The taxation of trades in assets [1]

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Abstract

When the asset market is incomplete, there typically exist taxes on trades in assets that are Pareto improving. The fiscal policy is anonymous, it is fully and correctly anticipated by traders, and it results in *ex post* Pareto optimal allocations; as such, it improves over previously proposed constrained interventions.

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1 Introduction

Ever since Arrow [1] and Debreu [7] stated definitively and demonstrated the theorems of classical welfare economics, the focus has been on possible sources of failure of the Pareto optimality of competitive equilibrium allocations. Complete markets in elementary securities or in contingent commodities allow the theorems of welfare economics to encompass economies with uncertainty, as in Arrow [2] or Debreu [9]. The absence of a complete asset market is a well-recognized reason for the Pareto suboptimality of competitive allocations.

Competitive equilibrium allocations in economies with an incomplete asset market are suboptimal in a strong sense: Pareto improvement is possible even under the restrictions implied by the incompleteness. Constrained suboptimality, defined in Diamond [10], was formally shown in Hart [23], and then proved robust or generic in Geanakoplos and Polemarchakis [18] and Citanna, Kajii and Villanacci [6], which extended the argument. Constrained suboptimality is a positive argument for intervention in competitive market economies, which is compelling when intervention is compatible with the structural characteristics that underlie the incompleteness; it is anonymous; it results in ex-post Pareto optimal allocations of commodities; and it is anticipated by traders in markets for assets.

Compatibility restricts alternative allocations, but it is hard to assess or make precise when reasons for the incompleteness of the asset markets are not made formally explicit; it is commonly taken to require that interventions take the asset structure as given. Anonymity economizes on information and complexity, and it circumvents incentive-compatibility constraints; ex-post optimality guarantees against further intervention or deviations; anticipation allows for repeated intervention.

A variety of intervention policies and corresponding notions of constrained suboptimality have been introduced in the literature, all compatible with the incompleteness of the asset market. The robust constrained suboptimality results obtained to date — Geanakoplos and Polemarchakis [18] for individual portfolio reallocations, Herings and Polemarchakis [24] for rationing in asset and spot commodity markets, Citanna, Kajii and Villanacci [6] for lump-sum taxes and transfers — fail at least one of the above-mentioned criteria: for instance, portfolio reallocations or lump-sum taxes and transfers are not anonymous, a fact emphasized in Kajii [27] , and rationing does not yield ex-post optimal allocations of commodities [1].

Here, the instruments of intervention are linear taxes or subsidies on the purchase of assets. The main result is that if the asset market is sufficiently incomplete, generically, there exist Pareto improving fiscal policies. The taxation of assets is anonymous; the resulting allocation of commodities is ex-post Pareto optimal, and there are no ex-post constraints on asset trades enforced by shutting down financial markets; intervention is compatible with the incompleteness

[1]The introduction of new assets or the alteration of asset payoffs in Cass and Citanna [5] and Elul [15], [16] preserves anonymity, but either it reduces incompleteness or it requires state-contingent policies.

of the asset market; and it does not require the announcement of future or state-contingent taxes or subsidies, which could be subject to credibility constraints.

The argument is easy to understand. With portfolio reallocation policies, as in Geanakoplos and Polemarchakis [18], individual asset holdings are confiscated and redistributed in order to control the state-contingent distribution of wealth. Here, a redistribution of portfolio holdings is induced through taxes, with possibly negative tax rates or subsidies, on trades in asset: asset holdings are indirectly controlled by creating a bid-ask spread –that, nevertheless, at times can be negative. The core of the technical argument consists in bypassing the nondifferentiabilities and nonconvexities that taxes and subsidies create.

Taxation has been extensively studied as a policy instrument. Since Dupuit [14], and through Hotelling [25], Boiteux [4] and Debreu [8], economists have established that commodity taxation, whether through a quantity tax or a tax ad valorem or excise with a rebate, generates a welfare burden on individuals at a competitive equilibrium, the "deadweight loss triangle" in textbooks.

The interest in taxation arises from the presence of distortions, such as externalities, public goods or, here, incomplete hedging opportunities [2]. The taxation of asset trades affects the holdings of assets and, as a consequence, the distribution of state-contingent wealth, which, in economies with multiple commodities, may affect relative prices in spot markets; indeed, Pareto improvements require changes in the relative prices of commodities, which rules out identical, homothetic state-contingent preferences that insulate spot commodity markets from variations in the distribution of wealth. In the presence of multiple commodities, with numéraire assets and for a generic choice of utilities and endowments, ad valorem taxes on asset prices and uniform rebates in fact have an *effective welfare-controlling* effect; this requires that the number of independent policy tools (the number of assets subject to taxation) be at least as large as the number of policy objectives (the levels of utility of individuals) and that state-contingent marginal valuations of revenue by individuals be sufficiently dispersed.

In the economies of this paper information is symmetric and the differential taxation of assets is designed to affect the distribution of state-contingent revenue; taxation applies to all trades, whether "speculative "or not [3].

Though taxation is anonymous, Pareto improving intervention requires information about the fundamentals of the economy — here, the preferences and endowments of individuals. It is then an issue whether the information re-

[2]Following Ramsey [30], several authors characterized optimal commodity taxes, among them Diamond and Mirrlees [11] and Guesnerie [21], while others, among them Feldstein [17] and Guesnerie [20] studied the possibility of tax reform — Auerbach [3] and Mirrlees [29] survey the literature. Also, Diamond and Mirrlees [12] looked into the implication of market incompleteness on optimal commodity taxes, and recently Geanakoplos and Polemarchakis[19] showed that also in economies with externalities, anonymous Pareto improving commodity taxes exist generically.

[3]There have been other arguments for the taxation of trades in assets. Tobin [33], [34] advocated the taxation of financial transactions and emphasized the impact of the financial system on macroeconomic performance. The argument there concerned, however, the taxation of speculative transactions and it was designed to curb destabilizing volatility in a dynamic context, as in Dow and Rahi [13] or Stiglitz [32].

quired for determining the welfare consequences of taxation can be obtained from market data, in particular from equilibrium prices. The argument in [28] is that market data, namely, the equilibrium prices of commodities and assets as endowments vary, suffices to identify the profile of utilities. Evidently, this is demanding, and the result only establishes that identification is possible "in principle."Further work should then investigate the implementation of Pareto-improving taxes on asset trades.

