Interbank Competition with Costly Screening*

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Abstract

We analyse credit market equilibrium when banks screen loan applicants. When banks have a convex cost function of screening, a pure strategy equilibrium exists where banks optimally set interest rates at the same level as their competitors. This result complements Broecker's (1990) analysis, where he demonstrates that no pure strategy equilibrium exists when banks have zero screening costs. In our set up we show that interest rate on loans are largely independent of marginal costs, a feature consistent with the extant empirical evidence. In equilibrium, banks make positive profits in our model in spite of the threat of entry by inactive banks. Moreover, an increase in the number of active banks increases credit risk and so does not improve credit market efficiency: this point has important regulatory implications. Finally, we extend our analysis to the case where banks have differing screening abilities.

1. Introduction

This paper explores the nature of equilibrium in the credit market under asymmetric information when banks are able to screen their customers. Information asymmetries are central to credit markets. There is nowadays a basic agreement among academics that banks exist because they monitor firms (Diamond, 1984; Gale and Hellwig, 1985; Holmström and Tirole, 1993). Hence, models of the credit market should incorporate a screening-monitoring role for banks. It is widely appreciated that introducing asymmetric information into models of the credit market yields equilibria with specific features. Contributions like those of Stiglitz and Weiss (1981), Sharpe (1990) and Rajan (1992) exhibit equilibrium phenomena such as credit rationing and *ex post* monopoly of information, which are absent from standard delivery-versus-payment markets.

Still, credit market models seem seldom to acknowledge the importance of screening. This creates an apparent schism between models of bank/firm contracts, where screening is central, and models of the credit market, where screening is typically non-existent. Fortunately, Broecker's (1990) model provides a bridge between the two by exploring credit market equilibrium when banks screen firms. However, in Broecker's equilibrium banks use mixed strategies to assign interest rates for loans. This is an unattractive feature for two reasons. First, it yields no empirical implications; second, it makes it difficult to study the comparative statics or the welfare properties of the model. Hence, although Broecker's elegant contribution is a step in the right direction, we think that it is worth devoting some effort to extending it.

The intuition behind Broecker's contribution is straightforward. When banks screen loan applicants, the order in which firms approach banks is important. Rational firms will apply firstly to the banks which post the lowest interest rates. As a result, a bank may ensure by lowering its interest rate that it has first choice from the population of loan applicants. So the bank simultaneously alters the price at which it lends, and the marginal cost of lending. Hence a bank may be able to profit by undercutting its competitors. Setting price equal to marginal cost in the traditional way may therefore not yield an equilibrium. This is at odds with standard microeconomic theory and hence opens new avenues for exploration. This is precisely the object of our paper.

Our work is primarily motivated by the divide between the theoretical justification for banks and current models of the credit market. In spanning the divide, we wish to address three specific points.

Firstly, mixed strategy equilibria are rather unsatisfying in the context of price competition. Once interest rates realize, banks would always want to change them immediately. For example, the bank with the lowest interest rate would always prefer to increase it. Moreover, the empirical evidence does not support the conclusion than bank loan rates move erratically all of the time. Finally, it is obviously impossiblee to derive empirical predictions from mixed strategy equilibria.

Secondly, we wish to understand whether the credit market should be thought of as a "natural monopoly". If so, credit market equilibria should be characterised by equilibrium profits. This question is related to the relationship between competition and financial stability, which has been investigated in models where banks have the choice of their riskiness levels (as for example in Matutes and Vives, 2000), but never in a set-up where the level of screening and of credit risk in the banking industry is endogenous because it depends on the number of banks an applicant is able to visit.

Finally, we wish to explore the potential for equilibrium credit rationing when the cost of bank screening is explicitly modeled.

Clearly, the answers to these points will inform regulatory attitudes towards credit market entry, and hence will have important policy implications. We address these issues in a model of a credit market in which banks face an adverse selection problem due to heterogeneity in firm repayment probabilities. We assume that banks have to rely upon active monitoring when responding to a firm's application for a loan. The monitoring technology is imperfect and independent across banks. Banks must account for the fact that their loan applicants may have already been rejected by other banks. In particular, a single bank offering the lowest interest rates will on average attract better applicants than banks charging higher interest rates.

We extend Broecker's framework by assuming that banks incur a screening cost which is increasing and convex in the number of applicants which they screen. A simple example would be a capacity constraint which renders it very costly (or just simply impossible) to screen all applicants. Our main result is that pure strategy equilibria exist for sufficiently convex screening costs. While this is interesting, it is perhaps unsurprising. With convex screening costs, undercutting one's competitors in order to gain market share and an improved applicant pool may be discouraged by the consequential increase in screening costs. It is less clearly obvious however that the equilibrium is characterised by indeterminancy, as banks can coordinaate on a number of different interest rates. This is an interesting feature of our model, as it implies that equilibrium is largely independent of marginal costs. This point is supported by empirical researcg: see, among others, Hannan and Berger (1991) and Mester (1994).

It is also worth emphasising the existence in our model of equilibrium profits for incumbent banks. This contrasts with Broecker's result that, with sufficiently many banks, the mixed equilibrium yields zero profits for all banks. Our model has a number of free entry equilibria, characterised by differing numbers of active banks. The more banks which decide to be active in equilibrium, the lower the average quality of borrowers and the higher the equilibrium interest rate charged by all of them. We show that welfare is decreasing in the number of active banks. The reason for this is that a bad project has more chance of securing a loan when there are more banks. Hence an increase in the number of active banks raises the equilibrium level of credit risk. Hence, our paper provides a new argument in support of the common statement that regulators should restrict entry to the banking sector. Moreover, we show in section 6 that a mild equilibrium refinement predicts that the maximal number of banks enters. This adds additional force to our policy suggestion.

Finally, we introduce some element of natural oligopoly to our model by examining the case where banks differ in their screening ability. We show that an inferior bank suffers losses whenever a superior bank charges the same or a lower interest rate. When interest rates are the same, the reason is that high quality borrowers will in the first instance approach the lender with the superior screening technology while low quality borrowers will approach the other bank.

Hence the bank with a comparatively weaker screening technology will also face an inferior pool of borrowers. When interest rates differ the bank with the weaker screening technology will face a weaker pool of borrowers because all have already been rejected by the bank with the stronger technology. We characterise the equilibria in section 7.

Our work is related to a number of papers. Firstly, as discussed at length above, Broecker (1990) models price competition amongst banks with a zero screening cost, and shows that the only equilibria are in mixed strategies. His results have been widely discussed in the lending literature (von Thadden, 2004) and extended by Dell'Ariccia, Friedman, and Marquez (1999) to analyze entry in the banking industry and by Marquez (2002) to explore the effect of increased competition on the quality of credit. Pure strategy equilibria obtain in our model for a sufficiently convex screening cost function. This result is slightly related to Riordan (1993), who proves the existence of pure strategy equilibrium when banks receive signals from a continuous distribution and interest rates are charged conditional on the signal (which allows price discrimination).

