Value-at-risk and market crashes

By Philip Hua and Paul Wilmott

If the Black—Scholes model and its extensions were the discoveries of the 70s and 80s, then Value-at-risk (VaR) models are the darlings of the 90s. These models have many uses within an organisation; for example, a risk manager may use VaR to allocate trading limits, senior management for asset allocation and regulators set and review capital reserves for the institutions. Whatever the uses, the essence of a VaR number is to act a benchmark for measuring how 'risky' the portfolio is across different business lines and products.

There are many VaR models currently on the market. Rightly or wrongly, they all try to give 'an estimate, within a given degree of confidence, of how much one can lose from one's portfolio over a given time horizon.' As an example of VaR, we may say that over the next two weeks, there is a 95% chance that we will lose no more than $10m.

If one ever believed in the VaR number as a defensive mechanism for risk control, then the events of the last few months should have raised serious doubts in the minds of the managers and regulators on the validity of using VaR models for market crash management. Indeed, some credit should be given to the Basle accord and the Capital Adequacy Derivatives II, for having the foresight to include the minimum multiplier of three in the capital reserve requirement to account for the VaR model shortcomings.

If the volatility of the *i*th asset is σ_i and the correlation between the *i*th and *j*th assets is r_{ij}, then the VaR for the portfolio consisting of M assets with a holding of N_i of the *i*th asset is

$$a\sqrt{dt\sum_{j=1}^{M}\sum_{i=1}^{M}N_iN_js_is_jr_{ij}}$$

Here δt is the horizon measured in years and a is the number of standard deviations from the mean assuming the losses are normally distributed. (Don't forget to multiply this by 3!)

The parameters in the VaR model are based on daily market events. Anyone who has invested any money in the stock market will tell you that during crashes, normal relationship between stocks do not hold. This behaviour is observed without exception in all crashes. Panic selling and flight-to-quality cause stocks to fall together and volatility to spike. This, coupled with illiquidity that prevents dynamic portfolio insurance, makes nonsense of every assumption in a VaR model. Ironically, it is the losses during these times that are of most concern to the senior management and the regulators.

Given that we do not believe in current VaR models during crash periods for the above reasons, can we develop something that measures and protects the firm from a market crash and remain sensibly practical? The answer is yes. Indeed, much of the elementary mathematics developed for risk management i.e. the usual 'greeks' for calculating market sensitivities can be used to reflect the true cost of a crash. Such new methodology for incorporating crash scenarios into a VaR calculation has come to be known as CrashMetrics

An alternative VaR for market crashes: CrashMetrics

Can we predict when a stock market crash will happen? Can we predict the size of the crash? The answer to both of these questions is 'no.' Can we predict the effect on a bank if a major crash does occur? This is much easier, and the effect is often extremely undesirable. Can some form of hedging ameliorate the potentially disastrous results of a market crash? We will describe how this can be done.

