Risk of Default in Latin American Brady Bonds

by

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Abstract

The 1989 Brady Plan, named after the former US Treasury Secretary Nicholas Brady, was the restructuring and reduction of several emerging countries' external debt into bonds with US Treasury bonds as collateral. So far no country has ever defaulted payments, yet the market value of these bonds is usually significantly lower than equivalent 'risk-free' bonds. Mexico was the first country to issue Brady bonds, in February 1990, and there soon followed other Latin American, Eastern European and Asian countries.

In this paper, we describe a stochastic model for the instantaneous risk of default, applicable to many fixed-income instruments and Brady bonds in particular. We make some simplifying assumptions about this model and a model for the riskless short-term interest rate. These assumptions allow us to find explicit solutions for the prices of risky zero-coupon bonds and floating rate coupons. We apply the model to Latin American Brady bonds, deducing the risk of default implied by market prices.

Introduction: One-factor interest rate models

This paper begins with a quick reminder of simple interest rate models, and then we concentrate on the default aspects of risky bonds.

The starting point for the pricing and hedging of fixed-income securities is often a stochastic differential equation for a short-term interest rate *r*:

$$dr = a(r,t)dt + b(r,t)dX \ ,$$

with *dX* being Brownian motion. Here α is the risk-neutral drift of the spot interest rate. This is often supplemented by equations for further factors, such as a long rate. From this equation can be derived the partial differential equation for the value *V*(*r,t*) of path-independent contracts:

$$V_t + \frac{b^2}{2} V_{rr} + aV_r - rV = 0. \qquad (1)$$

This equation follows from a hedging argument and amounts to valuing in a risk-neutral framework. This differential equation is accompanied by final conditions depending on the nature of the contract. If we use *Z*(*r,t,T*) to denote the solution of (1) for a zero-coupon bond maturing at time *T*, then *Z*(*r,T,T*)=1 for example, and using *W*(*r,t,T*) for a Floating Rate Coupon (FRC) then W(*r,T,T*)=*r* (if the floating rate is the spot rate and is paid at time *T*).

If the coefficients α and β take certain special forms, Equation (1) has special solutions in the cases of zero-coupon bonds and FRCs. These special forms are

$$\begin{aligned} a &= a(t) - b(t)r \\ b &= \sqrt{c(t) + d(t)r} \end{aligned},$$

when the solutions can be written as

$$\begin{aligned} Z(r,t,T) &= \exp(D(t,T) - E(t,T)r) \\ W(r,t,T) &= \exp(F(t,T) - E(t,T)r) + G(t,T)rZ(r,t,T) \end{aligned}.$$

Usually the functions *D, E*, *F* and *G* must be found as solutions of ordinary differential equations, although there are explicit solutions when all parameters are constant. See Wilmott, Dewynne & Howison (1993) (**LINK:www.oxfordfinancial.co.uk**), Ho & Lee (1986), Vasicek (1977), Cox, Ingersoll & Ross (1985) and Hull & White (e.g. 1990) for details of these models.

All of the above modelling can be applied to instruments having cash flows that are *guaranteed*. It is assumed that these cash flows, coupons and redemption values, are from a completely credit-worthy source, such as the US government, or underwritten in such a way that the income is certain. In practice, many bonds have no such guarantee. Perhaps they are issued by a company as a form of borrowing for expansion. In this case the issuing company may declare bankruptcy before all of the cash flows have been paid. Alternatively, they may be issued by a government with a record for irregular payment of debt. The Brady bonds issued by governments in emerging markets are priced with risk of default taken into account. (However, some of the interest or principal payments on Brady bonds are 'collateralised,' they are effectively guaranteed.) The risk of default and its effect on bond prices have been the subject of much discussion and many models, we mention here some of the most important.

The early models took the 'value of the firm' as a starting point, see Merton (1974) and Longstaff & Schwartz (1993) for examples. More recently has been the work on the 'instantaneous risk of default,' see later and Duffie & Singleton (1994), Lando (1994) and Schönbucher (1996). For a review of models see Cooper & Martin (1996).

