Information Acquisition in a Limit Order Market

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- How valuable is information in a financial market?
- In what way are markets informationally efficient?
- Hirshleifer effect: prices that reveal too much can destroy risk-sharing gains.
  - Information is not socially useful.
- Strategic Models: adverse selection deters trade
  - Gains to trade are fixed.
- How traders benefit from private information determines if they acquire it
  - Grossman-Stiglitz (1980) paradox: if prices are fully revealing, no one will acquire costly information.
- How investors trade on private information affects informationally efficiency
- We address these issues (numerically) in a limit order market.
  - How is information incorporated into market outcomes?
  - What effect does information acquisition have on equilibrium outcomes?
  - Which investors acquire costly information? and how does it affect their trading strategies?

What is information?

- Different types are valuable:
  - Information about the book (trading opportunities)
  - Publically available information — earnings announcements and news.
  - Private information (about idiosyncratic shocks) — illegal to trade on $\Longrightarrow$ appears in other prices.
- Information substitutes, and seen by a lot of traders

Key Results

- Informed investors tend to supply (rather than demand) liquidity.
- For a given acquisition cost, there are multiple equilibria, in which different sets of investors are informed.
  - Uninformed agents change strategies based on how many agents are informed.
  - Equilibria can be Pareto-ranked.
- Prisoner's dilemma: there exists an equilibrium such that:
  (i) it is a best response for all agents to acquire information.
  (ii) all agents are worse off than if no agent had acquired information.

Literature Review

- GE rational expectations models:
  - Grossman (1976)
  - Grossman and Stiglitz (1980)
  - Admati and Pfleiderer (1987)
  - Ausubel (1990)
- Noisy results:
  - Admati (1985) demonstrates that intuition obtained from models with single risky asset may not go through with multiple risky assets.
  - Barlevy and Veronesi (2000) show that CARA-normal phenomena may be reversed in more general models.
- Microstructure models with strategic traders:
  - Kyle (1985)
  - Taub, Bernhardt, and Seiler (2004)
  - Holden and Subrahmanyam (1992)
  - Back, Cao and Willard (2000)
  - Mendelson and Tunca (2004).
- Market maker sets prices equal to expected value of asset, conditional on all public information.
- Dynamic microstructure models.
  - Foucault, Kadan and Kandel (2003), Rosu (2004), Goettler, Parlour and Rajan (2004)

Model

- Market for single financial asset.
- Continuous time
- Poisson arrival (rate λ) process for new traders.
- Traders can post orders at discrete prices $p^0, p^1, \cdots$.
- At each t, each price has a backlog of outstanding orders, ℓ_t^i.
  - Collection of orders defines the limit order book
  - $L_t = \{\ell_t^0, \ell_t^1, \cdots\}$.
- Asset has a common value v_t (present value of future cash flows)
  - Innovation process has Poisson distribution with parameter μ.
  - Probability $\frac{1}{2}$ of v increasing/decreasing by 1 tick.
- Each trader has a type $\theta = \{\rho, \beta, I\}$.
- $\rho =$ continuous discount rate.
  - Not the time value of money, but a motive to trade early rather than later.
- $\beta =$ private value to trade.
- $I =$ information type $\in \{0, 1\}$: uninformed or informed.
  - Endogenously chosen in equilibrium
  - Informed traders observe contemporaneous value of v_t.
  - Uninformed traders observe v_t with a lag.
- At time t, value of 1 share to agent θ is $v_t + \beta$.

Trading

- On arrival at the market, a trader can submit one of:

  1. market order (order that executes against a previously posted limit order)
  2. a limit order (order to buy or sell at a particular price p^j)
  3. no order.

- Order may be either a buy or a sell.

  - Each trader allowed to trade one share of the asset (buy or sell is endogenous).

- If no immediate trade, trader stochastically re-enters the market. Re-entry time drawn from Poisson process.
- On re-entry, can change his order, or leave it in place.
- Trader leaves the market for ever after execution.

