higher risk aversion leads to a lower executive value. A CEO with much wealth outside the firm may value 10 year options at 80 % of the Black Scholes value (when $\rho = 0.75$), but the same options may only be worth 39 % say to an Internet company employee using $q_4 = 1/100$.

It is difficult to compare the values directly to other models, as we use different assumptions about risk aversion and the stock price distribution. It seems however, that they can be lower than those of Meulbroek (2001a). For NYSE firms with a quarter of the manager's portfolio in stock and β of 0.70, Meulbroek calculates ratios of 0.83 and 0.68 for options with a 3 and 10 year vesting period. If we loosely equate these to using the value q_4, then the ratios in this model are lower. However, this is very rough, and Meulbroek uses a different, non-utility based approach. The values reported by Hall and Murphy (2002) are lower than Meulbroek's but perhaps more similar to ours.

Table 2 reports the same ratio for stock compensation using the parameters from Table 1. The relationship between value and correlation is qualitatively similar to that of the option case. Lower correlation reduces the value to the executive and the corresponding ratio to the company cost.

For a particular ρ, β and q, the option and stock compensation becomes less efficient with increasing T. That is, with correlation 0.5, and risk aversion q_1, the ratio is 0.93 for a 3 year option but only 0.72 for a 10 year option. The value of the 3 year option is much closer to its company cost. Options are less efficient than stocks in terms of their executive value versus company cost. For the same parameters, the stock compensation has ratio 0.97 for $T = 3$ and 0.81 for $T = 10$. This is due to the convexity of the options.

We can now observe the effect of letting q vary via fixing risk aversion γ and varying the number of options held, see Figure 3. The per-unit-value decreases as λ increases, reflecting the fact that there is more risk associated with a larger number of stock options. A way to think about this is to consider the proportion of wealth that the options represent, as this quantity is changing with λ. In this case, if wealth is $\$150,000$ then a suitable risk aversion is $\gamma = 4/1.5 \times 10^7$ as used. If the executive is awarded $10,000$ options, at a cost of 63.8 cents each, then a proportion $6380/150,000$ is in options. The graph shows that in this case, the executive values the options at about 43 cents each. However, if the executive receives $100,000$ options, with cost \$ 63, 800, then this represents nearly half his wealth. He values the options at less than 20 cents each in this case.

4 The Effect of Risk on Option Value

By separating the firm-specific and market risks in the model we are able to examine the effect of each on value and gain insights into the nature of the relationship between risk and option value.

First we examine the effect of options on risk taking. By varying the stock volatility η, we can observe the effect on executive value. By fixing the correlation, varying η will have an impact on both firm-specific and market risks. Some thought tells us that increasing volatility will have two effects on value. First, firm-specific risk rises, and since this is unhedgeable and executives are risk averse, value falls. But volatility also has the effect of increasing value via the convexity of the call option payoff. Therefore we can expect a mixed effect on value, depending on which dominates. This will depend on the option moneyness and other parameters.

Figure 4 depicts this using options with various degrees of moneyness. The three solid lines represent the company cost for in-the-money, at-the-money, and out-of-the-money options, in order from highest to lowest. Each is increasing with volatility, since this only takes the convexity effect into account. The remaining three lines represent value to the executive for the same three options. For the out-of and at-the-money options, when η is low, the convexity effect dominates firm-specific risk and value is increasing. However, this reverses for higher volatility, leading value to decrease with volatility. The convexity is less important for the in-the-money option (the extreme case being stock with no convexity effect), and value is decreasing in volatility for the entire range. At high values of total volatility, the executive's risk aversion dominates any convexity effect, resulting in falling values.

It is commonly believed that executives have a desire to increase stock volatility, see Johnson and Tian (2000a) for instance. Executives are risk averse and for this reason they are given options to encourage them to take more risks. However, as we have seen, this does not necessarily hold true, as the executive's risk aversion to firm-specific risk counteracts and can outweigh the convexity effect. In models with only one asset, Lambert et al (1991), Detemple and Sundaresan (1999), Kulatilaka and Marcus (1994) and Hall (1998) recognize the value may fall with increased volatility. Carpenter (2000) uses a dynamic model of portfolio choice to show options may reduce risk taking incentives and Nohel and Todd (2001) find managers may not prefer an increase in volatility of stock returns. Our model is consistent with these theoretical findings and shows that executives given options generally do not have an interest in increasing stock volatility. This becomes even more apparent when we consider stock rather than option compensation. In this case, there is no convexity effect, and increasing volatility leads to a drop in value due solely to the risk aversion of the executive to unhedgeable risks.

Empirical evidence supporting the idea that options do not always encourage risk taking is available but limited. Firstly, Cohen, Hall and Viceira (2000) test for evidence of risk shifting behavior and find it to be a very small effect, regardless of direction. In the oil and gas industries, Rajgopal and Shevlin (2000) find managers are motivated to take on exploration risks but hedge oil price risk. Empirical evidence supporting our conclusion may also be found in Meulbroek (2000). She examines insider transactions in high volatility internet firms. The study finds negative information is not the primary motivation for insider selling. Rather, the evidence suggests executives place a high value on having a diversified portfolio and attribute a lower value to the options when volatility is high.

The second analysis we consider is varying correlation whilst keeping total volatility η constant. We considered this analytically earlier, and showed that the value to the executive increased with correlation. We think of positive correlation here, although analogous arguments can be given for negative correlation. We relate the effect of correlation on value to firm-specific and market risks. Figure 5 plots the value (per unit option) to the executive against correlation between the market and stock. The three lines represent three different options (from highest to lowest) in-the-money, at-the-money and out-of-the-money, with parameters given in the caption to Figure 5. Again, we fix the value of $q = \lambda\gamma$. We can think of this as fixing the risk aversion γ for the three different options and thus the number of options. Firstly observe that the value does indeed increase with correlation, for each option moneyness. This extends also to the case of stock. Altering correlation whilst keeping total volatility fixed changes the proportion of firm-specific and market risks. As correlation increases, less of the risk is firm-specific and more is market risk, leading to an increase in the option value to the executive. By varying firm risk as a proportion of total risk, we obtain a negative relationship between firm risk and value. Varying market risk as a proportion of total risk gives a positive relationship between market risk and value. These conclusions also hold if the executive receives the stock itself which can be thought of as a far in-the-money call option.

