Corporate Default Risk

Clarendon Lectures in Finance, 2004

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Three-Lecture Outline

1. Corporate default probabilities.
2. Pricing default risk.
3. Default correlation.

Corporate default probabilities.

A. Historical default patterns.

B. Default intensity.

C. Structural default models.

D. Term structure of default probabilities with firm-level and macroeconomic covariates.

A. Historical Default Patterns

A common but naïve measure of default probability for a firm or sovereign that is rated by an agency such as Moody's or Standard and Poors, is the average frequency with which obligors of the same rating have defaulted. For example, Figure 1 shows average one-year corporate default rates, by rating, for the years 1983-2002, as published by Moody's.

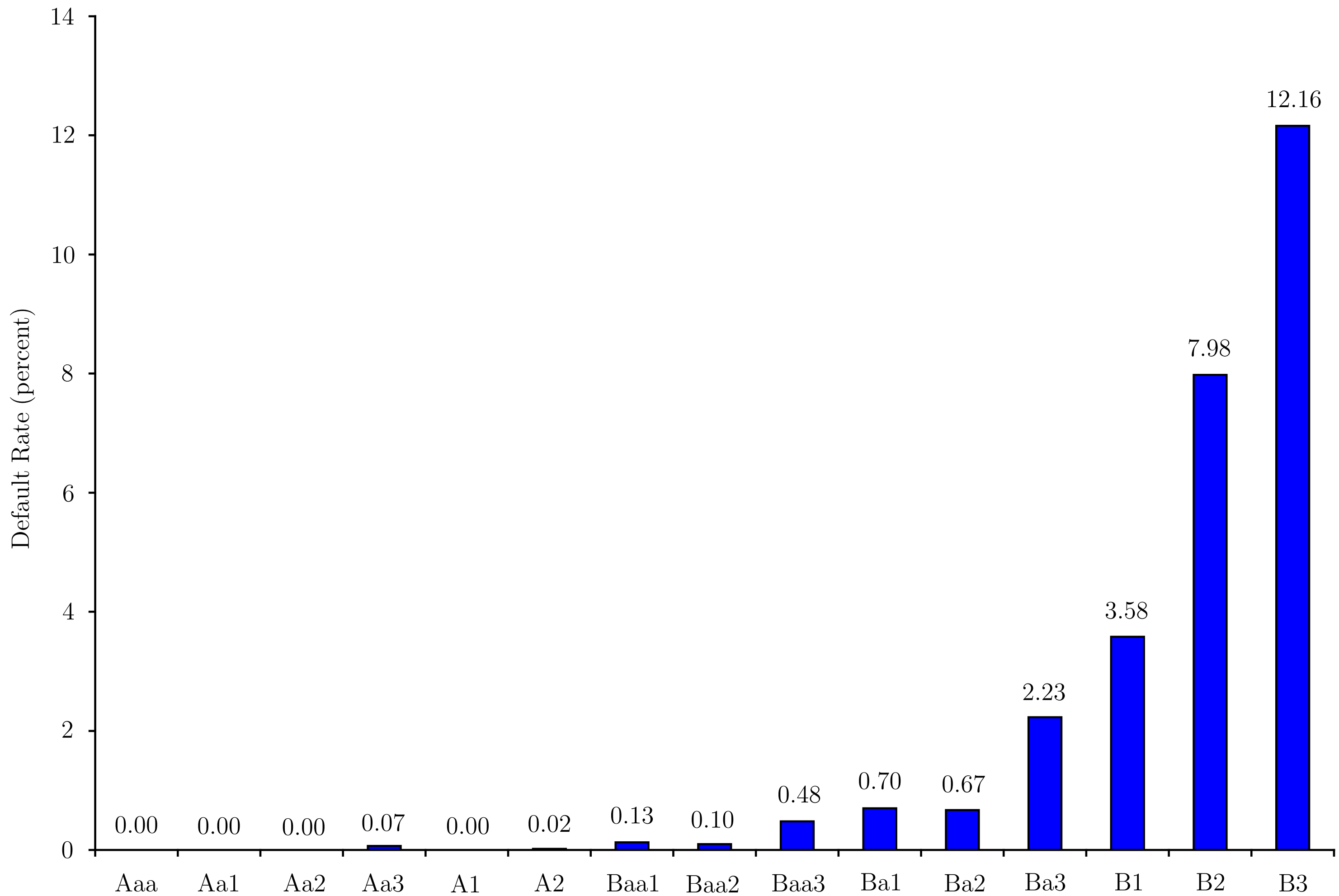


Figure 1: Default Rate by Moody's Modified Credit Rating. The rate for Caa rated bonds was 26.3%

Defining Default

Moody's defines a default to be any of:

- A missed or delayed payment of interest or principal (including delays within a grace period).
- A filing for bankruptcy (in the U.S., Chapter 11 or Chapter 7), or a legal receivership.
- A distressed exchange, including *(i)* re-structuring amounting to a diminished obligation, or *(ii)* an exchange for debt with the purpose of helping the borrower avoid default.

There has been debate in early 2001 over the exclusion of re-structurings from the list of default events covered by ISDA credit derivatives. More on this later . . .

Why Average Default Frequency is Naïve

- Credit ratings are not intended by rating agencies to be a measure of a firm's default probability over some time horizon.
- Credit ratings are *stable* measures of *relative* credit quality among firms. Firms are rated "through" the business cycle.
- Average default frequencies are merely that, "averages," and do not reflect newly available information as it arrives in the market.

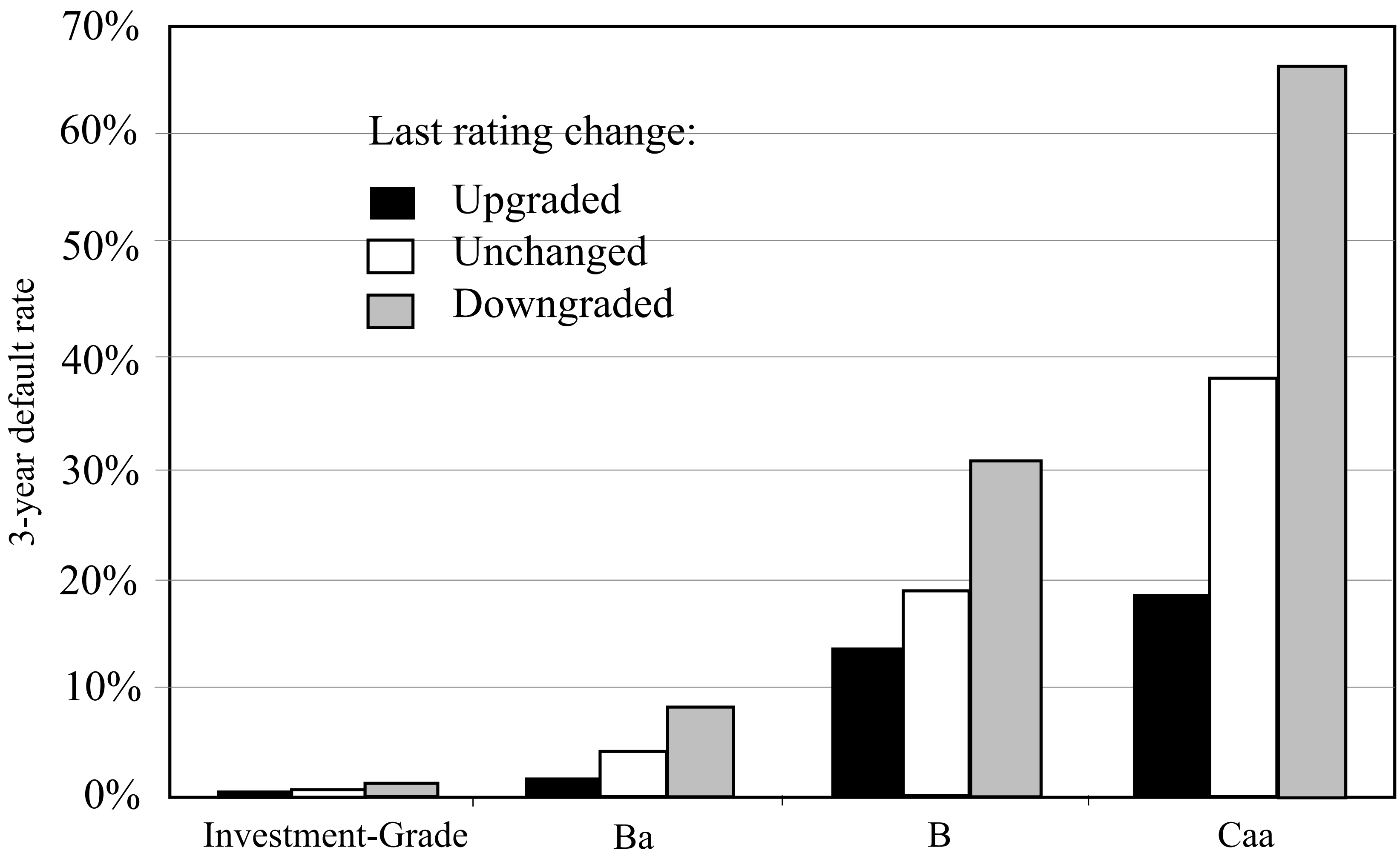


Figure 2: Upgrade-downgrade momentum (1996-2003 data). Source: Moody's, 2004.