The sorts of things that can happen

- An informed trader enters for the first time at time $t = 34.2$, for the first time when $v_t = 37.5$, and L_t given by:

Price	Depth	
39	-2	
38	-2	← *Ask*
37	3	← *Bid*
36	4	

- Suppose he places a buy order at 37; this is 4^{th} in the buy queue at this price.
- At some future random time, he re-enters the market (say this is time $t = 45$).
- Before he re-enters, the following events may happen:
  1. His priority in the queue at $p = 37$ improves.
  2. He executes at $p = 37$—another trader submits a market sell that executes against his order.
  3. His overall price priority decreases (e.g., the Bid moves to 38).
  4. v changes, to (say) 33.5.
- Execution time is random: If he executes before re-entry, he earns his payoff and leaves the market.
- Market fundamentals are random: v may change before execution.
- If no execution, when he re-enters the market, he examines status of his old order, and either leaves it in place, or submits a new order.

- Consider an informed trader who re-enters the market at time t.
- State space $= (s, a)$.
- $s =$ market state $= (L_t, v_t)$.
- $a = (p, q, x) =$ status of previous order.
- $p =$ price, $q =$ position in queue,
- $x =$ direction of order ($1 =$ buy, $-1 =$ sell, $0 =$ no previous order).

- Uninformed investor has expanded state space.
- Sees $s = (L_t, v_{t-\tau})$ at time t, knows a.
- Also sees net buy/sell transactions in the interval $[t-\tau, t]$.
- Forms an updated belief of v_t given observables.

- Suppose a trader j first enters at time τ, and buys a share at price p^i at time t.
- Overall payoff from this $= e^{-\rho(t-\tau)} \left(\beta_j + v_t - p^i\right)$.
- Traders are risk-neutral, maximize expected payoff.
  - Since they can re-enter, solve a dynamic program to decide optimal order.

- The Bellman equation, $J(s, a \mid \theta)$, is thus:

$$\max_{\tilde{a} \in \mathcal{A}(\theta,s,a)} \Big[\int_{w=0}^{\infty} \Big\{ \int_{\tau=0}^{w} \int_{v_\tau=-\infty}^{\infty} e^{-\rho\tau} \tilde{x}(\beta + v_\tau - \tilde{p}) \phi(\tau, v_\tau; s, \tilde{a}) f_v(v_\tau \mid v, \tau) dv_\tau d\tau$$

$$+ \quad (1 - F_{\widetilde{T}}(w \mid s, \tilde{a})) e^{-\rho w} \int_{(s', \tilde{a}') \in \mathcal{S} \times \mathcal{A}} J(s', \tilde{a}' \mid \theta) h(s', \tilde{a}' \mid s, \tilde{a}, w) d(s', \tilde{a}') \Big\}$$

Existence

- Restrict action space to be k ticks on either side of belief about v (in practice, no orders are submitted further away).
- Finite action space, countable state space (changes in state space happen after discrete intervals).
- Existence of Markov-perfect equilibrium follows from standard results (e.g. Reider, 1979).

Solving for equilibrium

- Numerical solution, along the lines of Pakes and McGuire (2001).
- Directly solve for value of each state.
- Usual guess-and-update procedure.
- To ensure Perfection: while solving, allow for small probability of trembles to update utilities of actions not taken.
- Take tremble probability to zero as we converge to solution.
- Only numeric uniqueness.

Usual convergence tests:

1. Den Haan & Marcet (1994): χ^2 test on
   $($ believed value $-$ actual value $)^2$.
2. Pakes and McGuire (2001):
   (i) Mean absolute error < 0.01
   (ii) Correlation between beliefs and actuals > 0.99.

Simulation
Parametrization

1. Rate of new trader arrival $= 1$ trader per unit of time.

2. Re-entry interval $= 6$ units of time, on average

3. On average, v changes every 12 units of time.

4. v lies between ticks—cannot trade at v.

5. $\rho = 0.05$.

6. β distribution:

β	-4	-2	-0.1	0.1	2	4
Probability	0.2	0.2	0.1	0.1	0.2	0.2

Analysis

- Compare market outcomes across different information structures and then endogenize information acquisition.
  1. All agents uninformed about v (i.e., observe with 24-trader lag).
  2. Agents with $|\beta| = 0.1$ informed about current v
  3. Agents with $|\beta| \in \{0.1, 2\}$ informed about current v
  4. All agents informed about current v.
- To calculate value of information, determine payoff to agents who deviate in information acquisition, and play optimally thereafter.

Order Placement Strategy

- On average, the $|\beta| = 0.1$ agents take the longest to trade.
  - Enter market most often.
- With asymmetric information, low $|\beta|$ agents take longer to execute.