Secondly, our results bear a direct relationship to the substantial literature which has argued that welfare is enhanced by allowing banks to obtain non-competitive rents, since this provides banks with the right incentives. This could be a result of horizontal differentiation (Chiappori *et al.*, 1995, and Matutes and Vives, 2000), because banks choose then to reduce the risk of the assets (Suarez, 1998, Matutes and Vives, 2000), or because they extract ex post rents from their lending relationships (Sharpe, 1990, Rajan, 1992, von Thadden, 1994). The possibility that competition may be inefficient is also acknowledged by Petersen and Rajan (1995), as in their model a more competitive banking market is not necessarily a more efficient one because competition makes it harder for young firms to build banking relationships and hence to obtain a loan.

In our model, proposition 1 shows that increased bank competition may be inefficient for an entirely different reason: namely, that more projects will be funded in equilibrium when there are more banks. As a result, credit risk will increase and so too will interest rates. Empirical support for these findings is provided by Shaffer (1998), who finds that *"Among mature banks, those operating in less concentrated banking markets experience significantly higher chargeoff rates for commercial loans and for total loans"* (p.389). Thus, the policy implication of our results is that it might be efficient to restrict entry into the credit market, as this reduces the number of chances for poor borrower to obtain funds.

Emons' (2001) work on house insurance is also related to our paper. He presents a model in which house owners can apply for insurance from multiple providers withou investing in safety measures. As a result, both prices and the number of damage claims are higher in competitive markets than in a monopolistic setting. In Switzerland and Germany some regions have a monopolistic insurance market, while others are competitive. Emons provides evidence from these markets which is consistent with his theory.

We present our analysis as follows. Section 2 describes a model of a credit market in which loan applications and loan screening are costly. Section 3 demonstrates the existence in our model of pure strategy equilibria for an exogenously given number of banks; section 4 provides a numerical example of the equilibrium, and comparative statics are derived in section 5. In sections 6 and 7 we analyze extensions to the respective cases with free entry and heterogeneous bank monitoring skills. Section 8 concludes.

2. The Model

Our model is a modified version of Broecker (1990). We consider a credit market with a continuum $[0, 1]$ of risk-neutral firms, each of which wishes to borrow \$1 to invest in a project which will return \$$X$ if it succeeds and \$0 if it fails. The success probability of type a firms is p_a and of type b firms is p_b, where $0 \leq p_a < p_b \leq 1$. A firm's type is its private information. The proportion $l \in (0, 1)$ of type a firms is common knowledge.

There are $N \geq 2$ risk neutral banks in the market which can raise any amount of funds at a unit interest rate. If a bank lends at an interest rate r to α type a firms and to β type b firms then its profit will be $\alpha\left(p_a \min\left(X, r\right) - 1\right) + \beta\left(p_b \min\left(X, r\right) - 1\right)$. Banks have access to an imperfect monitoring technology. The technology assigns each loan applicant to a category $C \in \{A, B\}$. We define

$$q\left(C|c\right) = P\left\{\text{Applicant is assigned to } C \in \{A, B\} \,|\text{Applicant's type is } c \in \{a, b\}\right\}.$$

We write q_c for $q\left(A|c\right)$ so that:

$$\begin{aligned} q\left(A|a\right) &= q_a, \qquad q\left(B|a\right) = 1 - q_a, \\ q\left(A|b\right) &= q_b, \qquad q\left(B|b\right) = 1 - q_b. \end{aligned}$$

We do not require, and we do not make, the simplifying assumption $q_a = 1 - q_b$ characteristic of a completely symmetric test.

We adopt Broecker's assumption that screening is informative but imperfect:

$$0 < q_b < q_a < 1. \tag{A1}$$

It is an easy matter to show as Broecker does that this implies that

$$\bar{p}_A < \bar{p} < \bar{p}_B,$$

where $\bar{p}_A$ and $\bar{p}_B$ are respectively the average success probability when there is only one bank of category A and B firms, and $\bar{p}$ is the average success probability of all firms. We adopt the following additional assumption from Broecker's paper, which implies that at least one bank will not make losses if it charges $r = X$ to category B firms:

$$\bar{p}_B X - 1 > 0. \tag{A2}$$

Broecker also assumes that $\bar{p}_A X - 1 < 0$, so that banks do not wish to lend to category A firms. We will make the slightly stronger assumption that banks do not want to lend to firms that are not screened at all. This assumption thus emphasizes the role of banks in credit markets:

$$\bar{p} X - 1 < 0. \tag{A3}$$

Broecker assumes that applying for a loan is costless, and that banks incur no screening costs. We depart from his model by allowing for application and screening costs:

$$\text{There is a cost } \varepsilon > 0 \text{ of applying for a loan;} \tag{A4$'$}$$

$$\text{The cost of screening } x \text{ applicants is } C(x), \text{ where } C(0) = C'(0) = 0, C'' \geq 0 \text{ and } C(1) < \bar{p}_B X - 1 \tag{A5$'$}$$

We introduce the ε cost of loan application so as to ensure that firms will prefer to apply sequentially for loans: this avoids duplication of screening costs. The upper bound on the maximal screening cost is sufficiently low to ensure that a single bank will profit from making loans to firms which pass the screening test.

The loan application process works as follows. Assumptions (A3) and (A1) together imply that banks will not lend to category A firms and so in the first stage banks simultaneously announce whether they wish to provide credit and, if they do so, the rate $r \in [0, X]$ at which they will lend to type B firms. Firms then make sequential applications for loans.[1]

We define social welfare in this model to be the total present value of all financed projects, less the total screening costs:

$$W(L) \equiv l\left(1 - q_a^L\right)(p_a X - 1) + (1 - l)\left(1 - q_b^L\right)(p_b X - 1) - LC(f(L)). \tag{1}$$

[1] In Broecker's original paper firms make simultaneous loan applications: our modification of his game is rendered necessary by assumption (A4$'$).

Broecker shows (Proposition 2.1) that under assumptions (A1) – (A3) and with no costs of loan application or of screening, this one-stage game has no equilibrium in pure strategies. In the following section we demonstrate that for appropriate functions $C(\cdot)$, the additional assumption (A4′) guaranteeing sequential loan applications is sufficient to overturn this result.

3. Existence of a Pure Strategy Equilibria

In this section we study the case were the number N of banks is fixed exogenously. We leave to section 6 the analysis of free entry.