In this document we discuss the subject of pricing bonds when there is risk of default. We now describe the modelling of the ‘instantaneous risk of default.’

The instantaneous risk of default

We can describe the instantaneous risk of default, p, as follows. If at time t the issuing company/government has not defaulted and the instantaneous risk of default is p then the probability of default between times t and $t+dt$ is $p\,dt$. Default is just a Poisson event, with intensity p.

We must now choose a model for p and then determine the value of risky debt based on this model.

The simplest example is to take p constant. In this case we can easily determine the risk of default before time T. We do this as follows. Let $P(t,T)$ be the probability that the company/country does not default before time T given that it has not defaulted at time t. The probability of default between later times t' and $t'+dt'$ is the product of $p\,dt$ and the probability that the company/country has not defaulted at time t'. Thus, we find that

$$\frac{\partial P}{\partial t'} = pP(t',T).$$

The solution of this with $P(T,T)=1$ is

$$e^{-p(T-t)}.$$

If there is no recovery in default and no correlation between the spot interest rate and default, the value of a zero-coupon bond paying \$1 at time T could be modelled by taking the present value of the *expected* cashflow. This results in a value of

$$e^{-p(T-t)}Z(r,t,T). \qquad (2)$$

where Z is the value of a riskless zero-coupon bond using either a quoted market price or whatever model is preferred. (Note that this does not put any value on the risk taken.)

This model is the very simplest for the instantaneous risk of default. It gives a very simple relationship between a risk-free and a risky bond. There is only one new parameter to estimate, p. To see whether this is a realistic model for the expectations of the market we take a quick look at the prices of Brady bonds. In particular we examine the market price of the Argentine Par bond.

Brady bonds were issued to restructure defaulted debt from emerging countries in Latin America, Eastern Europe, Asia and Africa. They are the most liquid emerging markets instruments. They are different from other bonds in that the principal and certain coupons are collateralised by the US or other first-world governments.

We describe the Argentine Par bond and others later in this paper. For the moment, we just need to know that this bond has interest payments and the final return of principal denominated in US dollars. Some of these cashflows are underwritten by the US government, with no real likelihood of default. If the above is a good model of market expectations with constant p then we would find a very simple relationship between interest rates in the US and the value of the Brady bond. To get the Brady bond value, take the market price of the US risk-free zero-coupon bond with the same maturity as one of the payments in the Par bond multiply by the future value of the cash flow, multiply again by expression (2) for the correct values of T-t and p and finally sum over all such cash flows. Conversely, the same procedure can be used to determine the value of p from the market price of the Brady bond; this would be the 'implied risk of default.' In Figure 1 we show the implied risk of default for the Par bonds of Argentina, Brazil, Mexico and Venezuela using the above algorithm and assuming a constant p.