β type	All Informed Equilibrium Entries	$|\beta| = 0.1$ Informed Equilibrium Entries	$|\beta| = 0.1$ Informed Deviation Entries
0.1	2.08	3.40	4.47
2	1.75	1.82	2.14
4	1.28	1.25	1.47

Table 1: **Average number of market entries by type.**

- Informed agents submit fewer market orders and uninformed agents submit more market orders

β	All Informed Equilibrium		$|\beta| = 0.1$ Informed Equilibrium		$|\beta| = 0.1$ Informed Deviation	
type	Market	Limit	Market	Limit	Market	Limit
0.1	0.56	0.98	0.38	1.76	0.63	2.41
2	0.40	0.78	0.44	0.88	0.46	1.00
4	0.57	0.51	0.62	0.48	0.60	0.56

Table 2: **Optimal Order Submissions Per Trader**

- Uninformed traders submit more conservative limit orders (relative to common value)

Informed	β	Limit Buys above v	$= v$	$< v$	Limit Sells below v	$= v$	$> v$
All	0.1	0.00	0.00	100.00	0.00	0.00	100.00
	2	50.04	0.00	49.96	0.00	0.00	100.00
	4	77.77	0.00	22.23	0.00	0.00	0.00
$\|\beta\| = 0.1$	0.1	0.00	0.00	100.00	0.00	0.00	100.00
	2	40.25	0.00	59.75	4.41	0.00	95.59
	4	59.55	0.00	40.45	0.00	0.00	0.00

Note: Numbers in table are % of total limit orders for each β.

Table 3: **Submission of Limit orders relative to v**

- Uninformed traders suffer from adverse selection (less favorable terms of trade)
- For market orders:

β type	All Informed	$\|\beta\| = 0.1$ Informed
0.1	-0.69	-1.10
2	-0.19	0.10
4	0.10	0.38

Table 4: **Difference between the transaction price and consensus value by β type**

Informational Efficiency

- Information here is potentially public, so we look at semi-strong form efficiency.
- Dynamic model; in addition to price, other observables also convey information.
  - Quotes versus transaction prices.
- Standard deviation of price (relative to v) decreases with number informed.
- Mean is zero (due to symmetry of model).

Measure		All Informed	$\mid \beta \mid \in \{0.1, 2\}$ Informed	$\mid \beta \mid = 0.1$ Informed	All Uninformed
$\widetilde{p} = p_\tau - v_\tau$	Std. Dev.	0.748	0.903	1.169	1.543
$\Delta\widetilde{p} = \widetilde{p}_{\tau+1} - \widetilde{p}_\tau$	Std Dev.	0.616	0.771	0.807	0.745

Table 5: **Transaction Frequency and the difference between transaction price and the common value**

- Prices do reflect information.

Speed of incorporation of information into price

- Consider isolated information events (changes in v).

Figure 1: **Response of Transaction Price after Information Event**

- Consider the first time (after information event) at which $|p - v| = 0.5$.
  - Again, decreases in number informed.

	All Informed	$\|\beta\| \in \{2, 0.1\}$ Informed	$\| \beta \|= 0.1$ Informed	Uninformed
Mean	2.503	2.727	3.762	5.550
Max	29.570	36.203	32.254	32.155

Table 6: **First time before** $|p - v| = 0.5$

- (Rate of transactions is the same across models, depends almost entirely on rate of new trader arrival).

Beliefs of Uninformed Traders

- Consider absolute value of $E(v|\text{observables})-v$ for uninformed traders.
- Again decreases with number of informed traders.

All Informed	$\|\beta\| \in \{2, 0.1\}$ Informed	$\|\beta\| = 0.1$ Informed	All Uninformed
0.39	0.44	0.65	1.04

Table 7: **Absolute Difference between Belief of Uninformed Traders and True Common Value**

- Information does get incorporated into price and market outcomes.
- Informed agents increase submission of limit orders.

Endogenous information acquisition

- Demand for information depends on improvement in welfare due to information.
- Varies by (i) β of trader
  (ii) strategies of other investors.
- Consider gross (i.e., without considering information acquisition cost) welfare with and without information.
- Look at (consumer surplus if informed − consumer surplus if uninformed).