Our analysis runs as follows. We firstly compute equilibrium bank profits, and also the profits which a bank obtains by deviating to a higher interest rate. In lemma 2 we demonstrate that no upward deviation will occur. This is because a higher interest rate will attract worse quality creditors. Under assumption (A3), which states that unscreened projects have negative present value, this lower quality is sufficient to discourage deviation. We then show that when screening costs are convex, the minimum bank breal-even interest rate is locally decreasing with the number of banks around $N = 1$. We establish in theorem 1 that this is sufficient to render undercutting unattractive and hence to prove the existence of a pure strategy equilibrium.

To compute the banks' profits first need to determine the number of firms which they screen in equilibrium, and the number of clients of type a and type b to which they will lend money. This is the object of the following lemma.

Lemma 1 *Consider a situation in which L banks offer the lowest interest rate. Let $f(L)$ be the number of firms screened by any of these banks, $h_k(L)$ the number of clients of type k $(k = a, b)$. Then*

$$\begin{aligned} f(L) &= \frac{1}{L}\left\{ l\left(\frac{1-q_a^L}{1-q_a}\right) + (1-l)\left(\frac{1-q_b^L}{1-q_b}\right)\right\}. \\ h_a(L) &= \frac{l(1-q_a^L)}{L} \\ h_a(L) &= \frac{(1-l)(1-q_b^L)}{L} \end{aligned}$$

Proof. The number of type a firms that are screened precisely k times $(k < L)$ is $l(1-q_a)q_a^{k-1}$ (as they are assigned to category A the first $k-1$ times, and then to category B). The number of type a firms that are screened exactly L times is lq_a^{L-1} (they are assigned the first $L-1$ times to category A). So the expected number of screenings for type a firms equals

$$l(1-q_a)\sum_{k=1}^{L-1} kq_a^{k-1} + lLq_a^{L-1} = l\frac{1-q_a^L}{1-q_a}$$

Similarly for type b firms the expected number of screenings equals

$$(1-l)(1-q_b)\sum_{k=1}^{L-1} kq_b^{k-1} + (1-l)Lq_b^{L-1} = (1-l)\frac{1-q_b^L}{1-q_b}$$

Given the symmetry between banks, the expected number of screenings equals $f(L)$.

Of all the applicants screened by a particular, bank only those who are assigned to category B receive a loan. This means that each bank has $l(1-q_a^L)/L$ clients of type a, and $(1-l)(1-q_b^L)/L$ clients of type b. □

Now let $L \leq N$ be the number of banks which are active in equilibrium. Using the notation of lemma 1, net of screening costs each active bank earns an expected profit $U(r, L)$ when they all charge an interest rate r:

$$\begin{aligned} U(r,L) &= h_a(L)(p_a r - 1) + h_b(L)(p_b r - 1) - C(f(L)) \\ &= \frac{l(1-q_a^L)}{L}(p_a r - 1) + \frac{(1-l)(1-q_b^L)}{L}(p_b r - 1) - C(f(L)). \end{aligned}$$

Suppose that a bank deviates and charges an interest rate $r' > r$, while its $L-1$ competitors continue to charge r. The deviating bank will screen the $lq_a^{L-1} + (1-l)q_b^{L-1}$ firms which are not awarded loans at the other banks. On average, it will give loans to $lq_a^{L-1}(1-q_a)$ firms of type a, and $(1-l)q_b^{L-1}(1-q_b)$ firms of type b. Its profits will therefore be given by the following expression:

$$V(r', L) = lq_a^{L-1}(1-q_a)(p_a r' - 1) + (1-l)q_b^{L-1}(1-q_b)(p_b r' - 1) - C(lq_a^{L-1} + (1-l)q_b^{L-1}).$$

We now define two types of break-even interest rate:

Definition 1

1. *The lowest interest rate at which the L banks charging it make nonnegative profits is the solution $r_0(L)$ of*

$$U(r_0, L) = 0\text{: } r_0(L) = \frac{l(1-q_a^L) + (1-l)(1-q_b^L) + LC(f(L))}{lp_a(1-q_a^L) + p_b(1-l)(1-q_b^L)}.$$

2. *When $L-1$ banks charge interest rate r, the lowest rate $r' > r$ at which another bank could extend loans and break even is*

$$r_0'(L) = \frac{lq_a^{L-1}(1-q_a) + (1-l)q_b^{L-1}(1-q_b) + C(lq_a^{L-1} + (1-l)q_b^{L-1})}{lq_a^{L-1}(1-q_a)p_a + (1-l)q_b^{L-1}(1-q_b)p_b}.$$

We now formulate an assumption regarding the quality of firms which receive opposing views from two screenings. Inequality (ES) below states that banks are unwilling to lend to such firms,[2] so that a bank will discard a positive result from its own screening if it knows that a competitor has already received a negative result for the same firm.

$$\frac{lq_a(1-q_a)p_a + (1-l)q_b(1-q_b)p_b}{lq_a(1-q_a) + (1-l)q_b(1-q_b)} X < 1. \tag{ES}$$

With a symmetric screening technology such that $1 > q_a = 1 - q_b > 1/2$, obtaining two opposite results is equivalent to having no test result at all. In this case, the left hand side of the inequality reduces to $\bar{p}X$ and the assumption (ES) is equivalent to assumption (A3). In general, inequality (ES) is satisfied when banks are relatively better at identifying type b than type a firms. It certainly holds in the limiting case where $1 > q_a > q_b = 0$ and type b firms are perfectly identified since in this case (ES) reduces to $p_a X < 1$. On the other hand, it is not satisfied when $1 = q_a > q_b > 0$ so that the screening technology perfectly identifies type a firms, since (ES) then reduces to $p_b X < 1$. Note that (ES) holds independently of L, a point that will greatly simplify the analysis.

Lemma 2 shows that under (ES), no bank has an incentive to charge a rate higher than its competitors.

Lemma 2 *Suppose that (ES) holds. Then* $r_0'(L) > X$ *(equivalently,* $V(X, L) < 0$*) for all* $L \geq 1$.