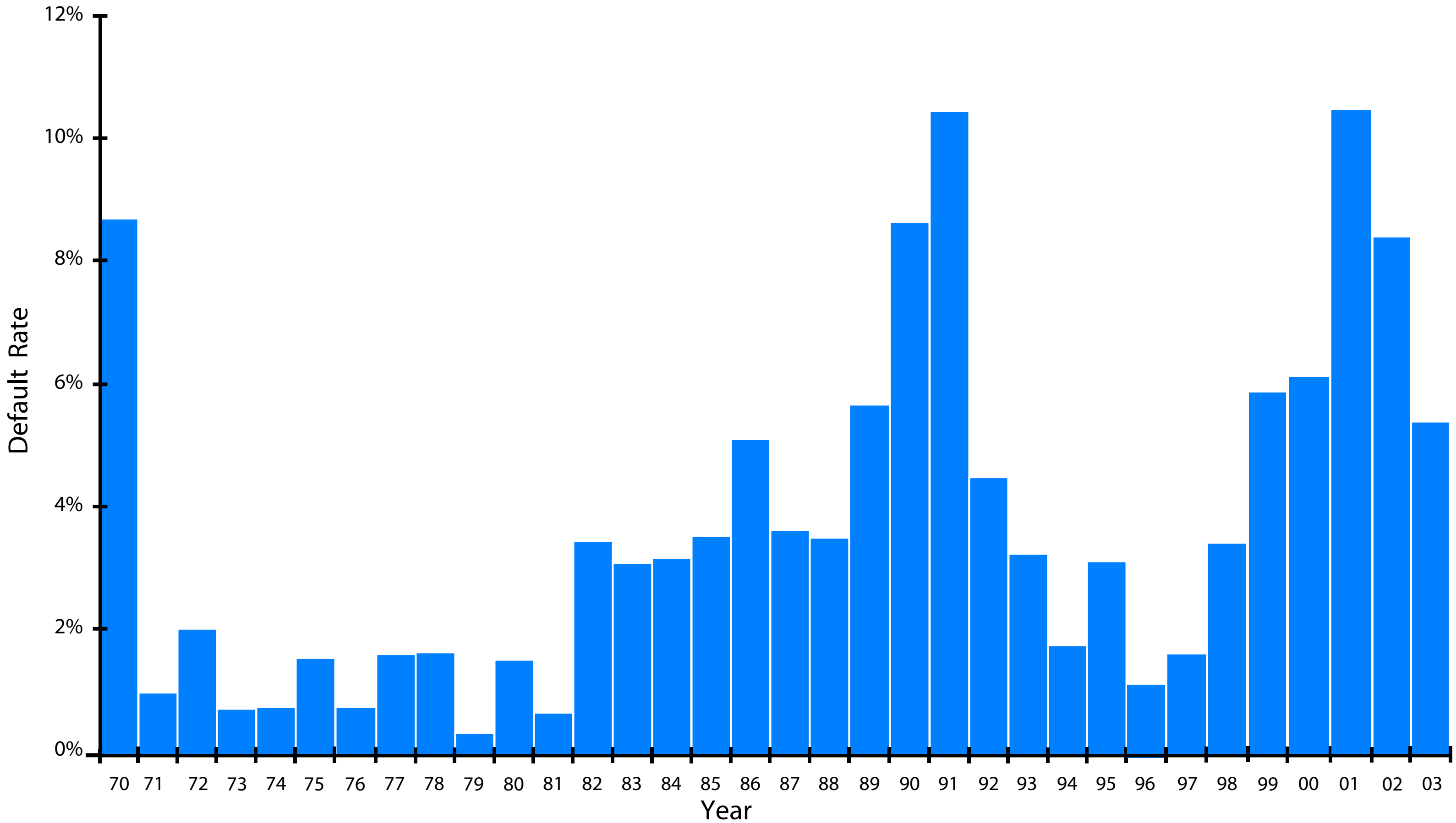


Figure 3: Rate of Speculative Grade Defaults (Moodys)

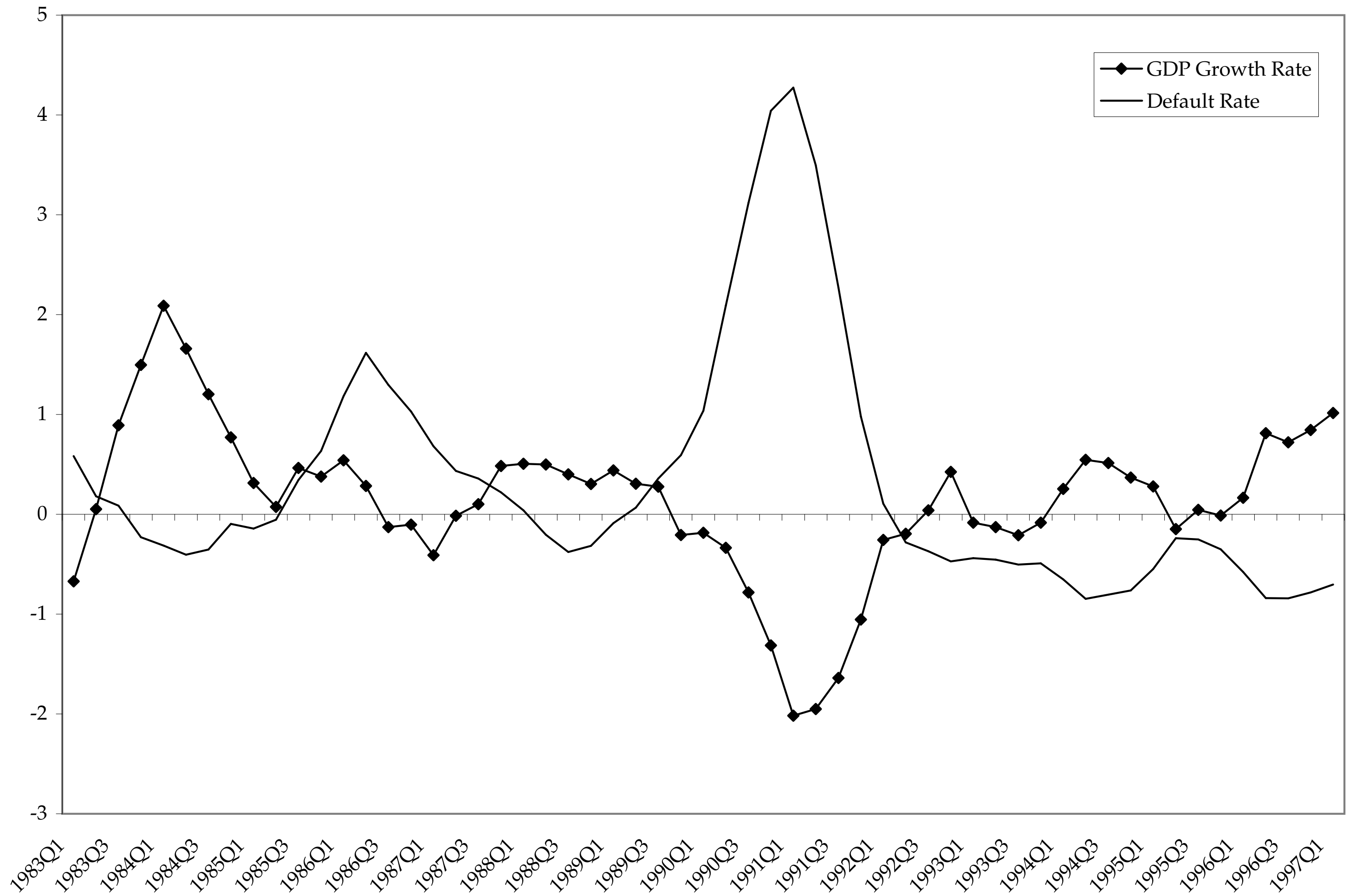


Figure 4: The business-cycle matters. Four-quarter moving averages of (standardized) speculative grade default rates (solid) and real GDP growth rates (diamonds) over the period 1983 through 1997. Source: Duffie and Singleton (2003).

