Default Correlation

Lecture 3, Clarendon Lectures in Finance, 2004

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Outline of Clarendon Lectures

1. Corporate default probabilities.
2. Pricing corporate default risk.
3. Default correlation.

Default Correlation

A. Overview

B. Copula models of default correlation.

C. Correlated default intensities.

D. Testing for doubly stochastic defaults.

E. Collateralized debt obligation modeling.

A. Overview

- There is currently concern about the pricing and risk management of products exposed to default correlation.
- Copula implementations treat default-event correlation only, and do not allow joint treatment of spread and default risk.
- Stochastic intensity models typically assume that default correlation is fully captured by intensity correlation, under the "doubly-stochastic" assumption.
- The doubly-stochastic assumption can be tested, given firm-level default probability estimates.
- Meanwhile, CDO and CDX-tranche pricing and rating is weak, and poorly integrated with spread-risk prices and risk analysis.

Default Event Correlation

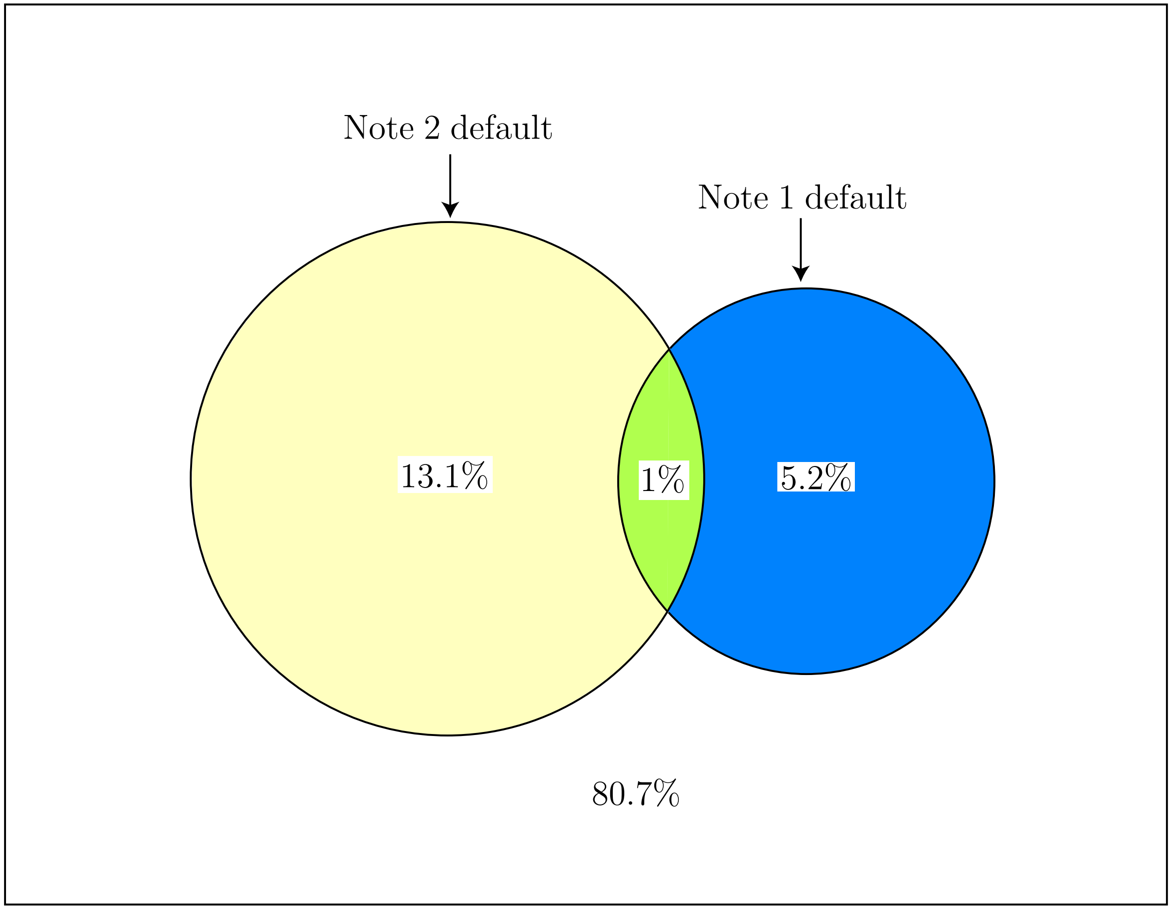


Figure 1: Default Event Correlation = 4.3%.

