A GENERAL FRAMEWORK FOR HEDGING AND SPECULATING WITH OPTIONS

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Abstract:
In contrast to their role in theory options are in practise not only traded for hedging purposes. Many investors also use them for speculation purposes. For these investors the Black-Scholes price serves only as an orientation, their decisions to buy, hold or hedge an option are also based on subjective beliefs and on their personal utility functions (in the widest possible sense). The aim of this paper is to present a general framework to include different types of investors, especially hedgers, pure speculators and speculators following strategies with bounded risk. We derive their subjective values of an option endogenously from the solution of their decision problems.

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1. Speculation, full and partial hedging : options as multipurpose securities

While the theory of financial markets typically treats options as an instrument suited for hedging purposes only (which is otherwise redundant) practitioners often use them in different ways. First, there is the pure speculator who buys options only for speculative reasons. He will buy an option only if his subjective value of it exceeds the current market price. The replication argument for deriving the Black-Scholes price has only minor importance for the actions of such a speculator (see also Korn and Wilmott (1996)). In contrast to him, the hedger who needs an option position for insurance reasons in his portfolio will strongly favour the Black-Scholes price. As a third type one could think of a speculator who has a partial hedging approach to bound his risk from speculating. His typical strategy could be of the form "Hold the option until it reaches a sufficiently high value, then hedge this position to insure the gain" or "Hedge a part of the option position, let the remaining part evolve freely (as long as it is favourable". The strategy could also involve a second criterion to hedge the whole position if the evolution of the underlying is totally different from the investor's expectations. Of course, one could further include the possibility to sell the option at the market again if the market price is favourable.

One reason for the attractiveness of trading in options is the fact that the derivative market is usually more liquid than the equity market and offers a great flexibility due to the vast number of different types of contracts traded. Also, agreement on prices is easier than in the equity market due to the acceptance of the Black-Scholes formula (and its appropriate adaptations to contracts different from European calls and puts) for pricing derivatives. Hence, if an investor has a subjective view on the value of an option contract he can always base his decision on a comparison between his subjective value and the Black-Scholes price.

In this paper we outline a framework that includes all the above mentioned types of investors and their strategies. The subjective value of an option will always be determined via the solution of an investment problem. By specifying different strategies of the investor, imposing constraints on the evolution of the (subjective) wealth process describing the investment problem and on measures for the riskiness of a strategy we will consider a variety of situations where the subjective option value can differ from the Black-Scholes price. In fact, the advantage of such an approach is that an investor can determine his personal value of an option depending on the way that he will use the option. Comparison to the traded price then tells him if the intended use of the option is reasonable or not.

Section 2 will contain the basic set-up of this investment problem and the study of some examples in the case with constant market coefficients. The extension of this situation to the case of a two-valued drift rate as in Korn and Wilmott (1996) is the subject of section 3. Here, we will also look at a situation where the subjective option value will be underpinned by the Black-Scholes price by introducing the possibility of a position closure. Numerical examples including a comparison of different option values and other features of our new approach will be given in section 4.

2. The subjective value of an option

In this section we start by considering a very basic financial market consisting of a bond and one risky asset with price dynamics given by

$$dB_t = B_t \; r\, dt \qquad B_0 = 1 \tag{2.1}$$

$$dS_t = S_t [\, \mu\, dt + \sigma\, dW_t \,] \qquad S_0 = s \tag{2.2}$$

where the market coefficients are all assumed to be constant and W_t is a one-dimensional Brownian motion. We look at an investor who holds an option on the risky asset with a final payoff $f(S_T)$. To hedge himself fully or partly against the uncertainty of this position he also trades in the underlying and in the bond. More precisely, his strategy is given by the portfolio $(1,-\Delta_t,-\varphi_t)$ at time t consisting of an option, $-\Delta_t$ units of the risky asset, and $-\varphi_t$ units of the bond. Let V(t,S) be the subjective value of the option at time t when the price of the risky asset is S. V is assumed to be sufficiently smooth. Define

