Room for a View

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There is no room in the classical Black-Scholes framework for the market view of an investor. The investor in derivatives needs to know the volatility of the underlying, that is the 'choppiness' of the market, but the direction is irrelevant. Suppose we have two stocks A and B having the same volatility, 20%, say. They both have a value 100 today and there are call options on these two stocks with strike of 100 and an expiry of six months. In the market these options will have the same value. If you were to invest in one of these call options, which would you choose to buy? If you are a pure speculator, we hope you will ask us 'In which direction are the two stocks expected to move in the next six months?' So, now we tell you that stock A has an expected return of 20% and stock B has an expected return of -20%. Are you indifferent between these two calls? Obviously not. Yet the pure delta hedger is. How can we reconcile these two positions?

In the perfect world, Black and Scholes set the price
The argument for the irrelevance of expected return goes something like this. In the perfect world of Messieurs B&S, delta hedging is possible. So let's sell an option for a little bit more than the B&S value, delta hedge until expiry, and collect a nice risk-free profit. Or, buy an option for a little bit less…collect a nice risk-free profit. Conclusion, anyone valuing a derivative at other than B&S's value will be handing out free money. This delta hedging is a completely risk-free exercise, and the principal of no-arbitrage says that such a situation cannot exist because arbitrageurs will quickly move in to exploit the 'mispricing' and, in the process, eliminate it. This is the basis of risk-neutral valuation, that the fair value of a derivative is the expected PV of all cashflows under a risk-neutral random walk. This principle applies whether the contract is an option on a stock, currency or commodity, and even in the fixed-income world.

Pay only as much as you think it's worth
The above no-arbitrage argument can be put in a simpler form: 'If you have two opportunities generating the same payment then they should have the same value'. There is not too much that can be said against this as long as you really do have two opportunities. For a speculator there are many reasons why delta hedging is no real alternative to holding an option position. Delta hedging involves trading at every time instant during the lifetime of the option. It can also result in huge amounts of transaction costs. And last but not least due to market imperfections (such as no frictionless trading, no infinite speed of transactions,…) it is simply not possible. Therefore, in judging the advantages of holding an option position another more general rule than the no-arbitrage rule serves as an aid for a speculator: 'Buy something if you think it's at least worth its price'. Our suggestion to decide if an option price offers you a good deal if you cannot delta hedge (or don't want to!) is to compare it with its expected PV with respect to your view.[1]

Best of both worlds
It is important to understand that taking your own view into account when making a decision whether or not to buy an option does not say that BS is wrong. Taking a view only helps you to decide if the BS price (which we assume to be the market price) offers you a good deal or not. As you cannot (or don't want to) delta hedge, the no-arbitrage argument is relevant for the market to settle the option price but not for you personally. Thus, there is no question of best of both worlds, as we do not argue against BS being the market price (or the correct theoretical option value), rather, we only want to add to the trader's

[1] See Korn & Wilmott (1996, 1997) for further details.

armoury and suggest ways in which his view of the market can be allowed for and, in particular, quantified.

Taking a view

If we express our own view on the 'worth' of an option position via the expected PV with respect to our own view this simply means that in computing it we assume we know the drift term of the price process of the underlying and compute the expected PV with respect to this price process and not with respect to the risk-neutral one.

Before describing possible models for the drift, we must discuss its measurement. What can we say? Measuring the drift of an asset is hard, harder even than measuring its volatility. And since the drift of the underlying has never been needed for *pricing* derivatives, very few people (other than fund managers) even try to measure it. On the other hand, all traders have a gut feeling for the direction of the underlying. They exploit this instinct either by pure speculation or, at the very least, by under/over hedging at times.

In order to make optimal use of the trader's instinct we will assume that he can translate his instinct into an estimate for the expected drift. This estimate will appear in our 'valuation' model. Here we put valuation in inverted commas to emphasise the distinction between our value, which can be used as a guide to what to buy and when to hedge, and the BS fair value at which the option can typically be bought and sold.

