From market games to real-world markets

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March 29, 2001

Abstract

This paper uses the development of multi-agent market models to present a unified approach to the joint questions of how financial market movements may be simulated, predicted, and hedged against.

We first present the results of agent-based market simulations in which traders equipped with simple buy/sell strategies and limited information compete in speculatory trading. We examine the effect of different market clearing mechanisms and show that implementation of a simple Walrasian auction leads to unstable market dynamics. We then show that a more realistic out-of-equilibrium clearing process leads to dynamics that closely resemble real financial movements, with fat-tailed price increments, clustered volatility and high volume autocorrelation.

We then show that replacing the 'synthetic' price history used by these simulations with data taken from real financial time-series leads to the remarkable result that the agents can collectively learn to identify moments in the market where profit is attainable. Hence on real financial data, the system as a whole can perform better than random.

We then employ the risk-control formalism of Bouchaud and Sornette in conjunction with agent based models to show that in general risk cannot be eliminated from trading with these models. We also show that, in the presence of transaction costs, the risk of option writing is greatly increased. This risk, and the costs, can however be reduced through the use of a delta-hedging strategy with modified, time-dependent volatility structure.

Paper presented at APFA2 Conference, Liege (2000)

1 Introduction

Agent-based models of complex adaptive systems are attracting significant interest across a broad range of disciplines [1]. An important application receiving much attention within the physics community, is the study of fluctuations in financial time-series [2]. Currently many different agent-based models exist in the 'econophysics' literature, each with its own set of implicit assumptions and

interesting properties [3] [4] [5] [6]. In general these models exhibit some of the statistical properties that are reminiscent of those observed in real-world financial markets: fat tailed distributions of returns, clustered volatility and so on. These models, despite their differences draw on several of the same key ideas; feedback, frustration, adaptability and evolution.

The Minority Game (MG) introduced by Challet and Zhang [7] offers possibly the simplest paradigm for a system containing these key features. Unlike the sophisticated model of Lux [3] there is no external noise process simulating information arrival. Nor is there any element of agents sharing local information as in the model of Cont & Bouchaud [4]. The MG simply comprises an odd number of agents N choosing repeatedly between the options of buying (1) and selling (0) a quantity of a risky asset. The resource level of this asset is finite and therefore the agents will compete to buy low and sell high. This gives the game its 'minority' nature; an excess of buyers will force the price of the asset up, consequently the minority of agents who have placed sell orders receive a good price at the penalty of the majority who end up buying at an over-inflated price. The MG agents act with inductive reasoning, using strategies that map the series of recent (binary) asset price fluctuations to an investment decision for the next time-step. In an attempt to learn from their past mistakes the agents constantly update the 'score' of their strategies and use only the most successful one to make their prediction.

The basic assumptions of this system are minimal but the resultant dynamics show a richness and diversity that has been the focus of much recent study . However, the MG as a realistic market model has many shortcomings:

- All agents trade at each time-step
- All agents trade equal quantities
- The system resource level is fixed
- Agent diversity is typically limited

Many of these as well as other interesting extensions (such as agents having the ability to learn of their own market impact [5]) have been studied separately and are discussed in [2]. This paper aims to jointly develop many of these extensions to the basic MG in an attempt to build a minimal and yet realistic market model.

The development and study of market models from a physicist's standpoint is motivated by the desire to learn what key interactions are responsible for phenomena observed in the real-world system, the financial marketplace. However, the scope for using such market models is not simply limited to qualitative phenomenological studies. The models may be extended or manipulated to explore quantitatively the emergence of empirical scaling laws. Alternatively, the approach to 'critical' self-organized, or stable states may be examined [13]. These are just a few of the uses which could be categorized as 'theoretical' study. What then can these models be used for on a more 'practical' or perhaps commercial level?

Recently we have been working on the possibility of using these market models in a similar way to the way in which a meteorologist may use a model of atmospheric dynamics; i.e. condition the models with observed data and let them run into the future to extract probabilistic forecasts. These forecasts may then be used for not only speculative gain but also for more insightful risk management and portfolio optimization. Section 2 of this text will expand on the idea of using the MG as a market model, detailing the extensions needed, Section 3 will then explore two different market-making mechanisms, assessing the resultant dynamics, Section 4 will detail how these models may be used for predictive purposes and Section 5 will focus on risk and portfolio optimization.

2 The MG as a market model

2.1 The Basic MG

As mentioned in the previous section, the MG formulation captures some of the behavioral phenomena that are thought to be of importance in financial markets; those of competition, frustration, adaptability and evolution. It is also a 'minimal' system of only few parameters:

N = Number of agents
m_i = 'Memory' of agent i
s_i = Number of strategies held by agent i

The *memory* of an agent is the number of bits of the most recent past global history that are used by a strategy in order to form a prediction. The agents are assigned their s_i strategies at the start of the game and are not allowed to replace them at any point. Each agent uses the historically most successful of her strategies to form a prediction, the predictions of all agents are then pooled and the global history is updated with the prediction of the minority group.

A single *strategy* maps each of the 2^m possible 'histories' to a prediction. Thus there are 2^{2^m} different possible binary strategies. However, many of the strategies in this space are largely similar to one another (i.e. are separated by a small Hamming distance). It has been shown [14] that the principle features of the MG are reproduced in a smaller *Reduced Strategy Space* of 2^{m+1} strategies wherein any two strategies are separated by a Hamming distance of either 2^m or 2^{m-1} (i.e. are *anti-correlated* or *un-correlated*). If the number of strategies in play i.e. $N.s$ is greater than 2^{m+1} then the game is said to be in the *crowded*' phase, in contrast $N.s \ll 2^{m+1}$ represents the *dilute* phase.

The properties of the crowded and dilute phases of the game are quite different and could be thought of as representing different regimes of a market. In the crowded phase there will at any one time be a large number of agents who are using the same (best) strategy and so will flood into the market as large groups, producing large swings in supply and demand and a consequently high volatility.

If the memory of the agents is larger such as to render $N.s \sim 2^{m+1}$ then the groups of agents using the same (best) strategy (*crowds*) will be smaller. There will also be groups of agents who are forced to use the anti-correlated (worst) strategy, these can be thought of as *anti-crowds* as they cancel the market action of the *crowds*. This cancellation effect causes a reduction in the market volatility. In the dilute phase it is very unlikely that any agents will hold the same strategies and so the market behaves more randomly and can be modelled well as a group of independent coin-tossers. A theory based on these crowding effects reproduces quantitative results for the market volatility in the basic and so called 'thermal' MG across the full range of parameters N,m,s. For more details of this the reader is referred to [12],[15]. This 'Crowd, Anticrowd Theory' may also be put to use in the formulation of an entirely analytical set of dynamical mapping equations that reproduce the MG [16]. These equations can be analyzed in several interesting limiting cases to unveil the dynamics underlying microscopic behavior in different regimes of the game. They may also be used in the analysis of approaches to unstable behavior in these types of games (and possibly the real market itself). Our preliminary studies have identified that there can be at least two different 'types' of build-up to a large movement (or 'crash'). Further work is currently underway to investigate the various 'types' of crash that can occur and their precursors.

2.2 The Grand-Canonical MG

In the basic MG agents must either buy or sell at every time-step. In a real market however, traders are likely to wait on the sidelines until they are reasonably confident that they are able to make a profit with their next trade. They will observe the market passively, mentally updating their various strategies, until their confidence overcomes some threshold value - then they will jump in and make a trade. We now demonstrate an extension to the basic MG which attempts to incorporate this general behavioral pattern.

The primitive binary agents of the basic MG keep a tally of the *virtual* score $r_{S,i}$ of each of their s_i strategies: $+1$ for a correct prediction and -1 for an incorrect prediction, and *virtual* in the sense that the strategy is scored whether it is played or not. They may also keep a tally of their own personal prediction success score r_i. It is reasonable that each agent i has a finite time horizon T_i over which these success scores are monitored; this is equivalent to a 'sunken losses' approach. We now make the simplest possible generalization which is to introduce a threshold value $r_{\min}$ in either r or r_S below which an agent would choose to not trade. In this case, the agent continues to update her strategy scores r_S but now adds a 0 to her personal score tally r. With this extension, the number of agents actively trading at each time-step N_{active} will vary throughout the game. This feature is reminiscent of the Grand-Canonical-Ensemble in statistical mechanics.

If an agent's threshold to play lies at the lower end of the range $-T \leq r_{\min} \leq T$ then we would expect the agent to play a large proportion of the time as her best strategy will have invariably scored higher than this threshold. Conversely,

for high $r_{\min}$, the agent will scarcely play at all. We would thus expect to see a transition occur between these two regimes at intermediate values of the threshold. Figure 1 shows the time-averaged number of active agents $\langle N_{active} \rangle$ and the standard deviation of this quantity as a function of $r_{\min}$ for a uniform population of $N = 101$ $m = 2$, $s = 2$ agents who record scores over $T = 50$ time-steps. Here $r_{\min}$, the threshold to play, is based on the agent's strategy score $r_{S,i}$ such that an agent only plays if $\max[\{r_{S,i}\}] > r_{\min}$. A similar transition effect is also seen if the threshold is based on prediction success score r_i.