$$X(t) = V(t,S_t) - \Delta_t S_t - \varphi_t B_t \tag{2.3}$$

X(t) is the wealth process corresponding to the above portfolio **if and only if** $V(t,S_t)$ is the market price of the option. Otherwise it is just a **subjective belief** of the wealth of the portfolio. The comparison of the subjective belief with the real market value will indicate if it is worth (in the subjective view of the investor) holding the portfolio or not. It is certainly no reasonable situation for an investor to hold a portfolio which he thinks is overpriced (at least for his intended use of the portfolio). Because we require the portfolio strategy to be self-financing we get

$$\begin{aligned} dX(t) &= \{V_t(t,S_t)+(V_S(t,S_t)-\Delta_t)\mu S_t + ½\, \sigma^2 S_t^2 V_{SS}(t,S_t) - r\varphi_t B_t \}dt + (V_S(t,S_t)- \Delta_t)\sigma S_t\, dW_t \\ &= \{V_t + (V_S - \Delta_t)\mu S_t + ½\, \sigma^2 S_t^2 V_{SS} - r(V- \Delta_t S_t- X(t)) \}dt + (V_S - \Delta_t)\sigma S_t\, dW_t \\ &=: \quad a(t)\, dt + b(t)\, dW_t \end{aligned} \tag{2.4}$$

where we have omitted the arguments for V and its partial derivatives in the second line. Of course, it is clear that V has to satisfy the terminal condition

$$V(T,S) = f(S) \tag{2.5}$$

where f should grow at most polynomial in S.

We now have a lot of possible choices of the "free parameters" to specify the portfolio and option valuation problem given by equations (2.3/4/5). In the following we will specify certain forms of the coefficients a(t) and b(t) and/or the strategy Δ_t (respectively φ_t) or impose constraints on them. This will lead to a partial differential equation (for short : pde) for V that thus determines the subjective value of the option. By using

$$\Delta_t = V_S - \frac{b(t)}{\sigma S_t} \tag{2.6}$$

the most general form of this pde would be

$$V_t + \tfrac{1}{2}\sigma^2 S^2 V_{SS} + rV_S S - rV = a(t) - rX(t) - \tfrac{\mu - r}{\sigma}\, b(t) \tag{2.7a}$$

$$V(T,S) = f(S) \tag{2.7b}$$

The right side of equation (2.7a) is the part that is different to the Black-Scholes equation. Its interpretation is highly dependent on the choice of a(t) and b(t). It can be seen best during the following examples. In these examples we show how certain goals, beliefs or strategies of an investor can be formulated in this approach.

Example 2.1 "Full hedging"

The requirement of a riskless portfolio, i.e. $b(t) \equiv 0$ for all $t \in [0, T]$, leads to

$$\Delta_t = V_S \qquad \forall\, t \in [0, T]\,.$$

For arbitrage reasons it is clear that we must also require a(t) = r X(t) which leads to

$$V_t + \tfrac{1}{2}\sigma^2 S^2 V_{SS} + rV_S S - rV = 0 \quad \forall\, t \in [0, T)$$
$$V(T,S) = f(S)$$

i.e. we have the Black-Scholes equation. Hence, our approach is a generalisation of the usual replication approach to option valuation. In this example the subjective value of the investor equals the Black-Scholes price because his only intention was hedging. The choice of x or X(T) in this example is more or less irrelevant. A choice of x as the initial wealth of our portfolio leads to a terminal wealth of $X(T) = x \exp(rT)$ (note that the numbers of bonds and shares to obtain a perfect hedge are already uniquely determined; a positive (negative) x corresponds to an additional long (short) position in the bond).

