# Thinking Ahead: Part I, the Decision Problem

by

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**Research Project:** Propose a tractable framework for the analysis of optimal, endogenously incomplete contracts between boundedly rational agents

When do contracts specify contingent action plans? when do they leave choices open and instead specify governance rules?

Kaplan and Stromberg (*RES* 2003): 213 investments in 119 companies by 14 Venture Capitalists between 86-99. Firms are in the information technology, telecommunications and software sectors,

No standard contracts $\longrightarrow$ optimal long-term contracting

KS's main findings:

1. VC contracts allocate separately cash flow rights, voting rights, board rights, liquidation rights, and other control rights,

2. Cash flow, future financing, voting and other control rights, are contingent on observable measures of performance:

   the VCs may obtain control from the entrepreneur if the firm's earnings fall below a pre-specified level,

   the entrepreneur may obtain more cash flow rights contingent on achieving certain performance targets (successful experiment, approval by FDA)

3. when the firm's performance is bad, VCs obtain full control; when it improves the entrepreneur gets more control rights; when the firm does very well the VCs retain cash flow rights but give up most of their control rights; the entrepreneur's cash flow rights also increase with performance.

4. VCs have less control in late rounds of financing

**Literature:**

- This research is part of a second generation of models of incomplete contracts (Battigalli and Maggi 2002, Bajari and Tadelis 2001, Hart and Moore 2004...) that develop theories of 'endogenous contractual tightness/incompleteness'.

- Contract incompleteness as chosen by contracting parties.

- Our approach motivated by Oliver Hart's view (1995, p81):
  *In reality, a great deal of contractual incompleteness is undoubtedly linked to the inability of parties not only to contract very carefully about the future, but also to think very carefully about the utility consequences of their actions. It would therefore be highly desirable to relax the assumption that parties are unboundedly rational.*

## First step: A Model of "Bandit Rationality"

- Simple model of boundedly rational decision maker (DM), based on time-costs of deliberating current and future decisions.

- Need a model where sometimes it is best for DM to **think ahead** about *utility consequences of their actions* and sometimes best to **defer this thinking to later.**

- General idea:

  - to specify an optimal state-contingent plan, decision-maker (or contracting parties) must think ahead and determine optimal action in each possible state of nature;

  - but, thinking is costly; in particular here, it takes time;

- We extend an approach first taken by Conlisk (1988) to model 'optimization costs' or 'bounded rationality':
  - more time spent 'thinking' improves accuracy of beliefs about optimal solution to a particular problem

- We model 'thinking' like 'thought experimentation'

- Thinking is like playing a 'multi-armed' *Bandit* problem

- Crucially, question is *when* to think? *ahead of* when, or *as* problems arise?

- trade-off between thinking ahead and thinking on the spot:
  - thinking ahead allows DM to be ready for any contingency.
  - being ready allows DM to react quickly to the realization of a state of the world.
  - reducing the time lag between state realisation and action is of some economic value.
    *But:*
  - thinking ahead about possible problem is useless if problem does not materialize.
  - as thinking takes time, another source of delays.
  - therefore may not be optimal to plan ahead about everything.

$\Longrightarrow$ Sometimes optimal thinking plan is to be ready for some contingencies but not for all.

In general, optimal thinking plan is such that the DM adopts a "step by step" approach where DM:

- singles out a subset of future decision problems and thinks these through first

- if thought experiment reveals good news, stops planning further and moves on

- prioritizes thinking by first thinking about most likely problems, easiest ones and those with best ex ante prospects.

- If environment sufficiently uncertain, no planning at all.

## Contracting

- **Optimal "incomplete" contract**: when contracting parties optimally think ahead, optimal contracts should specify in advance a stage contingent action plan, while when contracting parties optimally think on the spot, contracts should leave things open and rather specify a governance structure.

- **"Satisficing" contracts**: when parties are *satisfied* that the contract specifies enough substance, that they can go ahead with project.

- *Satisficing:* Contracts are generally incomplete, are more likely to leave for later determination what to do in less likely states, or when facing more difficult problems, or worst case scenarios...