2 Economies

The economy is of pure exchange, with finitely many individuals and commodities, two periods and uncertainty. Uncertainty is described by states of the world $s \in \mathcal{S} = \{1, \ldots, S\}$, with $S \geq 2$. Physical commodities are $l \in \mathcal{L} = \{1, \ldots, L\}$, also with $L \geq 2$. At a state of the world s, commodities are indexed by (l, s), and a bundle of commodities is a strictly positive real vector $x_s = (\ldots, x_{l,s}, \ldots)'$; across states of the world, a bundle of commodities is $x = (\ldots, x_s, \ldots)'$.

Individuals are $i \in \mathcal{I} = \{1, \ldots, I\}$, with $I \geq 2$. The preferences of an individual are described by the ordinal utility function u^i, with domain the consumption set of strictly positive bundles of commodities across states of the world; the endowment of the individual is e^i, a bundle of commodities across states of the world. The utility function is smooth, differentially strictly increasing: $Du^i \gg 0$, and differentially strictly quasi-concave: if $b \neq 0$ and $Du^i b = 0$, then $b' D^2 u^i b < 0$, while, along a sequence of consumption plans, $(x_n \gg 0 : n = 1, 2, \ldots,)$, if $\lim_{n\to\infty} x_n = x \not\gg 0$, then $\lim_{n\to\infty}(\|Du^i(x_n)\|)^{-1} x_n' Du^i(x_n) = 0$ [4]; the endowment is assumed strictly positive: $e^i \gg 0$.

The preferences of an individual may, but need not, admit a von Neumann-Morgenstern representation, (v^i, π^i), where v^i is a state-independent cardinal utility index, $\pi^i = (\ldots, \pi^i_s, \ldots)$ is a (subjective) probability measure on the set of states of the world, and $u^i = \mathrm{E}_{\pi^i} v^i$; alternatively, preferences may have an additively separable representation, $(\ldots, u^i, \ldots)$, where u^i_s, is a state-dependent cardinal utility index, and $u^i = \sum_{s\in\mathcal{S}} u^i_s$.

Commodities are traded in spot markets after the resolution of uncertainty. Prices of commodities at a state of the world are a row vector $p_s = (1, \ldots, p_{l,s}, \ldots) \gg 0$: commodity $l = 1$ is the numé raire, and prices are strictly positive; across states of the world, prices of commodities are $p = (\ldots, p_s, \ldots)$. At a state of the world, the value of the a bundle of commodities x_s at prices of commodities p_s is $p_s x_s$; across states of the world, the expenditures associated with a bundle of commodities x at prices of commodities p are $p \otimes x = (\ldots, p_s x_s, \ldots)$. The multiplicity of commodities and the diversity of individuals guarantee that spot markets for commodities are active.

Financial assets are $a \in \mathcal{A} = \{1, \ldots, A\}$, a finite, nonempty set. They are exchanged prior to the resolution of uncertainty, and they are employed to

[4]The boundary condition on the utility function is satisfied if the closure of the indifference surface through a consumption plan is contained in the interior of the consumption set, a stronger condition.

transfer revenue across states of the world, and they are in zero net supply. A portfolio of assets is $y = (\ldots, y_a, \ldots)'$. At a state of the world, the payoff of an asset is $r_{a,s}$, denominated in units of the numéraire commodity; across states of the world, the payoffs of an asset are $r_a = (\ldots, r_{a,s}, \ldots)'$. The asset payoffs at a state of the world s are $R_s = (\ldots, r_{a,s}, \ldots)$, and the matrix of asset payoffs is

$$R = (\ldots, r_a, \ldots) = (\ldots, R_s, \ldots)'.$$

The column span of the matrix of asset payoffs is $[R]$, the subspace of attainable reallocations of revenue across states of the world. Prices of assets are $q = (1, \ldots, q_a, \ldots)$. Assumptions on the payoffs of assets are standard: there are no redundant assets, $\dim[R] = A$; the asset market is active, $A \geq 2$; and the payoffs of asset $a = 1$ is positive, $r_1 > 0$.

The following additional assumption is strong, but it serves to yield a strong result: anonymous Pareto improving taxes — we discuss it in Section 5.

Assumption 1. *The asset market is sufficiently incomplete:*

$$\min\{A - 1, S - A\} \geq I.$$

An allocation of commodities is $\vec{x} = (\ldots, x^i, \ldots)$, such that $x^i \geq 0$ for every individual. Aggregate consumption is $x^a = \sum_{i \in \mathcal{I}} x^i$, while the aggregate endowment is $e^a = \sum_{i \in \mathcal{I}} e^i$. An allocation of commodities is feasible if $x^a = e^a$. An allocation of portfolios of assets is $\vec{y} = (\ldots, y^i, \ldots)$; the aggregate portfolio is $y^a = \sum_{i \in \mathcal{I}} y^i$. An allocation of portfolios of assets is feasible if $y^a = 0$. The excess demand of an individual is the row vector $z^i = (..., z^i_s, ...)$, where $z^i_s = x^i_s - e^i_s$.

The set of economies

Economies are identified by $\omega = (u, e) \in \Omega$, the utility functions endowments oh individuals. The space of utilities is endowed with the topology of C^2-uniform convergence over compact sets, while the space of endowments has the standard Euclidean structure.

Quadratic perturbations of utilities will be used to establish density in the main proposition. For an economy $\omega \in \Omega$ and given an equilibrium consumption plan x^*, a perturbed utility for an individual i is a function

$$u^i_{(x^{i*}, \rho, \varepsilon)}(x^i, M^i) = u^i(x^i) + (1/2)\varepsilon\rho(x^i)[(x^i - x^{i*})' M^i (x^i - x^{i*})],$$

where: x^{i*} is the equilibrium individual consumption plan; $\varepsilon > 0$ is a scalar; $\rho(x^i)$ is a bump function[5]; and M^i is a symmetric, LS–dimensional matrix — details on these perturbations are in Citanna, Kajii and Villanacci [6]. The second derivative of this function with respect to x^i is exactly equal to $D^2 u^i(x^i) + \varepsilon M^i$ in a small open neighborhood of the equilibrium allocation x^{i*}. The vector of quadratic perturbations is $M = (..., M^i, ...)$, while $u^i(x^i, M^i)$ denotes $u^i_{(x^{i*}, \rho, \varepsilon)}(x^i, M^i)$. A generic set of economies is an open and dense subset of Ω; a property holds generically if it holds for a generic set.

[5] Gullemin and Pollack [22], Chapter 1.