Darrell Duffie, Stanford University, June 2004

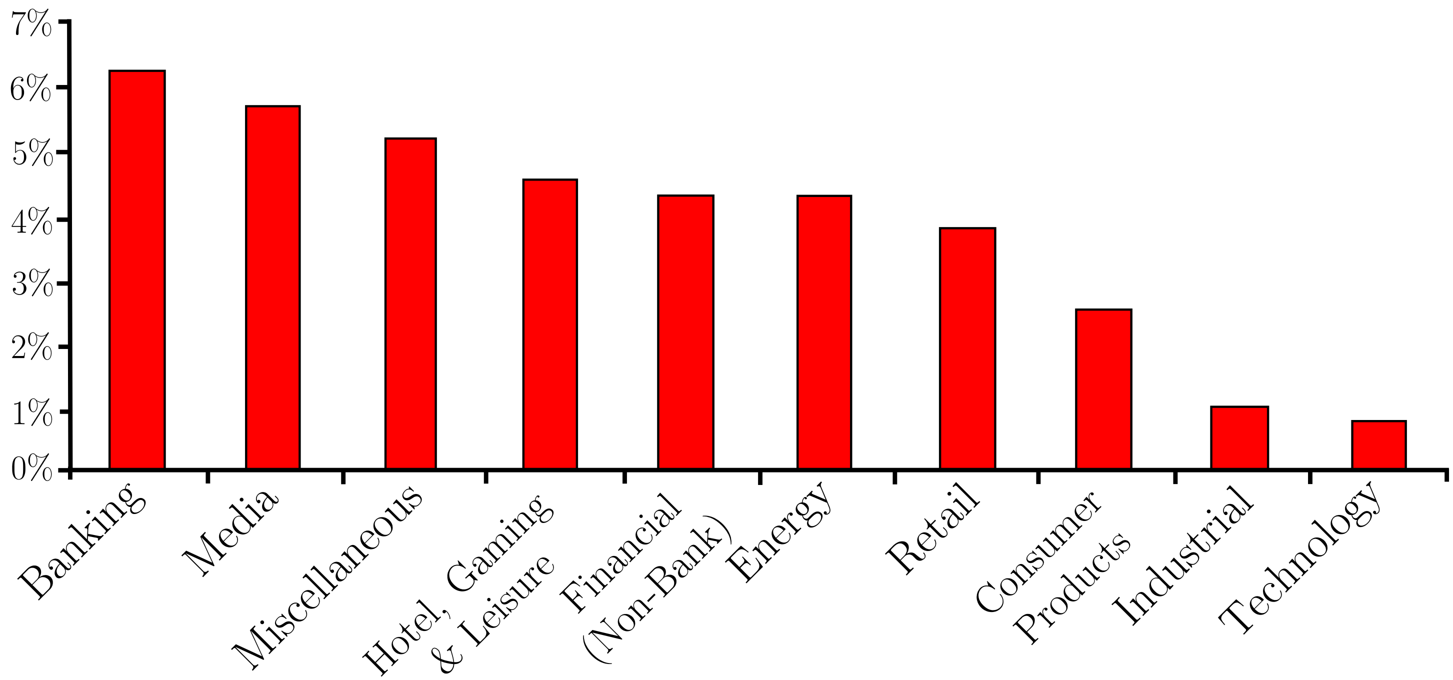


Figure 2: Empirical one-Year default-event correlations, average within sectors. Source: Moody's, 2000.

Example: CDS Index (CDX) Products

- CDS index products allow quick access to many (usually 100) names in one un-funded structured credit product.
- Entering as a seller of protection has the effect of entering 100 default swaps as a seller of protection on each.
- Trac-X and iBoxx, the two main competitors, merged, and CDX is the benchmark credit index.