Proof. Clearly,

$$r_0'(L) \geq \frac{lq_a^L(1-q_a) + (1-l)q_b^L(1-q_b)}{lq_a^L(1-q_a)p_a + (1-l)q_b^L(1-q_b)p_b} \geq \frac{lq_a(1-q_a) + (1-l)q_b(1-q_b)}{lq_a(1-q_a)p_a + (1-l)q_b(1-q_b)p_b} > X,$$

The first inequality follows from $C(\cdot) \geq 0$, the second inequality holds since

$$\frac{d}{dL}\left\{\frac{lq_a^L(1-q_a) + (1-l)q_b^L(1-q_b)}{lq_a^L(1-q_a)p_a + (1-l)q_b^L(1-q_b)p_b}\right\} = \frac{lq_a^L q_b^L\left(\ln q_a - \ln q_b\right)(1-l)(1-q_a)(1-q_b)(p_b - p_a)}{(lq_a^L(1-q_a)p_a + (1-l)q_b^L(1-q_b)p_b)^2} > 0. \tag{2}$$

so that $\frac{lq_a^L(1-q_a)+(1-l)q_b^L(1-q_b)}{lq_a^L(1-q_a)p_a+(1-l)q_b^L(1-q_b)p_b}$ is an increasing function of L, and the third follows directly from (ES). □

Equation (2) implies that when as in Broecker's paper, $C(x) \equiv 0$, the minimum breakeven interest rate $r_0(L)$ is increasing in the number of banks, L. However, for sufficiently convex cost functions this is untrue. In particular, $r_0(1) > r_0(2)$ if and only if

$$\frac{l(1-q_a) + (1-l)(1-q_b) + C(1)}{lp_a(1-q_a) + p_b(1-l)(1-q_b)} > \frac{l(1-q_a^2) + (1-l)(1-q_b^2) + 2C(f(2))}{lp_a(1-q_a^2) + p_b(1-l)(1-q_b^2)},$$

[2] This assumption was made by Broecker (1990) in the second part of his paper to show existence of a pure strategy equilibrium when banks can withdraw from the market after observing all interest rates offered.

which is equivalent to the following *convexity condition*:

$$\frac{lp_a(1-q_a)[(1+q_a)C(1)-2C(f(2))]}{lp_a(1-q_a)+(1-l)p_b(1-q_b)}+\frac{(1-l)p_b(1-q_b)[(1+q_b)C(1)-2C(f(2))]}{lp_a(1-q_a)+(1-l)p_b(1-q_b)}$$
$$> l\,(1-q_a)\,(1-q_b)\,(q_a-q_b)\,(1-l)\,(p_b-p_a)\,/(lp_a(1-q_a)+(1-l)p_b(1-q_b)). \quad \text{(C)}$$

Note that the right-hand side of (C) is strictly positive and independent of $C(\cdot)$. The left-hand site of the inequality is a weighted average of $(1+q_a)C(1)-2C(f(2))$ and $(1+q_b)C(1)-2C(f(2))$. If the weights used were l and $(1-l)$ respectively, this average would be exactly equal to zero in case of linear cost function $C(\cdot)$ (since $2f(2)=l(1+q_a)+(1-l)(1+q_b)$), but in the case of a strictly convex cost function the average is strictly positive.

In the limit case of perfect screening, with $q_a=1$ and $q_b=0$, condition (C) boils down to $C(1)>2C(f(2))$. Since $f(2)=\frac{1+l}{2}$, this reduces to $C(1)>2C(\frac{1+l}{2})$, stating again that linear costs would not satisfy the condition, and that a degree of convexity is required which is increasing in l.

We now consider the conditions for the existence of a symmetric pure strategy Nash equilibrium in an N bank economy. Since N is exogenously given, for large enough N banks will not earn positive profits if they all offer loans.[3] We therefore assume that banks can elect to be inactive in equilibrium, and that this earns them zero profits.[4]

A competitive equilibrium with L active banks is characterized by the following three conditions.

Firstly, lending at r should be individually rational for the banks, and borrowing at r should be individually rational for the firms:

$$X \geq r \geq r_0(L). \quad \text{(PS1)}$$

This implies in particular that $U(r,L)\geq 0$, so that no active bank prefers to be inactive.

Secondly, no active bank should wish to lower its interest rate slightly:

$$U(r,L)\geq U(r,1) \quad \text{(PS2)}$$

Once (PS1) is satisfied, a sufficient condition for (PS2) to hold is that $0\geq U(r,1)$, which can be restated as $r\leq r_0(1)$.

Thirdly, no active bank should benefit from an upward deviation, even to the highest possible interest rate X:

$$U(r,L)\geq V(X,L). \quad \text{(PS3)}$$

[3] This is even true if we ignore the screening costs: see Broecker (1990).

[4] Alternatively, we could weaken the equilibrium concept and modify (PS1) so as to allow banks to bear losses, which is in line with the idea that the number of banks is fixed in the short run and variable through entry/exit in the long run. The proof of existence is then simplified.

Finally, if $L < N$, so that there are inactive banks, three additional conditions have to be imposed. We require none of the $N - L$ inactive banks to choose to become active at a lower interest rate (PS4), at the same interest rate (PS5), or at a higher interest rate (PS6):

$$U(r,1) \leq 0 \tag{PS4}$$

$$U(r,L+1) \leq 0 \tag{PS5}$$

$$V(X,L+1) \leq 0 \tag{PS6}$$

These conditions can be expressed in terms of the break-even rate: (PS4) can be restated as $r \leq r_0(1)$, (PS5) as $r \leq r_0(L+1)$ and (PS6) as $r_0'(L) \geq X$.

We make a number of observations concerning (PS2), the condition which rules out downward deviation. Firstly, when $L < N$, (PS2) is implied by (PS1) and (PS4): if inactive banks are dissuaded by losses from undercutting active banks, then neither will active banks wish to undercut.

Now observe that $\frac{\partial}{\partial r}U(r,L) < \frac{\partial}{\partial r}U(r,1)$. It follows that when $r_0(L) > r_0(1)$, we have $U(r,L) < U(r,1)$ for any $r \geq r_0(L)$. Hence when $r_0(L) > r_0(1)$ the possibility of undercutting rules out an equilibrium with L banks. Finally, since $U(r,L)$ is linear in r, for any $L > 1$ with $r_0(L) < r_0(1)$ there exists a unique real number $\bar{r}_{L1}$ such that $U(\bar{r}_{L1},L) = U(\bar{r}_{L1},1)$. Equilibrium condition (PS2) can thus be rewritten as $r \leq \bar{r}_{L1}$.

Theorem 1 *Assume that (ES) holds. When screening costs are sufficiently convex (in particular, when (C) holds) a pure strategy Nash equilibrium exists in which more than one bank is active and where all active banks charge the same interest rate r. Moreover, in any pure strategy Nash equilibrium all active banks charge the same interest rate.*

Proof. From lemma 2, condition (PS3) is satisfied provided $U(r,L) \geq 0$. On the other hand, we know from the final condition in (A5′) that $r_0(1) < X$ and that $r_0(1) > r_0(2)$ (because of (C)).

Two cases are to be considered. Either (i) $r_0(1) \geq r_0(2) \geq ... \geq r_0(N)$ or (ii) there exists $L^* < N$ such that $r_0(L^*) < \min\{r_0(1), r_0(L^*+1)\}$.