3 Fiscal policy and equilibrium

Taxation of asset purchases is introduced in the economy. Rates of taxation or subsidy on the purchase of assets are $t = (\ldots, t_a, \ldots)$, with $t_a > -1$ for all $a \in \mathcal{A}$. Then, the purchase prices of assets are $(1+t) \otimes q = (\ldots, (1+t_a)q_a, \ldots)$. Taxing both purchases and sales does not entail any essential change; in order to ease notation, the focus here is on the taxation of purchases. The value of a portfolio of assets y at prices of assets q and rates of taxation t is $((1+t) \otimes q)y_+ - qy_-$, where $y_{a,+} = \max\{0, y_a\}$, while $y_{a,-} = \max\{0, -y_a\}$, and $y_+ = (\ldots, y_{a,+}, \ldots)$, while $y_- = (\ldots, y_{a,-}, \ldots)$. Note that no individual can simultaneously buy and sell an asset. This restriction is void when the tax t_a is nonnegative, but it is needed when the buyer receives a subsidy, or arbitrage opportunities could arise.

Aggregate fiscal revenue from the taxation of assets is $T = \sum_{i \in \mathcal{I}} (t \otimes q) y^i_+$. It is distributed across individuals according to a fixed scheme $\delta = (\ldots, \delta^i, \ldots) \gg 0$, with $\sum_{i \in \mathcal{I}} \delta^i = 1$ — the uniform distribution scheme is $\delta = (\ldots, (1/I), \ldots)$; the revenue of an individual is $\delta^i T$. Fiscal policy is conducted through the vector of tax rates $t \in \mathcal{T}$, where $\mathcal{T}$ is an open neighborhood of zero, of dimension A — aggregate fiscal revenue, T, is determined endogenously, as a residual.

With taxes, the optimization problem of an individual is

$$\begin{aligned} \max_{x,y} \quad & u^i(x), \\ \text{s.t.} \quad & ((1+t) \otimes q)y_+ - qy_- - \delta^i T \leq 0, \\ & p \otimes \left(x - e^i\right) \leq Ry. \end{aligned}$$

Definition 1. *A competitive equilibrium with fiscal policy t consists of a feasible allocation and prices of commodities and assets and fiscal revenue,* $(\vec{x}, \vec{y}, p, q, T)$*, such that, for every individual,* (x^i, y^i) *is a solution to the optimization problem.*

A competitive equilibrium in the standard sense corresponds here to a competitive equilibrium with inactive fiscal policy, $t = 0$. At a competitive equilibrium where fiscal policy is inactive and rates of taxation vanish, there is no fiscal revenue and the distribution scheme is immaterial.

3.1 Regularity and the existence of equilibria

We define, for every individual, the function F^i by

$$F^i = \begin{pmatrix} F^i_I \\ F^i_{II} \\ F^i_{III} \\ F^i_{IV} \end{pmatrix} = \begin{pmatrix} D_{x^i} u^i(x^i, M^i) - \lambda^i \otimes p \\ \lambda^i R - \mu^i q^i \\ ((1+t) \otimes q) y^i_+ - q y^i_- - \delta^i T \\ -p \otimes (x^i - e^i) + R y^i \end{pmatrix},$$

where

$$q_a^i = \begin{cases} (1+t_a)q_a, & \text{if} \quad y_a^i \geq 0, \\ \\ q_a & \text{if} \quad y_a^i < 0, \end{cases}$$

and $\lambda^i = (\ldots, \lambda_s^i, \ldots) \gg 0$ and $\mu^i > 0$ are Lagrange multipliers associated with the budget constraints across states of the world and the asset market, respectively, and $\lambda^i \otimes p = (\ldots, \lambda_s^i p_s, \ldots)$.

Across individuals, we define the function F^0 by

$$F^0 = \begin{pmatrix} F_V^0 \\ \\ F_{VI}^0 \\ \\ F_{VII}^0 \end{pmatrix} = \begin{pmatrix} \tilde{x}^a - \tilde{e}^a \\ \\ \tilde{y}^a \\ \\ T - (t \otimes q) y_+^a \end{pmatrix},$$

where $\tilde{x}^a$ is the aggregate demand for commodities and $\tilde{e}^a$ the aggregate endowment of commodities other than the numéraire across states of the world, $\tilde{y}^a$ is the aggregate demand for assets other than the numéraire, and $y_+^a = \sum_{i \in \mathcal{I}} y_+^i$.

Finally, we define the function F as $F = (\ldots, F^i, \ldots, F^0)'$; elements of the domain of the function are $(\xi, t, \omega) = (\vec{x}, \vec{y}, \lambda, \mu, p, q, T, t, u, e)$, where $\xi = (\vec{x}, \vec{y}, \lambda, \mu, p, q, T)$ are endogenous variables, with $\lambda = (\ldots, \lambda^i, \ldots)$, and $\mu = (\ldots, \mu^i, \ldots)$. The domain of endogenous variables is Ξ, an open set of dimension $N = (ILS + IA + IS + I + S(L-1) + (A-1) + 1)$, which coincides with the dimension of the range of the function F.

The zeros of the function F^i represent the Kuhn-Tucker conditions for the individual optimization problem when $t = 0$. For an economy $\omega \in \Omega$, a competitive equilibrium with inactive fiscal policy, $t = 0$, augmented with the associated Lagrange multipliers of the budget constraints of individuals, is then determined as a solution to the system of equations $F_{(0,\omega)}(\xi) = 0$. The argument in [18] applies and shows that competitive equilibria with inactive fiscal policy exist: $F_{(0,\omega)}^{-1}(0) \neq \emptyset$.

In general, F^i does not correspond to the Kuhn-Tucker conditions when $t \neq 0$: when $t_a < 0$, the budget constraint is not convex; while, in addition, if at a solution to the optimization problem of an individual, $y_a^i = 0$ for some a, then the optimum occurs at a kink and the equations that characterize the optimum are not smooth. However, both the potential lack of differentiability or the lack of convexity problems can be bypassed, as long as t is restricted to a neighborhood of $t = 0$, if the equilibrium with inactive fiscal policy is locally unique and all individuals trade all assets or $y_a^i \neq 0$. This condition, a key element of the analysis, holds generically as is summarized in the following standard lemma — items 1 and 2[6].

[6] The proof of Lemma 1 is standard and therefore we omit it; it follows immediately from the analogous argument in Geanakoplos and Polemarchakis [19]. We state item 3 here, but we use it only later, in the proof of density; it is this item that requires the condition $(S - A) \geq I$ that appears in Assumption 1, and we discuss it in Section 5.