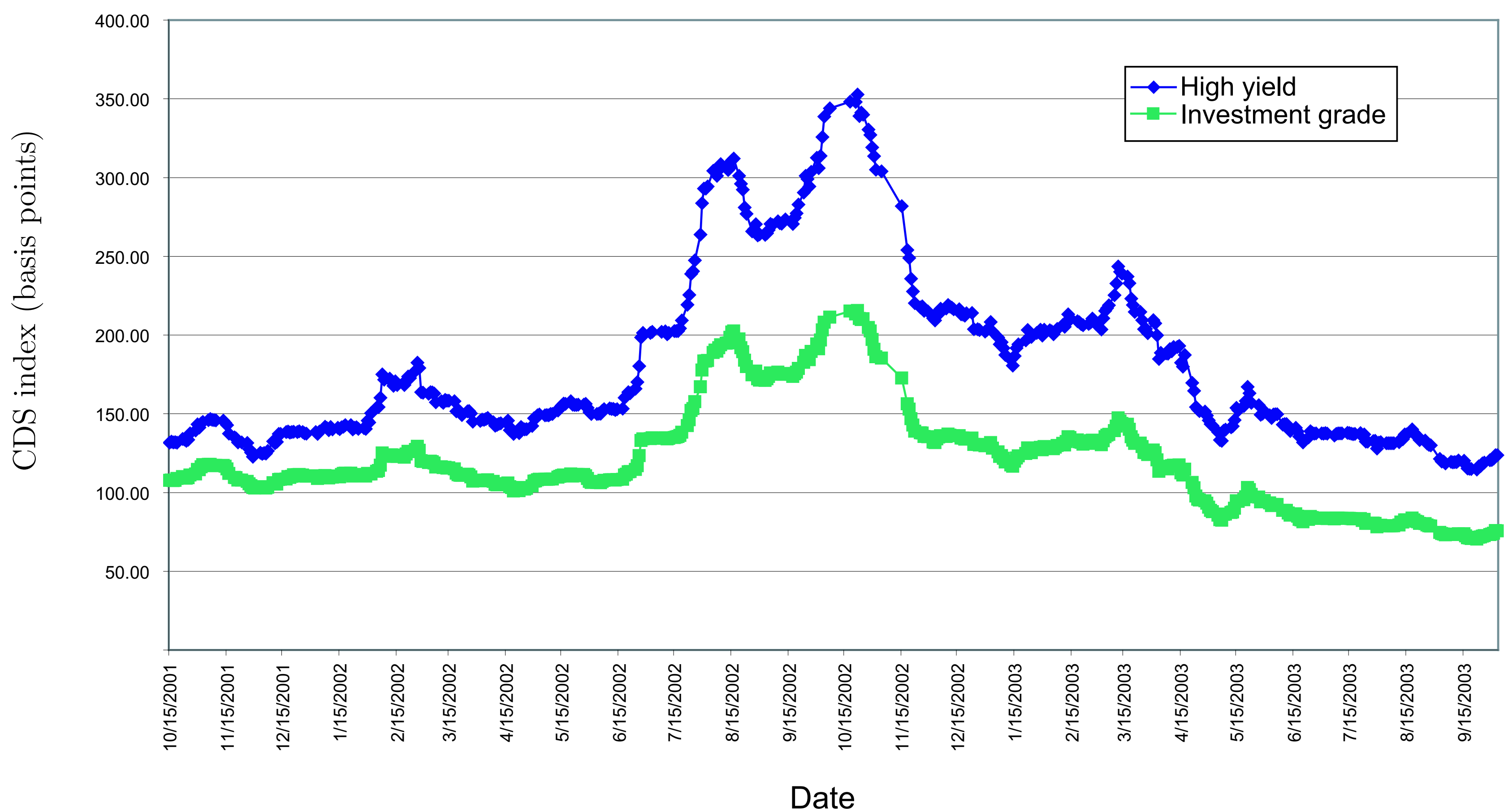


Figure 3: Trac-X NA, investment and high-yield 100-firm indices. Source: Morgan Stanley.