We can also try to isolate the effect of firm-specific risk on value by keeping market risk constant. Here, two forces are interacting, increasing firm-specific risk is reducing value, whilst the corresponding increase in total risk η will increase value via convexity. This convexity effect should have less of an impact than in Figure 4 however as market risk is constant. Figure 6 substantiates this claim. The lines from highest to lowest represent in-the-money, at-the-money and out-of-the-money options. Again q is fixed and we interpret this as having both the number of options and risk aversion fixed for the figure.

Basically, option value to the executive is decreasing with firm-specific risk, except for the case of out-of-the-money options. The convexity does have an effect at low values of firm-specific risk in this case. The at-the-money and in-the-money option values are decreasing in firm-specific risk over the whole range. The value to an executive receiving the stock itself would simply decrease with firm-specific risk, as there is no convexity effect.

A final graph will fix firm-specific risk and vary market risk $\eta\rho$. The same three options appear in Figure 7 as those used in Figure 6. As market risk increases, the value to the executive rises, at least for the range considered on the graph. For a stock, the value seems fairly flat with market risk. However, if we decompose this case, it is really a combination of effects in Figures 4 and 5. That is, increasing $\eta^2\rho^2$ whilst keeping firm-specific risk constant involves increasing both η and ρ. Thus for some parameter values, it is possible to conclude the value to the executive decreases with market risk, contrary to intuition.

Thus depending on which parameters are held fixed, differing but not inconsistent conclusions may be drawn about the effect of risks on the executive's valuation of the options.

5 Incentive Benefits of Compensation

We will now turn to the incentive benefits of stock based compensation. Following Hall and Murphy (2002), we consider incentives to be the options' effect on motivation of a manager to increase the company's stock price. That is, stock options provide incentives as the executive can influence the value of the options by his actions. The idea is that by giving stock based compensation to executives, they will act like an owner of the company. This is in fact one of the main reasons companies use stock based compensation. Of course, there are limitations to incentive provision in practice and evidence shows managers share of increases in firm value are about 3% on average [13]. Amongst other reasons, the cost inefficiency of options due to firm specific risk counteracts the benefit of options for incentive purposes. We will investigate these issues in this section. We use our model to obtain an explicit expression for 'true' incentives.

To measure incentives, a natural proxy is the derivative of the option value with respect to the stock price. Johnson-Tian (2000a) define incentives for traditional executive stock options as the derivative of the Black Scholes value with respect to the stock price. A larger number implies larger increases in executive wealth for a given increase in stock price and stronger incentives to exert effort to increase the price. They find an incentive to increase the stock price, which is consistent with with their use of the Black Scholes model. The same qualitative result is found by Agrawal and Mandelker (1987), Lewellan et al (1987) and Jensen and Murphy (1990).

However, just as the company cost overstates the value of the options to managers, the Black Scholes delta overstates their incentive effects. In common with Hall and Murphy (2000) and Jenter (2001), we define incentives as the natural extension of this, as the derivative of the utility value with respect to stock price. We obtain an explicit expression for incentives in our model. Using

[13] See Jensen and Murphy (1990)

the expression for value in (8), we differentiate to obtain

$$\frac{\partial}{\partial S} p^e(t,s) = -\frac{e^{-r(T-t)}}{\gamma(1-\rho^2)} \frac{W_s(t,s)}{W(t,s)}. \tag{10}$$

where the details of the expression for W_s are given in the Appendix, Section 9.2.

As shown in Figure 8, when these incentives are compared to the Black Scholes delta (the solid line), they are significantly lower. The plot shows incentives for an option with stock price 100, for two values of $q = \lambda\gamma$. We can think of this as representing incentives to two executives with the same number of options but different risk aversions. Incentives decrease with risk aversion level, the lower line uses the larger of the two parameters. The at-the-money difference between Black Scholes and our model incentives is as much as 0.34 for the chosen parameters. This difference is fairly stable over different strikes. Thus options give much less incentive to increase stock price than is reflected by the naive Black Scholes delta. This has also been noted by Hall and Murphy (2000), Jenter (2001) and Jin (2002). Another observation that can be made from Figure 8 is that incentives are greatest for in-the-money options (and the extreme case of stock) on a per unit basis. This holds true for both Black Scholes and the executive's true incentives modelled here.

6 Weighing up the Costs and Benefits of Stock Compensation

We have clearly shown that applying naively the Black Scholes model will overestimate incentives and give an unrealistic picture of the incentive effects to executives for both options and stock. However, what can we deduce about the incentives of stock versus option compensation and how should the option strike be set to give the desired level of incentives? To answer this question, we must consider both the cost and incentive benefits of the compensation. Options are efficient when their incentive benefits exceed their inefficiency cost, in terms of the deadweight cost or loss to the company.

As discussed, the incentives from giving stock compensation are greater than from options, and in-the-money options have higher incentives than out-of-the-money options. However, options are also worth less than stock, and thus it is cheaper in terms of cost to the company to grant equal number of options than stock. Thus, for the same amount of money, the company could grant a larger number of out-of-the-money options or fewer stocks, with differing incentives from each.

To make a fair comparison, we must vary the number of options or stock granted to give a constant cost to the company and compare the resulting incentives. This means we allow the company to grant fewer in-the-money options or stock or more out-of-the-money options, and compare to see which strike maximizes incentives. Equivalently, incentives could be fixed, and compensation chosen to minimize cost, as in Jenter (2001).