We consider first case (i): it is clear that it is an equilibrium for all N banks to charge $r_0(1)$ as (PS1) and (PS2) then hold, so does (PS3) and there is no inactive bank ($L = N$), so (PS4), (PS5) and (PS6) are trivially satisfied. In fact, the previous arguments apply to show that for any $r \in [r_0(N), \min\{X, \bar{r}_{N1}\}]$ all N banks charging r is an equilibrium.

Consider now case (ii). Let $r \in (r_0(L^*), \min\{r_0(1), r_0(L^*+1)\})$. It is obvious that the situation in which L^* banks charge r constitutes a pure strategy equilibrium: by the choice of

r conditions (PS1), (PS4), and (PS5) are satisfied. As observed before, this implies that also (PS2) is satisfied, and since $r_0'(L) > X$ for all L by Lemma 2, this implies that $V(X, L^*+1) < 0$ so that (PS6) is satisfied. Since $r > r_0(L^*)$, we have $U(r, L^*) > 0$ and (PS3) follows.

If in a pure strategy equilibrium two active banks charge different interest rates, the one who charges the higher interest rate makes negative profits because of Lemma 2. Hence, active banks must necessarily charge the same interest rates. □

Case (i) of the above proof corresponds to a symmetric equilibrium where all banks play the same strategy while case (ii) corresponds to an asymmetric one that combines some banks being active while others are inactive.

Theorem 1 rests upon a key difference between our set-up and Broecker's. Our convexity assumption (C) ensures that $r_0(L)$ is not an increasing function as in Broecker's paper, but that it has a minimum, either at $r_0(N)$ or $r_0(L^*)$. Quoting interest rates slightly above this minimum is a pure strategy equilibrium, as they are inferior to $r_0(1)$, so that undercutting is never optimal. At the same time, overcharging is never optimal under (ES).

Because it is always optimal for one bank to quote precisely the same rate as its competitors, the equilibrium interest rate is undetermined within the interval $[r_0(N), \min\{X, \bar{r}_{N1}\}]$ for the case with no inactive banks and $[r_0(L^*), \min\{r_0(1), r_0(L^*+1)\}]$ when only $L^* < N$ are active. We do not see the indeterminacy of interest rates as an obstacle or as a negative result. The specific characteristics of the credit market when screening is accounted for leads quite naturally to this result. Any coordination device which allowed us fully to characterize the equilibrium (Central Bank announcement, colluding to the highest possible interest rate, ...) would therefore be ad hoc.

One interesting implication of interest rate indeterminancy is that it partially disconnects credit from the marginal cost of funds. Only in the case of large shocks will some adjustment be necessary. As interest rates hit the lower bound r_0, or the number of active banks has to adjust, we may switch from one equilibrium to another one. In general, small changes in the model's parameters will produce small changes in the rates r_0 so that the interest rate r will remain in the same interval. Hence interest rates will not react immediately to changes in marginal costs, and in particular to changes in interbank rates. This is of interest, as it is consistent with the observation that interest rates for loans change only sporadically, a fact that has been widely documented in the empirical literature as the "stickiness of loan rates" (see, for instance, Hannan and Berger, 1991, or Mester, 1994), and which still lacks a fully satisfactory theoretical justification.

In addition to the equilibrium interest rate, the number of active banks is also somewhat indeterminant. For in case (ii) of the proof of theorem 1, in general more than one number L^* satisfies the condition $r_0(L^*) < \min\{r_0(1), r_0(L^*+1)\}$. In most cases the sequence $r_0(L)$ will reach a minimum at some $\underline{L}$. There is then an equilibrium at any $L^* \geq \underline{L}$, provided $r_0(L^*) < r_0(1)$. This does not mean that the number of active banks is unbounded. The following lemma establishes in fact that the number of active banks in equilibrium is bounded. Hence case (i) of the proof of theorem 1 cannot occur if N is sufficiently large.

Lemma 3

1. *Under (A5′),*

$$\lim_{L\to\infty} LC(f(L)) = 0$$

and thus

$$\lim_{L\to\infty} r_0(L) = \frac{1}{lp_a + (1-l)p_b} = 1/\overline{p}$$

2. *Under (A3) and (A5′) there exists M such that $r_0(M) \leq r_0(1)$ and $r_0(L) > r_0(1)$ for all $L > M$.*

Proof. (1) Set $\gamma = [\frac{l}{1-q_a} + \frac{1-l}{1-q_b}]$

$$\begin{aligned}
\lim_{L\to\infty} LC(f(L)) &= \lim_{x\to 0} \frac{1}{x} C(f(\frac{1}{x})) \\
&= \lim_{x\to 0} \frac{C(x[l\left(\frac{1-q_a^{1/x}}{1-q_a}\right) + (1-l)\left(\frac{1-q_b^{1/x}}{1-q_b}\right)])}{x} \\
&\leq \lim_{x\to 0} \frac{C(x\gamma)}{x} \\
&= \lim_{x\to 0} \frac{\gamma C'(x\gamma)}{1} = 0
\end{aligned}$$

Since under (A5′), $C'(0) = 0$ and $LC(f(L)) \geq 0$ for all L, the result follows.

(2) By assumption (A3), $1/\overline{p} > X$. Since $r_0(1) < X$ and $\lim_{L\to\infty} r_0(L) > X$, there exists M such that $r_0(M) \leq r_0(1)$ and $r_0(L) > r_0(1)$ for all $L > M$. □

We know from the observations after the equilibrium conditions (PS1) through (PS6) that there are no equilibria with L active banks when $r_0(L) > r_0(1)$. Hence, the lemma implies that in equilibrium certainly not more than M banks will be active. We will henceforth assume that there are at most M potential banks.

4. An Illustrative Example

We now demonstrate that the pure strategy equilibrium conditions (PS1) – (PS6) are compatible by considering numerical specifications for the parameters of the model. We also consider welfare and profits for this example.

Let $q_a = 0.75$, $q_b = 0.6$, $p_a = 0.3$, $p_b = 0.7$, $I = 0.4$, $X = 1.84$, and $C(x) = 0.02x^5$. It is easy to check that assumptions (A1)-(A3) are satisfied. It is also easily verified that the screening costs are so low that a single bank charging X would make positive profit. (Broecker, 1990, shows that this follows from (A2) in the case of no screening costs.) Straightforward calculations yield

$$\begin{aligned} r_0(1) &= 1.81818 > r_0(2) = 1.78885 > r_0(3) = 1.7854, \\ r_0(4) &= 1.79154 < r_0(5) = 1.80062 < r_0(6) = 1.80989 < r_0(1) < r_0(n) \text{ for } n \geq 7, \\ r_0(n) &> X \text{ for all } n > 11. \end{aligned}$$

We assume that there are $N \geq 12$ potential banks in the market. There are no pure strategy equilibria with one or two active banks, since any inactive bank would make strictly positive profits by mimicking an active bank. Furthermore, no pure equilibria exist where more than six banks enter, since these banks can only break even by charging an interest rate strictly above $r_0(1)$, in which case an inactive bank could make a positive profit by undercutting slightly below $r_0(1)$. On the other hand, for $L = 3$, 4, and 5, it is an equilibrium for L active banks to charge any interest rate $r \in [r_0(L), r_0(L+1)]$. It is also an equilibrium for siz banks to charge any interest rate $r \in [r_0(6), r_0(1)]$.