- **Satisficing contracts** tend to be *excessively complete:*

  - If possibility of future conflict, further need to protect some contracting parties, say the investor,

  - Tighter contracts better at doing that than contracts giving investor more control rights,

  - the higher the upfront investment costs the tighter the contract (more investment $\Longrightarrow$ more investor protection $\Longrightarrow$ more contractual completeness)

## The Decision Problem: A general framework

- an infinite horizon, discrete time problem

- an initial decision: take one of $n$ actions in the action set $A_0 = \{a_1, ...., a_n\}$

- future decision problems, contingent on the initial action choice $a_i$ and on the realized state of nature $\theta_j \in \Theta = \{\theta_1, ...., \theta_N\}$

- initial action may be taken at any time $t \geq 0$

- when an action is chosen at time $t$ DM receives an immediate payoff $\omega(a_i)$, and a state of nature is realized at time $t + \Delta_i$,

- DM must then choose another action $a_{ijk}$ from another set of actions $A_{ij} = \{a_{ij1}, ...., a_{ijm_{ij}}\}$

- DM may take this new action at any time $\tau \geq t + \Delta_i$

- when the action $a_{ijk}$ is taken DM obtains another payoff $\pi(a_i, a_{ijk}, \theta_j)$

- future payoffs are discounted, $\delta < 1$

Expected present discounted payoff of an action plan, with initial action $a_i$ chosen at date $t$ and subsequent actions $a_{ijk}$ chosen at dates $\tau_{ij} \geq t + \Delta_i$ is given by

$$\delta^t \omega(a_i) + \sum_{j=1}^{N} \delta^{t+\tau_{ij}} \pi(a_i, a_{ijk}, \theta_j) \mu_{ij}.$$

- DM knows the true payoff $\omega(a_i)$ but not the true payoff $\pi(a_i, a_{ijk}, \theta_j)$

- true payoff $\pi(a_i, a_{ijk}, \theta_j)$ can take any of the values $\pi(a_i, a_{ijk}, \theta_j, \eta_{ijh})$ in the finite set

  $\Pi_{ij} = \{\pi(a_i, a_{ijk}, \theta_j, \eta_{ij1}), ...., \pi(a_i, a_{ijk}, \theta_j, \eta_{ijy})\}$

- prior belief over those values given by $\nu_{ijk0h} = \Pr(\eta_{ijh})$

- before taking any action $a_{ijk} \in A_{ij}$ DM can learn more about the true payoff associated with that or any other action by engaging in *thought experimentation*

- we model thought experimentation as a *multi-armed bandit problem*:

– in any given period $t$ DM can ‘think’ about an action $a_{ijk}$ and obtain a signal $\sigma_{ijk}$ which is correlated with the true payoff parameter $\eta_{ijh}$

– upon obtaining this signal, DM revises her belief to $\nu_{ijkh} = \Pr(\eta_{ijh} \mid \sigma_{ijk})$

$\Longrightarrow$

- at $t = 0$ DM’ s decision problem is to decide whether to pick an action $a_i$ in $A_0$ right away or whether to think ahead about one of the future decision problems

- DM faces this same problem, with possibly updated beliefs from earlier thought experimentation, as long as she has not picked an action $a_i$

- when she has chosen an action $a_i$ and time $\Delta_i$ has elapsed a state of nature is realized

- DM' s decision problem is then again to decide whether to pick an action $a_{ijk}$ in $A_{ij}$ right away or whether to think about one of the actions in $A_{ij}$

- this general framework is restrictive in some respects:

  – we only allow for two rounds of action choice,

  – the action sets are finite,

  – the state-space is finite and,

  – learning through thought-experimentation can only be done for one action at a time

The framework, as described is clearly too general to be tractable at least in a first attempt!