Example 2.2 "Pure speculating"

Now we look at an investor who buys an option for speculative reasons only. He has no stock position and holds the unhedged option position, i.e. we obtain

$$\Delta_t = 0 , \quad b(t) = V_S \sigma S_t \quad \forall\, t \in [0, T] .$$

By requiring a(t) = rX(t) we get

$$V_t + \tfrac{1}{2} \sigma^2 S^2 V_{SS} + \mu V_S S - rV = 0 \quad \forall\, t \in [0, T)$$
$$V(T,S) = f(S)$$

for the subjective value V(t,S) in this case. For a simple call option this value is bigger than the Black-Scholes price iff the mean rate of stock return μ is bigger than the riskless interest rate r. V(t,S) then coincides with the simplest example for a subjective value of an option as defined in Korn and Wilmott (1996). Hence, our new approach also generalises the one in Korn and Wilmott (1996).

Example 2.3 "Speculating with position closure"

If we consider options with a terminal payoff that is not monotonous in the stock price then the subjective option value computed in the foregoing example need not be monotonous in the drift rate μ. Hence, this subjective value can be higher or lower than the corresponding Black-Scholes type price (which is the subjective value with μ equal to r) according to the current stock price. It is therefore reasonable for an investor whose decisions are (at least partly) based on the subjective option value that he will buy and hold an option only if the subjective value exceeds the Black-Scholes price (which we assume to be the market price). Thus, the investor would close his position by selling the option at the market at the first time instant when the subjective value and the Black-Scholes price coincide. As a result the pde of Example 2.2 for the subjective option value will be substituted by the free boundary problem

$$V_t + \tfrac{1}{2} \sigma^2 S^2 V_{SS} + \mu V_S S - rV = 0 \quad \forall\, t \in [0, T)$$
$$V(T,S) = f(S)$$
$$V(t,S) \geq BS(t,S) \quad \forall\, t \in [0, T)$$

where we only require V and its first derivatives to be continuous. Of course, this value is at least as big as the Black-Scholes price.

Example 2.4 "Partial hedging"

This example contains elements of Examples 2.1 and 2.2. By putting the constraints

$$\alpha V_S \leq \Delta_t \leq \beta V_S$$

on the stock position the investor cannot fully hedge the risk in his option position. He will now choose Δ_t to maximise the drift rate a(t). This leads to

$$\Delta_t = \begin{cases} aV_S , \; if \;\; V_S \geq 0 \\ bV_S , \; if \;\; V_S < 0 \end{cases}$$

The usual requirement of a(t) = rX(t) leads to the pde

$$V_t + \tfrac{1}{2}\sigma^2 S^2 V_{SS} + V_S S\left(\mu - (\mu - r)(\alpha 1_{\{V_S \geq 0\}} + \beta 1_{\{V_S < 0\}})\right) - rV = 0$$
$$V(T,S) = f(S)$$

which coincides with the Black-Scholes equation for $\alpha = \beta = 1$ and with the equation of Example 2.2 for $\alpha = \beta = 0$. By approximating the function

$$g(V_S) = \mu - (\mu - r)(\alpha 1_{\{V_S \geq 0\}} + \beta 1_{\{V_S < 0\}}$$

by a sequence g_n of functions which are bounded and Hölder continuous we get the existence of a unique solution to the above pde where g is substitued by g_n (see Karatzas and Shreve (1988), p. 368). But this is also the unique viscosity solution to this corresponding pde. By the well-known stability features of viscosity solutions there will also exist a unique viscosity solution of the limiting pde, i.e. the one above. Looking at the form of the pde one can easily deduce that for $\mu > r$ the subjective value V(t,S) will be bigger than the Black-Scholes price if we have $\alpha \leq 1 \leq \beta$.

However, at first sight it seems to be a contradiction that on one hand the investor chooses Δ_t to maximise the drift rate of X(t) while on the other hand we require a(t) to be equal to the riskless drift rate. The interpretation of this is as follows : the requirement of a(t) = rX(t) gives us the possibility to compare the performance of the resulting strategy with a riskless investment, and by choosing Δ_t in the above way we ensure that the pde for V(t,S) will be derived from an optimal action. More precisely, there is no other strategy delivering the required drift rate for a lower value of V(t,S). I.e. we avoid a possible overvalueing of the option by choosing a non-optimal strategy.