B. Default Intensity

- The intensity of default is the mean arrival rate of default, *conditioning on all current information,* measured in expected number of events per year.
- For example, an intensity of 0.16% (which was the Baa average default incidence for 1980-2000) is a mean arrival rate of 0.16 defaults per 100 obligor-years.
- The intensity of default adjusts over time as new information arrives into the market.
- For example, one can estimate the intensity model $\lambda_t = f(X_t)$, where X_t is a measure of the firm's leverage, or is a list of covariates linked to default including leverage, volatility, and macro-economic performance.

Point of Departure: Constant Intensity

- If a default time τ has a constant intensity λ, then

$$p(t) = P(\tau > t) = e^{-\lambda t}.$$

- The expected time to default is $1/\lambda$.
- The probability of default over the next time period of length Δ is approximately $\Delta\lambda$, for small Δ.
- The intensity is measured on a continuously compounding basis. For example, a constant default intensity of 0.04 corresponds to a one-year default probability of 3.92%, and a one-month default probability of about 0.33%. (The expected time to default is precisely 25 years.)

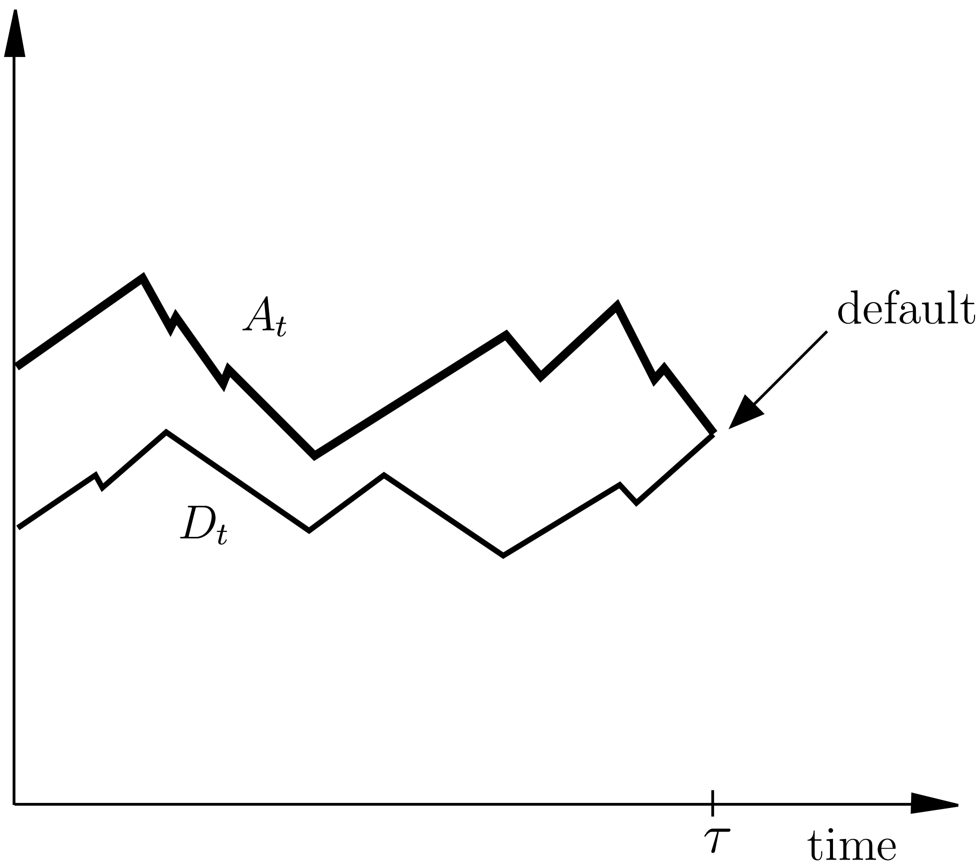
Random Variation of Intensity

- In reality, $\lambda(t)$ varies randomly with t, as credit-related information arrives in the market.
- We adopt the standard "doubly stochastic" model: Conditional on $\{\lambda_s : 0 \leq s < t\}$, default arrives according to a Poisson process with this time-varying intensity.
- The survival probability is thus

$$\begin{aligned} P(\tau > t) &= E[P(\tau > t \,|\, \{\lambda_s : 0 \leq s < t\})] \\ &= E\left[\exp\left(\int_0^t -\lambda(s)\, ds\right)\right]. \end{aligned}$$

C. Structural Default Models

Asset and Liability Values



Structural Models of Default

- Firms default when they cannot, or choose not to, meet their financial obligations.
- The classic Merton-Black-Scholes model considered a single liability. Solvency is tested only at the maturity date.
- The Merton-Black-Scholes model has been extended to allow for default at any time before maturity, to accomodate more complex capital structures, by Geske, Black-Cox, Fisher-Heinkel-Zechner, Leland, and others.