β type	All Informed	$\lvert\beta\rvert \in \{0.1, 2\}$ Informed	$\lvert\beta\rvert = 0.1$ Informed	All Uninformed
0.1	0.190	0.266	0.384	0.625
2	0.183	0.232	0.149	0.208
4	0.151	0.054	0.071	0.028

Table 8: **Welfare gain to being informed**

Observation 1 *If any agent acquires information, then* $|\beta| = 0.1$ *acquires information.*

- As expected, speculators have the greatest incentive to acquire information.

Observation 2 *The following are perfect Bayesian equilibria in the information acquisition game:*

$$c \in \begin{cases} [0, 0.15] & \textit{everyone acquires information} \\ [0.07, 0.22] & |\beta| \in \{0.1, 2\} \textit{ acquire information} \\ [0.19, 0.34] & |\beta| = 0.1 \textit{ acquires information} \\ [0.62, \infty] & \textit{no one acquires information} \end{cases}$$

- Possibility of multiple equilibria—ranges overlap.

Observation 3 *For* $c \in [0.07, 0.14]$ *there are three equilibria:*
(i) $|\beta| \in \{0.1, 2\}$ *acquire information*
(ii) Only $|\beta| = 0.1$ *acquire information*
(ii)All agents acquire information.

Further, for $c \in [0.19, 0.22]$*, there are two equilibria*
(i)$|\beta| \in \{0.1, 2\}$ *acquire information*
(ii)$|\beta| = 0.1$ *acquires information.*

Welfare ranking of equilibria

- Allocative efficiency? — Optimal incentive compatible mechanism is an open question.
- Consider two benchmarks for planner:
  (i) Frictionless benchmark W_f.
  - Suppose all agents in market at the same time.
  - Consummate all trades at price $= v$.
  - Agents with $\beta < 0$ are sellers, $\beta > 0$ are buyers.
  - Clearly IC.
  - Ignores all frictions.

  (ii) Naïve IC mechanism (W_ℓ)

  - LIFO rule for trading.
  - All trades occur at price $= v$.
  - Respects discounting, trader arrival sequence.
  - Also IC.

- Market does better than naïve mechanism, not as well as upper bound.

Frictionless: W_f	LIFO: W_ℓ	All Informed	$\lvert\beta\rvert = \{0.1, 2\}$ Informed	$\lvert\beta\rvert = 0.1$ Informed	All Uninformed
2.42	2.09	2.19	2.17	2.12	2.18

Table 9: **Welfare per trader, and benchmarks**

- Market is clearly IC (since it is an equilibrium).
  - High $\lvert\beta\rvert$ types have an incentive to reveal themselves and try to trade early.

β	Average time to execution
0.1	6.46
2	4.45
4	1.60

Table 10: **Time to execution when all agents are informed**

Welfare by trader type

- Payoffs net of information acquisition cost.

β type	All Informed	$\lvert\beta\rvert \in \{0.1, 2\}$ Informed	$\lvert\beta\rvert = 0.1$ Informed	All Uninformed
0.1	$0.518 - c$	$0.579 - c$	$0.689 - c$	0.472
2	$1.694 - c$	$1.704 - c$	1.582	1.676
4	$3.483 - c$	3.388	3.388	3.469

Table 11: **Net Payoffs for different information acquisition equilibria**

- For some range of costs, all traders worse off when all informed than when all uninformed.
- However, it is an equilibrium to acquire information.

- For the following cost ranges:

  (i) it is an equilibrium for some subset of agents to be informed

  (ii) all agents prefer (have higher welfare in) the regime in which no agent is informed.

Equilibrium	Cost Range
All Informed	$0.04 < c < 0.15$
$\lvert\beta\rvert \in \{0.1, 2\}$ Acquire information	$0.10 < c < 0.22$
0.1 informed	$0.21 < c < 0.34$

Table 12: **Cost ranges in which too much information is acquired in equilibrium.**

- Unlike the Hirshleifer effect, there is no change in the gains to trade as a result of information acquisition.
- Gains to trade depend solely on the β distribution.
- However, adverse selection due to information acquisition by others results in less favorable split of gains to trade.

- If information is costless, all agents are better off when all agents are informed, than when no agent is informed.

Conclusion

- Endogenous information acquisition equilibria exist.
- Information does find its way into market observables.
  - Characterize the time path
- Informed agents tend to submit more limit orders.
  - Take longer to execute
- Information may lead to all agents being worse off in equilibrium.