Lemma 1. *There exists a generic subset of economies* Ω^0*, such that, for every economy* $\omega \in \Omega^0$,

1. *the function* $F_{(0,\omega)}$ *is transverse to 0:* $\dim[D_\xi F_{(0,\omega)}] = N$; *and, at a competitive equilibrium with inactive fiscal policy,*

2. *every individual trades every asset:*

$$F_{(0,\omega)}(\xi) = 0 \quad \Rightarrow \quad y^i_a \neq 0, \quad a \in \mathcal{A},\ i \in \mathcal{I},$$

and

3. *the matrix*

$$\begin{pmatrix} \vdots \\ \\ \\ \lambda^i \otimes z^i \\ \\ \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ \\ \\ \ldots \lambda^i_s z^i_s \ldots \\ \\ \\ \vdots \end{pmatrix}$$

has full row rank, I.

In Lemma 1, as well as in the next lemma, genericity refers to perturbations only in endowments. By continuity, Lemma 1 implies that, for small enough t, the zeros of F represent competitive equilibria with fiscal policy t, and that the equations $F_{(t,\omega)}(\xi) = 0$ are smooth and determine, locally, the welfare impact of taxes on asset trades.

Lemma 2. *For every economy* $\omega \in \Omega^0$*, there exists an open set of fiscal policies,* $\mathcal{O}_\omega \subset \mathcal{T}$*, such that* $0 \in \mathcal{O}_\omega$*, and if* $t \in \mathcal{O}_\omega$*, then competitive equilibria* $\xi \in \Xi$ *with fiscal policy* t *for the economy* ω *exist and are obtained as solutions to the system of equations* $F_{(t,\omega)}(\xi) = 0$*. They are locally smooth functions of the fiscal policy parameters* ζ *and of the quadratic perturbations* M *:*

$$d\xi = -(D_\xi F)^{-1}(D_t F dt + D_M F dM).$$

Notice that, once the function F is restricted to the open and dense subset Ω^0, one gets as a consequence that $y^i_a \neq 0$ at all equilibria with $t = 0$, and there is no need to impose this condition as an additional restriction on the domain of F. While F may still be nondifferentiable for some combination (ξ, ω) in its domain, it will be smooth at those pairs (ξ, ω) for which $\omega \in \Omega^0$, $(t \in \mathcal{O}_\omega)$ and $F_{(t,\omega)}(\xi) = 0$, which is all that it is needed for the analysis. The reader should keep this in mind when openness of the subset of economies in Ω^0 where a Pareto-improving fiscal policy exist will be proved.

4 Pareto improving fiscal policy

Let $u(\vec{x}) = (\ldots, u^i(x^i), \ldots)$ be the vector of utilities associated with an allocation of commodities. An allocation of commodities, $\vec{x} = (\ldots, x^i, \ldots)$, is strictly Pareto superior to another, $\vec{x}' = (\ldots, x^{i\prime}, \ldots)$, if $u(\vec{x}) \gg u(\vec{x}')$.

Definition 2. *A feasible allocation of commodities is strictly constrained Pareto suboptimal if there exists a strictly Pareto superior, competitive equilibrium allocation with fiscal policy.*

Constrained interventions are restricted to the taxation of trades in assets and to the distribution of fiscal revenue.

Proposition (Constrained suboptimality) *Generically, every competitive equilibrium allocation with inactive fiscal policy is strictly constrained Pareto suboptimal. The fiscal policy that implements the strict Pareto improvement can be chosen to involve no fiscal revenue.*

The proof of the Proposition follows the reasoning developed in Citanna, Kajii and Villanacci [6]; that is,

$$\widetilde{\Phi}(\xi, t, \omega) = \begin{pmatrix} D_\xi F & D_t F \\ & \\ D_\xi u & 0 \end{pmatrix}$$

represents the derivative of the equilibrium system and of the utility vector at $t = 0$. An equilibrium is constrained suboptimal if the row rank of $\widetilde{\Phi}(\xi, t, \omega)$ is full [7]. Constrained suboptimality is nothing but a violation of the first order conditions for a vector maximum or it obtains when u is a submersion on the equilibrium set. The rank condition on $\widetilde{\Phi}(\xi, t, \omega)$ is indeed equivalent to showing that the system defined by

$$F_{opt}(\xi, b; t, \omega) = \begin{pmatrix} b_1 D\tilde{F} + b_2 Du \\ & \\ \|b\| - 1 \end{pmatrix} = 0,$$

where (b_1, b_2) is a vector of dimension $N+I$, has no solution (ξ, b) at $F_{(\overline{\zeta},\omega)}(\xi) = 0$, for any given ω. $D\widetilde{F}$ is nothing but an appropriately chosen submatrix of the derivative matrix DF. Hence if a planner were to choose t (a tax policy) to maximize the utility vector u (the social welfare) subject to $\widetilde{F} = 0$, the value $t = 0$ would not satisfy the first order conditions for an optimum: $b_1 Du + b_2 D\widetilde{F} = 0$ is not solved at $t = 0$, and hence a tax reform (a change in taxes and redistributions) would do better. Constrained suboptimality is equivalent to the existence of a feasible direction of tax reforms in the sense of Guillemin and Pollack [20].

[7] Citanna, Kajii and Villanacci [6], Proposition 1.

Constrained suboptimality holds for a generic subset of economies in Ω. In order to show density, and using the quadratic, finite-dimensional parametrization M of utility functions, it suffices to show[8] that the matrix $D_{b,M}F_{opt} = (D_bF_{opt}, D_MF_{opt})$ has full row rank.

5 Discussion

Market incompleteness The natural requirement that the number of policy instruments exceed the number of policy targets is the object of Assumption 1: instruments are taxes on trades in assets and the rebate, while targets are the utility levels of individuals at equilibrium. If Pareto improvement may involve fiscal revenue, it actually suffices that $A \geq I$; if intervention must satisfy fiscal balance, $T = 0$, the argument requires that $A \geq I + 1$.

Sufficient market incompleteness relative to the number of individuals, $S - A \geq I$, also in Assumption 1, is used to prove item 3 in Lemma 1. The matrix represents the relative commodity price effects of tax reforms. Indeed, for a marginal change of policy instruments or tax rates Δt, and fixing $\Delta q = 0$ and $\Delta y^i = 0$, the change in individual i's indirect utility induced via a relative spot prices change is $\Delta u^i = \left(\lambda^i Z^i\right) \otimes D_t p \Delta t$. Then, item 3 of Lemma 1 guarantees that there is sufficient variation of utilities due to these price effects, and asset trade taxation yields controllable utility changes. While Geanakoplos and Polemarchakis [18] used item 3, Lemma 1, it did not require $S - A \geq I$ since the no-arbitrage equations need not be satisfied by a direct portfolio reallocation policy.