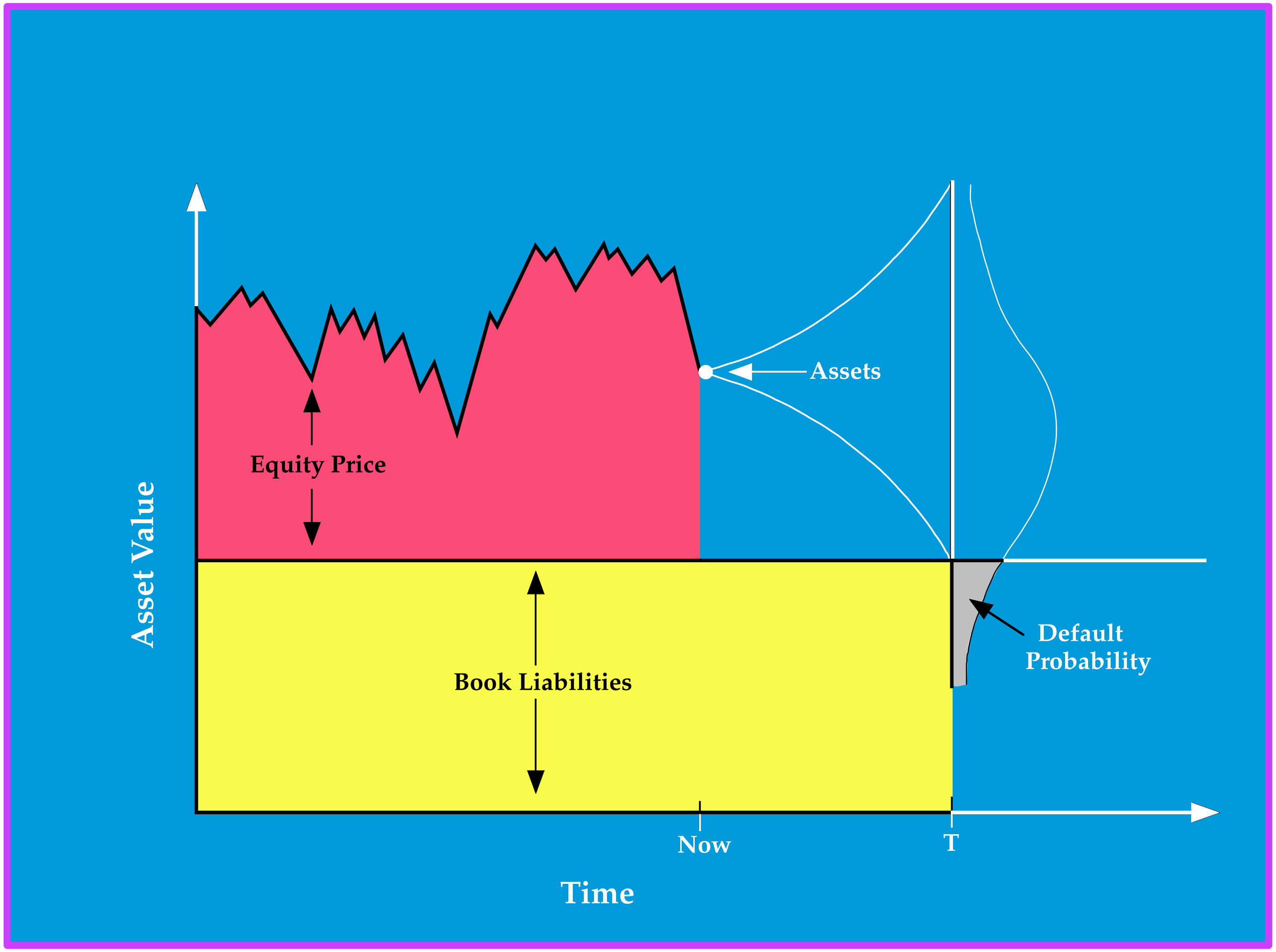


Figure 5: Merton-Black-Scholes model, book liabilities

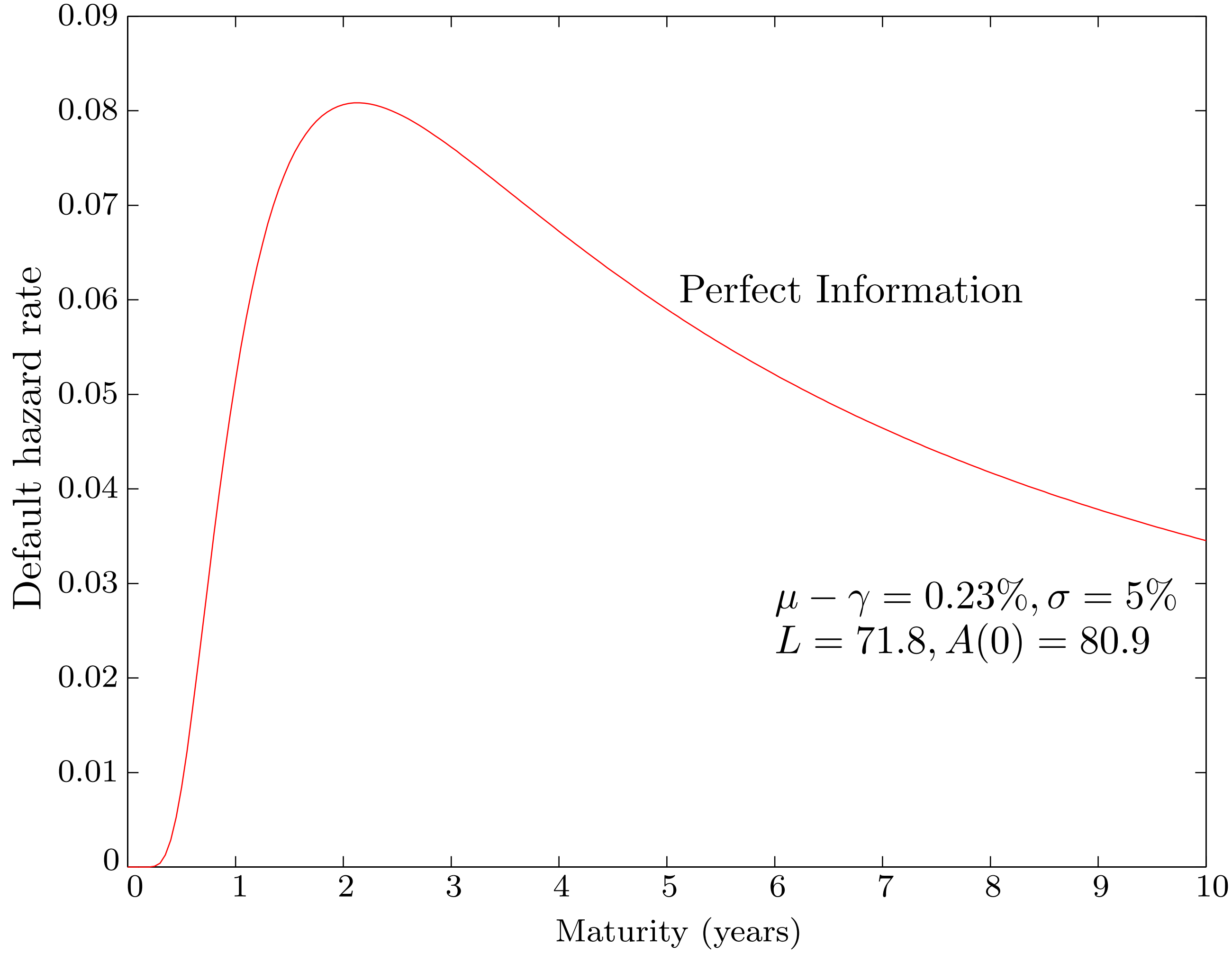


Figure 6: Forward default rate for first-passage model.

Imperfect Balance-Sheet Information

- The classical first-passage model assumes perfect knowledge of actual balance sheet.
- For a low-quality issuer, this results in unrealistically low default probabilities for short time horizons, and steeply rising forward default rates.
- Allowing for imperfect information generates more realistic term structures of forward default rates, but this is cumbersome for day-to-day business calculations.

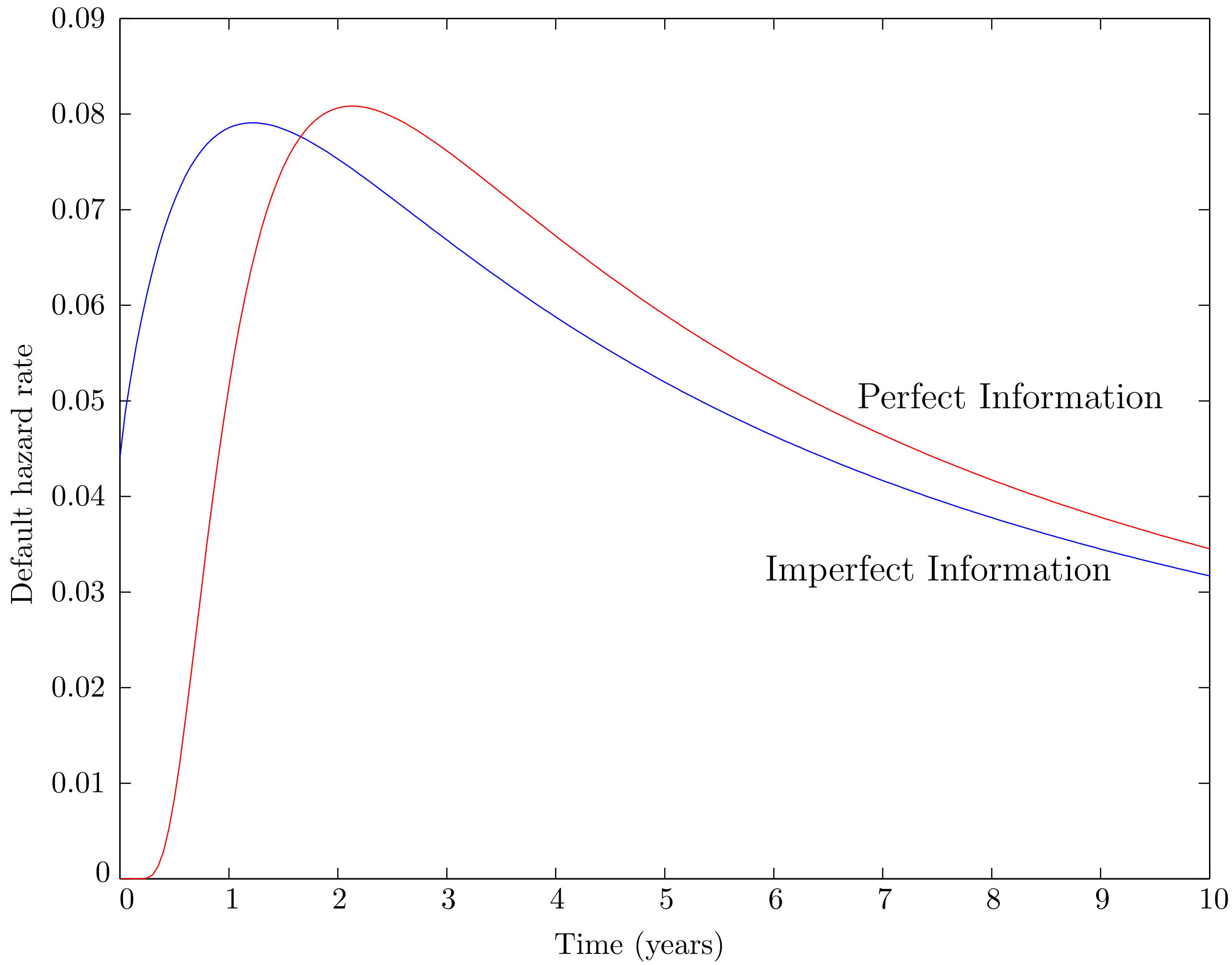


Figure 7: Imperfect asset information (noise level $a = 25\%$). Source: Duffie and Lando (2001).