The behavior of $\langle N_{active} \rangle$ can be reproduced to a coarse approximation by assuming that the strategy scores $r_{S,i}$ undergo independent binomial random walks:

$$r_S \sim 2Bin\left[T, \frac{1}{2}\right] - T$$

This gives:

$$\begin{aligned} \langle N_{active} \rangle &\approx N\left(1 - P\left[r_S < r_{\min}\right]^s\right) \\ \sigma^2\left[N_{active}\right] &\approx N\left(1 - P\left[r_S < r_{\min}\right]^s\right) P\left[r_S < r_{\min}\right]^s \end{aligned}$$

This approximation captures the essence of the transition mentioned in the paragraph above. However, the behavior of r_S is in reality far from that of a random walk. In the crowded regime r_S is strongly mean-reverting and in the dilute regime of the game it has a strong drift component, also the increments in individual strategy scores can be highly correlated. The approximation becomes better for $T \gg 2^m$, where many of these effects become averaged out.

With intermediate values for $r_{\min}$ this modified MG produces very interesting dynamics [17], for instance there can be moments of extreme illiquidity followed by a rush to the market causing huge swings in supply and demand. There are also noticeable 'ranging' and 'break-out' periods and other patterns familiar to market traders [18].

We now extend this model to allow $r_{\min}$ to be dynamic. Here each agent decides on her own threshold in a manner dependent on her current internal state variables. This allows an enhanced element of evolution within the model and more closely resembles behavioral models of markets wherein levels of confidence are time-dependent. We choose to make $r_{\min}$ a function of the agent's personal success rate r_i. Asserting that agents are rational and risk-averse implies that $r_{\min} > 0$ and that $\frac{dr_{\min,i}}{dr_i} \leq 0$ i.e. never play a strategy that has lost more times than won and take fewer risks if losing. Following basic utility theory we therefore arrive at: $r_{\min,i} = \max[0, -(r_i - \lambda.\sigma[r_i])]$ (where $\sigma[r_i]$ is the player's standard deviation of success and λ is their coefficient of risk-aversion). As agents' success rates vary in time, then so will their threshold values and we see an overall evolution towards a diverse population as shown in Figure 2.

This version of the 'Grand-Canonical' MG forms the basic framework for our development of a market model. The following subsection will outline the further necessary extensions to the model that, when combined, form our 'realistic' market model.

2.3 Agent diversity & wealth

It is a simple extension of the model developed above, to include agent heterogeneity in terms of wealth, investment size and investment strategy. As it stands, each trade made by an agent is the exchange of one quanta of a riskless asset for one quanta of a risky one, irrespective of the agent's wealth or the price of the asset. Also, agents always trade as 'value' investors, seeking to buy low and sell high at each time-step. We now generalize this framework to introduce a more realistic heterogeneity between investors.

We first allot each agent i a quantity of each asset, riskless $B_i[0]$ and risky $S_i[0]$. When a trade is made, it is made at the market price of $p[t] \pm \delta[t]$ where $\delta[t]$ corresponds to a spread raised by the marketmaker (the market-making mechanism is the subject of the next section). We now re-assert the assumption that investors are risk-averse and will thereforee trade amounts proportional to their absolute wealth. We also assume that the amount they trade will be in proportion to their confidence in the strategy they intend to use. It is helpful at this stage to define a measure of this confidence c_i.

$$c_i[t] = \frac{\max[r_{S,i}[t]] - r_{\min}[t]}{T_i}$$

thus $-2 < c_i < 1$ but the agent only plays if $c_i > 0$. Buy operations are then represented by:

$$\begin{aligned} B_i[t+1] &= B_i[t]\left(1 - c_i[t]\frac{p[t+1] + \delta[t+1]}{p[t] + \delta[t]}\right) \\ S_i[t+1] &= S_i[t] + \frac{c_i[t]\,B_i[t]}{p[t] + \delta[t]} \end{aligned}$$

and sell operations by:

$$\begin{aligned} B_i[t+1] &= B_i[t] + c_i[t]\,S_i[t]\,(p[t+1] - \delta[t+1]) \\ S_i[t+1] &= S_i[t]\,(1 - c_i[t]) \end{aligned}$$

Wealthy agents make large transactions and thus will have a high market impact (in a system where price movement size is an increasing function of order size, c.f. Equation 1) whereas poor agents effectively form a background 'noise' of small trades. Of course poor agents may grow rich or vice-versa. When agents have lost all their assets, they can no longer trade; this represents the bankruptcy of that agent. This situation happens extremely rarely in these models and so we have not sought to implement a system for the re-generation of new agents. Figure 3 shows the average distribution of agents' wealth as measured by $B[t] + S[t].p[t]$ (i.e. the probabilities are averaged over time).

As well as the diversity in agents' trade size, there can also be a diversity in investment strategy. Within the framework presented here, investment strategies can fall into the two broad classes; *value* and *trend*. A *value* investor

aims at each time-step to make a profit from buying low and selling high, a *trend* investor on the other hand aims to buy an upward moving asset and sell a downward mover. A population purely of value investors will have a minority-game character, a population of trend investors will create a majority-game of self-fulfilling prophecies. In general, the population of traders will be a combination of these types and thus the character of the market (minority or majority) is unclear. We are currently testing how the proportion of each investor type alters the global dynamics and stability of the market.

3 The Market-Making Mechanism

3.1 Walrasian auction

The simplest type of market-making process is that of a walrasian auction. This is a popular model in the economics community (and actually the system used in the London Metals Exchange). In a walrasian auction investors take part in a price setting process by submitting orders to buy or sell the risky asset based on a theoretical price. The level of this theoretical price is changed until the supply and demand for the asset exactly match and the market can be cleared, then the process repeats.

We can use our market model to simulate a simplified version of this process in the following way. First of all we assume that the supply and demand are in equilibrium at each time-step. The resulting equilibrium price for the risky asset then must be equal to the current demand-value of stocks sought divided by the number of risky assets offered. This gives:

$$p[t+1] = \frac{\sum\limits_{i,\text{Buyers}} c_i\,[t]\,B_i\,[t]}{\sum\limits_{i,\text{Sellers}} c_i\,[t]\,S_i\,[t]}$$

It is clear then that this process is unstable: if there are no buyers the price falls to zero and if there are no sellers it will rise to infinity! Even though these situations happen rarely in a run of the market model, the resulting dynamics still show an inherent instability and the fluctuations are excessive as well as exhibiting a strong anti-persistence. This situation arises because we are asserting that the buy and sell pressures are in equilibrium at each time-step. Of course this is far from the reality and we must extend the market-making mechanism to accommodate the real out-of-equilibrium process.

3.2 Non-equilibrium market

If the supply of risky assets does **not** exactly match the demand at each time-step then the market will either not clear, or the market-maker will take a position in the asset himself in order to fill the orders. In reality it is most likely that a combination of these scenarios occurs; the market maker will want to fill as many orders as possible and take the spread but he will not allow

himself to incur a large position. There are many ways in which this type of behavior could be modelled, we limit ourselves here to looking at one particular system.

Let us start by implementing the price setting rule of Bouchaud - Cont - Farmer [4],[6] :

$$p\left[t+1\right] = p\left[t\right] e^{\frac{Buys[t]-Sells[t]}{Liquidity}} \tag{1}$$

where: $\begin{matrix} Buys \\ Sells \end{matrix}\left[t\right] = \sum_{i \begin{matrix} \text{Buyers} \\ \text{Sellers} \end{matrix}} S_i\left[t+1\right] - S_i\left[t\right]$, and *liquidity* is a constant set by the market-maker. This rule prevents the market-maker being arbitraged but leaves his inventory ($B_M\left[t\right]$ and $S_M\left[t\right]$) as unbounded. Over many runs of such a market simulation we would expect the market-maker's mean inventory to be zero. What we really require on the other hand is that his mean inventory in a particular run be zero. We therefore propose the following extension to Equation 1:

$$p\left[t+1\right] = p\left[t\right] e^{\frac{Buys[t]-Sells[t]-S_M[t]}{Liquidity}} \tag{2}$$

This implies that if the market-maker is accruing a net long position in the risky asset, he'll start lowering the price in order to attract buyers into the market and vice-versa. This mechanism works remarkably well and we find that $S_M\left[t\right]$ under this new rule is strongly mean-reverting as shown in Figure 4.

With Equation 2 however the market maker can be arbitraged by the agents; the strategy buy, wait, sell or vice-versa will make money as long as enough agents do it at the same time. The agents in these systems learn to exploit this very quickly (an interesting result in itself) and the result is a negative drift to the market-maker's money $B_M\left[t\right]$. There are several mechanisms that the market-maker may exploit to overcome this; he can raise a spread or he can reduce the liquidity. We employ the first of these mechanisms, updating the spread proportionally to $-\frac{\langle B_M[t]\rangle}{\langle v[t]\rangle}$ where $v\left[t\right]$ is the volume of transactions defined as $v\left[t\right] = \sum_{i=1}^{N} S_i\left[t+1\right] - S_i\left[t\right]$. Here $\langle B_M\left[t\right]\rangle$ and $\langle v\left[t\right]\rangle$ are taken over a time-length T_M which is kept large compared with $\{T_i\}$ such as to average over local extreme behavior such as momentary illiquidity. This mechanism for raising a spread may not be highly efficient but it does maintain the market-maker's mean wealth close to the desired zero point by raising the spread if he starts losing money. The $1/v\left[t\right]$ dependence stabilizes this process somewhat by sharing the job of paying for the market-maker's deficit over the current number of market participants.

We now have a complete and arguably 'realistic' model and may begin to investigate its properties. We are at present looking at how different statistical properties of the model-market are dependent on its parameters. We seem to

find however that some statistical features are in general present over very large parameter ranges. These are the types of feature that are associated with 'real' markets: High excess kurtosis of returns with weak decay over time, volatility clustering, high volume autocorrelation etc. as shown in Figure 5.