## A simpler model:

- consider a long-term investment problem, with state-contingent returns and actions:

- **Project**:

  - requires initial funding of $I$

  - If investment is sunk at date $t$ then at date $t+1$ the project is in one of two equally states: $\theta \in \{\theta_1, \theta_2\}$

  - when $\theta_i$ is realized there are two possible actions: a *risky* and a *safe* one

  - The monetary payoff of the risky decision, $R$, is uncertain, $R \in \{\underline{R}, \bar{R}\}$

  - The monetary return of the safe action is certain, $\bar{R} > S > \underline{R}$

- $\nu = \Pr(R = \bar{R})$, independent across states

- **Agents:**

  - one entrepreneur (and later an investor);

  - entrepreneur (DM) can try to find out the right decision to take in state $\theta_i$;

  - as she thinks, there is a probability $\lambda$ per unit time that DM finds out the true payoff associated with the risky action;

  - with probability $(1 - \lambda)$ she learns nothing and must continue to think through the problem;

  - she can think through what should be done in state $\theta_i$ before investing and before the state is realized;

  - if the entrepreneur decides to think at date $t$ before investing she delays investment and returns

- **Timing:**
  - at date 0, DM decides to invest or to think;
  - at any date before investment has taken place the choice is the same as at date 0;
  - once investment has taken place, state $\theta_i$ is realized one period later
  - DM can either choose the risky or safe action or think;
  - if she decides to think at current period, action has to be postponed to next period

<u>**Assumption**</u>:

$$A_1 \ : \ \nu\bar{R} + (1-\nu)\underline{R} > S$$

$$A_2 \ : \ \delta S - I > 0$$

Importantly, we also assume no thinking cost associated with thinking over thinking over...

**The simple decision problem:**

The entrepreneur can decide to invest right away or she can try to think first about what decision to take if state $\theta_i$ occurs

- the expected present discounted payoff from thinking in state $\theta_i$ is given by

$$\underbrace{\lambda\delta\left[\nu\bar{R}+(1-\nu)\,S\right]}_{\substack{\text{PD payoff}\\ \text{from learning true}\\ \text{payoff in first round}\\ \text{of experimentation}}}+\underbrace{(1-\lambda)\delta^2\lambda\left[\nu\bar{R}+(1-\nu)\,S\right]}_{\substack{\text{PD payoff}\\ \text{from learning the true}\\ \text{payoff in second round}}}$$

$$+(1-\lambda)^2\delta^3\lambda\left[\nu\bar{R}+(1-\nu)\,S\right]$$
$$+\sum_{t=4}^{\infty}(1-\lambda)^{t-1}\delta^t\lambda\left[\nu\bar{R}+(1-\nu)\,S\right].$$

or, letting $\hat{\lambda}=\frac{\lambda\delta}{1-(1-\lambda)\delta}$,

$$\hat{\lambda}\left[\nu\bar{R}+(1-\nu)\,S\right]$$

**Assumption** : once in state $\theta_i$ and not knowing what is best to do DM prefers to think before acting:

$$\begin{aligned}\nu\bar{R} + (1-\nu)\underline{R} &\leq \hat{\lambda}\left[\nu\bar{R} + (1-\nu)\,S\right] \\ &\Leftrightarrow \\ \hat{\lambda} &\geq \hat{\lambda}_L \equiv \frac{\nu\bar{R} + (1-\nu)\underline{R}}{\nu\bar{R} + (1-\nu)S}\end{aligned}$$

$\Longrightarrow$

OUR FOCUS:

When do you want to learn what to do in state $\theta_i$?

If DM defers thinking to later, she expects:

$$V_L = -I + \hat{\lambda}\delta[\nu\bar{R} + (1-\nu)S]$$

Alternatively, she can "think ahead" before investing, she can think about one state or about both

suppose she thinks first about state $\theta_1$

if learns nothing, indifferent between continuing thinking about state $\theta_1$ or switching to state $\theta_2$

say she continues.... after a while she learns that in state $\theta_1$ she can expect some value $\pi_1$

should she continue thinking before investing?