In this paper, we will make an assumption made in efficient contracting theory, that when stock based compensation is given, the executive's base salary is reduced. There is evidence of this in practice, see footnote 21 in Hall and Murphy (2002). The relevant amount of cash to be deducted from salary is exactly the executive's valuation of the additional options or stock. If we think of the executive starting with \$100 000 cash, then the option grant must be such that the value of the options p^e plus cash pay total \$100 000. In this scenario, the executive is indifferent between additional stock or options (and less cash) and the status quo. This is equivalent to assuming the company and executive bargain efficiently over compensation, and essentially keeps the executive at the pre-grant expected utility.

We also fix the cost to the company of the compensation, which is the Black Scholes value of the options plus cash pay. The company varies the number of options or stock given to maximize incentives, keeping the cost to the company constant and the value to the executive the same as the pre-grant cash value. If we combine these two constraints, it is equivalent to keep fixed the difference in the Black Scholes and executive value of the compensation.

Returning to our illustration, assume the company paying the executive \$100 000 cash decides to grant stock or options. Say the company gives options with Black Scholes value \$10 000, but the executive values these at only \$6 000. The company gives the executive \$94 000 cash and the options, leaving the total value to him unchanged at \$100 000. The total cost to the company is now \$104 000. When deciding on whether to grant stock, in-the-money, at-the-money or out-of-the-money options, the company keeps the \$100 000 value to the executive constant and the \$104 000 cost to themselves constant. This is equivalent to keeping the difference of \$4 000 constant.

If we denote the executive value of this compensation by p^e_K, where the K denotes the dependence on strike, then we maximize incentives as follows:

$$max_K \frac{\partial p^e_K}{\partial S} \text{ such that } \lambda \left[BS^K - \frac{p^e_K}{\lambda} \right] = c \tag{11}$$

In the following analysis we choose c to be the cost when $\lambda = 1$ and $K = 100$. Then if out-of-the-money options are granted, λ can be greater than 1, whilst for in-the-money options, λ must be less than one. We will fix risk aversion, option expiry, μ, volatility of the market, riskless rate and initial stock value. The remaining parameters of stock volatility and correlation are varied in a number of ways. This enables us to investigate the conditions under which different option strikes may be optimal and the robustness of our conclusions.

Figure 9 fixes correlation and varies the stock volatility η. Lines represent various values of η ranging from 0.1 to 0.8. For most values of volatility, incentives are maximized by choosing stock ($K = 0$) rather than options. However, this is not the case for very low volatilities when it appears optimal to grant out-of-the-money options. This may be explained by examining the effect of volatility on the executive value, as illustrated earlier in Figure 4. For high volatility the difference in the Black Scholes and executive values for at-the-money and far out-of-the-money options is very similar. For example, the company could give one at-the-money option or 1.0175 options with strike 150. This is reflected in Figure 4 by noticing that for high volatility, the difference in Black Scholes and executive values is roughly constant. When volatility is reduced to $\eta = 0.1$, whilst both at-the-money and out-of-the-money Black Scholes values fall dramatically, the executive values behave quite differently. The at-the-money executive value actually increases to something very close to the Black Scholes value, consistent with the firm-specific risk effect outlined in the discussion of Figure 4. However, the out-of-the-money executive value drops when volatility is reduced, consistent with the impact of convexity dominating firm-specific risk for out-of-the-money options. This means the difference in Black Scholes and executive values is much larger for the out-of-the-money option than the at-the-money option and the company in this case can give 1.57 out-of-the-money options. This larger number of out-of-the-money options is enough to outweigh the smaller incentives per option, and out-of-the-money options become optimal when volatility is low. Thus it could be optimal for companies with very low stock price volatility to compensate with (more) out-of-the-money options. This may have implications for the practice of repricing out-of-the-money options, see Johnson and Tian (2000a). If a firm has very low stock price volatility then this analysis suggests that they should not reprice out-of-the-money options.

We also verified that this conclusion was robust to changes in risk aversion. Fixing the volatility at 0.45, we examined optimal incentives for various γ values. We found stock was still optimal across different risk aversion levels. Figure 10 illustrates this.

To investigate further the optimal form of compensation in the model, we consider three additional scenarios. First we fix stock volatility η and vary correlation. This implies total risk is fixed but the proportion of firm-specific and market risk changes. Figure 11 shows that the choice $K = 0$ is still optimal over a reasonable range for ρ.

Figures 12 and 13 fix market risk and firm-specific risk in turn, whilst varying the remaining risk. Again, we see taking $K = 0$ is optimal in both cases.

Thus for typical volatilities the model concludes that stock compensation is optimal and options are an inefficient means of creating incentives, in agreement with Jenter (2001) and Hall and Murphy

(2002). This conclusion is robust to changes in risk aversion, and is obtained under a variety of scenarios involving stock volatility and correlation. The static principal agent model of Hemmer et al (2000) also finds that introducing convex payoffs with CARA is non-optimal. Our result reinforces their conclusion and extends this to a continuous time model with trading in the market.

However, for very low volatilities, when correlation is held constant, the model gives the opposite conclusion that out-of-the-money options are optimal rather than stock. Other models do not get this effect as it is a function of the varying importance of convexity versus firm-specific risk across different strikes.

Our conclusion that stock is optimal does not explain why options (particularly at-the-money) are so prevalent in compensation packages. This could be explained by other factors which we do not account for making options more attractive for companies, including accounting and tax issues. Whilst this may explain the use of options in past years, with the recent likely changes to expensing of options, this attractiveness will diminish in the future. Under this scenario where there is no longer an advantage to using at-the-money options, our model concludes stock should be given as compensation for most companies with typical volatilities. With this, we may see more companies using stock rather than options to compensate executives in the future.

We can also analyze the effect of risk on incentives in our framework. The tradeoff between risk and incentives is at the center of agency theory. The principal agent model predicts that executive's pay-for-performance sensitivity is decreasing in the variability of the firm's performance. The empirical evidence testing this hypothesis is mixed. Aggarwal and Samwick (1999a) and Lambert and Larcker (1987) both find executives at riskier firms have lower incentive levels, supporting the conclusion of the principal agent model. Many papers however find no significant relationship, or even a positive one, see Table 1 in Prendergast (2000).