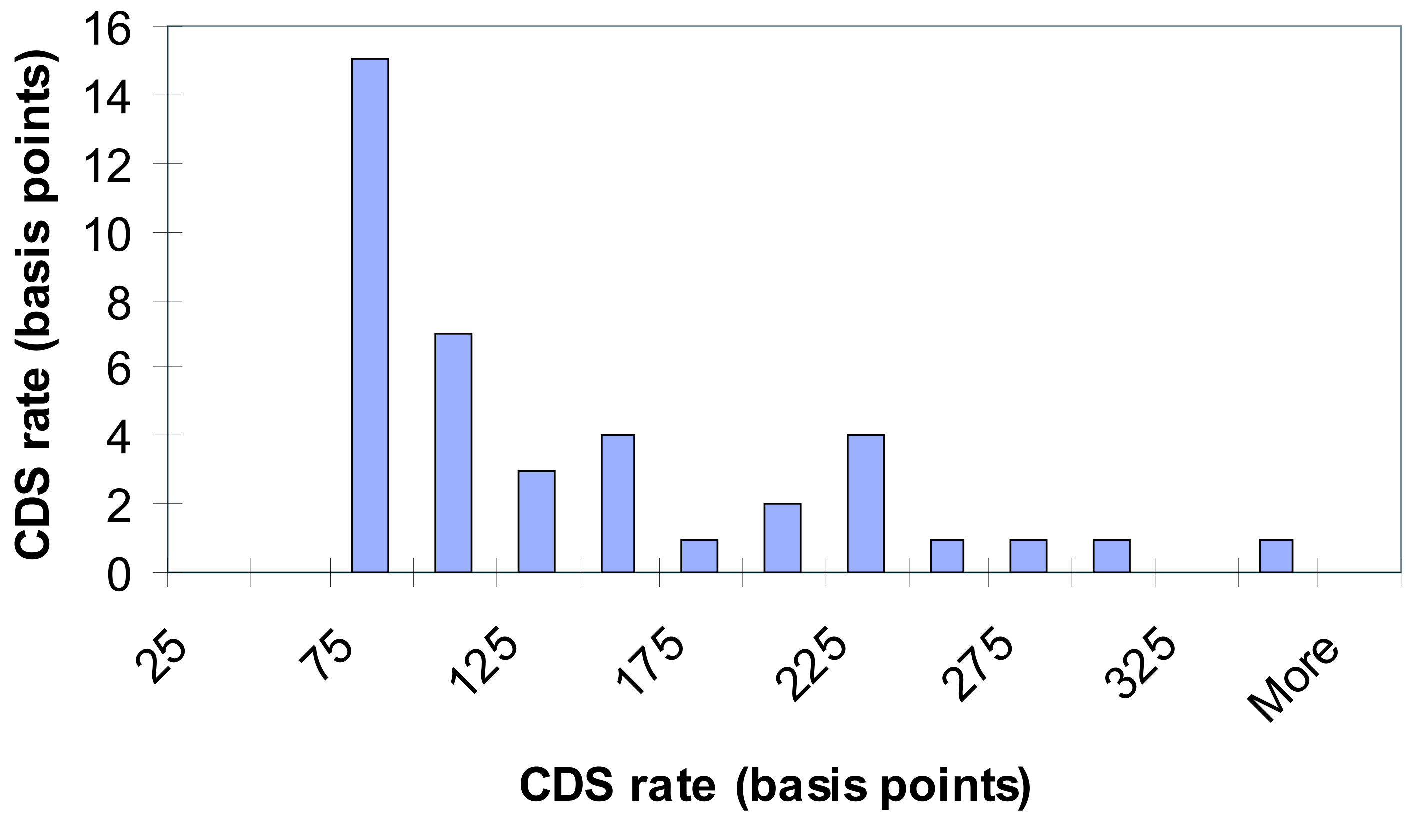


Figure 4: Histogram of underlying CDS rates, Trac-X NA high-yield, 100 firms, October 2, 2003. Source: Morgan Stanley.