Observe that the more banks are active in equilibrium, the higher is the equilibrium interest rate. The reason for this is twofold: on the one hand, a few banks charging a relatively high interest rate would provoke entry by additional banks; on the other hand, when many banks offer the same interest rate, average credit quality is quite low and hence the interest rate must be quite high to ensure that the banks break even.

The interest rate simply determines a transfer between firms and banks and hence does not affect social welfare $W(L)$. Total surplus depends only upon the number of active banks in equilibrium. With more active banks, more and on average worse projects get financed. This effect is welfare decreasing, but is partially offset by the convexity of the screening cost function. It can be easily calculated that the total welfare with 3, 4, 5, and 6 banks equals, $W(3) = 0.0318752$, $W(4) = 0.0279051$, $W(5) = 0.0226881$, and $W(6) = 0.0174316$, respectively. Hence, welfare is highest with the least number of banks.

We now consider bank profits. Obviously, these depend upon the equilibrium interest as well as the number of active banks. Since for $L \in \{3, 4, 5, 6\}$ it is an equilibrium for L banks to charge $r_0(L)$, there are always equilibria in which banks make zero profits. We focus upon the maximal equilibrium profits. Simple calculations yield maximal individual bank profits of 0.000815431, 0.00101624, 0.000888279, and 0.000689513 in the respective cases with 3, 4, 5, and 6 banks. The highest per-bank profit is obtained with 4 active banks. Joint profits are maximized with 5 active banks.

5. Comparative Statics

In this section we examine the effect of L upon social welfare and upon bank profitability for the following specific cost function:

$$C_S(s) = \begin{cases} 0, & s < 1 \\ c, & \text{otherwise} \end{cases} \tag{3}$$

We interpret $C_S(\cdot)$ as describing the costs of a capacity-constrained bank. A single bank is just able to screen all applicants but at a high cost. We assume that $c > 0$ is such that a single bank charging X will make small but positive profits and that condition (C) is satisfied so that existence of pure strategy equilibria is guaranteed as before: undercutting is optimal only when high interest rates are charged.

Proposition 1 *When screening costs are given by equation (3), welfare in equilibrium is a decreasing function of L.*

Proof. There is no equilibrium in which just one bank enters. Suppose that there exists an equilibrium in which $L > 1$ banks are active, and each charges r_L^*, and that there also exists an equilibrium with $L+1$ active banks, each of which charges r_{L+1}^*. Equilibrium conditions (PS5) and (PS1) imply that $r_L^* \leq r_0(L+1) \leq r_{L+1}^* \leq X$. The difference in total welfare between the two equilibria is

$$\begin{aligned} &W(L+1) - W(L) \\ &= l(p_a X - 1)(q_a^L - q_a^{L+1}) + (1-l)(p_b X - 1)(q_b^L - q_b^{L+1}) - (L+1)C(f(L+1)) + LC(f(L)) \\ &= V(X, L) + [C(lq_a^L + (1-l)q_b^L) + LC(f(L)) - (L+1)C(f(L+1))]. \end{aligned} \tag{4}$$

When screening costs are given by equation (3) and $L > 1$, the square bracketed term disappears. Hence

$$W(L+1) - W(L) = V(X, L) \leq 0,$$

where the inequality follows from (PS6) in case $r_L^* < X$: no inactive bank will enter and charge X when L banks charge $r_L^* < X$. In the case where $r_L^* = X$, then $r_0(L+1) = r_{L+1}^* = X$, which implies that $W(L+1) = 0$, as banks would make zero profit and firms would enjoy no surplus in the equilibrium with $L+1$ active banks. Obviously, also in this case we have that $W(L+1) - W(L) \leq 0$ □

As a technical aside, we note that, provided the square bracketed term in equation (4) above is small enough, proposition 1 can be generalised to more general cost functions. For the case of a linear cost function $C(\cdot)$, the square bracketed term is exactly zero. For a strictly convex cost function the square bracketed term is positive; however, the proposition goes through in this case provided $V(X, L)$ is sufficiently negative. For example, in the numerical example of section 4 welfare was decreasing in the number of active banks in equilibrium.

6. Entry

We now turn to the case where the number of banks is endogenous. We assume as usual that entry occurs until the point where an additional bank entering the market would make losses. In the canonical microeconomic model of entry in a delivery vs. payment type of market, this is equivalent to firms making zero profits in *the* equilibrium, up to an indivisibility. The analysis in our set-up is more complex, for a number of reasons. Firstly, bank profitability depends upon equilibrium credit risk, which in turn is determined by the number of active banks. Hence banks earn profits in the free entry equilibrium not because of indivisibilities or because of the strategic reaction of incumbents, but because the $(N+1)\,th$ bank's entry increases the costs of the N incumbent ones. Second, the number of active banks does not fully determine the equilibrium loan rate. We can therefore speak only of the equilibrium loan rate *range* and the equilibrium profit *range*. Since the equilibrium loan rate can be as low as $r_0(L)$, profits could be zero for any equilibrium number of banks.

To this point we have fixed the number of N of banks and allowed them to choose to become active. Although bankers correctly anticipate the number of active banks, they do not know for sure what this is at the time they set interest rates. Some banks may remain inactive and we have shown that this is necessarily the case for large N when screening is necessary to make nonnegative profits. In contrast, when entry is costly we would expect bankers to enter only if they (correctly) anticipate that they will be active.

We now assume that M potential banks first decide whether to enter the market at a small sunk cost F. Banks set interest rates and decide whether to be active only after it has been established how many banks enter. We thus exclude the possibility that a bank will use a hit-and-run strategy. This assumption is justified when interest rates can be changed more easily than banks can be established. In this set-up, a bank's strategy no longer consists of an interest rate r at which to provide loans to all applicants passing its screening test. Instead, it has to set an interest rate $r(N)$ which depends upon the number N of banks in the credit market.

The appropriate solution concept for our sequential entry model is that of subgame perfect equilibrium. We first characterise all subgame perfect equilibria. We show that all banks that enter will be active, but that the number of banks in the credit market is indeterminate. We then show that a very mild refinement of the equilibrium concept suffices to tie down the precise number of active banks.