The form of Δ_t can be interpreted as a form of "speculate until a sufficient gain is made, then save it"-strategy. As long as the (subjective) option value grows with the stock price the investor hedges as few as possible ("$\Delta_t = \alpha V_S$"), or otherwise expressed, tries to speculate on the maximum allowed level. If its value decreases the investor tries to hedge as much of the achieved position as possible ("$\Delta_t = \beta V_S$").

Example 2.5 "A mean-variance strategy"

While in the examples considered so far we have more or less concentrated on the risk in our portfolio (expressed by the diffusion coefficient of X(t)) we now look at both the diffusion and drift coefficient at the same time. To do this we formulate a mean-variance type problem (for the local parameters of the equation for X(t)) that consists of maximising

$$\begin{aligned} f(\Delta_t) &= a(t) - \delta\, b(t)^2 \\ &= V_t + \tfrac{1}{2}\sigma^2 S_t V_{SS} - r(V - \Delta_t S_t - X(t)) + (V_S - \Delta_t)\mu S_t - \delta\, ((V_S - \Delta_t)\sigma S_t)^2 \end{aligned}$$

where δ is a positive parameter describing the risk aversion of the investor. The maximiser of f can be easily computed as

$$\Delta_t^* = V_S - \frac{\mu - r}{\delta\sigma^2 S_t}$$

Note that such a strategy leads to a diffusion coefficient b(t) of the form

$$b(t) = \frac{\mu - r}{\delta\sigma}$$

which is constant (independent of the stock price !). The strategy Δ_t^* has the remarkable feature that it tends to V_S if the stock price is high while it will be strongly negative for small values of S_t. Comparison of this type of option investment to a riskless bond investment (i.e. requiring a(t) = rX(t)) leads to the following pde for the subjective value of the option.

$$V_t + \tfrac{1}{2}\sigma^2 S^2 V_{SS} + rV_S S - rV + \tfrac{1}{d}\left(\tfrac{m-r}{s}\right)^2 = 0$$

$$V(T,S) = f(S)$$

where the additional constants is just the product of the square of the mean-variance trade off and the inverse of the coefficient of risk aversion. Using standard argumnets (as for example presented in Friedman (1964)) one can again verify that there exists a unique solution to this pde. Compared to a riskless investment this strategy has a higher drift rate as long as μ is bigger than r. Thus, the intended use of the option delivers a higher drift rate than a riskless investment. Also, the form of the pde the subjective value of the option is higher than the Black-Scholes price.

Comparison of this strategy to a pure stock investment is not at all easy. On one hand, the diffusion coefficient can be higher or lower than σS_t (depending on the value of S_t), on the other hand there is also no clear ordering of the resulting drift coefficient.

3. Subjective option value and changing drift rates : A more complex view

In this chapter we allow an investor to have a more detailed view towards the future development of a stock price. While in the foregoing section he was forced to have a unique view expressed in the choice of a single drift and a single diffusion coefficient he is now allowed to think about possible changes in the future behaviour of the price movements. As in section of Korn and Wilmott (1996) we model these possibilities via a changing drift rate of the stock price. The drift coefficient μ_t is allowed to attain the values μ_1 and μ_2 with $\mu_1 \geq \mu_2$. Such an approach increases the flexibility for modelling. We assume that whenever the drift rate is μ_1 (resp. μ_2) then the intensity for a change of the rate to μ_2 (resp. μ_1) is $\lambda_1 dt$ (resp.$\lambda_2 dt$), i.e. the time until the next change of the drift parameter has an exponential distribution with parameter λ_1 (resp. λ_2), or in stochastic differential notation :