A possible explanation for the weak empirical link is that the literature focusses on total risk, when the effects of firm-specific and market risks are potentially very different. More seriously, the theoretical models also attempt to draw conclusions based solely on total risk, which (at least in the context of allowing the executive to trade) is not the only way to examine the link between risks and incentives. There is one exception to this. In a static principal agent framework, Jin (2002) finds optimal incentives decrease with firm-specific risk and are unaffected by market risk, when considering linear compensation. Empirically, he finds it is the firm-specific risk driving the relation between risk and incentives.

We can investigate these issues for option (nonlinear) compensation in a continuous time utility model. The model makes predictions about the optimal compensation structure and we may analyze how this changes with risk. Potentially different conclusions, not inconsistent with each other may be drawn if we consider firm-specific and market risks separately.

Figures 9-13 give information on how incentives vary with risk, where risk can be total, firm-specific or market. Note that now we compare the different lines whereas previously we compared different points on the same line. Option incentives decrease with total volatility when other parameters, including correlation are kept constant, see Figure 9. Optimal incentives (where incentives are maximized at $K = 0$) also decrease with total volatility, consistent with the conclusions of principal agent theory.

Figure 11 tells us option incentives increase with correlation, keeping total volatility fixed. Thus option incentives decrease with firm-specific risk and increase with market risk. This is also true for optimal incentives. Jin (2002) makes the same conclusion for firm-specific risk for stock incentives. Here, since we are holding total risk fixed, increasing market risk (via increasing correlation) is also simultaneously reducing firm-specific risk. Thus we cannot distinguish the true effect of market risk in this case.

When market risk is fixed, option incentives (and optimal incentives) decrease with firm-specific risk as illustrated in Figure 12. We now attempt to isolate the effect of market risk by holding firm-specific risk constant, refer to Figure 13. Option incentives (and optimal incentives) decrease with market risk when firm risk is fixed and for the chosen parameters. This conclusion differs from that made in Jin (2002) for linear compensation, that market risk has no effect. However our compensation and model assumptions are also very different to his. One explanation is that for stock compensation in our model, market risk does not affect value significantly, see Figure 7. This would also be consistent with the discussion surrounding Figure 4 on the lack of convexity effect for stocks. Thus our conclusion is consistent with Jin (2002) when we also just consider stock. For options, however, the effects are more complex. Figure 7 showed market risk could increase option value to the executive, and we commented that for other parameters and ranges it was possible that market risk decreased value. This effect appears to carry over to optimal incentives so that market risk decreases optimal incentives, at least for the examples we examine.

7 Implications for Relative Performance Based Compensation

A final area where our model can make a contribution is in understanding the role of relative performance based compensation. Principal agent theory in economics concludes that optimal contracts should display 'relative performance evaluation', see Holmstrom (1982). The prediction is that an executive will receive lower compensation if executives of rival firms deliver higher shareholder returns. There is a literature which examines whether executive plans exhibit implicit relative performance compensation and obtains mixed results. Gibbons and Murphy (1990) find support for this prediction, whilst Jensen and Murphy (1990), Aggarwal and Samwick (1999b) and Garen (1994) find little or no evidence of relative performance based plans in practice.

Explicit relative performance plans, on the other hand, involve options which are indexed to a market benchmark. One way in which this is done is where the index replaces the strike of the option.[14] The aim is to reward managers for the component of performance under their control by filtering out market wide performance. The idea is that executives should not be rewarded because of a bull market or penalized due to a bear market. Indexed options have not been popular in the US. In fact, Murphy (1999) finds that only 5 out of 1000 US firms granted non-traditional options in his data set. A major factor contributing to this lack of useage is probably US accounting rules which have expensed indexed but not traditional options granted at-the-money.

A question we can address therefore using the model in this paper, is whether, abstracting from these accounting issues, companies should use indexed options when compensating executives. Looking to the near future, when at-the-money options will probably be expensed in the US, what does our model say about the use of indexed options? We argue indexation does not improve efficiency of options in our model and thus US companies in the future do not need to introduce them.

If we consider an indexed option where the payoff is benchmarked to the market portfolio, then the market performance is being stripped out of the option to the executive. In this paper, executives receive call options on the company's stock, and trade the market portfolio to diversify away market risk exposure. Since they can continuously adjust their holdings in the market portfolio, they reduce exposure (if stock positively correlated with market) and thus achieve a similar effect to the indexed option. Effectively, the executive reduces his allocation in the market asset when there are unhedgeable idiosyncratic risks. This is true for many utility functions, see Gollier et al (1996) and Franke et al (2001).

To show this, an expression for θ^*, the optimal holdings in the market was derived in the Appendix (Section 9.3) and is given by

$$\theta^* = \frac{\mu - r}{\gamma \sigma^2 e^{r(T-t)}} + \frac{\eta \rho S W_s}{\gamma e^{r(T-t)} \sigma W (1-\rho^2)}.$$

[14]See Johnson and Tian (2000b) and Rappaport (1999). Meulbroek (2001b) has recently pointed out that typical indexed options where the exercise price changes to reflect benchmark index changes, do not remove the effect of the index from compensation. She proposes an alternative involving a benchmarked portfolio with a fixed strike.

The first term is the strategy which is undertaken in an exponential utility model when there is no option, see Merton (1969). In Section 9.2, we note that $W_s < 0$ and thus the second term in θ^* is negative, provided $\rho > 0$. Thus, the holding in the market is always lower with the option than would be optimal without the option. By making this adjustment, the executive is only exposed to the firm-specific risk of the company.

In the context of this model, there is no advantage to using relative performance compensation. A similar argument is put forward by Jenter (2001) and Meulbroek (2001b), however in Jenter's (2001) static framework, a manager cannot replicate indexation perfectly via trading in the market.