We have a lot of ideas to convey in this article, alternative 'valuation' models, optimal position closure, optimal hedging strategies and new models for the underlying. We introduce them one at a time, beginning with the simplest valuation model and a constant, known, drift.

Risk and expected return

Hold an unhedged position until expiry and the present value of your real expected payoff will be given by the solution of

$$\frac{\partial V}{\partial t}+\frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2}+\mu S\frac{\partial V}{\partial S}-rV=0 \quad \text{with} \quad V(S,T)=\text{payoff function.}$$

Here S is the price of the underlying, assumed to satisfy

$$dS=\mu S dt+\sigma S dX,$$

Time is denoted by t, σ is the volatility and μ is the constant drift, r is the risk-free interest rate. This unhedged position is risky. The risk can be measured by the standard deviation of the return

$$SD(S,t)=\sqrt{G(S,0)-V(S,0)^2}$$

where G(S,t) satisfies

$$\frac{\partial G}{\partial t}+\frac{\sigma^2 S^2}{2}\frac{\partial^2 G}{\partial S^2}+\mu S\frac{\partial G}{\partial S}=0 \quad \text{with} \quad G(S,T)=\mathrm{e}^{-rT}(\text{payoff function})^2.$$

The expected return and risk are the most basic quantitative tools for making decisions about the relative merit of investments. Use them to decide which option strategy (i.e. which combination of options) has the best risk/reward profile.

In Figure 1 we show the real expected PV, our 'value', for a call option with strike of 100 and with six months to expiry. The volatility of the underlying is 20% and the risk-free rate is 5%. The drift of the asset is 15%.

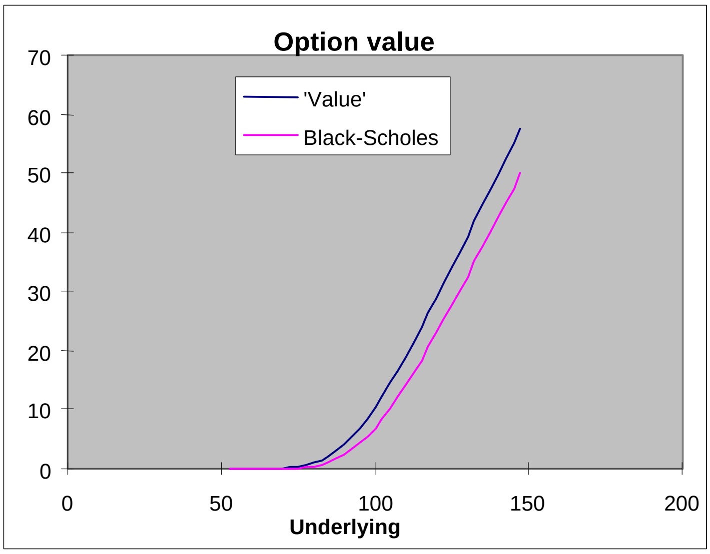

Figure 1: PV of real expected payoff and Black-Scholes value.

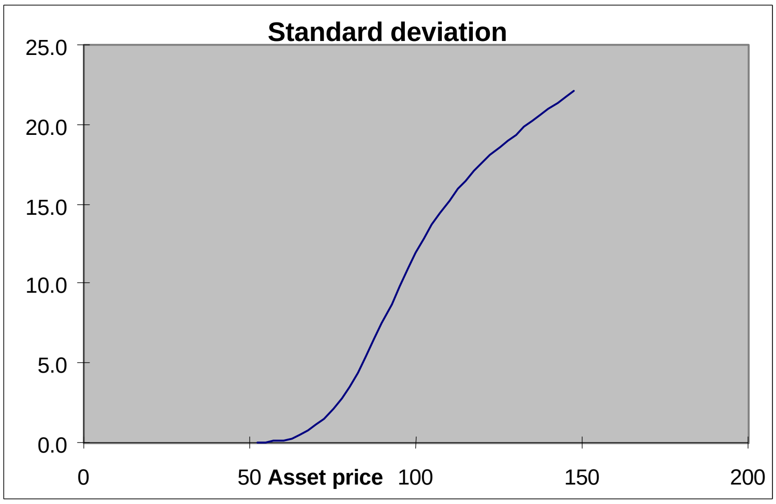

Figure 2: Standard deviation of PV of payoff.