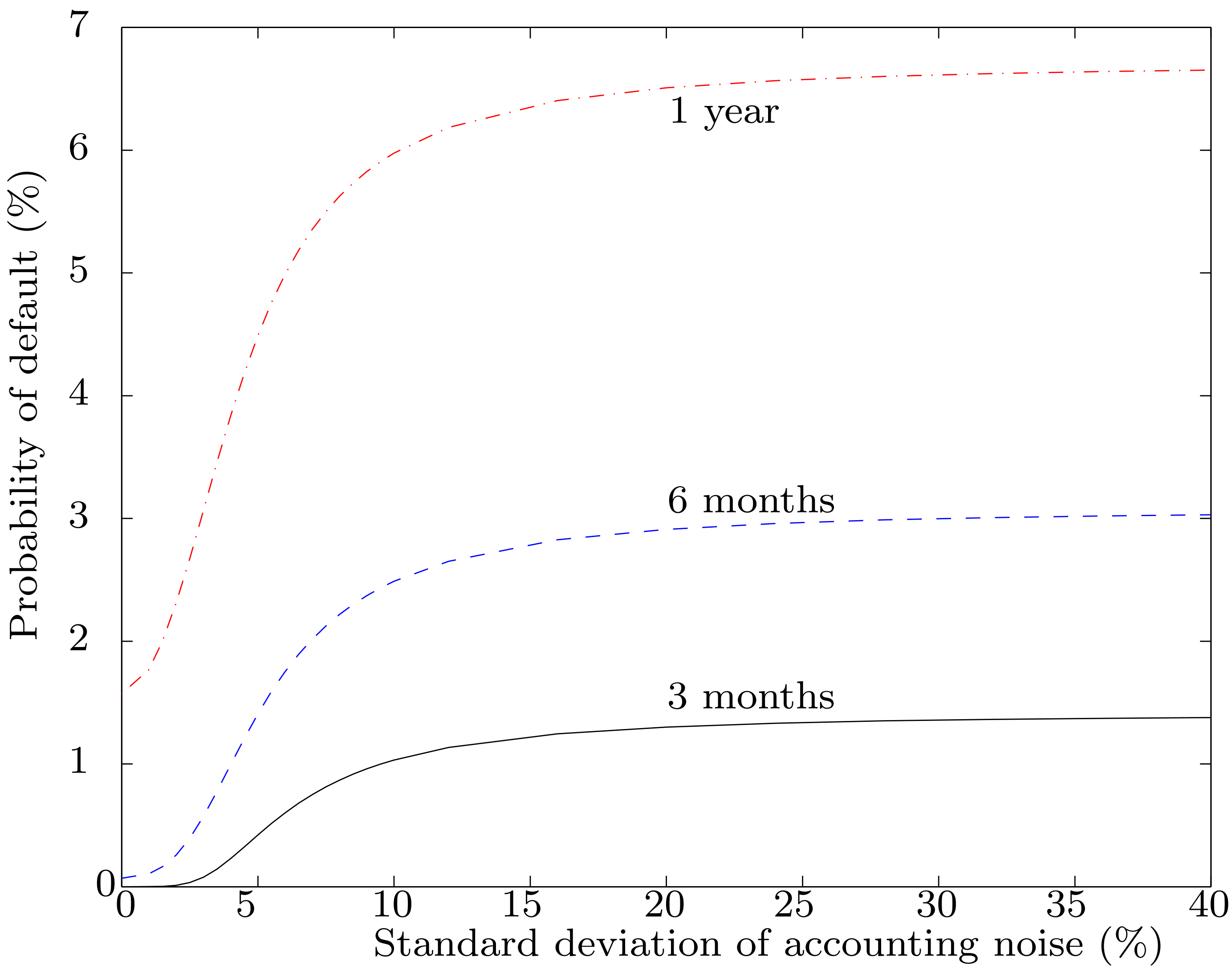


Figure 8: Effect of accounting quality Source: Duffie and Lando (2001).

Moody's KMV Estimated Default Frequency

- Asset value and volatility are computed jointly from a modified Black-Scholes options pricing model, treating equity as a call on assets struck at liabilities.
- The liability default boundary point is measured as short-term debt plus a fraction (half) of long-term debt.
- The "distance to default" is the number of standard deviations by which the expected asset value exceeds the default point.
- This firm's current *EDF* is the fraction of those firms in previous years with the same distance to default that actually did default within one year.

