Submission for 'Learning Curve'

Exercise Class

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Introduction
In this learning curve we explain the ideas behind the valuation of options with early exercise features, so called American options. We also aim to clarify some popular misconceptions about when an American option should be exercised. These misconceptions seem to be prevalent among both academics and practitioners.

How to *price* American options
If an option can only be exercised at expiry it is called European. If it can be exercised at any time prior to expiry then it is called American. Bermudan options have prespecified exercise dates which may be particular days or whole periods of time. Because they give the holder more rights, the American option is at least as valuable as an equivalent Bermudan option which in turn is at least as valuable as an equivalent European contract.

The idea behind valuing options with early exercise is to decide when the option should be exercised. Is there, in some sense, a best or optimal time for exercise? To correctly price American options we must place ourselves in the shoes of the option writer. We must be clear about the principles behind his strategy. From the modelling point of view we assume that the writer of the option is also hedging his option position by trading in the underlying asset. The hedging strategy is dynamic and referred to as 'delta hedging.' The position in the underlying asset in maintained delta neutral so as to be insensitive, to leading order, to movement of the asset. By maintaining such a hedge, the writer does not care about the direction in which the underlying moves, he eliminates all asset price risk. However, he does remain exposed to the exercise strategy of the option holder. If the writer makes an assumption about when the holder will exercise his option and this assumption turns out to be incorrect, this will have an impact on the writer's profit. Since the writer cannot possibly know what the holder's strategy will be, how can the writer reduce his exposure to this strategy?

The answer is simple. The writer assumes that the holder exercises at the *worst possible time for the writer*. He assumes that the option is exercised at the moment that gives the writer the least profit. This is often referred to as the optimal stopping time although as far as the writer is concerned it is the last thing he wants to happen. So, out of all the possible exercise strategies we must find the one that gives the option the least value to the writer or equivalently the highest value to the holder. This sounds very complicated but anyone who has implemented the binomial method knows that it is just a matter of adding one line of code to the program. That line of code simply tests at each node in the tree whether the theoretical option value is greater than the payoff, if it is not then the payoff is used instead, and this corresponds to a time at which the option should be exercised.

To summarize this section, the assumptions are that the option writer is delta hedging and prices the option at the highest possible value over all exercise strategies.

When should the holder exercise?
The holder of the option rarely delta hedges. Perhaps he has bought the option as a static hedge for the rest of his portfolio, or perhaps as a speculative investment. Either way it is unlikely that he is insensitive to the direction of the underlying asset. The initial assumption concerning the writer of the option does not apply to the holder. Should the holder therefore act in the 'optimal' way that follows from the two assumptions summarized at the end of the previous section?

Consider the very simplest scenario. You buy a call option because you believe that the underlying asset is going to rise significantly. If you are correct you will make a substantial return. If there are no dividends on the underlying then it is 'theoretically' never optimal to exercise before expiry. We put the word theoretically between inverted commas because the 'theory' is only relevant to someone who is delta hedging. Now suppose that the stock does indeed rise, but the economic situation makes you believe that a sudden fall is imminent. What should you do? The obvious solution is to sell the option and lock in your profit. But this may not be possible, for example if the option is OTC. The only way of locking in the profit may be to exercise the option early. The 'theory' says don't exercise, but if the stock does fall then you lose the profit. At this stage it is important to remember that the theory is *not relevant to you*.

The writer and the holder of the option have different priorities, what is optimal to one is not necessarily optimal to the other. The holder of the option may simply have a gut feeling about the stock and decide to exercise. That is perfectly valid. Or he may have a stop-loss strategy in place. He may even have a complex utility maximization strategy. Anyway, it is highly unlikely that his exercise time will correspond to that calculated by the writer of the option.

The exercise strategy should depend on whether or not you are delta hedging
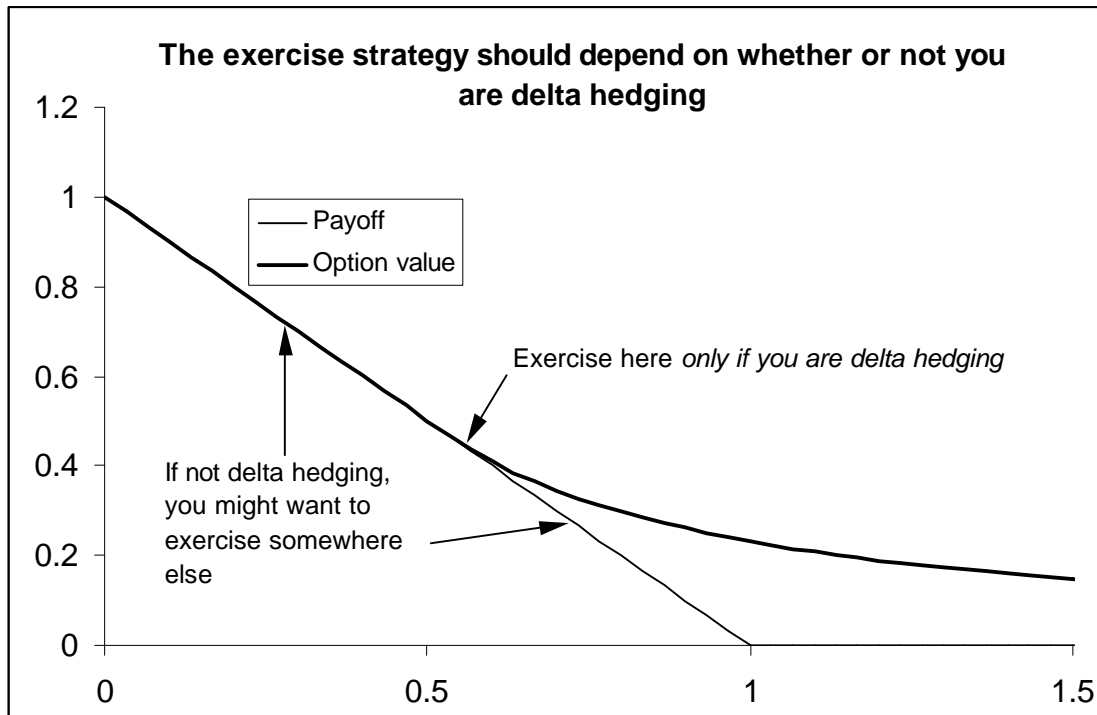
The figure shows the payoff and theoretical value for an American put option some time before expiry. If you are delta hedging there is an optimal asset value at which to exercise. If you are not delta hedging…

How does the writer feel about exercise?
The writer has received a sum of money in exchange for the option. That sum of money was calculated assuming that the option holder exercises at a certain optimal time. This optimal exercise strategy gives the option its highest theoretical value. The writer receives this maximum amount *even though the holder may exercise at any time*.

It is clear that the writer can never lose. The worst that can happen to him is that the option is exercised at this theoretical optimal time. But this has already been priced into the premium he received. On the other hand, if the holder exercises at some other time he can only benefit. How does the writer feel about exercise? At worst he has no feelings, at best he is very happy!

Conclusion
Since there is a clever but complex theory behind the exercise of American options there has arisen some misunderstanding about the optimal time to exercise. It is commonly believed that there is only one optimal time. This is far from being true. We hope that this article clears up some of the confusion. Finally, we would like to make the observation that if you want to sell someone an option and early exercise doesn't add too much to the theoretical value then you should always make it American. This gives you the possibility of a surprise windfall profit if the holder exercises at an unexpected time.

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