$$d\mu_t = (\mu_2 - \mu_{t-})\, dN_t^{(1)} + (\mu_1 - \mu_{t-})\, dN_t^{(2)} = \sum_{i=1}^{2} (\mu_i - \mu_{t-})\, \{d\tilde{N}_t^{(i)} + \lambda_i dt\}$$

where $N_t^{(i)}$ ($\tilde{N}_t^{(i)}$) is a (centered) Poisson process with intensity λ_i, i = 1,2. If we now assume that the subjective option value V (as constructed in section 2) can also depend on the current value μ_t of the drift parameter then we have the following representation :

$$\begin{aligned} dV(t,S_t,\mu_t) = [\, & V_t+\mu_{t-}SV_S+\tfrac{1}{2}\sigma^2S^2V_{SS}+\{V(t,S_t,\mu_{t-}+(\mu_2-\mu_{t-}))-V(t,S_t,\mu_{t-})\}\lambda_1 \\ & + \{V(t,S_t,\mu_{t-}+(\mu_1-\mu_{t-}))-V(t,S_t,\mu_{t-})\}\lambda_2\,]\, dt \;+\; \sigma SV_S dW_t + \\ & + \{V(t,S_t,\mu_{t-}+(\mu_2-\mu_{t-}))-V(t,S_t,\mu_{t-})\}d\tilde{N}_t^{(1)} \\ & + \{V(t,S_t,\mu_{t-}+(\mu_1-\mu_{t-}))-V(t,S_t,\mu_{t-})\}d\tilde{N}_t^{(2)} \end{aligned} \tag{3.1}$$

We refer the interested reader to Korn and Wilmott (1996) for a detailed discussion of the impact of the investor's beliefs on the choice of the parameters λ_i. Before we look at the examples presented in section 2 in our new framework we introduce a useful notation. Let V^i be the subjective option value if the current value of μ_t is μ_i. Then equation (3.1) simplifies to

$$\begin{aligned} dV^i(t,S_t) \;=\; [\, & V^i_t+\mu_i SV^i_S+\tfrac{1}{2}\sigma^2S^2V^i_{SS}+\{V(t,S_t,\mu_i+(\mu^*-\mu_i))-V^i(t,S_t)\}\lambda_i]\, dt \\ & + \sigma SV^i_S dW_t +\{V(t,S_t,\mu_i+(\mu^*-\mu_i))-V^i(t,S_t)\}d\tilde{N}_t^{(i)} \end{aligned} \tag{3.2}$$

with $\mu^* = \begin{cases} m_1 & , \textit{if } i = 2 \\ m_2 & , \textit{if } i = 1 \end{cases}$. We therefore introduce the notation $X^i(t)$ with the obvious interpretation. The stochastic differential equations for $X^i(t)$ then have the form

$$dX^i(t) = a^i(t)\, dt + b^i(t)\, dW_t + c^i(t)\, d\tilde{N}_t^{(i)} \tag{3.3}$$

with

$$a^i(t) = V^i_t+\mu_i S(V^i_S-\Delta_t) + \tfrac{1}{2}\sigma^2S^2V^i_{SS}+\{V(t,S_t,\mu_i+(\mu^*-\mu_i))-V^i(t,S_t)\}\lambda_i - r\varphi_t B_t \tag{3.4}$$

$$b^i(t) = (V^i_S - \Delta_t)\sigma S_t \tag{3.5}$$

$$c^i(t) = V(t,S_t,\mu_i+(\mu^*-\mu_i))-V^i(t,S_t) \tag{3.6}$$

Of course, the final condition for V^i should be independent of the value of the drift rate :

$$V^i(T,S) = f(S) \tag{3.7}$$

We can again derive partial differential equations characterising the subjective option value(s) V^i, i = 1, 2, by specifying certain forms of the coefficients $a^i(t)$,