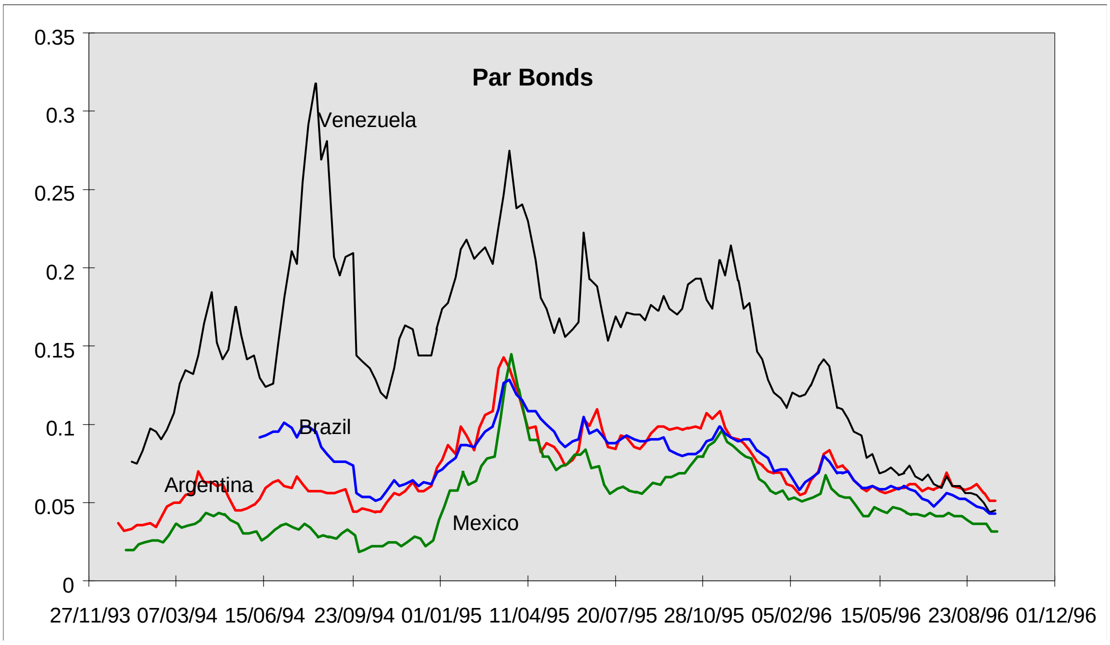

Figure 1: The implied risk of default for the Par bonds of Argentina, Brazil, Mexico and Venezuela, assuming constant p.

In this simple model we have assumed that the instantaneous risk of default is constant (different for each country) through time. However, from Figure 1 we can see that, if we believe the market prices of the Brady bonds, this assumption is incorrect: the market prices are inconsistent with a constant p. This is our motivation for the next model, the stochastic risk of default model. Nevertheless, supposing that the figure represents, in some sense, the views of the market (and this constant p model *is* used in practice) we draw a few conclusions from this figure before moving on.

The first point to notice in the graph is the perceived risk of Venezuela, which is consistently greater than the three other countries. Venezuela's risk peaked in July 1994, nine months before the rest of South America, but this had absolutely no effect on the other countries.

The next, and most important, thing to notice is the "Tequila effect" in all the Latin markets. Before December 1994 we can see a constant spread between Mexico and Argentina and a contracting spread between Brazil and Argentina. The Tequila crisis began with a 50% devaluation of the Mexican peso in December 1994. Markets followed by plunging. The consequences were felt through all the first quarter of 1995 and had a knock-on effect throughout South America. In April 1995 the default risks peaked in all the countries apart from Venezuela, but by late 1996 the default risk had almost returned to pre-Tequila levels in all four countries. By this time, Venezuela's risk had fallen to the same order as the other countries.

Stochastic risk of default

To improve this model, and make it consistent with observed market prices, we now consider a model in which the instantaneous probability of default is itself random. We assume that it follows a random walk given by

$$dp = g\,(r, p, t)dt + d\,(r, p, t)dX_1,$$

with interest rates given

$$dr = \alpha(r,t)dt + \beta(r,t)dX_2.$$

There is a correlation of ρ between dX_1 and dX_2. It is reasonable to expect some interest rate dependence in the risk of default, but not the other way around. To value our risky zero-coupon bond now we construct a portfolio with one of the risky bond, with value *H(r,p,t)* (to be determined), and short Δ of a riskless bond, with value *Z(r,t)* (satisfying our earlier bond pricing equation):

$$\Pi = H(r,p,t) - \Delta Z(r,t).$$

(We have dropped the dependence on the maturity date for clarity.)

In the next timestep either the bond is defaulted or it is not. There is a probability of default of *p dt*. We must consider the two cases: default, and no default in the next timestep (see Figure 2 for a schematic diagram illustrating the analysis below). We take expectations to arrive at an equation for the value of the risky bond.