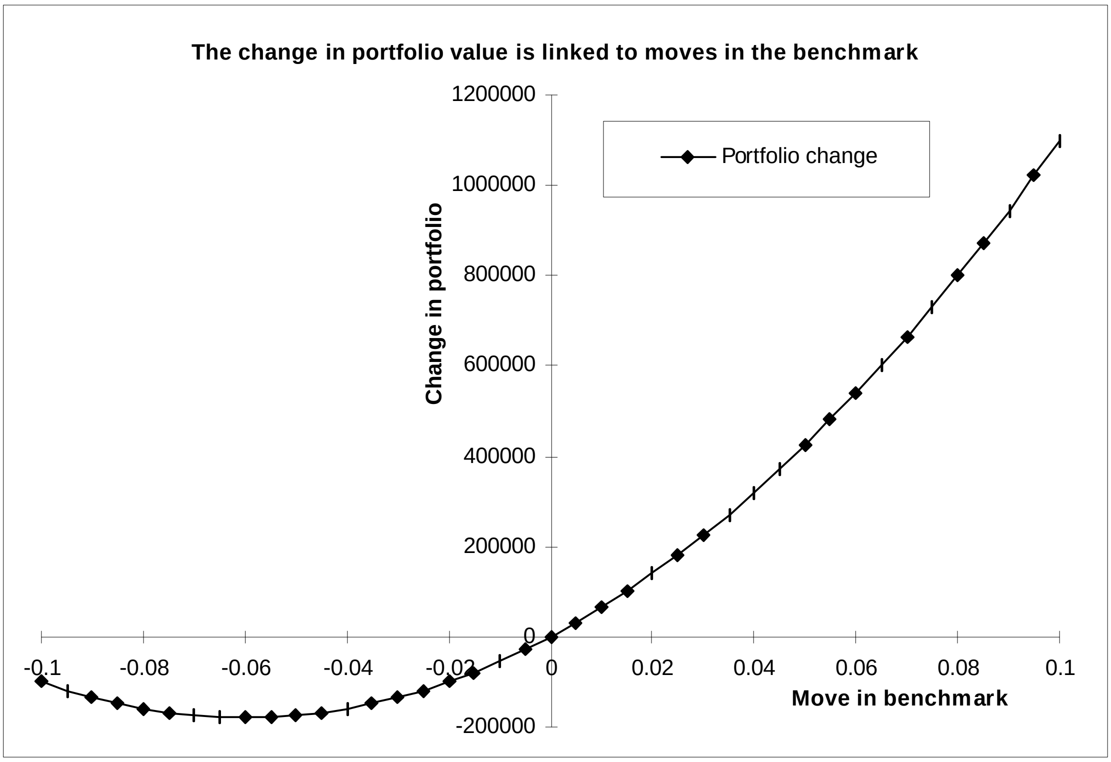
Our first observation is that stock market crashes are special, they do not conform to typical market conditions. The sizes of the moves are much larger than usual and the concept of correlation becomes more straightforward. With few exceptions, when there is a market crash all assets move together, correlations become one. For this reason, crashes are easier to analyze than normal market conditions, we no longer have to worry about measuring correlations.

The second observation is that the large asset moves during a crash mean that the moves in the values of derivatives cannot be approximated by the option's delta, at the very least we must include the option's convexity or gamma.

Finally, we should not be concerned with a probabilistic description of VaR, after all we are talking about the possible collapse of a bank. Of more natural concern in such a situation is the worst-case scenario.

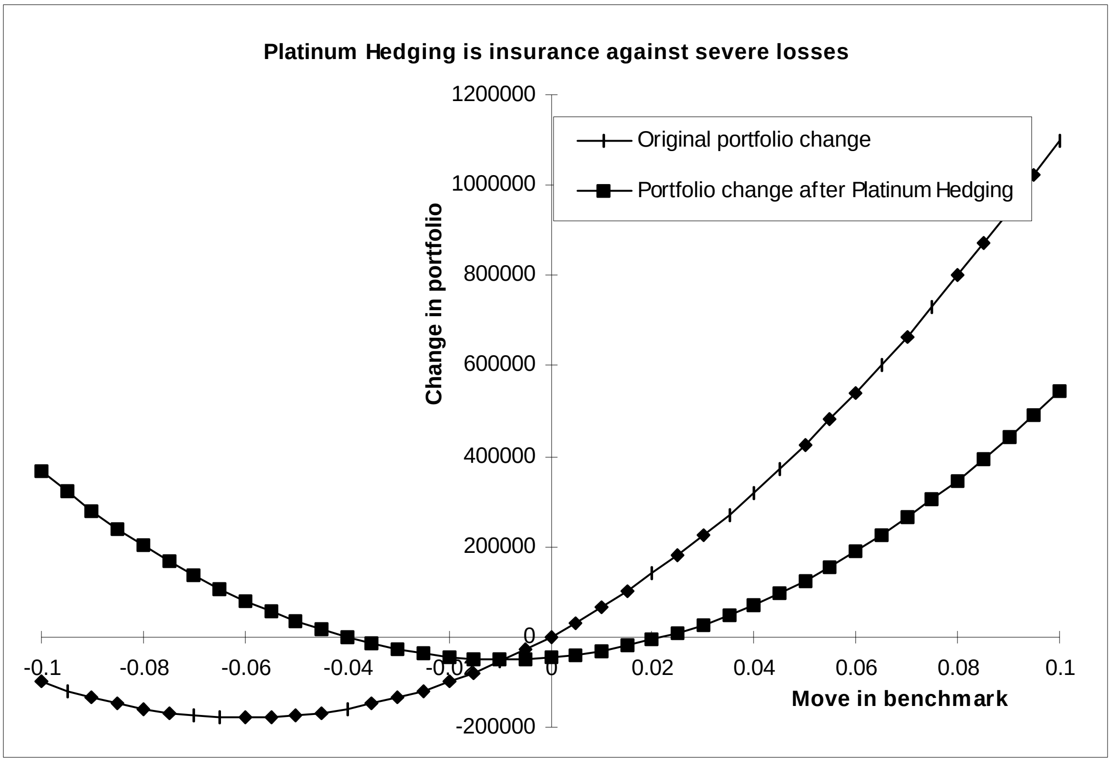
We are not going to go into details here, but the mathematics of CrashMetrics is of the simplest kind. There are four components to this VaR measure:

1. During crashes all financial instruments are perfectly correlated to one (or more) benchmarks. This benchmark may be the S&P500 or a representative bond yield. It can even be a more general credit quality index. If δI is the return on this index then the return on the ith asset is $\delta S_i = \kappa_i \ \delta I$. The k_i are called crash coefficients and are used to relate the changes of the 'greeks' in the individual assets to the index. All other parameters that we cannot measure accurately, such as correlation and volatility play no part here.
2. Changes in value of a portfolio of financial instruments, including derivatives, can be approximated via the usual deltas, gammas and cross gammas. Because all assets are correlated during extreme market moves the portfolio change is easily related to the change in the benchmark. In the first figure below is shown a typical plot of this relationship. The slope at the origin is known as the crash delta and is a special combination of all the deltas on each underlying asset. The curvature or convexity at the origin is known as the crash gamma and is also a special combination of all the gammas.
3. From this curve we can calculate the worst-case scenario. This is the most that can be lost by the portfolio during a market crash. This is a very robust measure of VaR. Another output is the worst-case margin that would be required. This last point is particularly important in view of the recent Long-Term Capital Management fiasco.



But what can we do if this measure of VaR, or margin, is unacceptable? We do not believe in dynamic hedging for crash protection. Instead, we statically hedging using other traded, so called 'Platinum Hedging.' This brings us to the final component of CrashMetrics:

4. Options can be bought or sold in an optimal fashion to ameliorate the worst-case VaR or margin call. The purchase of such insurance is not without cost, of course. This cost can be represented by the bid-offer spread on the options chosen for the Platinum Hedging. This is clearly seen in the following plot. This shows our original portfolio and the new statically hedged portfolio. Note that the worst-case portfolio move is significantly less damaging than originally. Furthermore, the curve no longer goes through the origin, we have paid out an insurance premium, the bid-offer spread, to guarantee a better worst-case scenario.



Not only does CrashMetrics provide a more robust measure of VaR it also contains within it the mechanism for improving the VaR in an optimal fashion. Moreover, all the parameters that we endeavour, but fail, to measure accurately such as volatiities and correlations play no role here. The only parameters needed are the crash coefficients. Initial results suggest that these are more stable than correlations.

A dataset containing estimates of the crash coefficients for some of the stocks in the S&P500, FTSE100, Nikkei 225, Dax, Hang Seng etc. with each index as a benchmark can be downloaded free of charge from www.wilmott.com. The same site contains a very detailed description of the full CrashMetrics methodology.

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