4 Prediction from market-models

The market-models introduced in Section 2 consist of a population of adaptive agents who attempt to predict the future movement of an asset price. Recently, we have been investigating the accuracy of these predictions when the synthetic self-generated global history of asset movements is replaced with a real financial time series.

The first step in this process is to generate binary information from the given financial time-series. This can be done in many ways in order to investigate the predictability of different aspects of the movement. We choose here to examine the sign of movements and hence our information history $h[t]$ becomes:

$$h[t] = H[p_{\text{real}}[t] - p_{\text{real}}[t-1]]$$

where $H[x]$ is the Heaviside function. If $p_{\text{real}}[t] = p_{\text{real}}[t-1]$ then we assign $h[t]$ a 0 or 1 randomly. Before we begin to look at how the agent-models perform with this new information set, let us first examine some of its properties. The agents examine chunks of the information set h of length m bits in order to make a prediction. If we look at the occurrence rate of $m+1$ length bit-strings we can therefore infer the success rate of strategies. For example Figure 6 shows the occurrence probability of 4-bit strings; i.e. 3 memory bits ($m = 3$) and one prediction bit. The bit-strings are enumerated by their decimal value e.g. $0011 \rightarrow 3$. We can infer that the strategy $\{10101010\}$ (which is the $m = 1$, $\{10\}$ i.e. anti-persistent strategy) will have the highest success rate as 000 is more often followed by a 1, 001 by a 0 etc.

As we decrease the sampling rate on our data-set so as to look at the signs of price increments over longer periods, we find that the most successful strategy becomes less well defined and tends to swap regularly. It is no longer the case that a simple anti-persistent strategy is the best. Also as we increase the memory m and look at longer bit-strings, we find that the 'information content' of the bit-string occurrence histograms gets 'washed away' in the mixing of low m probabilities. This implies that the most dominant physical process is a low m process. Figure 7 shows these two effects by examining the excess standard deviation of the bit-string distributions i.e. $\sigma^{\text{real}}_{Bitstring}/\sigma^{\text{random}}_{Bitstring}$ where $\sigma^{\text{random}}_{Bitstring} = \frac{\sqrt{L(2^{m+1}-1)}}{2^{m+1}}$, where L is the length of the data-set.

In the agent simulations, $h[t]$ plays the same role as before, with strategies and agents being scored for prediction success in the same fashion as detailed in Section 2. Of course now the feedback has been removed from the model, it bears more resemblance to a system of genetic algorithms. The key important difference though is the fact that this system of independent agents has a large

built in frustration: the agents aren't allowed to replace poorly performing strategies. Although this at first appears to be a handicap, it can in fact be a strength. In systems where there is not necessarily a 'correct' strategy to employ, there is an advantage in having many currently non-optimal strategies in play as this gives greater adaptability. We have compared the prediction success of these types of model with those employing simple Bayesian update of the probability of a given outcome for a given history and found the former to be much more powerful. Figure 8 shows the time-series of the \$/Yen FX-rate between 1990-99, below this is a plot of the cumulative non-compounded profit attained from using the agent model's predictions to trade hourly. The trading strategy employed is simply to put the original investment amount on either the \$ or the Yen side of the market and take it off again at the end of the hour, banking the profit in a zero interest account. This is clearly an unrealistic strategy as transaction costs would be penalizing, however it is used in order to demonstrate simply that the agent-model performs better than random (around 54% prediction success rate) [1]. The two profit lines on Figure 8 represent two different uses of the independent predictions of the agents. The lower line corresponds to the case where the investment is split equally between all agents, the upper line is for the case where the agents' predictions are pooled together with a non-linear function. This demonstrates that the population as a compound entity can perform much better than the sum of its individual parts. This kind of phenomena has been termed 'collective intelligence' in the past.

Arguably the most interesting phenomena of models such as the MG arise from the strong feedback mechanism. In replacing the self-generated $h[t]$ with an external process we disable that feedback. The system is still however able to function as a weak predictor. It appears that the prediction success rate can be raised by invoking again a feedback within the system. It is probable that this feedback forces a more efficient learning process to take place. These effects are the subject of our current, ongoing studies.

We have hence demonstrated the success of the agent-based models in direct prediction of the sign of the next price increment. However, we can also implement the models in a different way by 'training' them on historical data of a particular asset movement and then using the artificial market-making process to run the models forward into the future. If this is done with an ensemble of such models, each having a different initial allocation of strategies, we can form a distribution of likely future asset price levels. Typically the resulting distributions are fat tailed and can have considerable skewness quite in contrary to more standard economic models. This information can not only be of use in speculation but also in risk control and portfolio management.

[1] We have run these models with randomly generated information histories $h[t]$ and were able to reject the null hypothesis that the mean prediction success rate with real data was random.

5 Risk management

5.1 Implied future risk from agent-models

The control of risk in financial investment should be of equal importance to the realization of profit. Most current theories of risk control rely on the implicit assumption that future behavior of the market will be like its past behavior. This assumption is continually being brought into question when banks and investors seem to be 'caught out' by events that past distributions seemed to imply were impossible. There thus may be room here for risk-control models that rely more on possible emergent future behavior than on historic data.

Using agent-based models in the way mentioned at the close of Section 4 gives us distributions for likely future price levels based on what microscopically might happen. This may be just the type of forward-casting model that could be of use here. We must first however develop a framework within which we can use the type of information that these models give us. Much of risk-control concerns itself with the use of derivative instruments, we therefore follow this direction but take pause to note that a similar methodology can be used for analyzing any portfolio of assets.

Several years ago Bouchaud and Sornette developed a framework for examining and controlling the risk inherent in writing derivative contracts [19]. This formalism explicitly deals with future asset movements in a probabilistic, path-dependent fashion i.e. does not rely on any random-walk model etc. This makes the formalism ideal for combining with the forward-casting agent-models.

The formalism examines the variation in future wealth ΔW_T from holding a certain portfolio, for example short one euro-call contract of price C_0 maturity T and strike X and long $\phi_t[S_t]$ hedging assets in the underlying which is at price S_t at time t:

$$\Delta W_T = C_0 - \max[S_T - X, 0] + \sum_{t=0}^{T} \phi_t[S_t](S_{t+1} - S_t) \ . \tag{3}$$

The variance of this wealth process (which is used as a measure of risk) is then found analytically for a general underlying movement. For our models, this can be done in a Monte-Carlo fashion using each member of the model ensemble to generate a ΔW_T. Doing this we could also look at other measures of risk such as VAR etc. This process generates a more insightful measure of risk based on likely future microscopic behavior.

The control of this risk is the next issue. Bouchaud and Sornette's variance of the wealth process can be minimized with respect to the hedging strategy $\phi_t[S_t]$. The full details are given in [20]; the result is a risk-minimizing 'optimal strategy' given by:

$$\phi_t[S_t] = \int_X^\infty \frac{(S_T - S_t)\langle \delta S_{S_t, t \to S_T, T} \rangle}{\langle \delta S_t^2 \rangle} P[S_T | S_t] \, dS_T \tag{4}$$

Using the forward-casting agent-models we obtain $P\left[S_T|S_t\right]$ (the probability of the underlying moving from value S_t to S_T) by counting the number of members of the (large) model ensemble that cast paths passing near both these two values (price space S is discretized for this purpose). Similarly $\langle \delta S_{S_t,t\rightarrow S_T,T} \rangle$ is found as the mean increment at time t of paths passing near S_t and S_T, $\left\langle \delta S_t^2 \right\rangle$ is simply the mean squared increment at time t of all paths. The resulting reduction in risk when using this 'optimal strategy' with historical distributions is well documented [20]; similar effects are obtained when using the agent-models' future-cast distributions. The important difference to note is that the risk being minimized is now the microscopically derived future risk rather than a measure assuming the continuity of past behavior.

5.2 Transaction costs

We now digress slightly and examine the effect of transaction costs on the risk control process discussed in the previous paragraphs. Bouchaud and Sornette's formalism is easily couched in discrete time, accounting for the fact that continuous trading is un-physical due to transaction cost and brokerage inefficiencies. However, transaction costs themselves have not explicitly been accounted for in the wealth process, therefore their effect on risk-control cannot be gauged. We address this point here by adding a term to equation 3 in order to include a general transaction cost structure.

$$\Delta W_T \rightarrow \Delta W_T + \sum_{t=0}^{T} k_1 + \left(k_2 + k_3 S_t\right) \left|\phi_t\left[S_t\right] - \phi_{t-1}\left[S_{t-1}\right]\right|$$