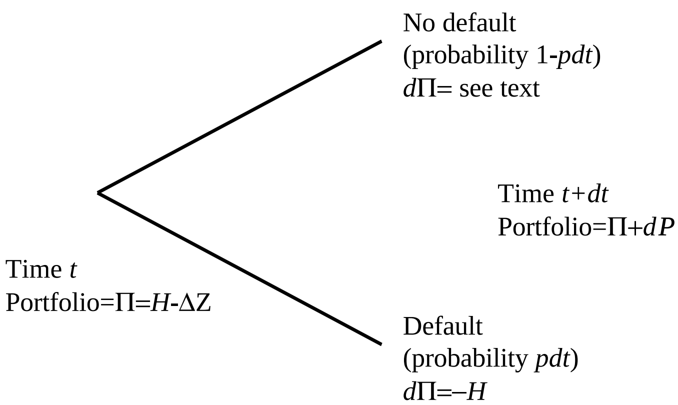

Figure 2: A schematic diagram showing the two possible situations: default and no default.

First, suppose that the bond is not defaulted, this has a probability of (1 - *p dt*). In this case the change in the value of the portfolio during a timestep is

$$d\Pi = \left(\frac{\partial H}{\partial t} + \frac{\beta^2}{2}\frac{\partial^2 H}{\partial r^2} + \rho\beta\delta\frac{\partial^2 H}{\partial r \partial p} + \frac{\delta^2}{2}\frac{\partial^2 H}{\partial p^2} \right) dt$$
$$+ \frac{\partial H}{\partial r}dr + \frac{\partial H}{\partial p}dp - \Delta\left(\frac{\partial Z}{\partial t} + \frac{\beta^2}{2}\frac{\partial^2 Z}{\partial r^2} \right)dt - \Delta\frac{\partial Z}{\partial r}dr.$$

As usual, choose Δ to eliminate the risky *dr* term.

On the other hand, if the bond defaults, with a probability of *p dt*, then the change in the value of the portfolio is

$$d\Pi = -H.$$

We are now faced with the problem of finding a valuation equation. There are two obvious choices. We could take real expectations with respect to the default process, or try to hedge the default. The latter approach will result in a market price of risk term for the default and further parameters to estimate or fit. We will adopt the expectations approach.

Taking expectations and using equation (1) for the riskless bond, we find that the value of the risky bond satisfies

$$\frac{\partial H}{\partial t} + \frac{b^2}{2}\frac{\partial^2 H}{\partial r^2} + rbd\,\frac{\partial^2 H}{\partial r \partial p} + \frac{d^2}{2}\frac{\partial^2 H}{\partial p^2}$$
$$+a\,\frac{\partial H}{\partial r} + g\,\frac{\partial H}{\partial p} - (r+p)H = 0. \tag{3}$$

This equation has final condition

$$H(r,p,T) = 1$$

if the bond is zero coupon with a principal repayment of \$1. The equation is the same, but the final condition different, for an FRC. Note that we could easily have assumed that a percentage, say, of the coupon is paid at maturity in the case of default. This is easy to model but probably unrealistic in the case of Latin American Brady bonds. When they default it is more likely that, instead of a coupon at maturity, the holder might be given a relatively valueless new bond with no short-term coupons. And this tends to be how the default is perceived in the country of issue.

As a check on this result, return to the simple case of constant *p*. In the new framework this case is equivalent to $\gamma=\delta=0$. The solution of Equation (3) is easily seen to be

$$e^{-p(T-t)}Z(t,T).$$

as derived earlier.

Some special cases and yield curve fitting

We mentioned in the Introduction that some spot interest rate models lead to explicit solutions for bond prices, for example the Vasicek model, the Cox, Ingersoll & Ross model and in general the affine model with four time-dependent parameters. We can find simpler equations than the two-factor diffusion equation for the value of a *risky* bond in the above framework if we choose the functions α, β, γ, δ carefully. For a full description of these and other interest rate models see Rebonato (1997).

We have already discussed the choice of α and β. We choose the general model discussed above but with *b*, and *c* independent of time and *d*=0.

For simple exponential solutions of (3) to exist we also require γ and δ^2 to be linear in *r* and *p*. The form of the correlation coefficient is more complicated so we shall choose it to be zero. Let us choose

$$g \;=\; f + gr - hp$$

and

$$d \;=\; j\sqrt{p}.$$

(This is not the most general system having simple solutions.)