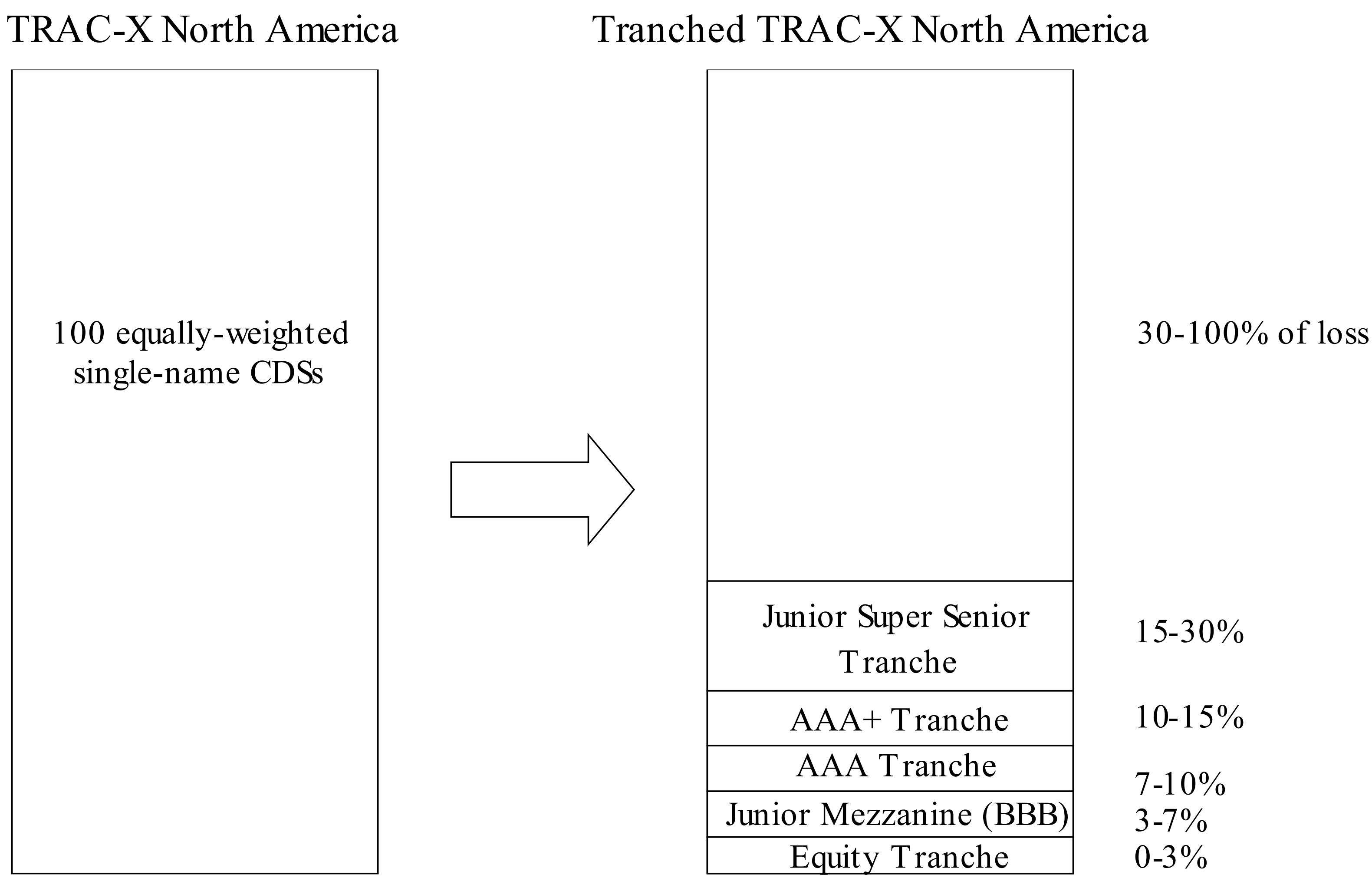


Figure 5: Tranched Trac-X NA Attachment Points and Ratings

Table 1: TriBoxx Tranche PV01 Estimates. February, 2004. Source: Citigroup.

	Mid-Spread (bp)	One-Year Carry (MM)	PV01 (MM)	Efficiency PV01/Carry
iBoxx	56	56	5.6	0.10
9%-12%	53	53	12.2	0.23
6%-9%	113	113	24.6	0.22
3%-6%	343	343	70.0	0.20
0%-3%	1,765	1,765	84.7	0.05

B. Copulas

- Copulas specify correlation for random variables, such as default times, whose individual probability distributions have already been determined.
- Copulas are especially convenient for simulation.
- We will explain some severe limitations to the copula approaches that have been applied to default correlation.