Since $S > A$, Assumption 1 implies that $LS > I$, a condition used in Geanakoplos and Polemarchakis [18] to establish the constrained inefficiency of financial equilibrium when the planner changes asset holdings directly and must take into account the initial asset prices [9]. Kajii [26] shows that Pareto improvement through anonymous direct asset reallocations can be obtained only if indeed the planner must take into account the initial asset prices, but this condition is not sufficient, and gives a counterexample. Here, anonymity is obtained with the additional condition of sufficient incompleteness, and suggests that, once no-arbitrage equations as well as initial asset prices are taken into account, but with the slack provided by taxes, portfolio reallocations are anonymous.

Taxation of both purchases and sales of the numéraire asset gives more degrees of freedom, and it allows one to derive generic Pareto improvements under the less stringent condition $\min\{A, S - A\} \geq I$ — the proof mimics that of the Proposition and we omit it. Evidently, the taxation of both purchases and sales for an asset other than the numéraire is redundant, since the asset price can adjust as the tax rates change and, thus, effectively nullify one of the tax instruments in each market.

[8] Citanna, Kajii and Villanacci [6], Proposition 3.

[9] That is, portfolio reallocations must be balanced not only in terms of the units of assets redistributed, but also in value, where their value is computed at the initial equilibrium prices.

The following example illustrates the effects of asset trade taxation. There are two individuals, two consumption goods and two assets. The intertemporal von Neumann - Morgenstern utility function of an individual is

$$u^i = x_{1,1} + \delta^i \mathrm{E}_{s>1}\left[(\alpha^i)\ln x_{1,s} + (1-\alpha^i)\ln x_{2,s}\right], \quad \delta^i > 0, \quad 0 < \alpha^i < 1,$$

and his endowment is $e^i = (e^i_{1,1}, (e^i_{1,2}, e^i_{2,2}), \ldots, (e^i_{1,S}, e^i_{2,S})) \gg 0$.This is a slight deviation from the general specification, as utility for consumption at $s = 1$ is quasi-linear in the numéraire commodity at $s = 1$, where there is no utility for good $l = 2$. Optimal consumption at $s > 1$ is then given by $x^i_{1,s} = \alpha^i m^i_s$ and $x^i_{2,s} = (1-\alpha^i)m^i_s$, where $m^i_s = e^i_{1,s} + p_{2,s}e^i_{2,s} + r_s y^i$ is the revenue of the individual after the resolution of uncertainty. Then, using commodity market clearing we get spot equilibrium prices

$$p_{2,s} = \frac{1}{\sum_i \alpha^i e^i_{2,s}}\left[\sum_i [(1-\alpha^i)e^i_{1,s} - \alpha^i r_s y^i]\right].$$

Importantly, since equilibrium in the market for assets requires that $\sum_i y^i_a = 0$, the distribution of holdings of assets and, more generally, the distribution of revenue, affect relative prices only if individuals are heterogeneous: $\alpha^1 \neq \alpha^2$. The indirect utility function is

$$\tilde{u}^i = e^i_{1,1} - \sum_a q^i_a y^i_a + \delta^i \mathrm{E}_s\left[\ln m^i_s - (1-\alpha^i)\ln p_{2,s}\right] + const.,$$

where q^i_a has been previously defined. Since the utility function is quasi-linear, it is not necessary to specify the distribution of fiscal revenue T. The marginal utility of revenue for the individual at a state of the world is

$$\lambda^i_s = \frac{1}{m^i_s},$$

and

$$\frac{\partial \tilde{u}^i}{\partial y^i_a} = -q^i_a + \delta^i \mathrm{E}_{s>1}\lambda^i_s r_{a,s},$$

while

$$\frac{\partial \tilde{u}^i}{\partial q_a} = -y^i_a \quad \text{and} \quad \frac{\partial \tilde{u}^i}{\partial p_{2,s}} = -\delta^i \lambda^i_s\left[\frac{(1-\alpha^i)}{p_s}m^i_s - e^i_{2,s}\right].$$

The optimization of individuals in the market for assets requires that

$$\frac{\partial \tilde{u}^i}{\partial y_a} = 0 \quad \text{or} \quad -q^i_a + \delta^i \mathrm{E}_s \lambda^i_s r_{a,s} = 0.$$

Given the rates of taxation of assets, t_a, after substitution, the equilibrium conditions reduce to a system of two polynomial equations in two unknowns, y^2_1 and y^2_1. *???* With inactive fiscal policy, the equations that determine equilibrium are

$$\delta^1 \mathrm{E}_{s>1}\lambda^1_s r_{a,s} = \delta^2 \mathrm{E}_{s>1}\lambda^2_s r_{a,s}, \quad a = 1, 2.$$

For generic values of the preference parameters and endowments of individuals, at equilibrium every individual trades every asset: $y_a^i \neq 0$, and, as a consequence, the equilibrium is locally a smooth function of the rates of taxation of assets, t_a. Since

$$d\tilde{u} = \begin{pmatrix} d\tilde{u}^1 \\ \\ d\tilde{u}^2 \end{pmatrix} = \begin{pmatrix} \frac{\partial \tilde{u}^1}{\partial t_1} & \frac{\partial \tilde{u}^1}{\partial t_2} \\ \\ \frac{\partial \tilde{u}^2}{\partial t_1} & \frac{\partial \tilde{u}^2}{\partial t_2} \end{pmatrix} \begin{pmatrix} dt_1 \\ \\ dt_2 \end{pmatrix},$$

Pareto improving taxes exist if the matrix

$$\begin{pmatrix} \frac{\partial \tilde{u}^1}{\partial t_1} & \frac{\partial \tilde{u}^1}{\partial t_2} \\ \\ \frac{\partial \tilde{u}^2}{\partial t_1} & \frac{\partial \tilde{u}^2}{\partial t_2} \end{pmatrix}$$

has full row rank or, since here the matrix is square, if the determinant does not vanish. But,

$$\frac{\partial \tilde{u}^i}{\partial t_a} = \sum_{a'} \left[\frac{\partial \tilde{u}^i}{\partial y^i_{a'}} \frac{\partial y^i_{a'}}{\partial t_a} + \frac{\partial \tilde{u}^i}{\partial q^i_{a'}} \frac{\partial q^i_{a'}}{\partial t_a} \right] + \sum_s \frac{\partial \tilde{u}^i}{\partial p_s} \frac{\partial p_s}{\partial y^2_a} \frac{\partial y^2_a}{\partial t_a}$$

and, by direct substitution at $t_a = 0$,

$$\frac{\partial \tilde{u}^i}{\partial t_a} = -\sum_{a'} y^i_{a'} \frac{\partial q^i_{a'}}{\partial t_a} - \mathrm{E}_{s>1} \lambda^i_s \left[\frac{(1-\alpha^i)}{p_s} \tau^i_s - e^i_{2,s} \right] \frac{\partial p_s}{\partial y^2_a} \frac{\partial y^2_a}{\partial t_a}.$$