8 Conclusion

This paper proposes a continuous time utility model to value stock and option compensation, under the assumption that executives cannot trade in the stock of the company. Trading in the market ensures that executives may diversify away market risk, but remain exposed to firm-specific risk.

The model exhibits the well known result that the cost of the compensation to the company (its market value) overstates the value that executives place on it, due to restrictions faced by executives in trading their own company's stock. However unlike many one asset models, we can examine the effect of market and firm-specific risks separately on value. By distinguishing between the two types of risk, it is clear that the drop in value compared with Black Scholes or company cost is due to the executive not being able to trade away firm-specific risk. There are two opposing forces at play. On the one hand, the board aims to tie managers reward to company performance. However, this exposure reduces the private value they place on the stock based compensation as they are risk averse and exposed to firm-specific risk.

Our findings have a number of implications for future research. Firstly, empirical studies often rely on the Black Scholes option value to represent both company cost and value to the executive. This paper (and many before it) show that this is not a good approximation for the value to the executive, and models with more realistic assumptions are required. Similarly, studies relying on the Black Scholes delta to represent incentives are introducing a substantial bias.

Secondly, one of the reasons options are granted to executives is to encourage them to take on more risky projects. However, our model shows that this will not always be achieved due to their risk aversion towards unhedgeable firm-specific risk.

A third area where our results have implications is the optimal form of compensation. Economic justification is given suggesting companies in the future should compensate with stock rather than options. This will be important if the likely scenario of option expensing eventuates in the US. Studies examining the compensation mix before and after these accounting changes would be able to supply evidence (for or against) our hypothesis. Our model highlighted that for companies with very low stock price volatilities, granting stock may not be optimal. This could be explored further and empirically tested in the future. The model also concludes indexed options are not optimal, so this could also be tested after the accounting changes.

Following on from this, the paper also makes conclusions about the relationship between risks and optimal incentives. We find optimal incentives decrease with firm-specific risk but may increase or decrease with market risk, depending on which parameters are held fixed. Empirical testing of these conclusions would involve an extension of the work of Jin (2002) to cover non-linear payoffs. (check!)

A future direction for related research would be to incorporate American style exercise into the framework of the paper. Most executive stock options are American, with an initial vesting period when they cannot be exercised. It may be possible to examine the exercise decision in this continuous time utility set-up.

9 Appendix

9.1 Derivation of Value Function

From (3) the value function is written as

$$\begin{aligned} V(t, X_t, S_t; \lambda) &= \sup_{(\theta_u)_{u\geq t}} \mathbb{E}_t[U(X_T + \lambda(S_T - K)^+)] \\ &= -\frac{1}{\gamma} e^{-\gamma X_t e^{r(T-t)}} g(T-t, \log S_t) \end{aligned}$$

where $g(0, \log s) = e^{\lambda\gamma(S-K)^+}$ and for $U(x) = -\frac{1}{\gamma}e^{-\gamma x}$. We know V is a supermartingale under any strategy θ and a martingale under the optimal strategy. Applying Ito's formula to V and optimising over values of θ gives

$$-\dot{g} + (\nu - \frac{1}{2}\eta^2)g' + \frac{1}{2}\eta^2 g'' - \frac{(\sigma\rho\eta g' + (\mu - r)g)^2}{2\sigma^2 g} = 0 \tag{12}$$

a non-linear pde. The optimal strategy is given by

$$\theta^* = \frac{\mu - r}{\gamma\sigma^2 e^{r(T-t)}} + \frac{g'\eta\rho}{\gamma g\sigma e^{r(T-t)}}. \tag{13}$$

Using the approach in Henderson and Hobson (2002a), we consider the transformation $g(\tau, z) = e^{\alpha\tau}G(\tau, z + (\delta - \eta^2/2)\tau)^b$. If we take $b = (1-\rho^2)^{-1}$, $\alpha = -\frac{(\mu-r)^2}{2\sigma^2}$, and $\delta = r + \eta(\frac{\nu - r}{\eta} - \frac{\rho(\mu-r)}{\sigma})$ we get the heat equation

$$\dot{G} = \frac{1}{2}\eta^2 G''$$

subject to $G(0, z) = e^{-\gamma\lambda(e^z - K)^+/b}$.

The solution is

$$G(\tau, x) = \mathbb{E}G(0, x + \eta Z_\tau)$$

for Brownian motion Z and hence

$$g(T-t, \log S_t) = e^{-(\mu-r)^2(T-t)/2\sigma^2}[\mathbb{E}^0 e^{-\lambda\gamma(1-\rho^2)(S_T-K)^+}]^{(1-\rho^2)^{-1}} \tag{14}$$

giving (4).

We define $W(t, S) = \mathbb{E}^0 e^{-\lambda\gamma(1-\rho^2)(S_T-K)^+}$ and hence using (14) obtain the relationship

$$g(T-t, \log S) = e^{-(\mu-r)^2(T-t)/2\sigma^2} W(t, S)^{(1-\rho^2)^{-1}}.$$

Differentiating and substituting into the pde (12), gives the following pde for W

$$\dot{W} + \frac{1}{2}\eta^2 S^2 W_{ss} + SW_s\delta = 0 \tag{15}$$

with $W(T, S) = e^{-\lambda\gamma(1-\rho^2)(S-K)^+}$.