If does, she can expect to get:

$$V_E^1 = \hat{\lambda}\left(-I + \delta\left(\frac{\pi_1}{2} + \frac{x}{2}\right)\right)$$

If she does not:

$$V_L^1 = -I + \delta \left( \frac{\pi_1}{2} + \frac{\hat{\lambda} x}{2} \right)$$

We have:

$$\begin{aligned} \Delta^1 &\equiv V_L^1 - V_E^1 \\ &= (1-\hat{\lambda})(\delta \frac{\pi_1}{2} - I) \end{aligned}$$

**Lemma 1:** *Thinking ahead about state $\theta_2$ is dominated by thinking on the spot in that state if* $\delta \frac{\pi_1}{2} - I \geq 0$.

Two effects behind Lemma 1:

a) if state 2 occurs, it would have been best to think ahead and save $(1-\hat{\lambda})I$

b) but state 1 may occur instead and DM already knows what to do there. So thinking about state 2 postpones the time at which returns in that state can be obtained for nothing and costs

$$(1-\hat{\lambda})\delta\frac{\pi 1}{2}$$

TRADE-OFF:

More planning reduces lag between time at which investment costs are paid and time at which returns are realized.

More planning delays the whole project, maybe unnecessarily.

Importantly, this trade-off is affected by what DM learns about state $\theta_1$

**How much planning ahead?**

Should DM start to think at all or just go ahead with the investment (the *think on the spot* strategy).

**Lemma 2:** *If DM chooses never to think about state $\theta_2$ irrespectively of what she learns about state $\theta_1$ then it is best for her not to think at all.*

$\Longrightarrow$

Three possible thinking strategies:

- "think on the spot": defer all thinking.

$$V_L = -I + \hat{\lambda}\delta[\nu\bar{R} + (1-\nu)S]$$

- "complete planning": think ahead about both states

$$V_E = \hat{\lambda}^2 \left[-I + \delta(\nu\bar{R} + (1-\nu)S)\right]$$

- "step by step" strategy: think about state $\theta_1$ and continue exploring iff $\pi_1 = S$.

$$V_S = \hat{\lambda}[\nu \left( -I + \frac{\delta}{2}\overline{R} + \frac{\delta}{2}\hat{\lambda}\left[ \nu \bar{R} + (1-\nu)S \right] \right) +$$

$$(1-\nu)\hat{\lambda}\left( -I + \frac{\delta}{2}S + \frac{\delta}{2}\left[ \nu \bar{R} + (1-\nu)S \right] \right)]$$

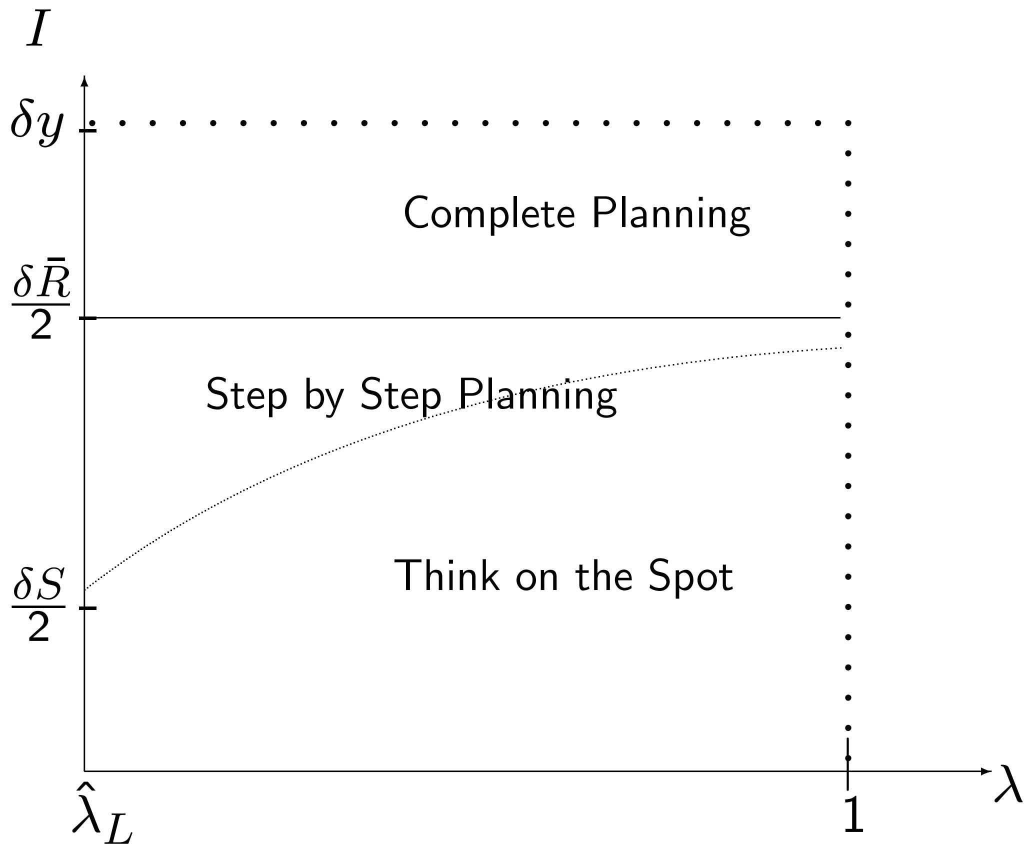
**Proposition 1:** *Under assumption $A_1$, $A_2$ and $A_3$, the solution to the entrepreneur's decision problem is as follows: the entrepreneur prefers the "think on the spot" strategy if and only if:*