In the absence of relative price effects: $\partial p_s / \partial y^2_a = 0$, the argument for Pareto improving taxes fails since at equilibrium $\sum_i \partial \tilde{u}^i / \partial t_a = 0$. But, generically in the parameters, the system of polynomial equations that determine equilibrium augmented by the polynomial equation that determines a vanishing determinant for the matrix of coefficients has no solution; equivalently, at equilibrium, the determinant does not vanish and Pareto improving taxes exist.

Negative tax rates Pareto improving taxes cannot be guaranteed to be nonnegative, and they are guaranteed to be negative when we impose the fiscal balance requirement $T = 0$ [10]. Negative asset trading tax rates limit their decentralization through markets, as certain trades must then be prohibited in order to avoid unbounded tax arbitrage profits. Here, we ban these arbitrage trades by definition of $y_{a,+}$ and $y_{a,-}$, since no individual can be simultaneously on both sides of an asset market. The following numerical example shows that sometimes Pareto improving tax rates can be negative even if T is not restricted to be zero.

[10] We thank an anonymous referee for pointing out to us that negative taxes are found in the U.S. tax code: for example, when a worker's contribution to a 401(k) plan is deducted at the marginal tax rate in effect when he works, but later withdrawals are taxed at the lower marginal tax rate in effect in retirement.

In the example, there is no aggregate risk, utilities are von Neumann-Morgenstern, homothetic, and individuals are heterogeneous both in endowments and preferences. Setting $I = 2 = A$ and $S = 4$, the Proposition applies assuming that tax revenues are not restricted to be zero, since $S - A = 2 \geq I = 2$ and $A = 2 \geq I = 2$, which is enough as discussed above. Further, $L = 2$, and utilities are of the form

$$u^i(x^i) = \sum_s \pi_s^i(\alpha_s^i \ln x_{1,s}^i + (1 - \alpha_s^i) \ln x_{2,s}^i),$$

with $\pi^i = \pi$, for every individual. Heterogeneity in α's guarantees that spot commodity prices depend on the wealth distribution, not just on aggregate wealth. For every commodity and every state of the world, $\sum_i e_{l,s}^i = 1$. The asset payoff vectors are $r_1 = (1, 0, 0, 0)'$ and $r_2 = (0, 1, 1, 1)'$. Asset 2 is a discount bond.

With no taxes, the individual budget constraint is $q_1 y_1 + q_2 y_2 = 0$. Buying asset 1 then corresponds to selling asset 2, i.e., borrowing money, while selling it corresponds to buying asset 2 or lending. Hence, while taxes are on the purchases of assets, this can be interpreted as taxing purchases and sales of asset $a = 2$ only. Then, *other things equal*, a tax $t_1 > 0$ corresponds to an increase in the borrowing rate (from $1/q_2$ to $(1 + t_1)(1/q_2)$), while a tax $t_2 > 0$ corresponds to a decrease in the lending rate (from $1/q_2$ to $[1/(1 + t_2)](1/q_2)$). The actual change in the rate depends of course on the resulting after-tax q_2.

For normalization purposes, $q_1 = 1$ and $p_{1,s} = 1$ at every state of the world. In the example, the parameters of the economy are set as follows:

$$\pi = (5, 1/3, 1/3, 1/3), \qquad \alpha^2 = \mathbf{1} - \alpha^1,$$

$$\alpha_s^1 = \begin{cases} 1/3 & s \text{ odd}, \\ 2/3 & s \text{ even}, \end{cases} \qquad e_s^1 = \begin{cases} (1/2, 3/10) & s \leq 2, \\ (3/10, 1/2) & s > 2. \end{cases}$$

At the computed no tax equilibrium $y_2^1 = 0.0153$ and $q_2 = 0.1844$ –so $i = 1$ is a lender and $i = 2$ a borrower– and utilities are $u^1 = -4.9463$ and $u^2 = -2.8891$. After taxes of $t_1 = -0.16$ and $t_2 = 0.07$ are introduced, the new equilibrium has a price of $q_2 = 0.1842$, unchanged y_2^1 (obviously, y_1^1 changes), and utilities of $u^1 = -4.9462$ and $u^2 = -2.8890$.[11] Changes in endowments do not affect the

[11] We compute equilibria by first solving for optimal consumption, which is $x_{1,s}^i = \alpha_s^i m_s^i$ and $x_{2,s}^i = (1 - \alpha_s^i) m_s^i / p_{2,s}$, where $m_s^i = e_{1,s}^i + p_{2,s} e_{2,s}^i + r^s y^i$ is state-contingent income. Then, the commodity price $p_{2,s}$ and, therefore, Lagrange multipliers λ_s^i are computed as a function of 1's holdings of asset $a = 2$, and of the price of this second asset, y_2^1, q_2. Since Lagrange multipliers μ^i can also be determined as a function of y_2^1, q, equilibrium computation boils down to solving by norm minimization the system of no arbitrage equations for asset $a = 2$ and the two individuals, in the unknowns y_2^1 and q_2, with positivity constraints on consumption through the individual rationality constraints. Programming was done using the 'fmincon' routine of MatLab v. 6.0. Numerical errors are below $1/10^9$.

sign of tax changes, and Pareto improving taxes may be robustly negative. To guarantee that $t \in O_\omega$, tax rates and corresponding utility changes are nonzero, but small. With a change in certainty equivalent of less than $1/10^3$, this example is obviously not meant to try and assess the quantitative importance of our results, which remains to be assessed.