9.2 Calculation of Incentives

We begin with (8) and differentiate to obtain (10):

$$\frac{\partial}{\partial S}p^e(t,s) = -\frac{e^{-r(T-t)}}{\gamma(1-\rho^2)}\frac{W_s(t,s)}{W(t,s)}.$$

Using the pde (15) for W we can develop an expression for W_s. Setting $z(t,s) = W_s(t,s)$ we see z satisfies

$$\dot{z} + \tfrac{1}{2}\eta^2 S^2 z_{ss} + S z_s(\eta^2 + \delta) + \delta z = 0$$

with $z(T,s) = -\lambda\gamma(1-\rho^2)I_{(s\geq K)}e^{-\lambda\gamma(1-\rho^2)(s-K)^+}$. Define $\frac{d\mathbb{P}^z}{d\mathbb{P}^0} = e^{\eta\bar{B}^0 - \frac{1}{2}\eta^2 t}$ where $\bar{B}^0 = \rho B^0 + \sqrt{1-\rho^2}W$ is $\mathbb{P}^0$ Brownian motion and $W^z = \bar{B}^0 - \eta t$. Then S is an exponential Brownian motion with volatility η and drift $(\delta + \eta^2)$ under $\mathbb{P}^z$. Now we may write (using the Feynman-Kac representation, see Karatzas and Shreve (1991))

$$z(t,s) = W_s(t,s) = e^{\delta(T-t)}\mathbb{E}_t^{\mathbb{P}^z}(-\lambda\gamma(1-\rho^2)I_{(S_T\geq K)}e^{-\lambda\gamma(1-\rho^2)(S_T-K)^+}) \tag{16}$$

and thus combining (16) with (10), we have an explicit expression for incentives. Note that as $W_s < 0$, $\frac{\partial}{\partial S}p^e$ is always greater than zero.

9.3 Optimal Strategy

From (13) we have an expression for θ^* in terms of g. Using the above relationship between g and W, we see $g'/g = SW_s/(W(1-\rho^2))$ and

$$\theta^* = \frac{\mu - r}{\gamma\sigma^2 e^{r(T-t)}} + \frac{\eta\rho S W_s}{\gamma e^{r(T-t)}\sigma W(1-\rho^2)}. \tag{17}$$

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	$T = 3$				$T = 5$				$T = 10$			
ρ	q_1	q_2	q_3	q_4	q_1	q_2	q_3	q_4	q_1	q_2	q_3	q_4
0.25	0.92	0.87	0.81	0.55	0.85	0.79	0.71	0.43	0.68	0.6	0.5	0.26
0.5	0.93	0.90	0.84	0.60	0.88	0.82	0.75	0.48	0.72	0.64	0.55	0.30
0.75	0.96	0.93	0.90	0.71	0.92	0.88	0.83	0.60	0.80	0.74	0.65	0.39
0.90	0.98	0.97	0.95	0.85	0.96	0.94	0.91	0.76	0.89	0.85	0.78	0.56
0.95	0.99	0.98	0.97	0.91	0.98	0.97	0.95	0.85	0.93	0.90	0.86	0.68
1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 1 - Ratio of executive value to company cost for at-the-money option compensation. We vary expiry (in years), $q = \lambda\gamma$, and correlation between stock and market. Parameters: $q_1 = 1/1000$, $q_2 = 4/2500$, $q_3 = 4/1500, q_4 = 1/100$. We take volatilities and risk premium from the CRSP stock price data (1998) as used in Meulbroek (2001a). Take $\sigma = 0.22$ the volatility of the NYSE index, $\eta = 0.45$ the average volatility of NYSE firms, risk premium $\mu - r = 0.075$ and $S_0 = K = 100$.

	$T = 3$				$T = 5$				$T = 10$			
ρ	q_1	q_2	q_3	q_4	q_1	q_2	q_3	q_4	q_1	q_2	q_3	q_4
0.25	0.96	0.94	0.9	0.76	0.92	0.88	0.83	0.63	0.78	0.72	0.64	0.42
0.5	0.97	0.95	0.92	0.79	0.93	0.90	0.85	0.67	0.81	0.75	0.67	0.46
0.75	0.98	0.97	0.95	0.85	0.96	0.94	0.90	0.75	0.86	0.81	0.75	0.55
0.90	0.99	0.99	0.98	0.92	0.98	0.97	0.95	0.86	0.92	0.89	0.85	0.68
0.95	0.99	0.99	0.99	0.96	0.99	0.98	0.97	0.91	0.95	0.94	0.90	0.77
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Table 2: Ratio of executive value to company cost for stock compensation. Parameters as for Table 1, except $K = 0$.

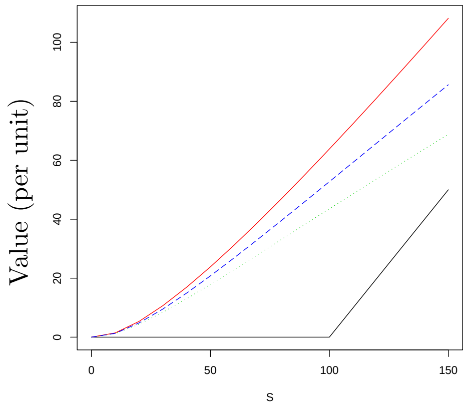

Figure 1: *Executive Value Lines for option compensation, expressed as Value per unit option. The top line is the risk neutral value or company cost and the lowest line is the option payoff. The two inner lines represent the value to the executive using the model, the higher line uses* $q = \lambda\gamma = 1/1000$ *whilst the lower line uses* $q = \lambda\gamma = 4/1500$. *Parameters are* $T = 10, K = 100, \rho = 0.80, \mu = 0.10, \eta = 0.45, \sigma = 0.35, r = 0.05,$ *and* $\delta = r$.

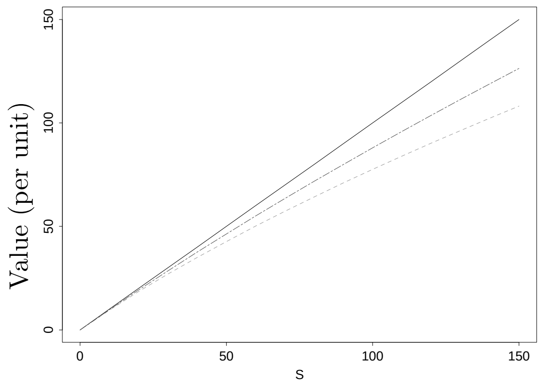

Figure 2: *Executive Value Lines for Stock compensation expressed as Value per unit option. The top line is the company cost and the two lower lines represent the value to the executive under the model. The higher line uses* $q = \lambda\gamma = 1/1000$ *whilst the lower line uses* $q = \lambda\gamma = 4/1500$. *Parameters are the same as those used in Figure 1, with* $K = 0$.