$$I \leq \frac{\hat{\lambda}\delta\left[ x - \nu \frac{\overline{R}}{2} \right]}{1 + \hat{\lambda} - \nu\hat{\lambda}}.$$

*Otherwise the entrepreneur will think before investing, either adopting:*

*- the "step by step" planning strategy if* $\frac{\hat{\lambda}\delta\left[ x - \nu \frac{\overline{R}}{2} \right]}{1 + \hat{\lambda} - \nu\hat{\lambda}} \leq I \leq \frac{\delta\overline{R}}{2}$.

*- a "complete planning" strategy where she thinks about both states before investing if* $\frac{\delta\overline{R}}{2} \leq I$.




Key points:

- higher values of investment favor more planning ahead;

- An increase in $\lambda$ is a good thing whatever strategy the entrepreneur chooses but it favors more the "think on the spot" strategy

**COMPARATIVE STATICS:**

The role of Uncertainty:

Take $\mu$, the probability that state $\theta_1$ is realized, larger than .5

No change to payoffs of the "thinking on the spot" and "complete planning" strategies.

If DM goes for a "step by step" approach:

- should start thinking about state $\theta_1$ : cost of thinking ahead is identical, expected benefit is bigger.

- Lemma 1 implies that it is more likely that DM stops after learning about state $\theta_1$:

If thinks again before investing:

$$V_E^1(\mu) = \hat{\lambda}\left(-I + \delta\left(\mu\pi_1 + (1-\mu)x\right)\right)$$

If not:

$$V_L^1 = -I + \delta\left(\mu\pi_1 + (1-\mu)\hat{\lambda}x\right)$$

so that:

$$\Delta^1 = (1-\hat{\lambda})(\mu\delta\pi_1 - I)$$

increasing in $\mu$.

**Proposition 2:** *Suppose that* $\mu = \frac{1}{2} + \varepsilon$.

- *Whenever some thinking ahead takes place, it is best to start thinking about the most likely state* ($\theta_1$).

- *The payoff associated with:*
  *- the "complete planning" strategy is unchanged,*
  *- the "thinking on the spot" strategy is unchanged,*
  *- the "step by step" strategy where the entrepreneur thinks first about the most likely state goes up.*

- More uncertainty ($\mu \longrightarrow \frac{1}{2}$): either complete planning or no thinking at all.

## The role of Complexity:

Suppose decision in state $\theta_1$ is less "complex": $\lambda_1 > \lambda_2$.