6 Proofs

Lemma 2 For any given $\omega \in \Omega^0$, one parametrizes the utility using M, the quadratic perturbation term, and considers the associated finite-dimensional parametrization (M, e) where $M = 0$ at the initial ω. $F^{-1}_{(0,\omega)}(0) \neq \emptyset$ for all ω and Lemma 1, item 1 (regularity) allow the application of the Implicit Function Theorem to claim that the system of equations $F_{(t,\omega')}(\xi) = 0$ has a solution, where $(\xi, t, \omega') \in \mathcal{N}_{(\overline{\xi},0,\omega)}$, an open neighborhood centered at $(\overline{\xi}, 0, \omega)$, with $\overline{\xi} \in F^{-1}_{(0,\omega)}(0)$. Standard compactness of $F^{-1}_{(0,\omega)}(0)$ implies that $\#F^{-1}_{(0,\omega)}(0) = K < \infty$, and so the existence of finitely many such neighborhoods, $\mathcal{N}_{(\overline{\xi}_k,0,\omega)}$, one for each $\overline{\xi}_k \in F^{-1}_{(0,\omega)}(0)$, $k \in K$. Projecting $\mathcal{N}_{(\overline{\xi}_k,0,\omega)}$ onto $\mathcal{T}$, and taking the intersection over k, one gets an open set $O'_\omega \subset \mathcal{T}$ around $t = 0$, such that $F_{(t,\omega)}(\xi) = 0$ has finitely many, locally unique smooth solutions $\xi_k(t;\omega)$ for each $t \in O'_\omega$.

To establish that a competitive equilibrium with fiscal policy exists, we want to show that, for $t \in O_\omega \subset O'_\omega$, the unique solution to the optimization problems of individuals obtains at $F_{(t,\omega)}(\xi) = 0$.If not, there exists a sequence $\{t_n\}_{n=1}^{\infty} \subset O'_\omega$ with $\lim_{n\to\infty} t_n = 0$ and, for each n, individuals $i_n \in \mathcal{I}$ whose optimization problem with fiscal policy t_n at prices p_n, q_n have portfolios $\tilde{y}^{i_n}_n$ different from the portfolios $y^{i_n}_n$, where p_n, q_n and $y^{i_n}_n$ are part of ξ_n such that $F_{(t_n,\omega)}(\xi_n) = 0$. Passing to a subsequence if necessary, $i_n = i$ for some $i \in \mathcal{I}$, and $\tilde{y}^i_{a,n'}\tilde{y}^i_{a,n''} \geq 0$, *and* $y^i_{a,n'}y^i_{a,n''} \gg 0$, *all* $a \in \mathcal{A}$; such a subsequence exists since the sets of individuals and assets are finite. The solutions to the optimization problems of individuals converge, or $\lim_{n\to\infty} \tilde{y}^i_n = \tilde{y}^i$, *by compactness of the budget set at* t_1 *and the fact that* ...Similarly, $\lim_{n\to\infty} y^i_n = y^i$, where y^i is part of ξ such that $F_{(0,\omega)}(\xi) = 0$, by continuity of $\xi(t;\omega) = (\xi_k(t;\omega))_{k\in K}$; and $y^i_a \neq 0$,for all $a \in \mathcal{A}$ by Lemma 1, item 2. Since at $t = 0$ the budget set of the individual is convex, and utility is strictly concave, $\tilde{y}^i = y^i$, so that $\tilde{y}^i_a y^i_a > 0$,all $a \in \mathcal{A}$. At any n, if $\tilde{y}^i_{a,n}y^i_{a,n} \geq 0$, all $a \in \mathcal{A}$, then, by definition of the optimization problem with fiscal policy and by strict concavity of utility, $\tilde{y}^i_n = y^i_n$, a contradiction with $\tilde{y}^i_n \neq y^i_n$; then, there exists $\bar{a}$ with $\tilde{y}^i_{\bar{a},n}y^i_{\bar{a},n} < 0$. Taking limits, we have $\tilde{y}^i_{\bar{a}}y^i_{\bar{a}} \leq 0$, again a contradiction, establishing the existence of competitive equilibrium with fiscal policy for $t \in O_\omega$. □

Constrained suboptimality The vector of coefficients in $F_{opt} = 0$ is $b = (b_1, b_2) = (\alpha, \beta, \gamma, \delta, \epsilon, \theta, b_2)'$. With these coefficients, $F_{opt} = 0$ writes as the system of equations

$$\begin{array}{llr} \alpha & \alpha^i D^2 u^i - \gamma^{i\backslash} P + \delta \widetilde{I} + b_2^i D u^i = 0, \quad \text{all } i & (i) \\ \beta & \gamma^i W + (0\ \varepsilon) = 0, \quad \text{all } i & (ii) \\ \gamma & -\alpha^i (0\ P') + \beta^i W' = 0, \quad \text{all } i & (iii) \\ \delta & \sum_i (\alpha^i \Lambda^i + \gamma^{i\backslash} Z^i) = 0, & (iv) \\ \varepsilon & \sum_i (\mu^i \beta_a^i - \gamma^{i,0} y_a^i) = 0, \quad \text{all } a > 1 & (v) \\ \theta & -\sum_i (1/I) \gamma^{i,0} + \theta = 0, & (vi.a) \\ & \sum_i [\beta_a^i \mu^i q_a I(y_{a,+}^i) - \gamma^{i,0} q_a y_{a,+}^i] + \theta q_a \sum_i y_{a,+}^i = 0, \quad \text{all } a & (vi.b) \\ b_2 & -\gamma^{i,0} + \theta = 0, \quad \text{all } i & (vi.c) \\ & b_2' b_2 - 1 = 0, & (vii) \end{array}$$

where $\gamma^i = (\gamma^{i,0}, \gamma^{i\backslash})$, and the following notation has been used: $\widetilde{I} = diag[0\ I_{L-1}, ..., 0\ I_{L-1}]_{S(L-1)\times SL}$ is an $S(L-1) \times SL$ block-diagonal matrix formed by S blocks, each corresponding to an $(L-1) \times L$ –dimensional matrix, with a first column of zeros and the $L-1$ –dimensional identity matrix; $P = diag[p_1, ..., p_S]_{S\times SL}$, $Z^i = -diag[z_1^{i\backslash}, ..., z_S^{i\backslash}]_{S\times S(L-1)}$, $\Lambda^i = -diag[\lambda_1^i(0, I_{L-1})', ..., \lambda_S^i(0, I_{L-1})']_{SL\times S(L-1)}$ and $\mu^i Q_i = -\mu^i diag[q_1 I(y_{1,+}^i), ..., q_A I(y_{A,+}^i)]_{A\times A}$; a backslash, \, on a variable denotes that the first component has been deleted, and $I(y_{a,+}^i) = 1$ if $y_a^i > 0$, and is zero otherwise.