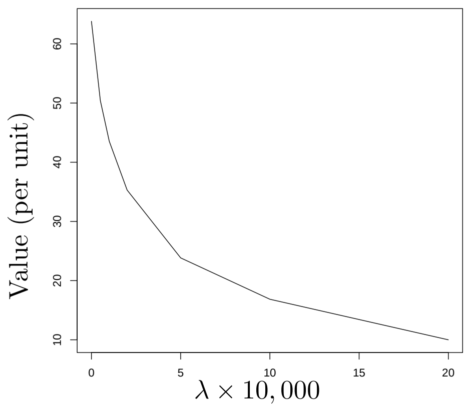

Figure 3: *Per unit value for at-the-money option compensation as a function of the number of units of option held. Risk aversion is fixed to be* $\gamma = 4/1.5 \times 10^7$. *Parameters are* $S_0 = K = 100,\ T = 10, \rho = 0.80, \mu = 0.10, \eta = 0.45, \sigma = 0.35, r = 0.05,\ \delta = r$.

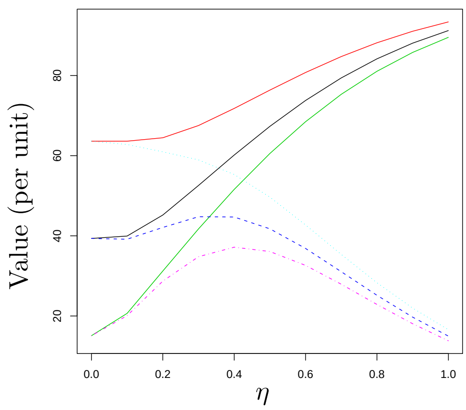

Figure 4: *Effect of volatility on value, fixing* $\rho = 0.8$. *Graph for three options: the three increasing solid lines are, from highest to lowest, in-the-money* ($K = 60$), *at-the-money* ($K = 100$), *out-of-the-money* ($K = 140$) *Black Scholes values. The executive value for the same three options are represented by the dashed lines, with the same ordering highest to lowest. Risk aversion* $q = \lambda\gamma = 4/1500$ *is used and* $S_0 = 100$. *Other parameters are those used in Figure 1.*

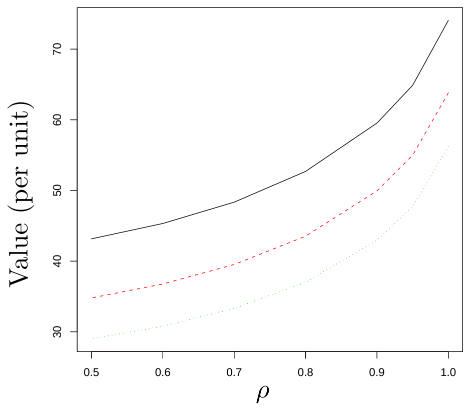

Figure 5: *Effect of correlation on value to the executive, fixing* $\eta = 0.45$*. The three lines represent in-the-money (*$S_0 = 100, K = 60$*), at-the-money (*$S_0 = 100 = K$*) and out-of-the-money (*$S_0 = 100, K = 140$*) options from highest to lowest. Each uses* $q = \lambda\gamma = 4/1500$*. Parameters are* $T = 10, \mu = 0.10, \sigma = 0.35, r = 0.05,\ \delta = r$*.*

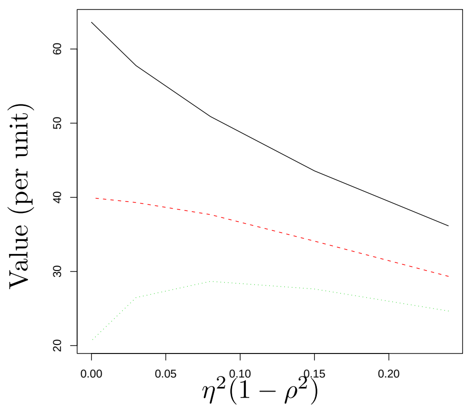

Figure 6: *Effect of firm-specific risk on option value, fixing market risk. The three lines represent in-the-money (*$S_0 = 100, K = 60$*), at-the-money (*$S_0 = 100 = K$*) and out-of-the-money (*$S_0 = 100, K = 140$*) options from highest to lowest. Each uses* $q = \lambda\gamma = 4/1500$*. Other parameters are those used in Figure 1.*

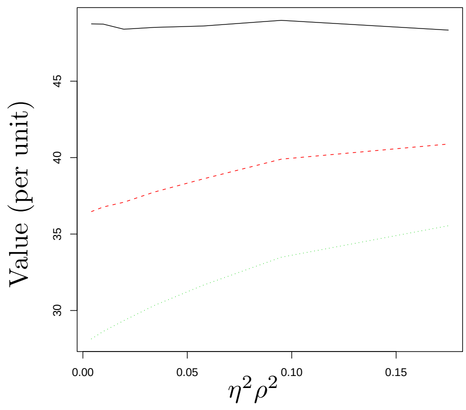

Figure 7: *Effect of market risk on option value, fixing firm- specific risk. The three lines represent in-the-money ($S_0 = 100, K = 60$), at-the-money ($S_0 = 100 = K$) and out-of-the-money ($S_0 = 100, K = 140$) options from highest to lowest. Each uses $q = \lambda\gamma = 4/1500$. Other parameters are those used in Figure 1.*