$b^i(t)$, $c^i(t)$ and / or the strategy Δ_t (respectively φ_t). By noting that relation (2.6) is still valid as

$$\Delta_t = V^i{}_S - \frac{b^i(t)}{\sigma S_t} \tag{3.8}$$

we get the following general form of these pde

$$V^i{}_t + ½\, \sigma^2 S^2 V^i{}_{SS} + r V^i{}_S S - r V^i = a^i(t) - r X^i(t) - \tfrac{\mu - r}{\sigma}\, b^i(t) - \lambda_i c^i(t) \tag{3.9a}$$

$$V^i(T,S) \;\; = f(S) \tag{3.9b}$$

Due to the special form of $a^i(t)$ and $c^i(t)$ these equations form a coupled two-dimensional sytem. Just remind yourself of the relations

$$V(t,S_t,\mu_1+(\mu^*-\mu_1)) = V^2\,(t,S_t) \quad V(t,S_t,\mu_2+(\mu^*-\mu_2)) = V^1\,(t,S_t) \tag{3.10}$$

We now look at the examples already presented in section 2. Also, we will examine the effect of position closure at the first time when the subjective value falls below the Black-Scholes price of the option.

Example 3.1 "Full hedging"

The full hedging requirement would now force the coefficients $b^i(t)$ and $c^i(t)$ to be identical to zero for all $t \in [0, T]$ and $i = 1,2$. Hence, we should have

$$\begin{aligned} V^2\,(t,S) &= V^1\,(t,S) \quad \forall\, (t,S) \in [0, T] \times (0, \infty) \\ \Delta_t &= V^i{}_S \qquad\;\; \forall\, t \in [0, T]\,. \end{aligned}$$

Therefore, we end up with a single pde for $V = V^1 = V^2$ and the (local) no arbitrage requirement $a^i(t) = r\, X^i(t)$ which leads to the Black-Scholes equation. As a special consequence one could say that a total hedging requirement leaves no space for having a non-unique view on the future price dynamics (or at least : this view does not enter into the option value).

Example 3.2 "Pure speculating"

In this case where the investor has no stock position and holds the unhedged option position we obtain

$$\Delta_t = 0\,, \quad b^i(t) = V^i{}_S \sigma S_t \quad \forall\, t \in [0, T]\,.$$

By requiring $a^i(t) = r X^i(t)$ and using relation (3.10) we get the following system

$$\tfrac{\partial}{\partial t} V^1 + ½\, \sigma^2 S^2 \tfrac{\partial^2}{\partial S^2} V^1 + \mu_1 S \tfrac{\partial}{\partial S} V^1 - r V^1 + \lambda_1 (V^2 - V^1) = 0 \tag{3.11}$$

$$\tfrac{\partial}{\partial t} V^2 + ½\, \sigma^2 S^2 \tfrac{\partial^2}{\partial S^2} V^2 + \mu_2 S \tfrac{\partial}{\partial S} V^2 - r V^2 + \lambda_2 (V^1 - V^2) = 0 \tag{3.12}$$

(with the obvious final conditions) which was already reported in Korn and Wilmott (1996).

Example 3.3 "Speculating with position closure"

As a variant of the foregoing example we now introduce the possibility that the investor sells the option at the first time where its subjective value $V(t,S_t,\mu_t)$ falls below (in the sense of lower or equal to) the Black-Scholes price. Note, that due to the discontinuities in the paths of the stock drift parameter process the subjective option value need not be continuous in time along its paths. This new feature will turn the pde system (3.11/12) into one with a free boundary condition

$$V^2,\ V^1 \geq BS(r,t,S) \tag{3.13}$$

where we require V^2, V^1 and its first derivatives to be continuous. Although the argument in Example 2.3 for introducing the possibility of closing the position is valid here too, the discontinuities in price paths are a different feature which is only possible in a non-constant drift rate setting. We will compare the two cases of a constant and a two-valued drift rate in Section 4.