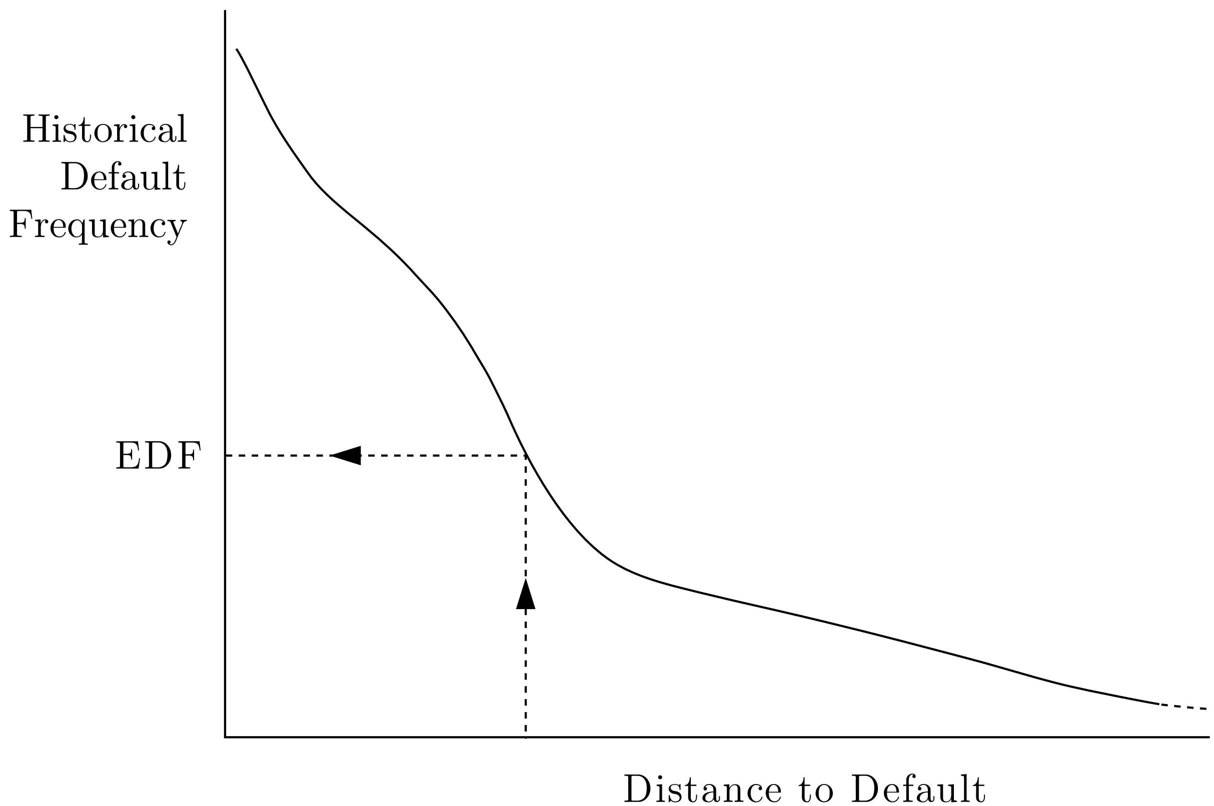


Figure 9: Mapping Distance to Default to EDF, using Historical Default Frequency

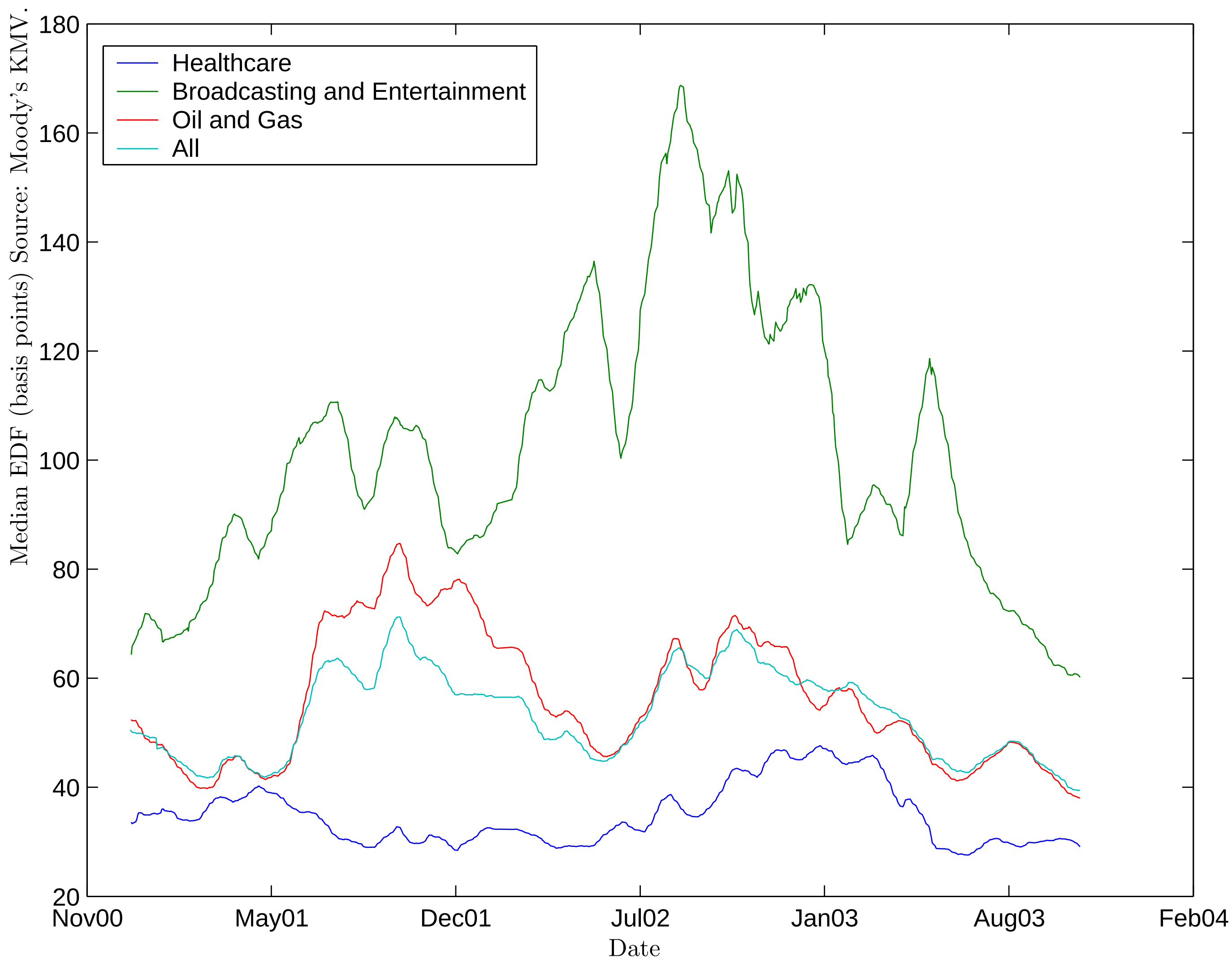


Figure 10: Sector median EDFs

D. Term Structure of Default Probability

Using Firm and Macro Covariates

- Joint work with Ke Wang.
- U.S. industrial machinery and instruments sector (870 firms).
- Data: Compustat, CRSP, Bureau of Economic Analysis.
- 1971 - 2001 quarterly observations (26,365 firm-quarters).
- Events:
  - Failures (70): Chapter 11 or Chapter 7 bankruptcy.
  - Other exits (468): merger, acquisition, privatization and others.

Econometric Strategy

- For a covariate vector $Z_i(t)$ of firm-specific and macro-economic variables, default intensity of firm i:

$$\lambda_i(t) = \Lambda(Z_i(t), \beta)$$

- Time-series model for covariates $\{Z_i(t) : t \geq 0\}_{i=1}^{n}$ with parameter γ.
- Doubly-stochastic property implies joint maximum-likelihood estimation of (β, γ) by separate MLE estimation of β (duration approach) and γ (time-series approach).
- MLE of survival probability to T:

$$P_{\hat{\gamma},\hat{\beta}}(\tau > T) = E_{\hat{\gamma}}\left(e^{-\int_0^t \Lambda(Z_i(t),\hat{\beta})\, dt}\right).$$

Intensity Estimation Results

- Default Intensity

$$\lambda_{i,t} = \exp\left(\beta_0 + \beta_1 D_{i,t} + \beta_2 Y_t\right)$$

β	Constant	Distance to Default D	Personal Income Growth Y
$\hat{\beta}$	-4.1983	-0.4432	-0.4616
s.e.	(0.2473)	(0.0591)	(0.1389)

MLE Time Series Estimation for Covariates

$$Z_{i,t} = (Y_t, D_{i,t})$$

- Personal Income Growth:

$$Y_t = 0.65(1.89 - Y_{t-1}) + 0.88\,\epsilon_t^Y \tag{1}$$

- Distance to default of firm i:

$$D_{i,t} = 0.11(\hat{\theta}_{Di} - D_{i,t-1}) + 0.96\,\epsilon_{i,t}^D \tag{2}$$

- Normal ϵ_t, with estimated correlations $\text{corr}(\epsilon_{i,t}^D, \epsilon_t^Y) = 0$, $\text{corr}(\epsilon_{i,t}^D, \epsilon_{j,t}^D) = 0.068$