We firstly define another break-even interest rate:

Definition 2 *Let $r_F(L)$ denote the interest rate at which L banks are able to just recover their setup cost. i.e., $U(r_F(L), L) = F$.*

(i) Characterization of pure strategy subgame perfect equilibria

We first consider the second stage when N banks have entered, with $1 \leq N \leq M$. Since setup costs are sunk the equilibrium conditions for a pure strategy equilibrium in this subgame are exactly those outlined in the previous section. If $N = 1$ the unique bank will obviously set $r = X$. If $N \geq 2$, $L \leq N$ banks will set the same interest rate r while the remaining $N - L$ banks choose to stay inactive. The interest rate r must satisfy conditions (PS1) through (PS6).

We now determine conditions for the existence of a pure strategy subgame perfect equilibrium in which exactly L^* banks enter. Since an inactive bank would deviate and save the setup cost F, all of these L^* banks must be active on the equilibrium path of play. At the equilibrium interest rate r, each bank must cover its setup costs: that is $U(r, L^*) \geq F$, or equivalently $r \geq r_F(L^*)$. Since all L^* banks are active, equilibrium conditions (PS4) through (PS6) have no bite. Hence the equilibrium interest rate can be anything between X and the minimal interest rate needed to break even, net of setup costs.

Finally, if $L^* < M$ we need a restriction on the interest rate $r(L^* + 1)$ which the active banks would set in the hypothetical case where more than one bank enters. This rate must be set to discourage entry by an $(L^* + 1)\,th$ bank. If all banks use the same strategy this implies that $U(r(L^* + 1), L^* + 1) \leq F$. Alternatively, in an asymmetric equilibrium with $L^* + 1$ entrants a proper subset would be active and would set an interest rate which ensured that the remaining banks (including the $(L^* + 1)\,th$) chose to remain inactive. A possible, but not unique, threat to support the equilibrium would be to set the interest rate equal to $r_0(L^*+1)$.

We have proved the following characterization of pure subgame perfect equilibria.

Theorem 2 *Assume that (ES) holds. When screening costs are sufficiently convex (in particular, when (C) holds) a pure subgame perfect equilibrium exists. For any $L^* \geq 2$ and any $r \leq X$ for which $U(r, L^*) \geq F$ and $U(r, L^*) \geq U(r, 1)$, there is a subgame perfect equilibrium in which L^* banks enter and then charge r. Further, in a subgame perfect equilibrium it is also possible that just one bank enters and then charges the monopolistic interest rate X, threatening to charge the break-even interest rate $r_0(2)$ when another bank enters.*

(ii) Equilibrium selection: determining the number of banks

In neither our original model of banking competition with a fixed number of banks, nor in the model with endogenous entry, is either the number of active banks or the interest rate uniquely determined. We can therefore view competition between banks as a double coordination problem: how many banks should enter, and which interest rate should they set?

At first sight the indeterminancy appears more severe in the case of endogenous entry since the equilibrium interest rate with more than one bank can go all the way up to $\min\{X, \bar{r}_{M1}\}$. Moreover, monopolistic credit markets are also possible in this set-up. However, there is an important difference between the equilibria of the two models. With a fixed number of banks almost all equilibria are strict, in the sense that any deviation from the equilibrium strategy leads to a strict loss for the deviator. All such equilibria are therefore very stable and robust.

In contrast, none of the equilibria in the model with endogenous entry is strict. For example, consider a subgame perfect equilibrium in which $L^* \geq 2$ banks enter and set $r(L^*) = r$, $r(L) = r_0(L)$ for $L \notin \{1, L^*\}$ and $r(1) = X$. For any bank which enters in equilibrium the only optimal interest rate is the one set by all other banks. In this sense, the equilibrium is strict. But banks have much more freedom in the unreached subgames: they can change their interest rate off the equilibrium path when $L \neq L^*$ banks enter. This multiplicity of best replies for any bank destabilizes many (subgame perfect) equilibria. In order to select between the equilibria we apply the concept of a minimal curb set.

Definition 3 (Basu and Weibull, 1991) *A non-empty cartesian set of pure strategy profiles $Y = Y_1 \times \cdots \times Y_M$ is a* curb set *if for all $y \in Y$ and for any strategy z_i of player i that is a best reply against y, $z_i \in Y_i$. Y is a* minimal curb set *if it is a curb set and no strict subset $Y' \subset Y$ is a curb set.*

A minimal curb set can be viewed as the set-valued extension of a strict Nash equilibrium. While strict Nash equilibria are not guaranteed to exist, Basu and Weibull (1991) have shown that in games with compact strategy sets and continuous payoffs, minimal curb sets do exist. Moreover, any minimal curb set contains the support of at least one Nash equilibrium. Hurkens (1995) shows how boundedly rational players may learn to play strategies from a minimal curb set.

In our model, payoffs are not continuous as banks can sometimes improve their profits by slightly undercutting their competitors. To avoid the problem of non-existence of best replies we assume that interest rates have to be chosen from a finite but fine grid $\{0, \Delta, 2\Delta, \cdots, K\Delta\}$ where $\Delta > 0$ is small and $K\Delta = X$.[5] We show that in any minimal curb set all banks enter and then all charge the same interest rate which allows them to obtain strictly positive profits.

Theorem 3 *For each $r \in (r_F(M), \min\{X, \bar{r}_{M1}\}]$, there exists one minimal curb set Y^r. The set Y^r consists of all pure strategies that tell the bank to enter and charge r when exactly M banks enter. There are no other minimal curb sets.*

Proof. Fix $r \in (r_F(M), \min\{X, \bar{r}_{M1}\}]$ and let the set Y^r consist of all pure strategies that tell the bank to enter and charge r when exactly M banks enter.

First, it is clear that the set Y^r is closed under rational behavior as all best replies are exactly the strategies to enter and charge r when all banks enter. By the same argument, the set is a minimal curb set.

Second, we need to show that there are no other minimal curb sets. Suppose on the contrary that there is a different minimal curb set Y. Let $y \in Y$ be a pure strategy profile and let L be the number of banks that enter according to this profile. We will show that $Y \cap Y^r \neq \emptyset$ for some Y^r. This then implies that $Y^r \subset Y$ which proves the statement. We will use induction by L.

First consider $L = 0$. Clearly, a best reply for any bank i against y is to enter and charge X when it is the only bank to enter and to charge r if all banks enter (and to charge for example also r when a different number of banks enter). Clearly, the profile of best replies is an element of Y^r.