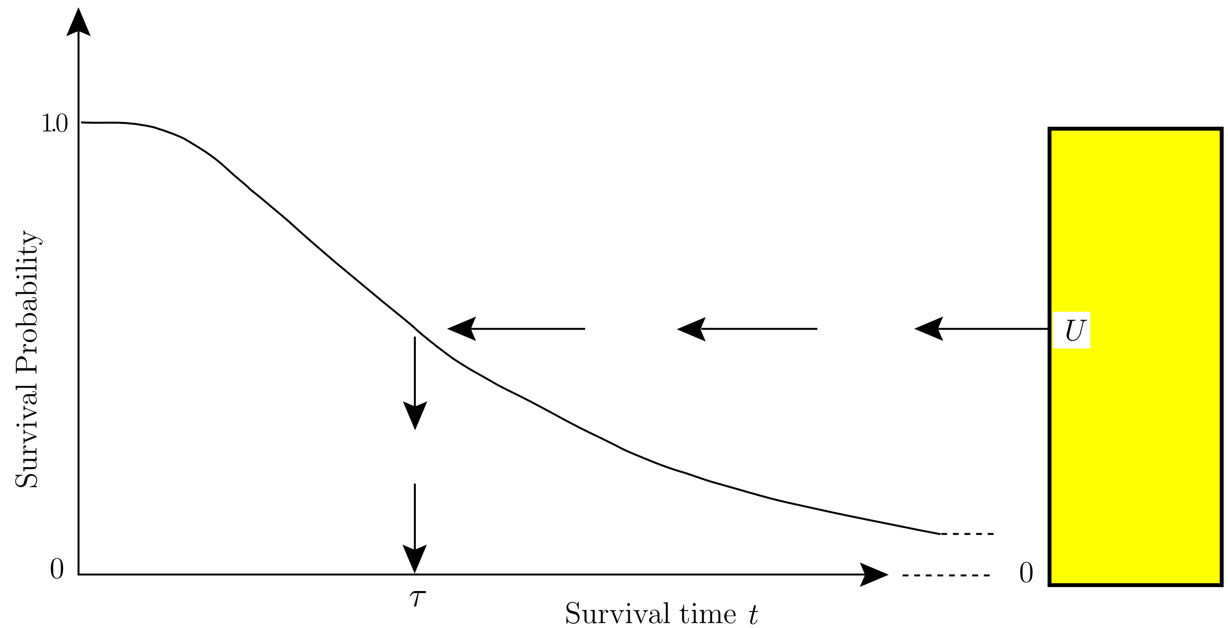


Figure 6: Simulating Default Time by Inverse CDF Method

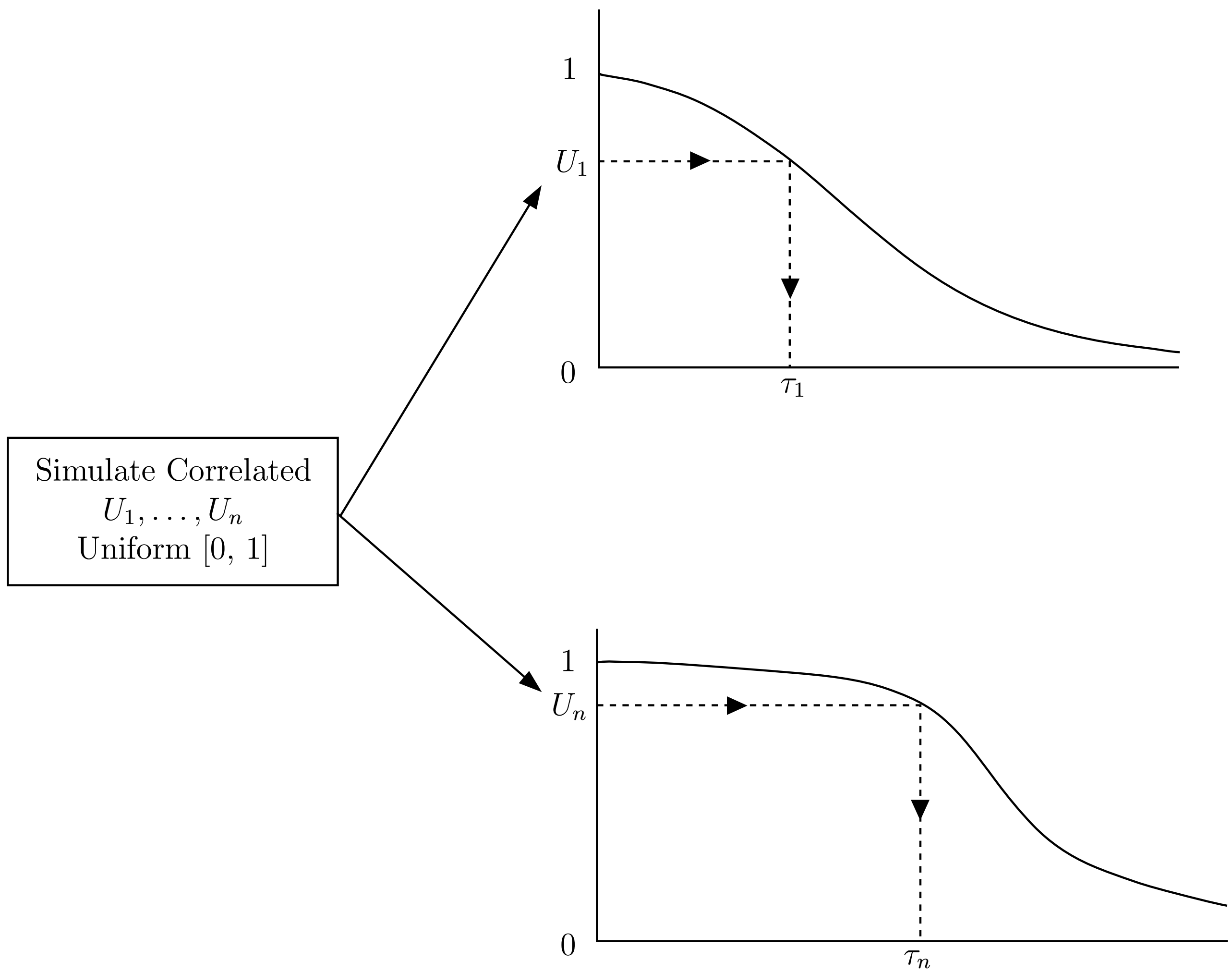


Figure 7: Copula-based simulation of correlated default times

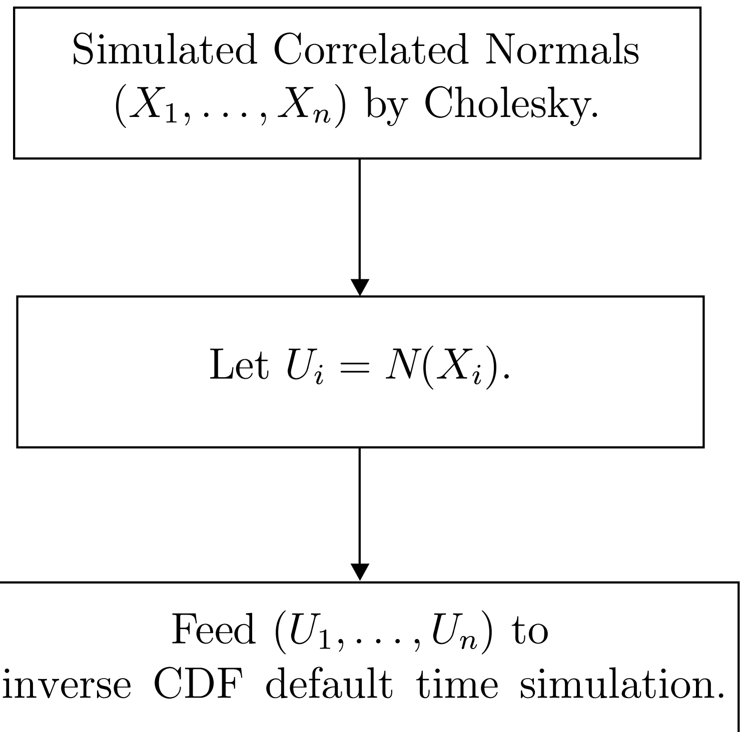


Figure 8: Using Correlated Gaussians to Simulate Correlated Uniforms. Here, $N(x)$ is the probability that a standard-normal is less than x.