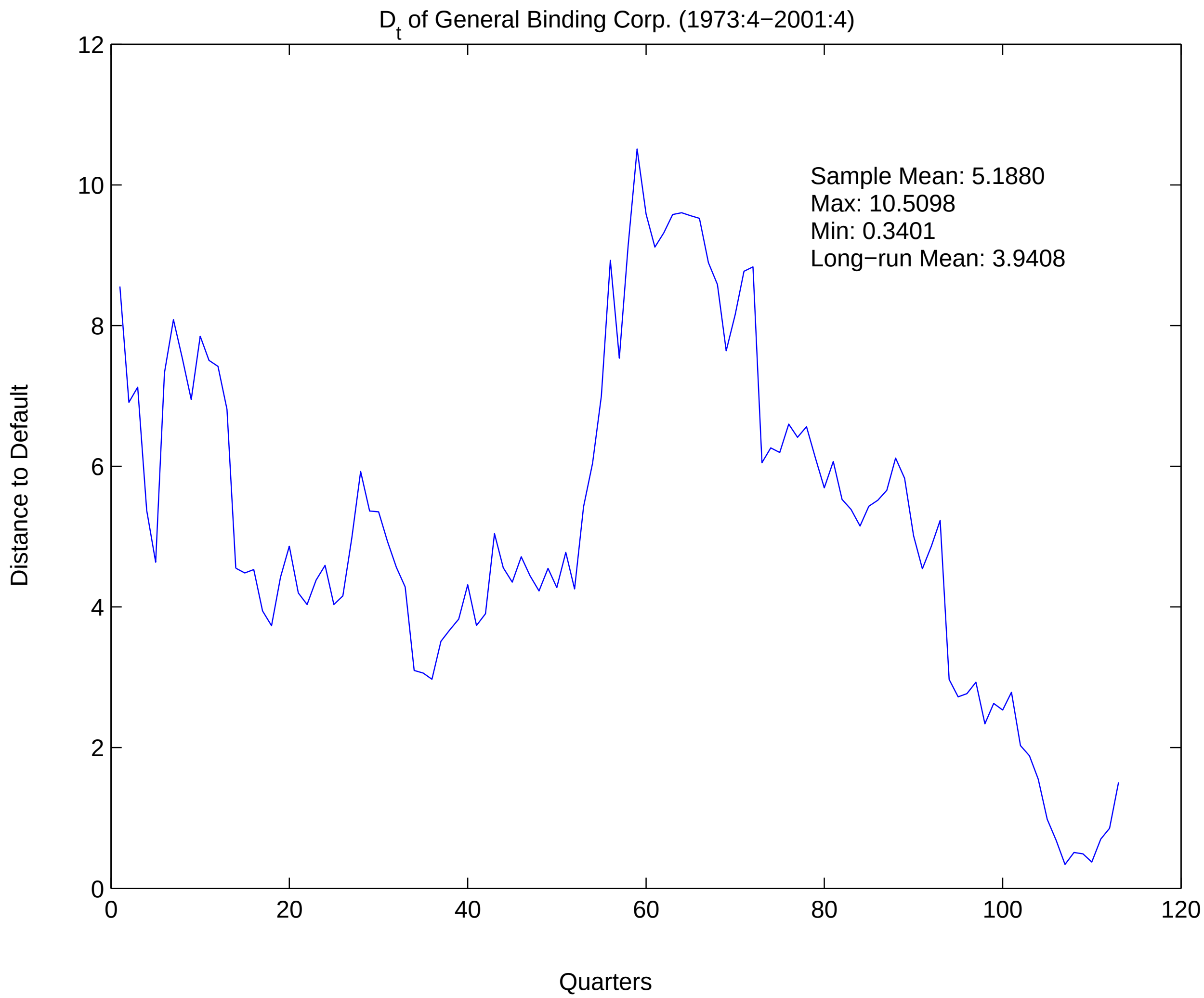


Figure 11: Distance to default, General Binding Corporation

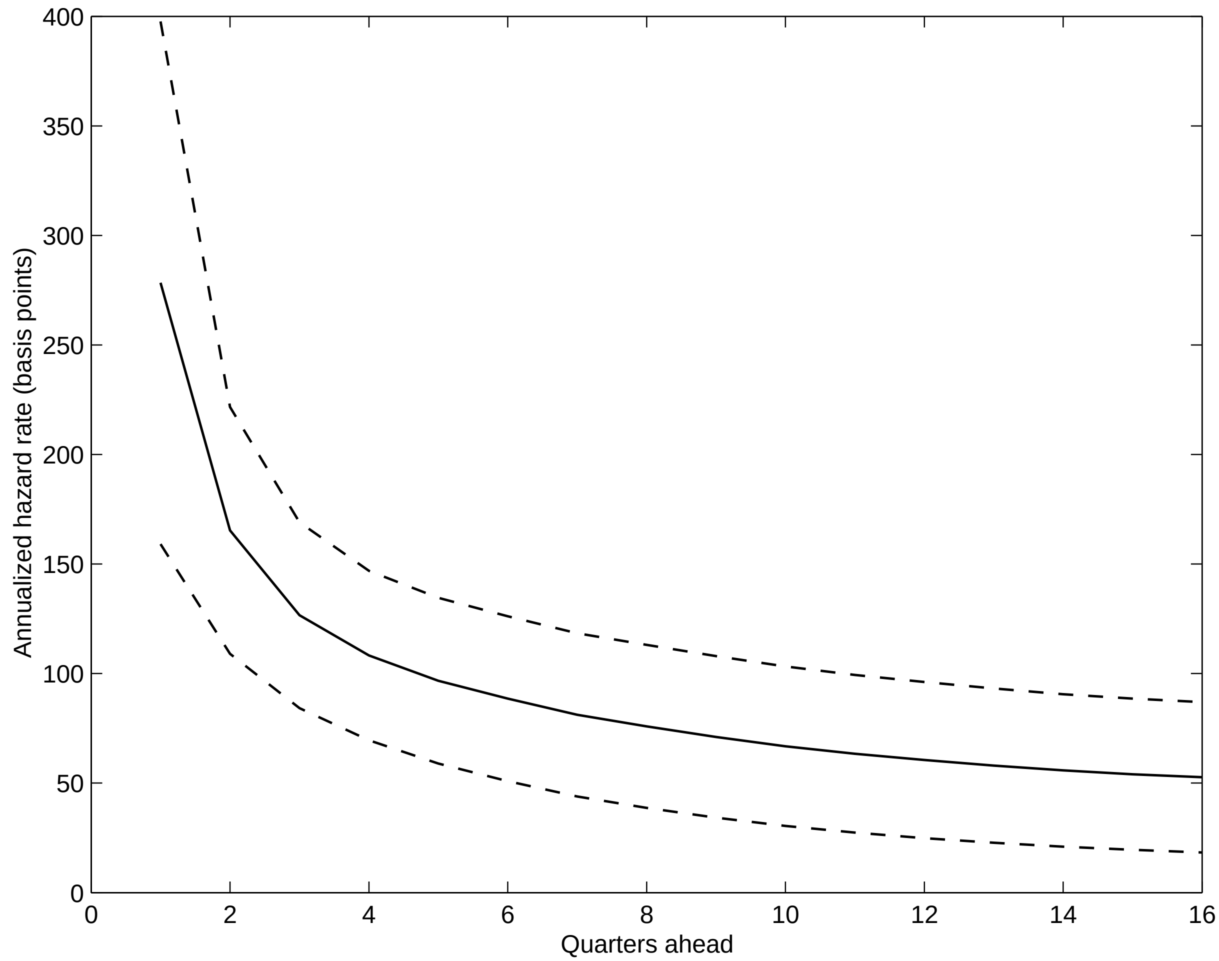


Figure 12: Estimated term structure of default hazards for General Binding Corporation, conditioning on covariates in December, 2001.

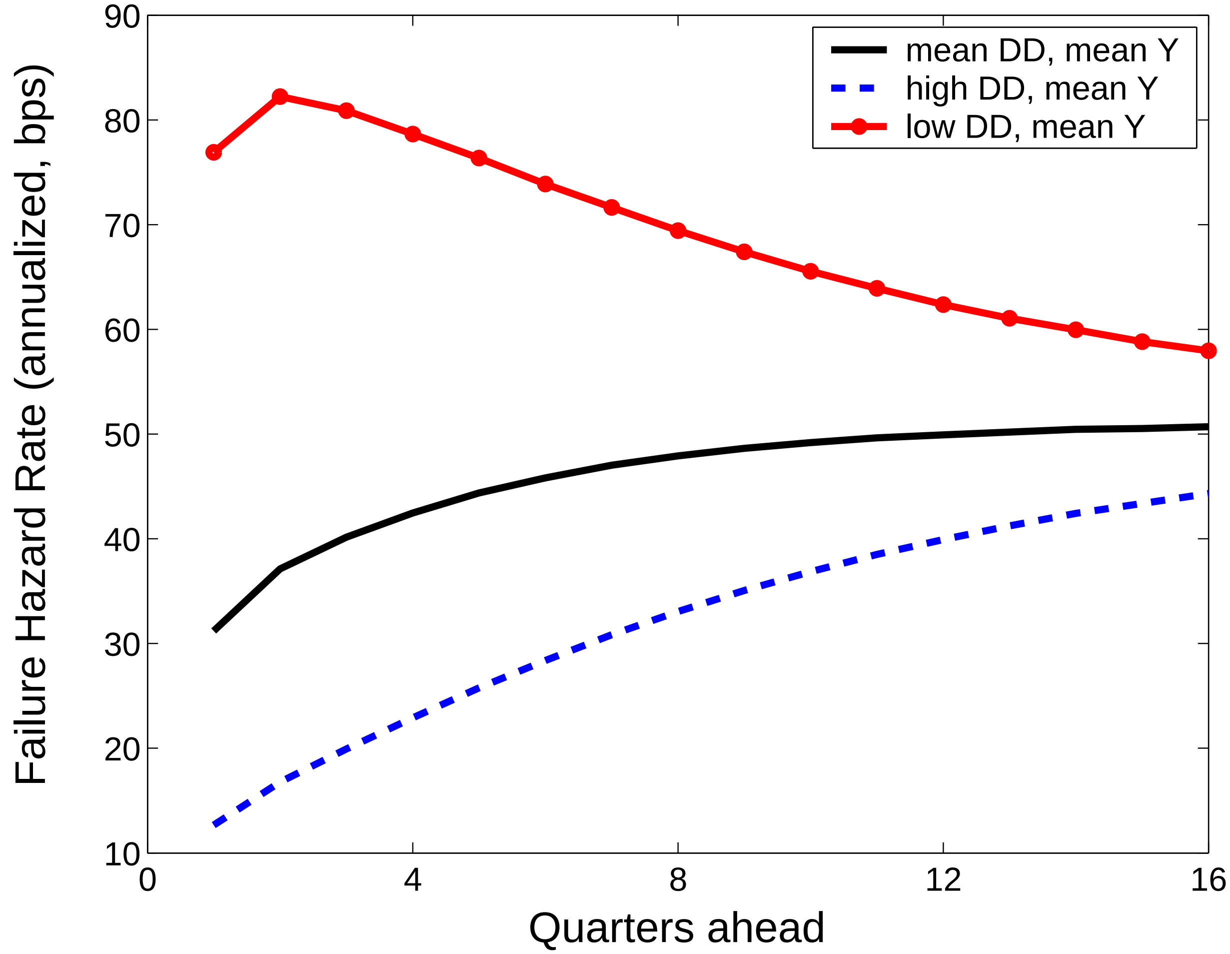


Figure 13: U.S. personal income growth at long-run mean, GBC's distance to default at three hypothetical levels.