The PV of the standard deviation of the payoff is shown in Figure 2.

Exploiting the market, closing the position

In the above we have assumed that we must take the market (BS) price to enter into our option position. Why stop there? We can also sell it back at the market price if that value exceeds, for example, our PV of expected payoff. (The value at which we sell it back could even have a different volatility, incorporating bid-offer spread and a view on the market's perception of volatility. Whether the market's view is correct or not is irrelevant!) This strategy of optimally closing the position is modelled by the addition of the constraint

$$V(S,t) \geq M(S,t)$$

to the differential equation for V. In this inequality, the function M is the market's value for the option, e.g. the BS value. This makes the optimal position closure problem mathematically identical to the American option free boundary problem.

To hedge or not to hedge

We may find ourselves with an option position that was appealing when initiated but which has gone horribly wrong due to adverse movements in the underlying. If we can close the position as explained above, all well and good, but what if we can't or if the bid-offer spread on the position makes entering and closing a position too costly? One possibility is to begin to delta hedge. Theoretically, this locks in any profit or loss made so far. Let's refine this strategy to the limit: don't hedge when the expected movement is favourable, hedge when it is unfavourable. We arrive at the following non-linear equation for the PV of expected payoff

$$\frac{\partial V}{\partial t} + \frac{s^2 S^2}{2}\frac{\partial^2 V}{\partial S^2} + \left(m - (m-r)\mathrm{H}\left(\frac{\partial V}{\partial S}\right)\right) S\frac{\partial V}{\partial S} - rV = 0 \quad \text{with} \quad V(S,T) = \text{payoff function}$$

where

$$\mathrm{H}(\Delta) = \begin{cases} 0 & \text{if} \quad \Delta > 0 \\ 1 & \text{if} \quad \Delta < 0 \end{cases}$$

(We have assumed that $\mu > r$.)

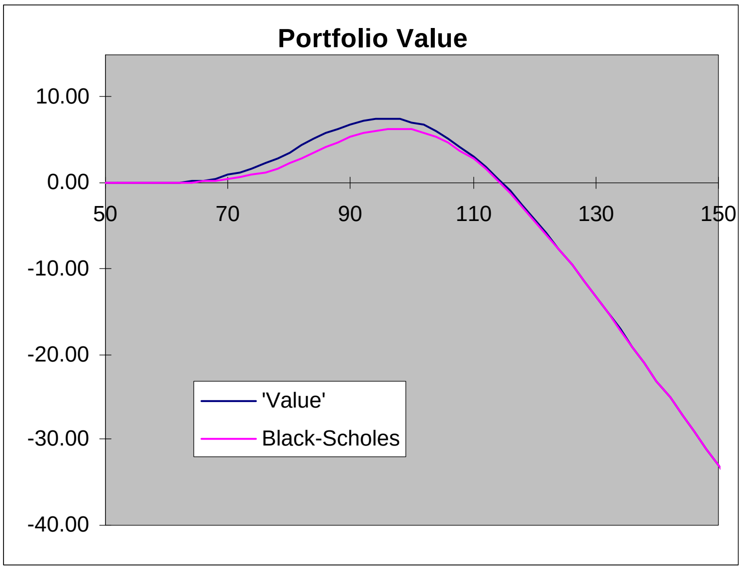

Figure 3: Hedging with a view. New strategy and Black-Scholes value.