Payoff under "complete planning" strategy gives:

$$V_E = \hat{\lambda}_1 \hat{\lambda}_2 \left[\delta x - I\right]$$

Payoff under "thinking on the spot":

$$V_L = -I + (\hat{\lambda}_2 + \hat{\lambda}_1)\frac{\delta x}{2}$$

Compared to before, take $\hat{\lambda}_2 = \hat{\lambda} - \varepsilon$, $\hat{\lambda}_1 = \hat{\lambda} + \varepsilon$

$\Rightarrow$ A "compounding" effect pushing towards less planning.

If “step by step”:

- Option to think only once $\Longrightarrow$ DM should think first about easy problem: if lucky, no need to crack the difficult one.

**Proposition 3:** *Suppose that* $\hat{\lambda}_1 = \hat{\lambda} + \varepsilon$ and $\hat{\lambda}_2 = \hat{\lambda} - \varepsilon$.

- *The payoff associated with:*
  *- the “complete planning” strategy goes down with* $\varepsilon$,
  *- the “thinking on the spot” strategy is unchanged,*
  *- the “step by step” strategy where the entrepreneur thinks first about the simple problem (in state* $\theta_1$*) goes up (resp. down) with* $\varepsilon$ *if* $I < I_m$ *(resp.* $I > I_m$*).*

- *The larger the difference between the two decision problems, the more attractive is the “step by step” approach.*

## The role of State Heterogeneity.

Suppose that state $\theta_1$ has better prospects than state $\theta_2$, e.g: $\nu_1 > \nu_2$.

For instance, $\nu_1 = \nu + \varepsilon$ while $\nu_2 = \nu - \varepsilon$.

With equally likely states, the payoff of thinking on the spot and complete planning are unchanged.

Different for step by step strategy: if DM starts thinking about state $\theta_1$, she is more likely to think ahead only about one state than if she starts thinking abouty state $\theta_2$.

This is an improvement as it speeds up the time when DM may get a high return.

**Proposition 4:** *Suppose that either* $\nu_1 = \nu + \varepsilon$, and $\nu_2 = \nu - \varepsilon$ or that $S_1 = S + \varepsilon$, $S_2 = S - \varepsilon$, while $\overline{R}_1 = \overline{R} + \varepsilon$ and $\overline{R}_2 = \overline{R} - \varepsilon$.

- *Whenever some thinking ahead takes place, it is best to start thinking about the high payoff state ($\theta_1$).*

- *The payoff associated with:*
  *- the "complete planning" strategy is unchanged,*
  *- the "thinking on the spot" strategy is unchanged,*
  *- the "step by step" strategy where the entrepreneur thinks first about the high return state goes up.*

Entrepreneur stops planning on good news, so best to increase the chances of being in this position.

## Team Thinking:

2 identical Entrepreneurs with ability $\lambda_T$.

Take $\lambda_T$ such that: $\lambda_T(2 - \lambda_T) = \lambda$

Payoff associated with the "thinking on the spot" strategy is unchanged.

If think ahead, best to diversify tasks.

**Proposition 5:** *Suppose that there are two identical agents who can think about what decisions to take, with ability $\lambda_T$ so that $\lambda_T(2 - \lambda_T) = \lambda$.*

- *The payoff associated to:*
  *- the "complete planning" and the "step by step" strategies are higher than with a single agent*,
  *- the "thinking on the spot" strategy is unchanged,*

- *There is more planning ahead than in the single individual case.*

Deferring thinking eliminates the benefit from diversification.

**The N state Model:**

Take $N$ equally likely states, with identical payoffs.

**Lemma 2bis:** *If irrespective of what DM learns about state $\theta_i$,she does not think ahead about any other state before investing, then best not to think about $\theta_i$.*

If she invests next no matter what, the exact payoff of state $\theta_i$ has no informational value. So if she prefers to continue experimenting now, she will do so later and vice versa.

**Lemma 3:** *It is never optimal to stop thinking ahead on learning bad news and to continue thinking ahead on learning good news.*

Compare to an expected payoff of $x_i$ in state $i$, learning bad (resp. good) news makes entrepreneur relatively more patient (resp impatient).