With these choices for the functions in the two stochastic differential equations we find that the solution of (3) with $H(r,p,T)=1$ is

$$H = \exp\left(A(t,T) - B(t,T)r - C(t,T)\,p\right)$$

where *A*, *B* and *C* satisfy

$$\frac{dA}{dt} = fC + a(t)B + \frac{dB^2}{2}$$

$$\frac{dB}{dt} = 1 - gC + bB + \frac{cB^2}{2}$$

and

$$\frac{dC}{dt} = 1 + hC + \frac{j^2C^2}{2}$$

with $A(T,T)=B(T,T)=C(T,T)=0$. In some cases these equations can be solved explicitly (although only in terms of special functions), but in others they must be solved numerically. Such a solution will of course be much quicker than the numerical solution of the two-factor diffusion equation.

There are several requirements for the parameters if both the risk-adjusted interest rate and the probability of default are to stay positive. These requirements are

$$d \ge 0, \quad a \ge \frac{c}{2} + \frac{bd}{c}, \quad f \ge \frac{j^2}{2} - \frac{gd}{c}.$$

Because we have allowed the spot interest rate model to have some simple time dependence we have the freedom to fit the yield curve. By this we mean that we can choose one of the time-dependent functions in the stochastic differential equation for *r* so that an output of the model is the yield curve as given by the market. This yield-curve fitting is easiest for the Vasicek model, since it can be done completely analytically. Unfortunately, this model does not satisfy the positivity requirements above. This may or may not matter; indeed, there are probably more important reasons for criticising the model, but even these may be outweighed by the practical importance of such a simple and flexible model.

There is one very special case that we take advantage of below. If the random walks for the interest rate and the risk of default are uncorrelated, and there is no other coupling between these two variables, then we can decouple the risky bond into the product of a riskless bond and a factor involving the risk of default.

A case study: Latin American Brady bonds(LINK:http//www.bradynet.com)

The Brady Plan, created by former US Treasury Secretary Mr Nicholas Brady, began in 1989. The plan consists of repackaging commercial bank debt into tradable fixed income securities. Creditor banks either lower their interest on the debt or reduce the principal. Debtor countries, in exchange, are committed to make macroeconomic adjustments. Most Brady bonds are dollar denominated with maturities of longer than 10 years and either fixed or floating coupon payments. There are 15 countries currently in the Brady Plan.

Brady Bonds have the following characteristics.

Par Bonds: 30 year fixed rate bonds with semi-annual coupons and bullet amortisation. The full principal and the next (rolling) two/three interest coupons are guaranteed by US government bonds of similar maturity.

Discount Bonds: 30 year floating rate bonds paying libor + 13/16 semi-annually and bullet amortisation. The full principal and the next (rolling) two/three interest coupons are guaranteed by US government bonds of similar maturity.

Floating Rate Bonds: 12 year floating rate bonds paying libor + 13/16 and varying amortisation semi-annually.

(N.B. The Par and Discount in Venezuela have some dependence on oil prices.)

As complicated as they may seem, these bonds are priced in the same manner as regular bonds. The yield however, has been the subject of many debates. A common method of calculating the yield on the risky part of the instrument is by stripping the guaranteed coupons and obtaining a "Strip yield" which will represent only the part for which the local government is liable.

Now, we will use the above model to examine whether these bonds are priced correctly by the markets. Note that we always take into account that the guaranteed part is risk free.

From time-series data for real risky bond prices and a suitable model, such as described above, we can calculate the value of the instantaneous risk of default for each data point that is needed for the model to give a theoretical value equal to the market value of the bond. This number is called the 'implied instantaneous risk of default' and plays a role in default risk analysis that is similar to that played by implied volatility for options: it is used as a trading indicator or as a measure of relative value.

The models were chosen as follows.

The spot rate model

We assume that the spot interest rate is uncorrelated with the risk of default, and that there is no other coupling between these two variables.