In Figure 3 we show the result of this hedging strategy. The portfolio contains two call options struck at 90 and short three calls at 100, all have expiry in six months.The volatility is 20%, the risk-free interest rate 5%, the drift is 15%.

More complex models

The above models are quite sophisticated and can be used to derive optimal strategies from a trader's insight. What they don't capture is the common experience of 'changing one's mind'! You buy a call spread, expiring in three months, thinking that the market is going up but not too far. One month into the contract market conditions change (or you 'refine' your opinion of the market) and are now saddled with a contract you don't want. How much do you expect to make now, and will the market rally?

The simplest model that captures this situation, and uses meaningful parameters, is the two-valued drift. The drift is currently high at μ^+ but may fall to μ^-, with the change of drift states modelled as a Poisson process with intensity α^+ for a drop in drift and α^- for a rise. The PV of expected payoff now has two values V^+ and V^- representing the PV when you begin in the higher drift state and lower drift state respectively. These functions satisfy

$$\frac{\partial V^+}{\partial t} + \frac{s^2 S^2}{2}\frac{\partial^2 V^+}{\partial S^2} + m^+ S\frac{\partial V^+}{\partial S} - rV^+ + a^-(V^- - V^+) = 0$$

and

$$\frac{\partial V^-}{\partial t} + \frac{s^2 S^2}{2}\frac{\partial^2 V^-}{\partial S^2} + m^- S\frac{\partial V^-}{\partial S} - rV^- + a^+(V^+ - V^+) = 0$$

with $V^+(S,T) = V^-(S,T) = \text{payoff function.}$

In Figure 4 we show the PV of expected payoffs when in the higher and the lower drift states. The Black-Scholes value is also shown. The option has six months to expiry and a strike price of 100. The volatility is 20%, the risk-free interest rate 5%, the higher drift is 15% and the lower drift 0%. The Poisson intensities are 1, for higher to lower, and 0.5, for lower to higher.

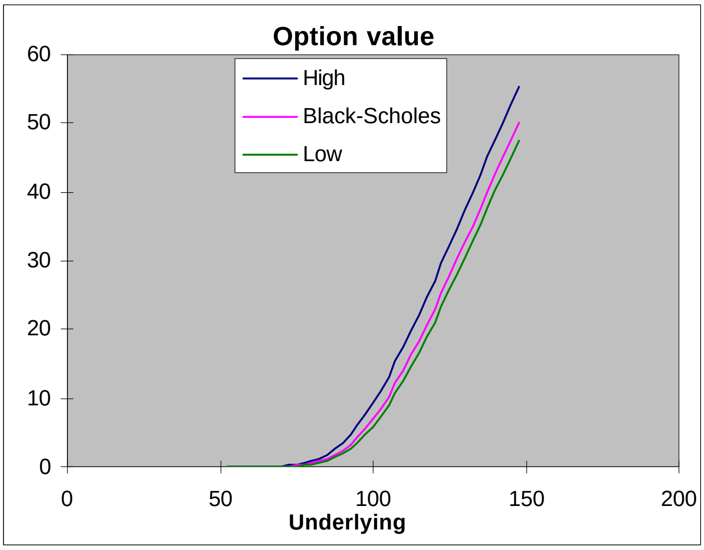

Figure 4: Option 'value' in high and low drift states, and the Black-Scholes value.

If we were to permit early closure, we would find that the value in the lower state was identical to Black-Scholes. In other words, we would sell as soon as we believed the asset to be in a sideways trend. The standard deviation of the payoff can also be calculated.

Final thoughts and conclusion

Whatever the risk-neutral valuation says about the price of an option, a speculator must have a view, whether on the direction of the underlying or something more subtle such as the implied or actual volatility. In this article we have taken that view and put it to good (i.e. optimal) use. We have only skimmed the surface of possible approaches to incorporating the view. There are many other possibilities, for instance we have said nothing about how to use the models for choosing which option or portfolio of options the speculator should buy.

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