We now again proceed to find the variance of this wealth process as a gauge of risk. We find that the approximation of $\left|\phi_t\left[S_t\right] - \phi_{t-1}\left[S_{t-1}\right]\right| \approx \frac{\partial \phi_t}{\partial S_t} \left|\delta S_t\right|$ holds reasonably well as the time dependence of $\phi_t\left[S_t\right]$ is weak. This allows us to formulate an analytical correction term to Bouchaud and Sornette's expression for risk (full details will be presented elsewhere):

$$\begin{aligned} R \rightarrow R + \sum_{t=1}^{T} & \left(\begin{array}{c} \int_{-\infty}^{\infty} \left\langle \delta S_t^2 \right\rangle \left(k_2 + k_3 S_t\right)^2 \\ \times \left(\frac{\partial \phi_t}{\partial S_t}\right)^2 P\left[S_t|S_0\right] dS_t \end{array} \right. \\ & \left. - \left(\begin{array}{c} \int_{-\infty}^{\infty} \left\langle \left|\delta S_t\right| \right\rangle \left(k_2 + k_3 S_t\right) \\ \times \frac{\partial \phi_t}{\partial S_t} P\left[S_t|S_0\right] dS_t \end{array} \right)^2 \right) \\ + \sum_{ti \neq tj} & \int\int_{-\infty}^{\infty} \left(\begin{array}{c} \left\langle \left|\delta S_{ti}\right| \right\rangle \left\langle \left|\delta S_{tj}\right| \right\rangle \\ \times \left(k_2 + k_3 S_{ti}\right)\left(k_2 + k_3 S_{tj}\right) \\ \times \frac{\partial \phi_{ti}}{\partial S_{ti}} \frac{\partial \phi_{tj}}{\partial S_{tj}} P\left[S_{ti}|S_0\right] \\ \times \left(P\left[S_{tj}|S_{ti}\right] - P\left[S_{tj}|S_0\right]\right) \end{array} \right) dS_{ti} dS_{tj} \end{aligned} \tag{5}$$

The first line of equation 5 represents the sum of independent transaction cost variances whereas the second line represents the covariance between transaction costs. The covariance terms become very large as we execute more transactions. This non-local behavior leads to a divergence of the risk as we go toward

continuous time as shown in Figure 9. Clearly if we are to minimize risk now the answer is not to simply re-hedge more often.

The minimization of risk with respect to a choice of hedging strategy $\phi_t[S_t]$ is now highly complex and in general path-dependent as might be expected from equation 5. However, we may use pertubation theory to obtain approximate solutions. We find that the risk and transaction costs are reduced greatly using a volatility correction to equation 4 of the form:

$$\left\langle \delta S_t^2 \right\rangle \rightarrow \gamma[t] \left\langle \delta S_t^2 \right\rangle$$

The form of $\gamma[t]$ as a function of time is amusingly that of a smile, much like the volatility correction in strike price to the Black-Scholes delta that is implied by equation 4 itself. The origins of these two 'volatility smiles' are of course very different. Using this correction, for portfolios where transaction costs are likely to be high, we see a dramatic reduction in the risk and also in the absolute transaction costs. Figure 10 demonstrates this for a particular option.

6 Conclusion

We have presented here a development from the basic minority game, to a full market model. We have attempted to capture the behavioral aspects of market-making and agent-participation in a thorough and yet simplistic fashion. From this model we have then shown behavior reminiscent of 'real' financial asset movements with fat-tailed distributions of returns, clustered volatility and high volume autocorrelation.

We then moved on to show how these types of agent-based models perform in a predictive capacity when we replace the self-generated synthetic asset-price history with a real financial asset movement. We showed that as independent entities, the agents were able to function in a manner similar to an inefficient genetic algorithm and thus exploit the residual information present in the asset movement's sign. We then went on to show that when combined as a population, the agents were able to perform as a much stronger predictor, suggesting an element of collective-intelligence. We then outlined another manner in which ensembles of these models can be used to forecast future asset-price levels in a probabilistic manner.

Lastly, we showed how output from the agent-models could be used in a portfolio management setting in order to measure and control risk. We went on to demonstrate that the addition of transaction costs to Bouchaud and Sornette's formalism for risk management led to a greatly increased risk for high-frequency trading. We then presented a volatility correction to the 'optimal strategy' that could be used to reduce this excess risk and also reduce transaction costs.

Our aim is to develop a general understanding and framework for investigating and exploiting financial markets based on microscopic models of agent interactions. It is hoped that the work presented here represents positive and significant steps toward this goal.

We thank A. Short for many useful discussions.

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Figure 1: Mean and standard deviation in the number of active agents N_{active} (game parameters $N = 101$, $m = 2$, $s = 2$, $T = 50$)

Figure 2: Distribution of threshold values $r_{\min}$ after 6000 time-steps (game parameters $N = 151$, $m = 3$, $s = 2$, $T = 50$, $\lambda = 0.07$)

Figure 3: Time averaged PDF of agent's wealth as measured by $B[t] + S[t]\,p[t]$ (Game parameters $N = 151$, $m = 3$, $s = 2$, $T = 50$, $\lambda = 0.07$). Original allocation of wealth; $B[0] = 1000\$$, $S[0] = 100$, $p[0] = 10\$$.

Figure 4: Market-maker's stock $S_M[t]$ over 6000 turns (Total stock in market 15100)

Figure 5: Price & Volume Statistics for a single run of the market simulation (parameters $N_{value} = 101$, $N_{trend} = 50$, $m = 3$, $s = 2$, $T = 20$, $\lambda = 0.07$)

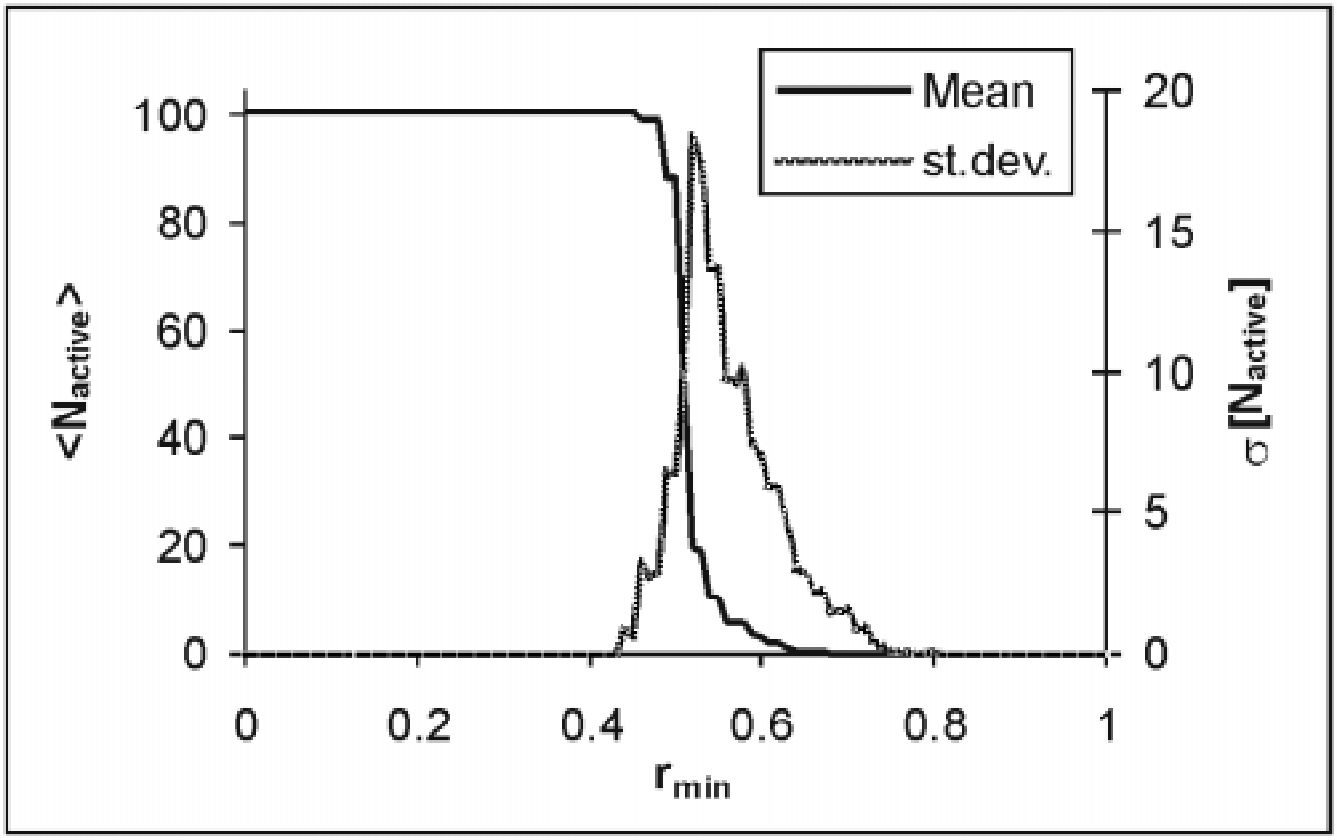
Figure 6: Occurence probability of 4-bit strings in the price-sign history $h[t]$ generated from 10 years of hourly \$/Yen FX-rate data