Copulas don't handle mark to market risk

- Let $V_i(t)$ denote the market value of bond i at time t.
- We will want to calculate $P(V_1(t) + \cdots + V_n(t) \leq k)$, or the price of an option on $V_1(t) + \cdots + V_n(t)$.
- The joint revaluation risk should include both correlated default and correlated uncertain changes in spreads.
- Nobody has yet cracked this with a copula.
- Absent this, how can copula-based modeling be integrated into standard basket-product pricing and risk-management?

Copulas for Default Times: Advantages

- **Data flexibility.** For example, with 500 names to track, one can model all individual models of default risk, one at a time, then, for each new application involving a small subset of names, one can layer in correlation with the copula.
- **Simulation.** Rather than laboriously simulating each name's default intensity path and then drawing defaults, one can directly simulate default times from survival functions.
- **Contagion.** Copulas easily introduce contagion effects.
- **Generality:** By Sklar's Theorem, there is a copula to go with any joint distribution of random variables, so nothing is ruled out.

C. Correlated Default Intensities

- Doubly-stochastic: Conditional on the path of the default intensity processes $\lambda_1, \ldots, \lambda_n$ of the n names, the respective default times $\tau_1, \ldots, \tau_n$ are independent Poisson arrivals at these intensities.
- This means that the only source of default correlation is correlation in the default intensities.

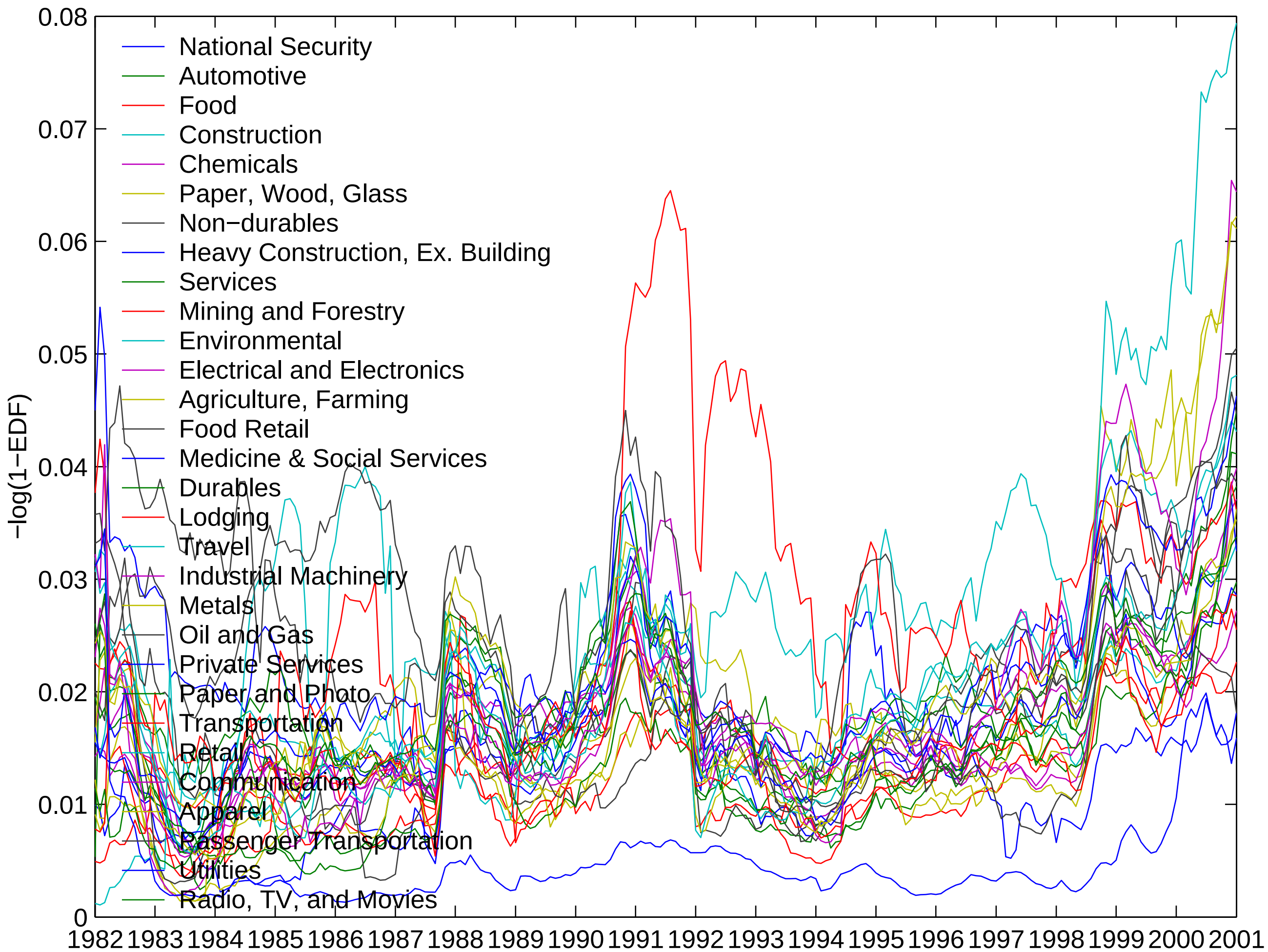


Figure 9: Sector-average intensities (Source: Moody's PD data).