**Theorem (Satisficing):** *For $N$ small enough, DM adopts a step by step strategy whereby she thinks ahead about some states, continues to do so upon learning bad news and invests only once she has accumulated enough good news.*

**Proposition 6:** *Consider $N$ equiprobable states. If $N \geq \frac{\delta S}{\delta S - I}$ the optimal thinking strategy is to invest right away and to think on the spot.*

**Corollary:** *The entrepreneur's payoff is weakly decreasing in $N$.*

All previous comparative statics results extend: optimal to think first about more likely states, better states, less complex states...

Guiding Principle: the entrepreneur wants to be in the position where investing is best thing to do and becomes optimal to stop thinking ahead. What is the fastest way to be there?

Link with Psychology literature: can well-known psychological traits be ‘explained’ by appealing to “bandit” rationality?

- Psychological traits are systematic (directed) mistakes

- Bounded rationality should generate unbiased mistakes

But in our model, to the external observer the optimal behavior of decision maker exhibits systematic patterns (although not mistakes):

- Exploration heuristics: look at a few states, invest when enough good news.

- Salience: look at big things first.

- Optimism: look at good case scenario first.

What is the right benchmark for systematic mistakes?

**CONTRACTING:**

Consider a similar model as before but where 2 people are involved:

- could be an investor who finances part of the investment while entrepreneur pays the rest.

- could be a worker who executes the decision and the entrepreneur can instruct him to choose a particular action...

- Case 1: no conflict of interest between these two parties.
- Case 2: different preferences over ex post decisions but thinking strategy is verifiable.
- Case 3: conflict and thinking strategy non verifiable.

Case 1: consider employer-employee relationship. Employee executes action. Employer does the thinking and can instruct employee.

When optimal to think ahead, action to be taken in $\theta_i$ can be specified in contract.

When optimal to defer thinking, employer keeps authority (i.e. residual right of control when contract is silent about what to do) in these states.

$\Longrightarrow$ Authority is more prevalent in environment of greater uncertainty, and is associated with more complex problems, less likely events etc...

Caveat: spot contracts?

Case 2: consider a finance relationship where the entrepreneur is unable to self-finance $I$.

Suppose not all income is transferable.

In particular, assume that only $\overline{R} - b$ is transferable in that state of the world (moral hazard ...)

Take $\overline{R} - b < S$ : a conflict of interest.

For simplicity, suppose that it is known that in state $\theta_1$, the optimal decision is risky.

Consider the case where:

$$\frac{\delta}{2}\left(\overline{R} - b\right) + \frac{\delta}{2}\widehat{\lambda}\left[\nu S + (1-\nu)\left(\overline{R} - b\right)\right] < I$$

Then the entrepreneur cannot follow the strategy of thinking on the spot and get financed.

What to do?

- Commit to choose $S$ more often? by contract or by giving control to investor.

Best to do it in state $\theta_2$ only, without thinking.

Compare to decision problem, a welfare loss:

$$L = \frac{\delta}{2} q_2^* (\widehat{\lambda} x - S)$$

- Think more ex ante?

Contract specifies a credit line up to $I$ and that before investing the entrepreneur has to come up with hard information about efficient decision. Contract stipulates this "business plan" will be followed.

Could be that Investor's budget constraint is now satisfied:

$$-I + \frac{\delta}{2} \left( \overline{R} - b \right) + \frac{\delta}{2} \left[ \nu S + (1 - \nu) \left( \overline{R} - b \right) \right] > 0$$

Two effects:

- align costs and returns in time and thus relax budget constraint

- but also allows the entrepreneur to fine tune investor's returns.

$\Longrightarrow$ Contracting, and the need to satisfy budget constraint of investor at minimum cost, creates a new value of information and therefore pushes towards more ex ante thinking.

Case 3: Conflict of interest and thinking strategy non verifiable.

Now ex post the entrepreneur has no incentive to think: just want a chance to take $b$.

Strategy of thinking ahead is also affected: if investor commits to a credit line, entrepreneur has no incentive to think ahead.

But investor can insist on signing contracts that fully specify the decision to be taken later.

Creates ex ante incentives to think.

Again pushes towards more thinking ex ante.

$\Longrightarrow$ Contracts are excessively complete.