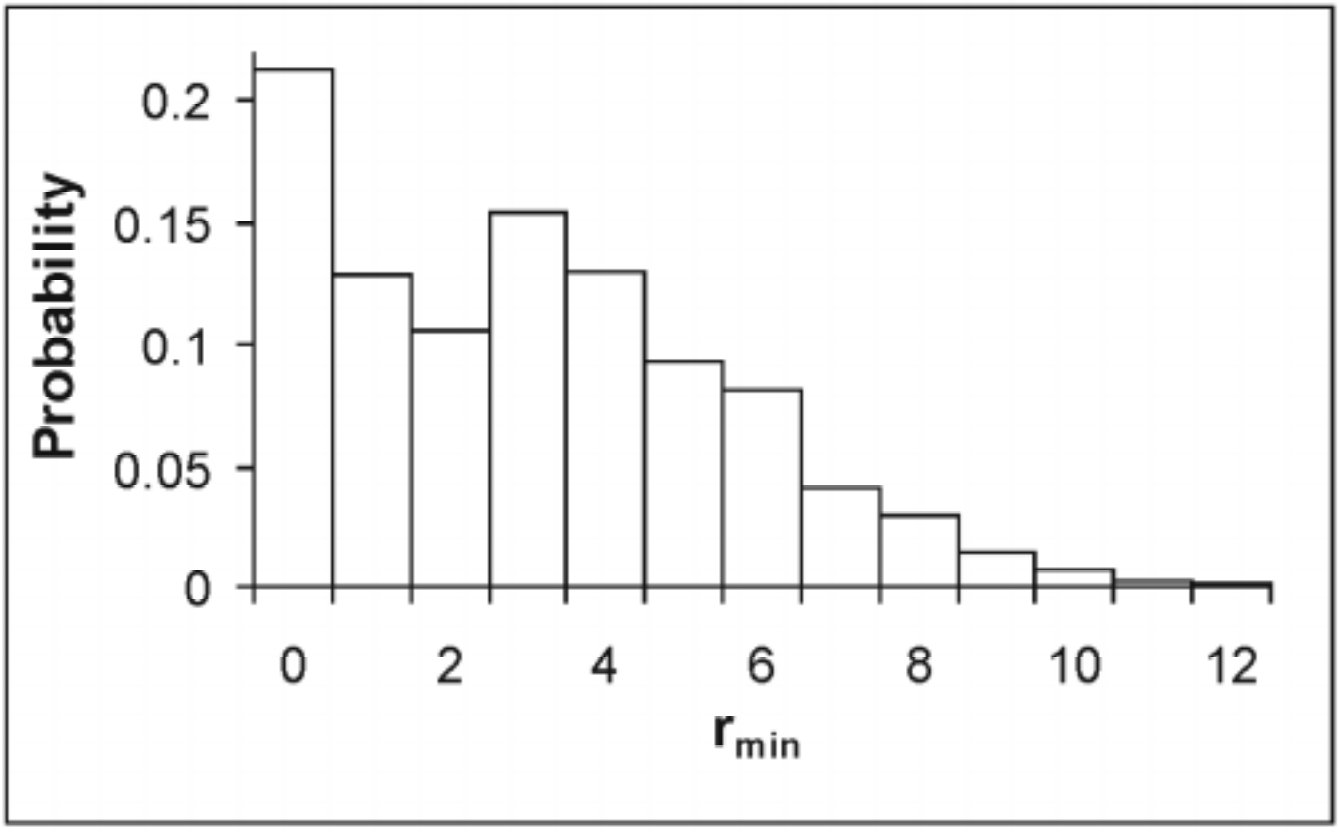
Figure 7: $\sigma^{\text{real}}_{Bitstring}/\sigma^{\text{random}}_{Bitstring}$ as a function of price increment length for $m = 2, 5, 8$ (dataset \$/Yen FX-rate between 1990-99)

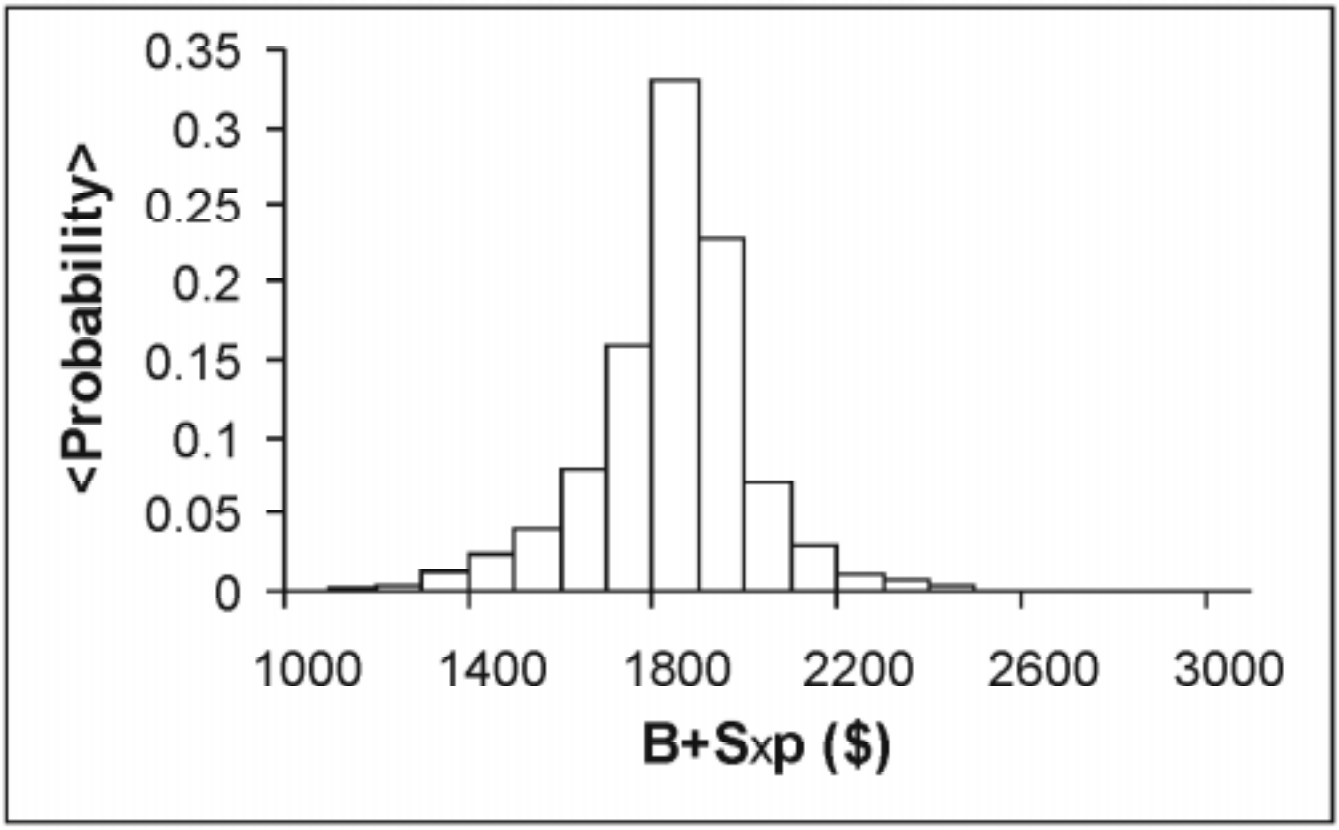
Figure 8: \$/Yen FX-rate 1990-99 (top) and cumulative non-compounded profit from using agent predictions both independently and collectively (bottom)

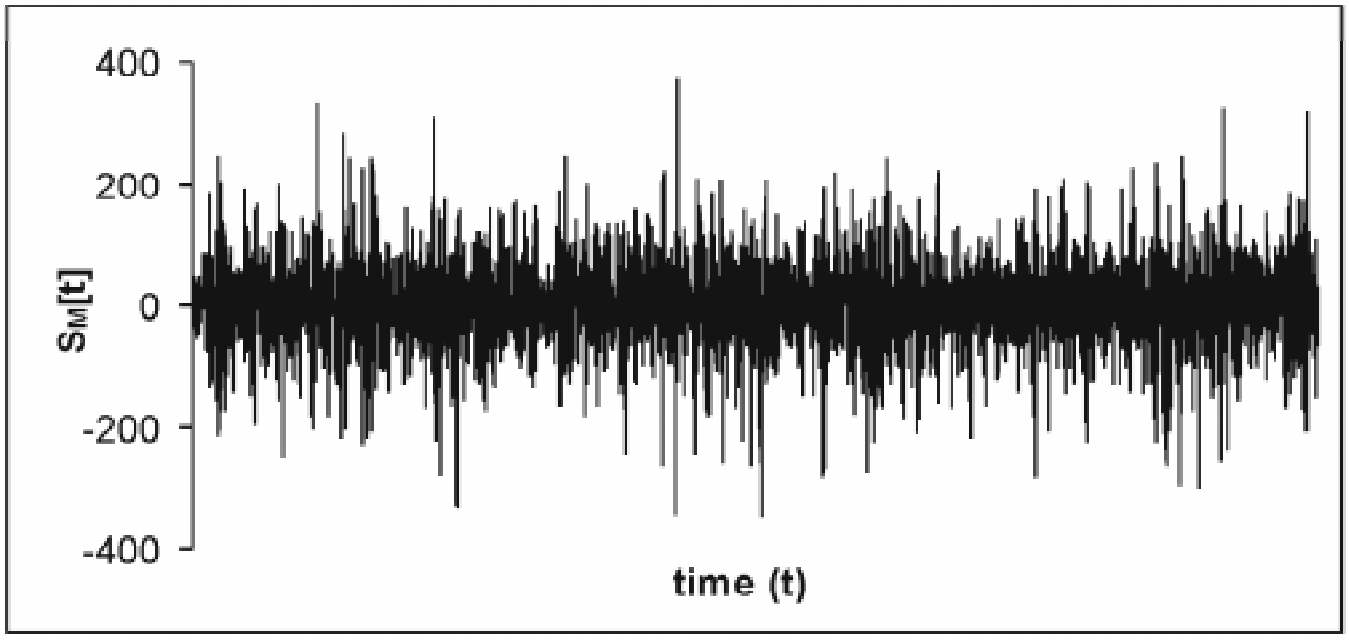
Figure 9: Standard deviation of the wealth process (risk) as a function of trading time (length of time between trades). 30-day at-the-money european option vol=7.37p/day.

Figure 10: Simulated distribution of wealth for portfolio short one 30-day euro-call, at the money, vol=7.37p/day and long $\phi_t[S_t]$ of the underlying with transaction costs at $k_3 = 5\%$. $\phi_t[S_t]$ according to Black-Scholes Delta (top) and with modified volatility as described in the text (bottom).