The default model

The risk of default was assumed to satisfy

$$dp = (f - hp)dt + j\sqrt{p}dX_1.$$

The risk of default is mean reverting to approximately the level *f*/*h*. (We have not included any interest rate dependence in this because, provided $f > j^2/2$, this precludes the possibility of negative risk of default.) The speed of this reversion is determined by *h*. For Argentina, *h, f* and *j* were chosen as in the table below.

	h (speed of mean reversion)	*f*/*h* (approx. level to which risk reverts)	*j* (volatility parameter)
Argentina	0.5	9%	.03

The choice/determination of these parameters is of some interest. There are two approaches we can adopt. One is to choose parameters based on plausibility or common sense. The other is to try and fit them so that the implied risk of default time series is consistent with the parameters. The former is not entirely satisfactory, and somewhat arbitrary. The latter is both very complicated and time-consuming, and, in a sense, assumes that the market already 'knows about' the model we are using. For that reason we have taken a combined approach. We try to fit as well as we can, without worrying too much about the precision.

The parameters reflect a 'memory' of two years in the drift rate ($h = 0.5$) and have similar levels for the mean risk and volatility as measured by the constant *p* model. Finally, the value for *p* was chosen daily so that the market price of the bonds and their theoretical price coincided. Because of the assumption that the interest rate model and the risk of default model are uncoupled, the price of a risky coupon is just the product of the risk-free present value multiplied by a default factor. This makes yield curve fitting very simple. Having found a time series for the implied risk of default we compared some of its statistical properties with the theoretical results given below.

The steady-state distribution for p is given by the probability density function

$$\left(\frac{2h}{j^2}\right)^{2f/j^2} \frac{1}{\Gamma\left(2f/j^2\right)} p^{-1+2f/j^2} e^{-2hp/j^2},$$

where $\Gamma(.)$ is the gamma function. Thus the average value of p is

$$\frac{j^2}{2h}\frac{\Gamma\left(1+2f/j^2\right)}{\Gamma\left(2f/j^2\right)}$$

and the standard deviation

$$\frac{j^2}{2h}\sqrt{\frac{\Gamma\left(2+2f/j^2\right)}{\Gamma\left(2f/j^2\right)}-\left(\frac{\Gamma\left(1+2f/j^2\right)}{\Gamma\left(2f/j^2\right)}\right)^2}.$$

We iterated on the values for the parameters to get a mean and standard deviation, and a volatility, for the time series that was as close as possible to the theoretical values. We were able to get a mean and standard deviation that were within 25% of the theoretical values and a volatility that was within 50%. To expect to do any better with such a simple model would be highly ambitious.

The period chosen (end December 1993 to end September 1996) is a particularly exciting one because of the Tequila Effect and it could easily be argued that there was a dramatic change of market conditions (and hence model parameters) at that time. However, we have kept the same parameters for the whole of this period since it was risk of default causing the Tequila effect and should therefore be accounted for in these parameters.