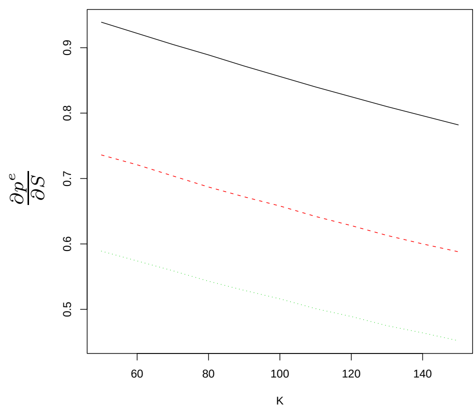

Figure 8: *Option Incentives (the change in the executive value (per option)p^e/λ for a 1 unit change in the stock price). The solid line corresponds to the risk neutral delta whilst the two lower lines use the utility model with $q = \lambda\gamma = 1/1000$ and $q = \lambda\gamma = 4/1500$ (lowest line). Parameters are those used in Figure 1 with $S = 100$.*

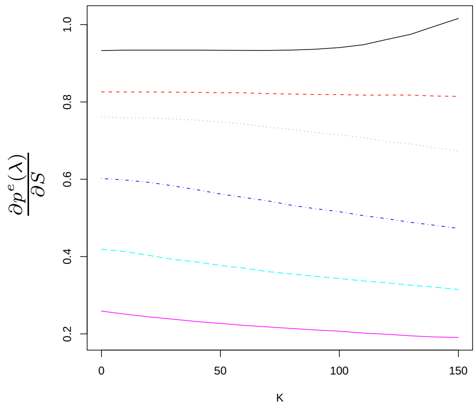

Figure 9: *Incentives - Stock vs Options. Each line represents a different value for volatility η. From highest to lowest, $\eta = 0.10, 0.18, 0.27, 0.45, 0.62, 0.8$. Each uses $\gamma = 4/1500$, $\rho = 0.8$, $S_0 = 100$ and other parameters same as Figure 1.*

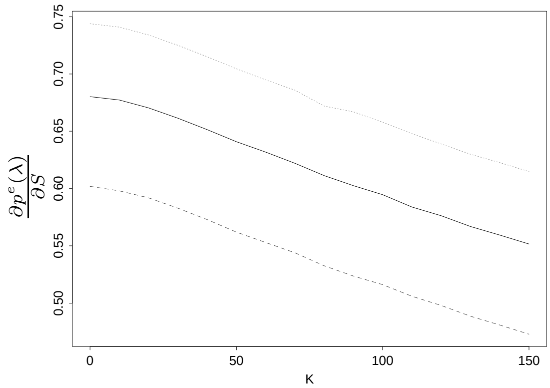

Figure 10: *Incentives - Stock vs Options, keeping $\eta = 0.45$ and $\rho = 0.8$. From highest to lowest, $\gamma = 1/1000$, $\gamma = 4/2500$ and $\gamma = 4/1500$. $S_0 = 100$ and other parameters same as Figure 1.*

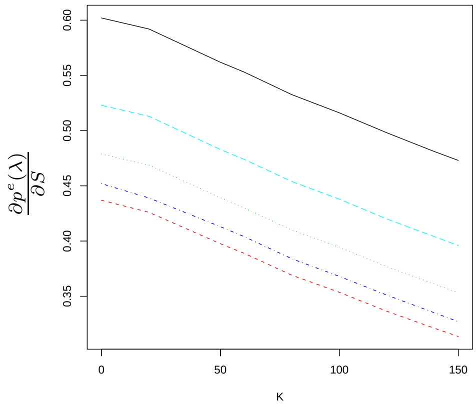

Figure 11: *Incentives - Stock vs Options, fixing* $\eta = 0.45$ *and varying correlation. From highest to lowest the lines use correlations of 0.8, 0.65, 0.5, 0.35, 0.2. Each uses* $\gamma = 4/1500$, $S_0 = 100$ *and other parameters same as Figure 1.*

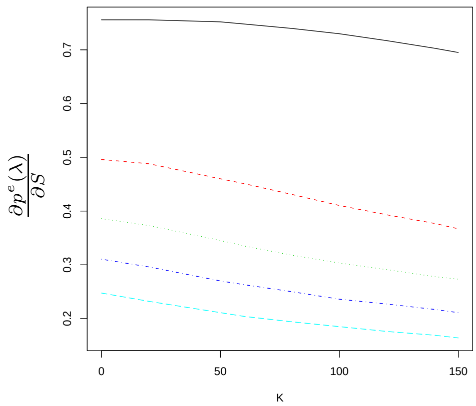

Figure 12: *Incentives - Stock vs Options, fixing market risk and varying firm-specific risk* $\eta^2(1-\rho^2)$. *From highest to lowest, the lines use values for* $\eta^2(1-\rho^2)$ *of 0.03,0.15,0.24, 0.33, 0.43. Each uses* $\gamma = 4/1500$, $S_0 = 100$ *and other parameters same as Figure 1.*

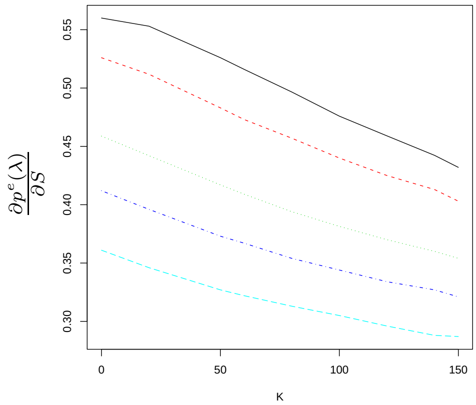

Figure 13: *Incentives - Stock vs Options, keeping firm-specific risk* $\eta^2(1-\rho^2)$ *fixed and varying market risk* $\eta^2\rho^2$*. From highest to lowest, the lines use* $\eta^2\rho^2$ *values of 0.05,0.175,0.31,0.42,0.54. Each uses* $\gamma = 4/1500$*,* $S_0 = 100$ *and other parameters same as Figure 1.*