Remarks on existence and uniqueness

Example 3.4 "Partial hedging"

This example contains elements of both the foregoing ones. By putting the constraints

$$\alpha\, V^i_S \leq \Delta_t \leq \beta\, V^i_S$$

on the stock position the investor cannot fully hedge the risk in his option position. He will now choose Δ_t to maximise the drift rate a(t). This leads to

$$\Delta_t = \begin{cases} aV^i{}_S\,,\ if\ \ V^i{}_S \geq 0 \\ bV^i{}_S\,,\ if\ \ V^i{}_S < 0 \end{cases} \qquad \text{if } \mu(t) = \mu_i$$

The usual requirement of a(t) = rX(t) leads to the following system of pdes

$$V^1{}_t + ½\,\sigma^2S^2V^1{}_{SS} + V^1{}_S S(\mu_1 - (\mu_1\text{-}r)(\alpha 1_{\{V^1{}_S \geq 0\}} + \beta 1_{\{V^1{}_S < 0\}})) - rV^1 + \lambda_1(V^2 - V^1) = 0$$

$$V^2{}_t + ½\,\sigma^2S^2V^2{}_{SS} + V^2{}_S S(\mu_2 - (\mu_2\text{-}r)(\alpha 1_{\{V^2{}_S \geq 0\}} + \beta 1_{\{V^2{}_S < 0\}})) - rV^1 + \lambda_2(V^1 - V^2) = 0$$

$$V^i(T,S) = f(S)$$

for the subjective option values $V^i(t,S)$ as a generalisation of the pde of Example 2.4 . Again, the effect of introducing the more general model for the drift rate will be demonstrated via some numerical examples in Section 4.

Remark about existence/uniqueness in the sense of viscosity solutions.

Example 3.4 "A mean-variance strategy"

Picking up the mean-variance example from Section 2 we try to maximise

$$f^i(\Delta_t) = a^i(t) - \delta\, b^i(t)^2 - \eta\, c^i(t)^2$$
$$= V^i_t + \tfrac{1}{2}\sigma^2 S_t V^i_{SS} - r(V^i - \Delta_t S_t - X(t)) + (V^i_S - \Delta_t)\mu_i S_t - \delta\,((V^i_S - \Delta_t)\sigma S_t)^2$$
$$- \eta\,(V(t,S_t,\mu_i+(\mu^*-\mu_i)) - V^i(t,S_t))^2$$

with positive parameters δ, η describing the risk aversion of the investor towards (stock price) diffusion risk and (option value) jump risk. The maximiser of f has the same form as in the mean-variance example of the last section. The jump risk has no impact, and we have

$$\Delta_t^* = V^i_S - \frac{\mu_i - r}{\delta\sigma^2 S_t}$$
$$b^i(t) = \frac{\mu_i - r}{\delta\sigma}$$

i.e. the diffusion coefficient of the subjective option value is piecewise constant (independent of the stock price !). Comparison of this type of option investment to a riskless bond investment (i.e. requiring $a^i(t) = rX(t)$) leads to the following system of pdes for the subjective option values V^i

$$V^1_t + \tfrac{1}{2}\sigma^2 S^2 V^1_{SS} + rV^1_S S - rV^1 + \frac{1}{d}\left(\frac{m_1 - r}{s}\right)^2 + \lambda_1(V^2 - V^1) = 0$$
$$V^2_t + \tfrac{1}{2}\sigma^2 S^2 V^2_{SS} + rV^2_S S - rV^2 + \frac{1}{d}\left(\frac{m_2 - r}{s}\right)^2 + \lambda_2(V^1 - V^2) = 0$$
$$V^i(T,S) = f(S)\ .$$

4. Numerical Examples

In this section we will give some numerical examples for comparing the different subjective option values which were proposed in the foregoing sections. We will also look at the effect of the introduction of a two-valued drift rate process in the particular situations that were leading to the proposed subjective option values.

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