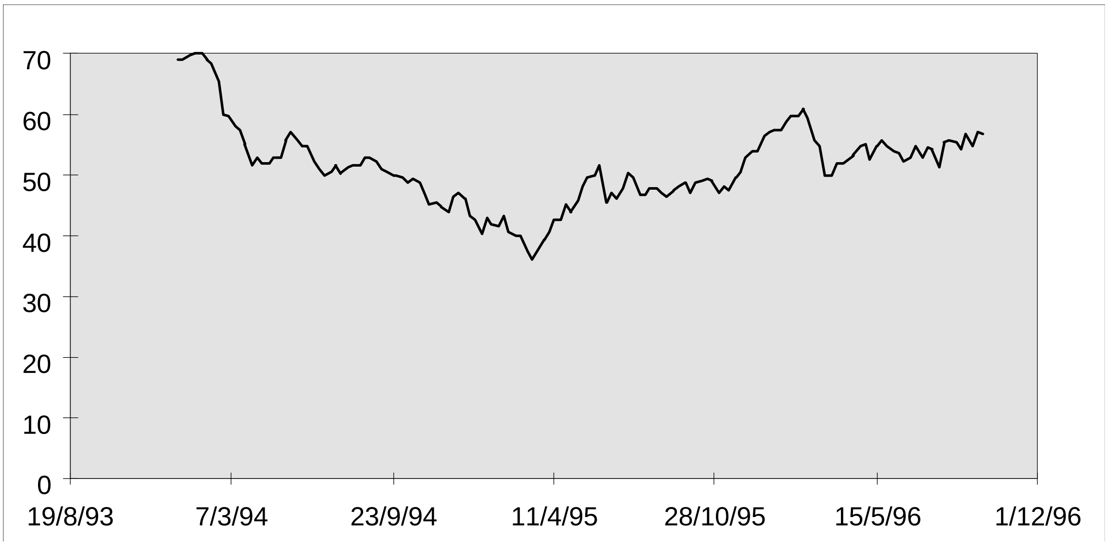

Figure 3: Market price of the Argentinean Par bond from end December 1993 to end September 1996.

The Tequila effect took place in December 1994 but its consequences lasted much longer, in some countries up to three and four months. In the case of Argentina, we can observe the minimum price of the Par bond at the end of March 1995, dipping below $35. Since then a steady recovery can be seen.

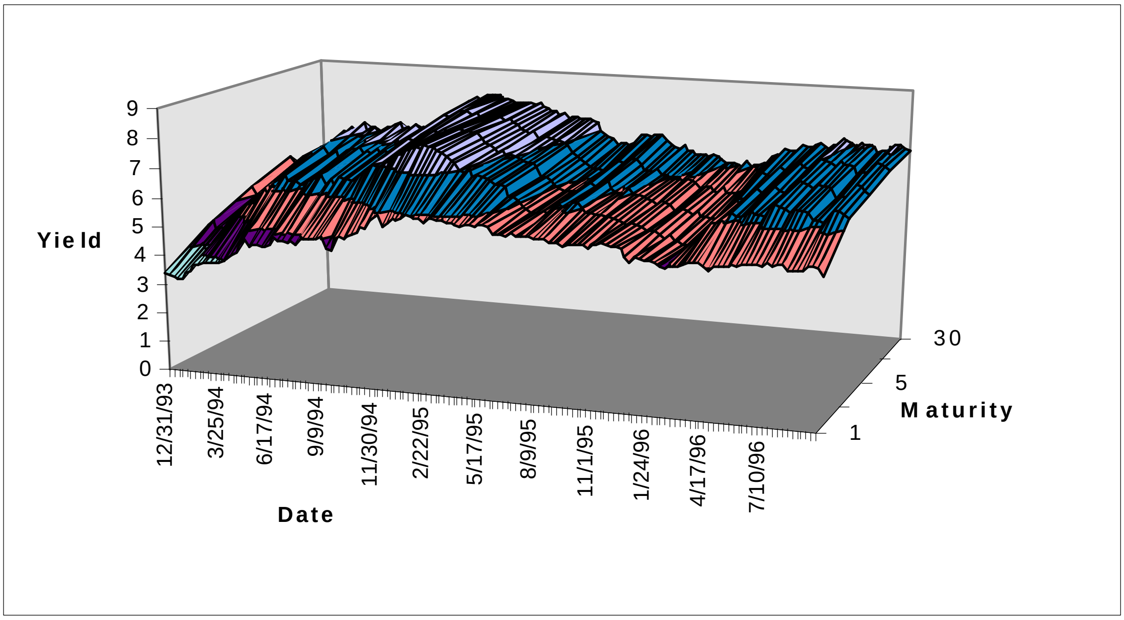

Figure 4: US yield curve from end December 1993 to end September 1996.

We can observe that the Tequila effect was accompanied by a sharp increase in long rates in the US, which knocked Brady bond prices even further. The highest long rate over this period was 8% and it occurred in March 1995.