For I objectives (the utility vector) and one budget constraint (equation (VII)), there are a total of $(1 + A)$ instruments, (T, t); hence the requirement that $(1 + A) \geq (1 + I)$, or $A \geq I$; for fiscal revenue to vanish, $T = 0$, the requirement is that $A \geq 1 + I$.

Equation (vii) must be true for otherwise, in a generic set of economies, this would contradict the regularity of the original incomplete markets equilibrium, Lemma 1, item 1. The first column displays the matching of variables to equations. Of equations (vi), only equations $(vi.a)$ and $(vi.b)$ should be counted when fiscal policy allows for fiscal revenue; only $(vi.b)$ should be counted when $T = 0$. The number of equations, at least $(N + A)$, is greater than the number of variables b under Assumption 1 and the remaining variables ξ are matched by the equations $F = 0$, in number N. Therefore, that system, generically, has no solution as long as $D_{b,M} F_{opt}$ has full rank, as previously stated.

The quadratic finite-dimensional parametrization of utility used to compute $D_M F_{opt}$ allows one to perturb the Hessian of the utility function without altering its gradient at any equilibrium point. For an economy $\omega \in \Omega^0$, equilibria are locally finite, by Lemma 2, so that this construction is well-defined.

It is now possible to demonstrate the results concerning constrained suboptimality. To this end, we need an additional lemma.

Lemma 3. *Constrained suboptimality is dense: for a dense subset of economies, Ω^{**} of Ω^0, $F_{opt} = 0$ has no solution.*

Proof The proof is split in two cases, according to whether or not utility perturbations are effective — case a and case b, respectively.

Delete equations $(vi.c)$ and, possibly, some equations $(vi.b)$, reducing the number of equations to I.

Case a ($\alpha^i \neq 0$, for all i): One perturbs equations (i) using M^i; equations (ii) using $\gamma_i^{\setminus}$ — this is possible by no redundancy: $\dim[R] = A$; equations (iii) using $\alpha^{i,s,1}$, all s, i, and equations (iv) using $\alpha^{i,s,l}$ some i, all (s, l) with $l \neq 1$.

From Lemma 1, item 2:

i for each a, there exists $i(a)$ with $y_a^{i(a)} > 0 : I(a) \equiv \{i \in I : y_a^i > 0\} \neq \varnothing$;

ii for each a there exists $i'(a)$ with $y_a^{i'(a)} < 0 : I'(a) \equiv \{i \in I : y_a^i < 0\} \neq \varnothing$;

It follows that one can use β_a^i with $i \in I'(a)$ to perturb equations (v); to perturb equation $(vi.a)$ one can use θ, and choose β_a^i with $i \in I(a)$ to perturb the a-th equation $(vi.b)$. Equation (vii) one perturbs by using b_2^i, for some i. The rank of $D_{b,M}F_{opt}$ is full.[12]

Case b ($\alpha^i = 0$, some i): To fix ideas, and without loss of generality, $i = 1$. Then, taking $l = 1$ and combining equation (i), $b_2^1 D_1 u^1 = \gamma^{1\setminus}$, with the first order conditions for $i = 1$, $D_1 u^1 = \lambda^1$, one obtains $\gamma^{1\setminus} = b_2^1 \lambda^1$. Therefore, (i) holds only if $\delta = 0$. Similarly, from (ii) and the first order conditions, $\gamma^{1,0} = b_2^1 \mu^1$, and $\varepsilon = 0$; from no redundancy and (iii), $\beta^1 = 0$.

It is immediate that, for $i > 1$, $Du^i \alpha^i = 0$ (again one uses (iii) and the first order conditions), while, if $\alpha^i \neq 0$, $\alpha^i D^2 u^i \alpha^{i\prime} = 0$, contradicting differential strict quasi-concavity of u^i. Thus, $\alpha^i = 0$ all i, and $\gamma^i = b_2^i(\mu^i, \lambda^i)$ (i.e., γ^i is colinear to (μ^i, λ^i)), $\beta^i = 0$ for all i. Substitution into the system of equations yields that (iv) becomes $\sum_i b_2^i \lambda^i Z^i = 0$, or equivalently $(b_2^1, \ldots, b_2^I)(..., \lambda^i \otimes z^i, ...)' = 0$. By Lemma 1, item 3, the matrix on the right has full rank I and therefore (iv) implies $b_2 = 0$, a contradiction to (vii), or $b_2' b_2 = 1$. Hence $\alpha^i = 0$, some i, cannot be (or there is no solution to the system of equations in this case).

When fiscal revenue is assumed to vanish, $T = 0$, the argument is similar. Delete equation $(vi.a)$, and equations $(vi.c)$. Some equations $(vi.b)$ can possibly

[12]When double-sided taxation of asset $a = 1$ is introduced, the rate of tax on sales $\bar{t}_1$ as instrument and the equation

$$\textstyle\sum_i [\beta^{i,1} \mu^i I(y_{1,-}^i) + \gamma^{i,0} y_{1,-}^i] + \theta \sum_i y_{1,-}^i = 0, \quad (vi.\bar{b})$$

are added to the system $F_{opt} = 0$. One can use $\beta^{i,1}$ with $i \in I(1)$ to perturb equation $(vi.b)$, and $\beta^{i,1}$ with $i \in I'(1)$ to perturb equations $\left(vi.\bar{b}\right)$, and another equation can be deleted (i.e., another instrument t_a can be dispensed with). Notice that adding equation $(vi.\bar{b})$ for $a > 1$ creates a redundancy with equation (v), and the system cannot be perturbed. This is consistent with our discussion of ineffectiveness of double-side taxation.

be deleted reducing them to $I + 1$. For Case a, equations (i) through (v) are perturbed as when $T \neq 0$ is allowed; assuming without loss of generality and possibly after relabelling that the equations $(vi.b)$ left are those corresponding to $a \leq I + 1$, they are perturbed using β_a^i, with $i \in I(a)$ for each $a \leq I$, while the last equation $(vi.b)$ is perturbed using θ. Equation (vii) is perturbed using $b^i_,$ some i. Case b is dealt with exactly as above.

Proof of the Proposition Lemma 3 then established density of the constrained suboptimality property, and all is left to show is that Ω^{**} is open in Ω^0. But this is a trivial exercise, since properness of the natural projection for the system $F(\xi, t, \omega) = 0$ of equilibrium equations at $t = 0$ is already known — Citanna, Kajii and Villanacci [6], Lemma 1. This concludes the proof. $\square$

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