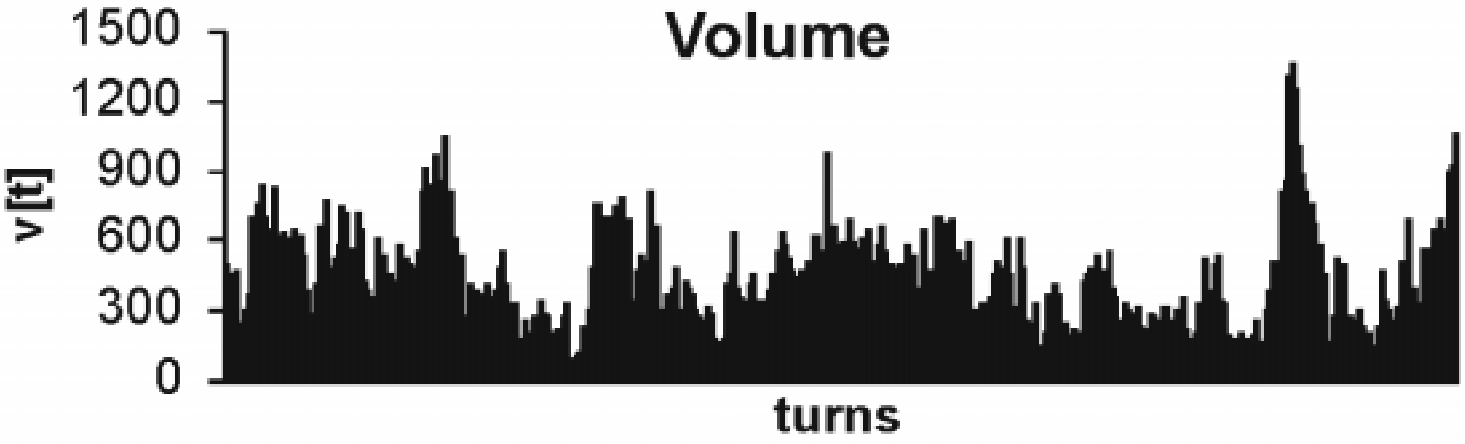


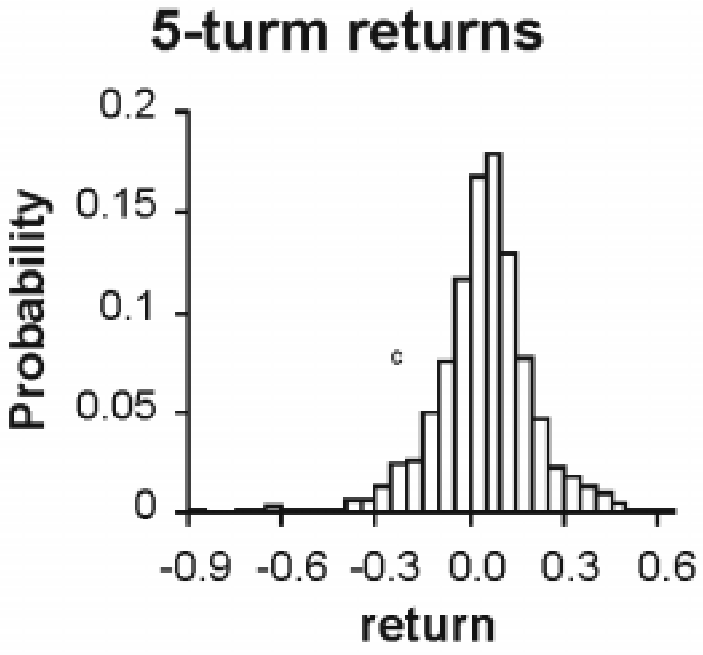


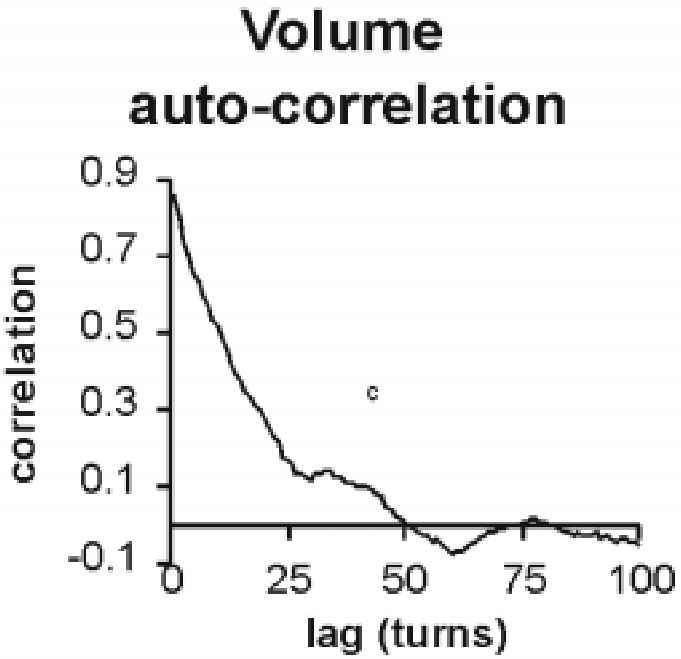


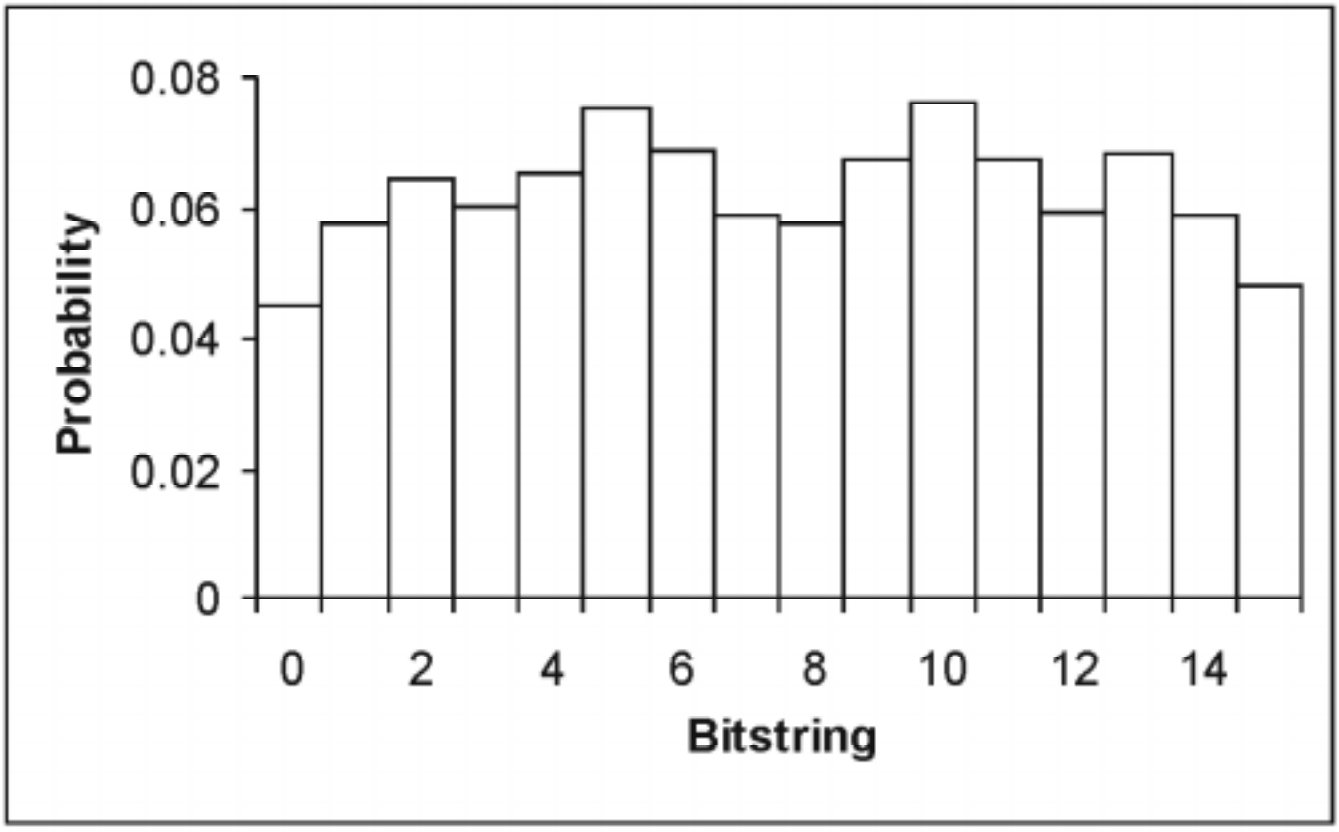


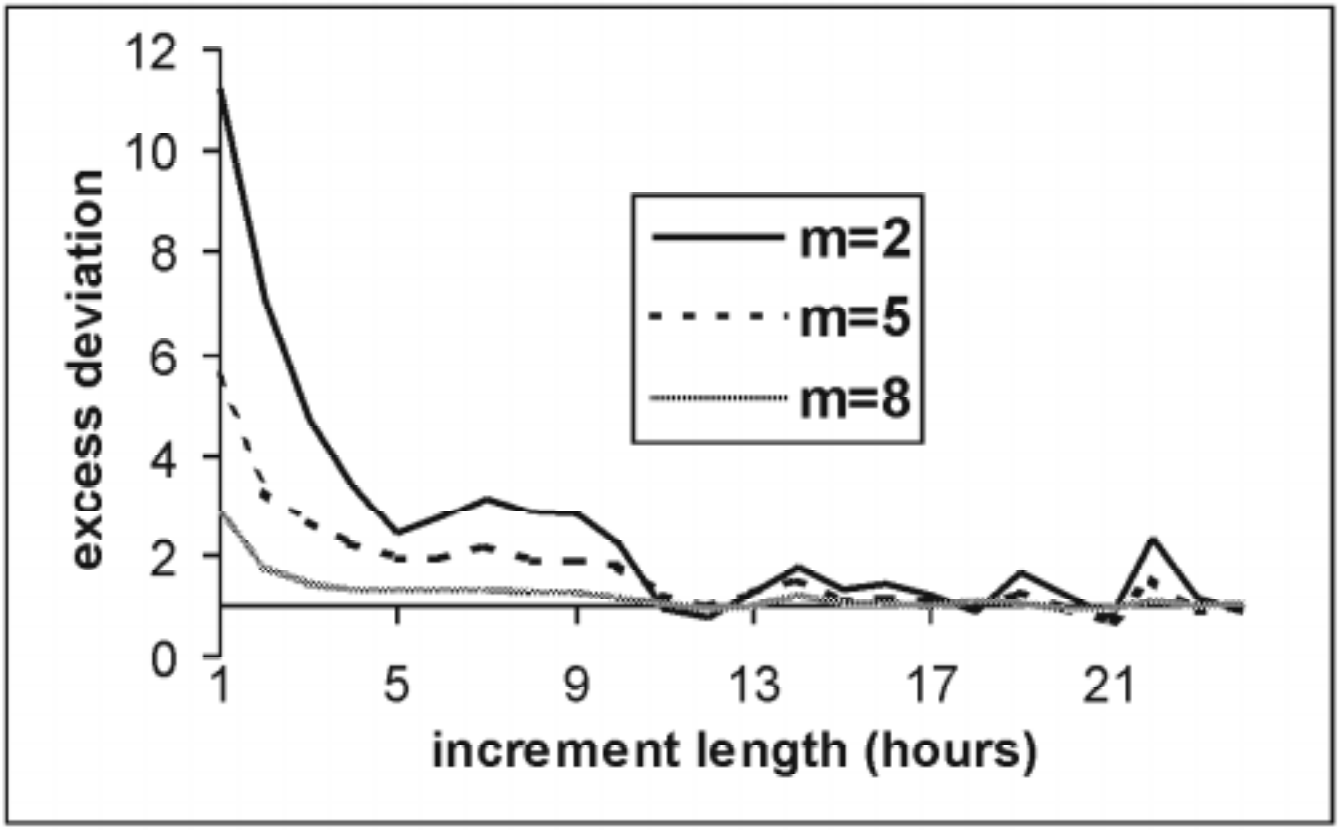

Price
11
10
9
8
p[t] ($)
turns


Volume
1500
1200
900
600