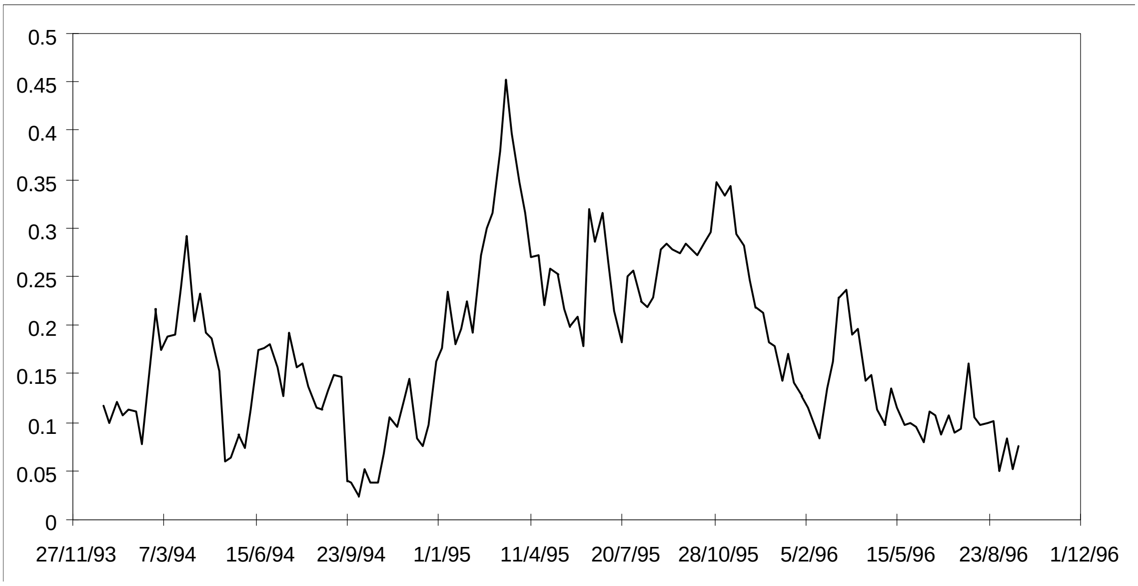

Figure 5: The implied risk of default for the Argentinean Par, see text for description of stochastic model.

In Figure 5 we show the implied instantaneous risk of default for Argentinean Par bonds over the period end December 1993 to end September 1996. As expected, the highest probability of default took place at the end of March 1995 when the Tequila Effect was at its worse and long rates at their highest. Since then there has been a steady, but obviously not monotonic, decrease in the risk of default implied by this model.

Conclusions

In this paper we have presented a model for the instantaneous risk of default, and found some explicit solutions for risky bond prices. We have applied the model to the Argentinean Par bond to derive the implied risk of default from just before the Tequila Effect until quite recently. Future work will concentrate on examining other Argentine bonds and bonds from other Latin American countries.

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References

Cooper, I & Martin, M (1996) Default risk and derivative products. Applied Mathematical Finance **3** 53-74

Cox, J, Ingersoll, JE & Ross, S (1985) A theory of the term structure of interest rates. Econometrica **53** 385-407

Duffie, D & Singleton, KJ (1994) Econometric modelling of term structures of defaultable bonds. Working Paper, Graduate School of Business, Stanford Univ.

Ho, TS & Lee, SB (1986) Term structure movements and pricing interest rate contingent claims. Journal of Finance **41** 1011-1029

Hull, J & White, A (1990) Pricing interest rate derivative securities. Rev. Fin. Stud. **3** 573-592

Lando, D (1994) On Cox processes and credit risky bonds. Working Paper, Inst. of Mathematical Statistics, Univ. of Copenhagen

Longstaff, FA & Schwartz, ES (1994) A simple approach to valuing risky fixed and floating rate debt. Working Paper 22-93 Anderson Graduate School of Management, Univ. of California at Los Angeles

Merton, RC (1974) On the pricing of corporate debt: the risk structure of interest rates. Journal of Finance **29** 449-470

Rebonato, R (1997) *Interest-rate Option Models*, Wiley

Schönbucher, PJ (1996) The term structure of defaultable bonds. Univ. of Bonn Working Paper

Vasicek, O (1977) An equilibrium characterisation of the term structure. Journal of Financial Economics **5** 177-188

Wilmott, P, Dewynne, JN & Howison, SD 1993 *Option Pricing: Mathematical Models and Computation*, Oxford Financial Press(**LINK:www.oxfordfinancial.co.uk**)

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