Next consider $L = 1$. A best reply for the one bank that enters against the strategy profile y is to enter and charge X if it is the only bank, to charge $r_F(2)$ if two banks enter, and to charge r if M banks enter. Hence, this strategy must be in the curb set. A best reply of any

[5] Alternatively, we could choose to work with the concept of sets closed under better replies, introduced by Ritzberger and Weibull (1995).

other bank against this strategy is to use exactly the same strategy, that is, to enter and charge $r_F(2)$ if two banks enter and to charge r if M banks enter. It follows that $Y \cap Y^r \neq \emptyset$.

Let us now make the induction step: assume that for curb sets Y containing strategy profiles in which 0, 1, ..., or $L < M$ banks enter, we know that $Y \cap Y^r \neq \emptyset$ for some r. Now consider a curb set Y with a strategy profile y in which $L + 1$ banks enter.

First, consider the case where not all banks charge the same interest rate when all enter. In this case the bank which charges the highest interest rate will make a loss. Its best reply is either (i) decide not to enter, (ii) decide to enter but match the lowest interest rate or (iii) decide to enter and undercut the lowest interest rate by Δ. Whatever the best reply is, it must be contained in the curb set. By repeating the argument repeatedly we will either obtain a strategy profile y' in which only L banks enter, or we obtain a strategy profile y'' in which all $L + 1$ banks charge the same interest rate from which no banks desires to deviate. In the first case we can apply the induction hypothesis to show that $Y \cap Y^r \neq \emptyset$ for some r. In the second case we have obtained a Nash equilibrium in the curb set in which $L + 1$ banks enter and then charge some interest rate r'. If $L + 1 = M$, we are done. If $L + 1 < M$ we need to do one additional step. A best reply for each of the $L+1$ banks is to enter and charge r if $L+1$ banks enter, charge r if M banks enter and charge $r_F(L + 2) + \Delta$ if $L + 2 < M$ banks enter. These best replies must be contained in the curb set. But the best reply for each of the inactive banks against these latter strategies is then to enter and use the same interest rate strategy. Again, it follows that $Y \cap Y^r \neq \emptyset$ for some r. □

We summarize our findings as follows. The only (subgame perfect) equilibria that are contained in minimal curb sets are those in which the maximal number of banks enter and charge an interest rate which allows them to make strictly positive profits while not giving any banks an incentive to deviate. All such interest rates are possible and plausible and we cannot distinguish between them without making further, ad hoc, assumptions.

We do not see the indeterminacy of interest rates as an obstacle or as a negative result. We believe it is in line with observed interest rate stickiness. If we were able to select a unique interest rate as the only plausible one, then this interest rate would necessarily depend in a specific way on the parameters of the model. So the equilibrium interest rate would immediately adjust in response to a small shock to these parameters.

Since we are actually able to predict only that the maximal number of banks should enter and charge an interest rate which yields a positive profit, small shocks to the model parameters would most likely not affect the interest rate. Hence, interest rate adjustment is necessary in our model only in response to large shocks. This is consistent with the observation that interest rates change only sporadically.

7. Heterogeneous Monitoring Skills

In this section, we consider an extension of our model in which some banks are endowed with a better monitoring technology than the others. Specifically, we assume that these superior banks have a monitoring technology $\hat{q}_a$, $\hat{q}_b$ which satisfies the following assumptions:

$$\begin{aligned} \hat{q}_a &> q_a; \\ \hat{q}_b &< q_b. \end{aligned}$$

Superior banks will thus reject less good (type b) firms and reject more bad (type a) firms than inferior banks do. The adverse selection effect is thus stronger in the presence of superior competitors. On the other hand, the adverse selection effect is less strong for superior banks than it is for inferior banks. This is most easily seen in the extreme case that superior banks have perfect monitoring technologies.

The existence and characterization of pure strategy equilibria will obviously depend on the number of inferior and superior banks, on how much better the superior technology is, and on the exact screening cost. In this Section we will assume that screening costs are given by (3) so that both superior and inferior banks have capacity constraints. Let us assume that firms know the quality of monitoring of each bank. When all the banks charge the same interest rate, firms will apply to the bank where they have the highest probability of being granted a loan. Hence, all type b firms will in first instance apply to the superior banks, while type a firms will exhaust the supply of inferior banks before applying to a superior bank. Hence, the inferior banks suffer from a "cherry-picking externality" (Morrison and White, 2004). If condition (ES) holds, an active inferior bank will make losses whenever a superior bank charges the same or a lower interest rate. Hence, in a pure strategy equilibrium either no inferior bank is active or two or more inferior banks charge the same low interest rate while one or more superior banks charge a much higher interest rate.

We are able to show that equilibria of the first type will certainly exist, provided that at least two banks have the superior technology.[6] Namely, applying the construction of Section 3 applied to a model in which all banks have the superior monitoring technology, one finds that

[6] The second type of equilibrium, in which inferior banks charge low interest rates and superior banks charge high interest rates, may exist in very special cases. However, we were not able to find parameters consistent with the model and satisfying this condition.

a number of superior banks coordinating on the same interest rate below $r_0(1)$ will constitute a pure strategy Nash equilibrium in this adjusted model. Obviously, inferior banks can do no better than remain inactive.

Theorem 4 *Suppose that there are at least two banks with superior technology* $(\hat{q}_a, \hat{q}_b)$ *with* $\hat{q}_a > q_a$ *and* $\hat{q}_b < q_b$. *Suppose furthermore that inequality (ES) holds when* q_a *and* q_b *are replaced by* $\hat{q}_a$ *and* $\hat{q}_b$*, respectively. Then there exists a pure strategy equilibrium in which only superior banks are active, and all pure strategy equilibria are of this type.*

Proof. The proof is similar to that of theorem 1. □

This straightforward extension of theorem 1 is of interest for several reasons. First, the equilibrium we are envisaging is one where the number of active banks is limited by the access they have to the superior technology. Second, if the acquisition of the superior technology is the result of learning by doing, then it fully justifies Shaffer's (1998) idea of a winner's curse in the market for credit.

8. Conclusion

In this paper we explore credit market equilibrium when banks perform costly screening. When the screening technology is sufficiently accurate and sufficiently convex, we are able to prove that pure strategy equilibria exist. This complements Broecker's (1990) result that there is no pure strategy equilibrium when screening is costless. Moreover, equilibrium prices in our model are the result of a coordination process. This implies the existence of positive equilibrium profits as well as a disconnect between marginal cost and prices.

Finally, a substantial literature has argued that welfare is enhanced by allowing banks to extract rents from their lending relationships. The prior literature has typically argued that this rent is required to satisfy a screening incentive compatibility constraint. We approach this problem from an alternative perspective. Proposition 1 shows that, in general, an increase in the number of banks will decrease welfare. The reason in our model is that a higher number of banks raises the probability that a bad creditor will obtain a loan after visiting all of them. Our work therefore provides a new argument to support the policy recommendation that entry to the credit market should be restricted.

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