300
0
v[t]
turns

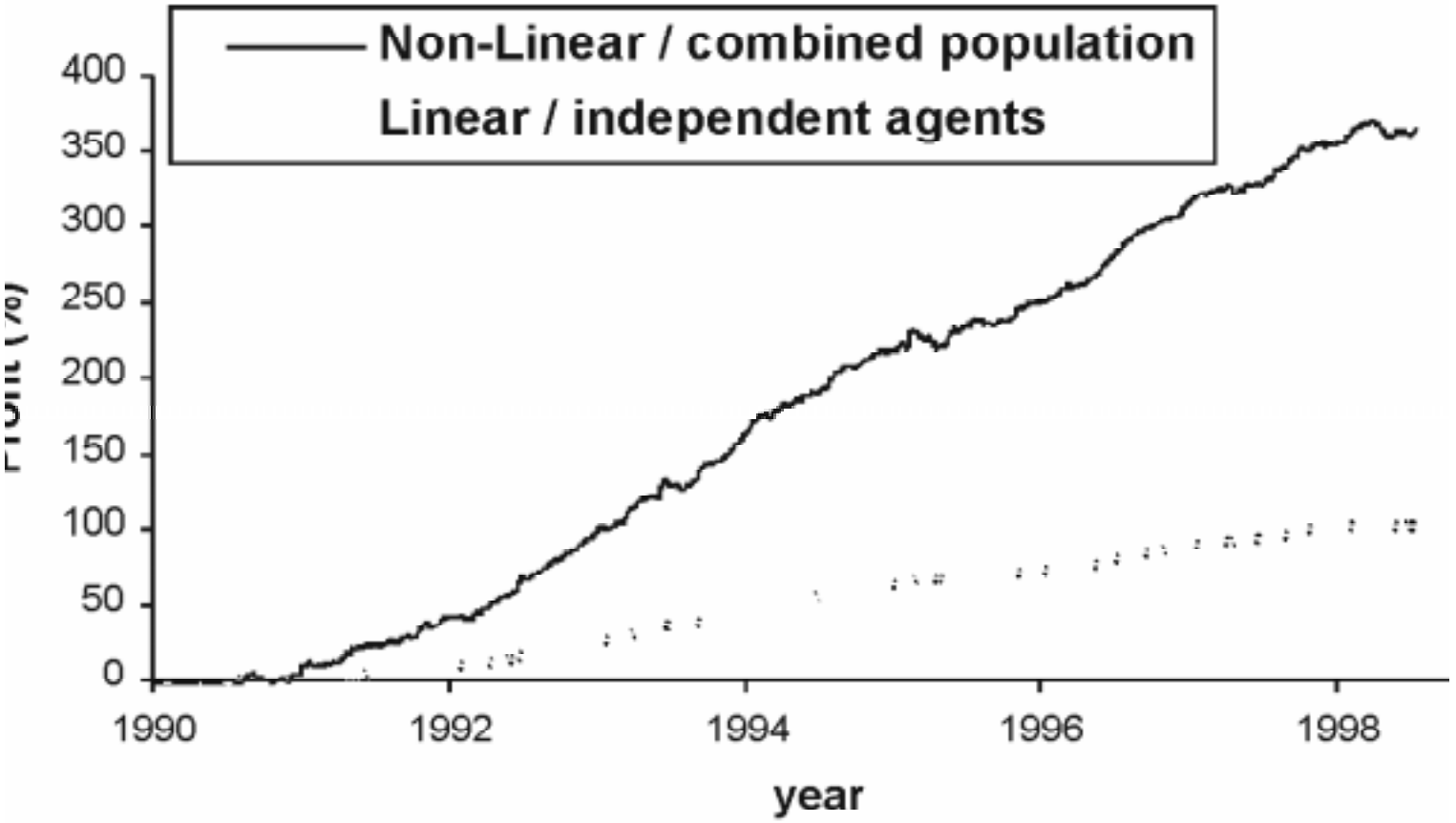

5-turm returns
0.2
0.15
0.1
0.05
0
Probability
-0.9 -0.6 -0.3 0.0 0.3 0.6
return

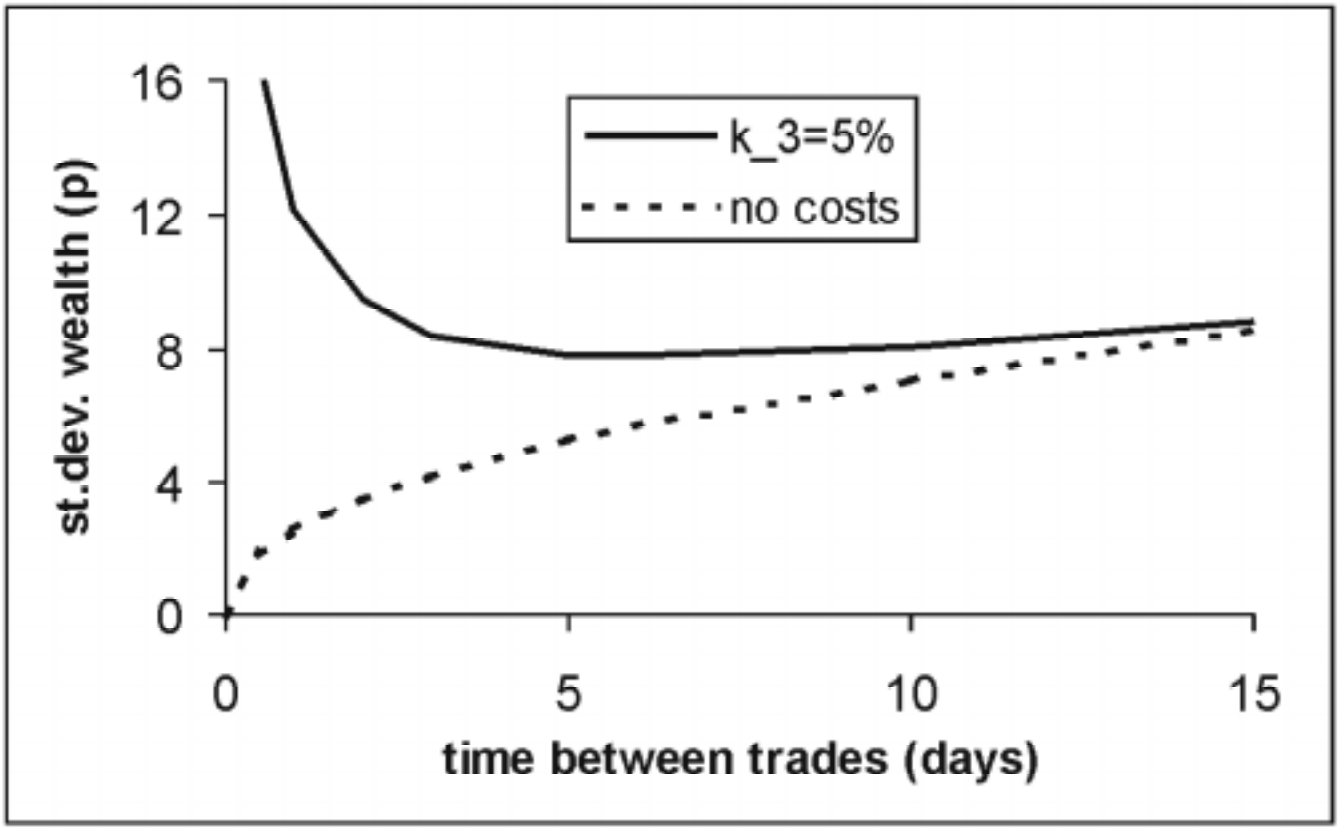

Volume
auto-correlation
0.9
0.7
0.5
0.3
0.1
-0.1
correlation
0 25 50 75 100
lag (turns)