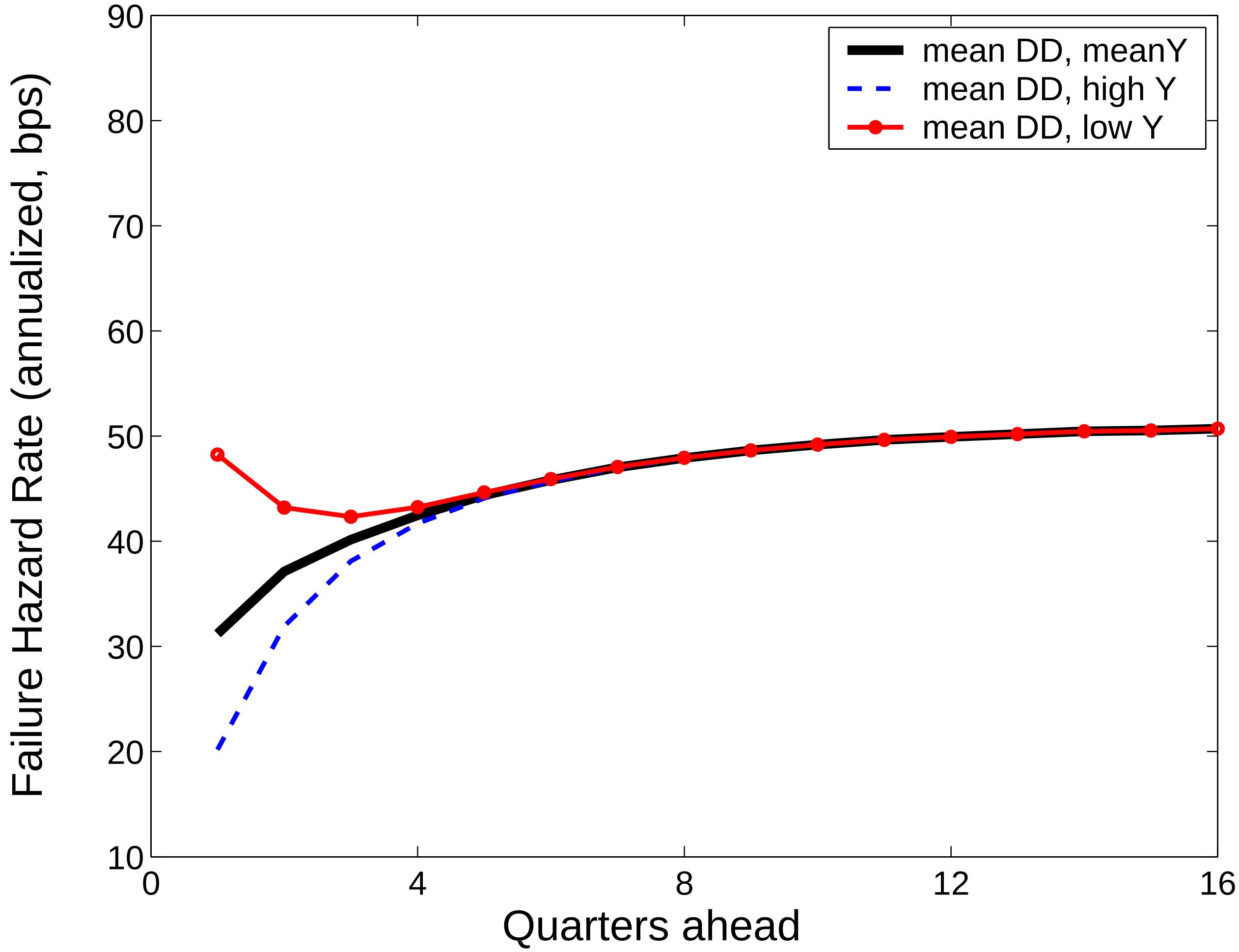


Figure 14: Distance to default at mean, personal income growth at three hypothetical levels.

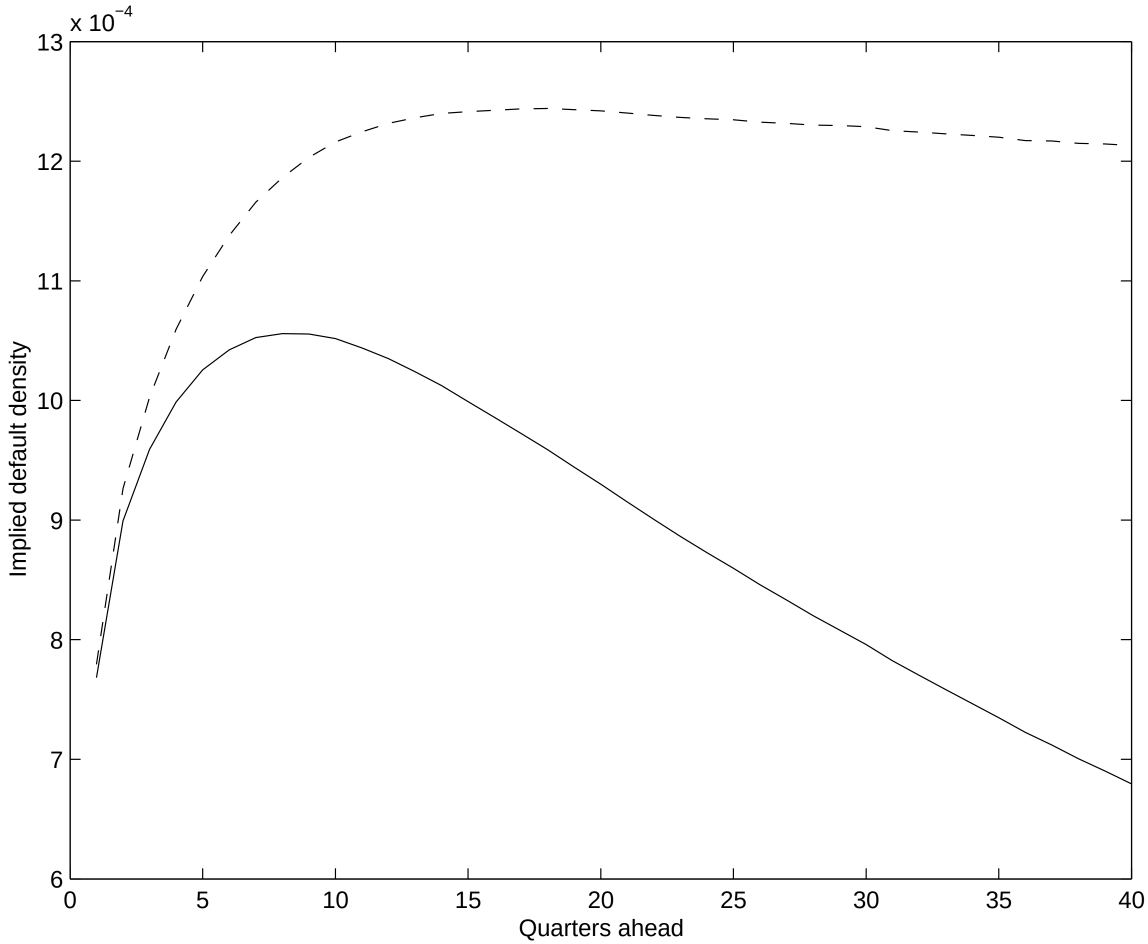


Figure 15: Failure-time density of GBC. Solid line: Considering merger survivorship; dashed line: ignoring merger survivorship.

- Merger Intensity

$$\mu_{i,t} = \exp\left(\alpha_0 + \alpha_1 D_{i,t} + \alpha_2 Y_t\right)$$

α	Constant	Distance to Default D	Personal Income Growth Y
$\hat{\alpha}$	−3.9770	0.0138	−0.1732
s.e.	(0.1343)	(0.0129)	(0.0608)