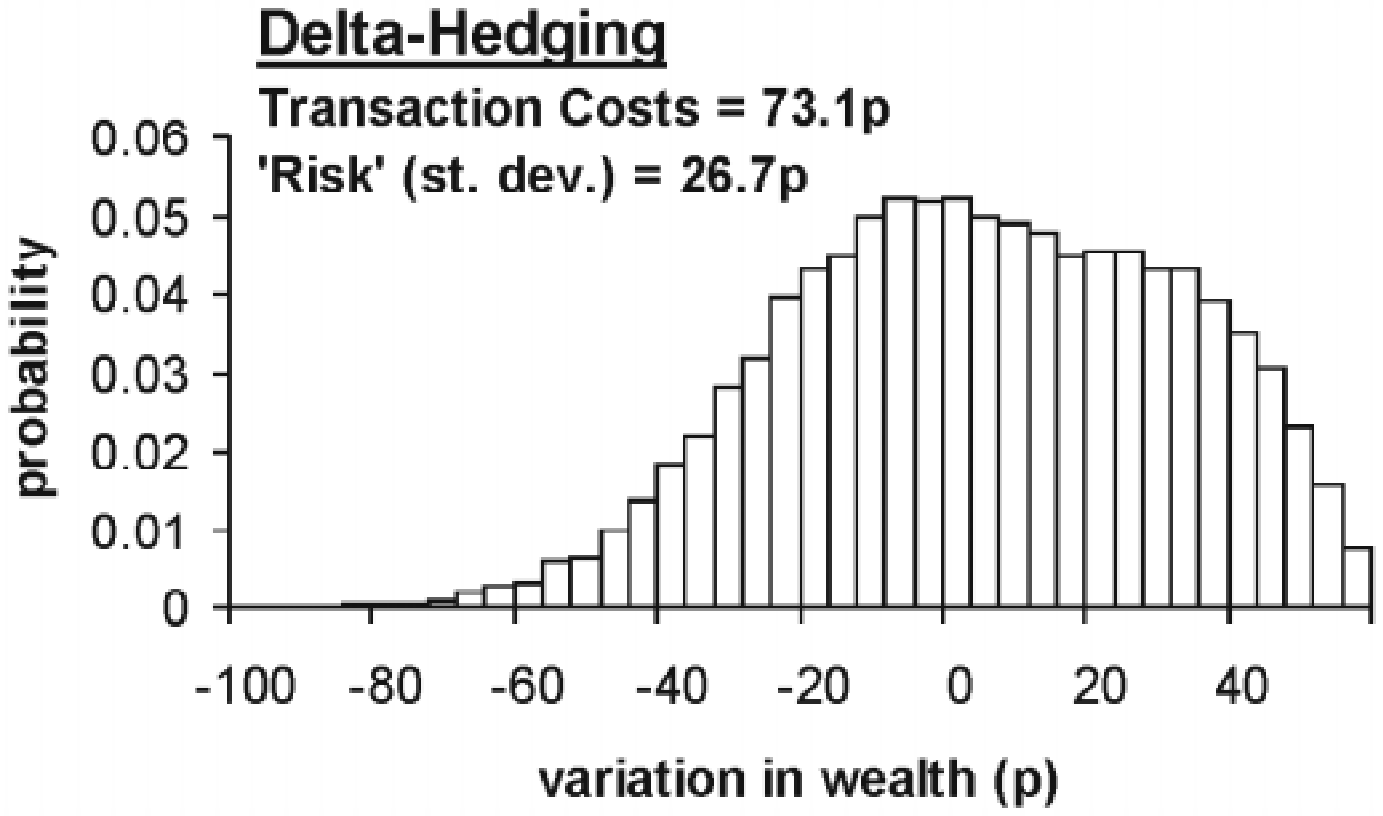


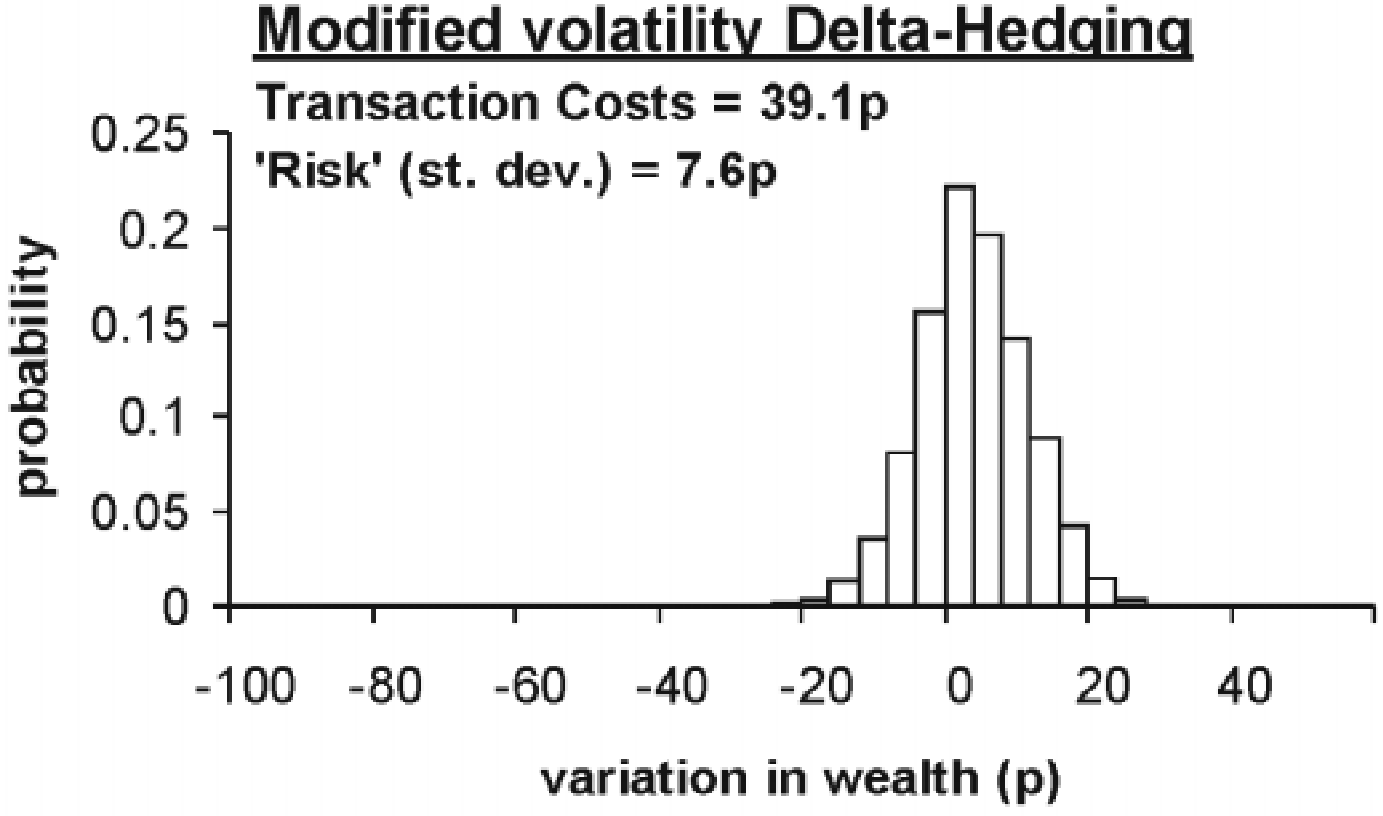



$/Yen FX
170
150
130
110
90
70
1990
1992
1994
1996
1998
year


Non-Linear / combined population
Linear / independent agents
400
350
300
250
200
150
100
50
0
1990
1992
1994
1996
1998
year




Delta-Hedging
Transaction Costs = 73.1p
'Risk' (st. dev.) = 26.7p
probability
0.06
0.05
0.04
0.03
0.02
0.01
0
-100
-80
-60
-40
-20
0
20
40
variation in wealth (p)


Modified volatility Delta-Hedging
Transaction Costs = 39.1p
'Risk' (st. dev.) = 7.6p
probability
0.25
0.2
0.15
0.1
0.05
0
-100
-80
-60
-40
-20
0
20
40
variation in wealth (p)