Additional Channels of Default Correlation

The doubly-stochastic property rules out:

- Frailty: Incompletely observed default covariates. (Recent examples may include Enron and Worldcom.)
- Contagion: The default of one firm causes the default of another. (Example: Penn Central, 1971.)

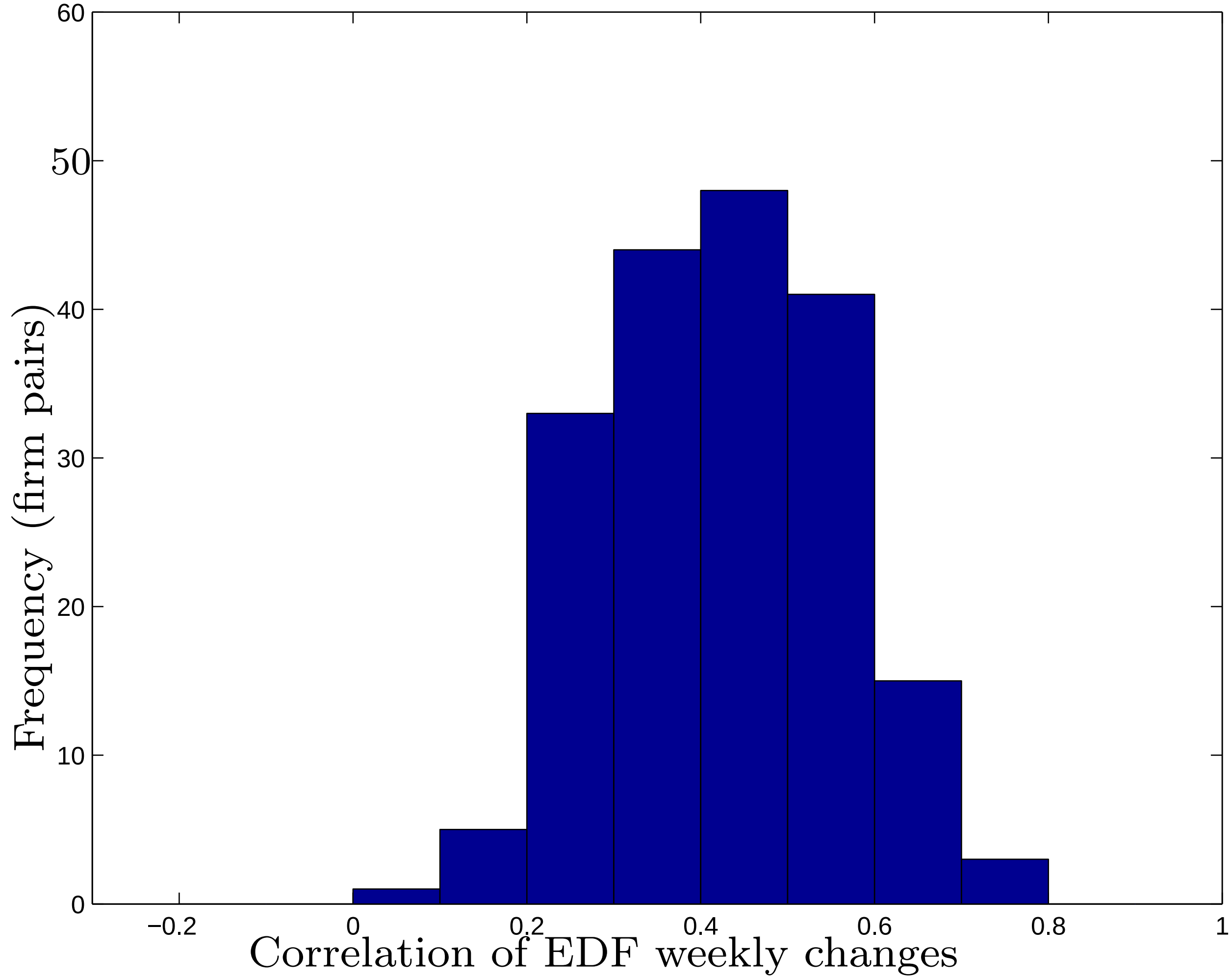


Figure 10: Distribution of Correlations of weekly % changes in Moodys KMV EDFs, Oil-Gas sector.

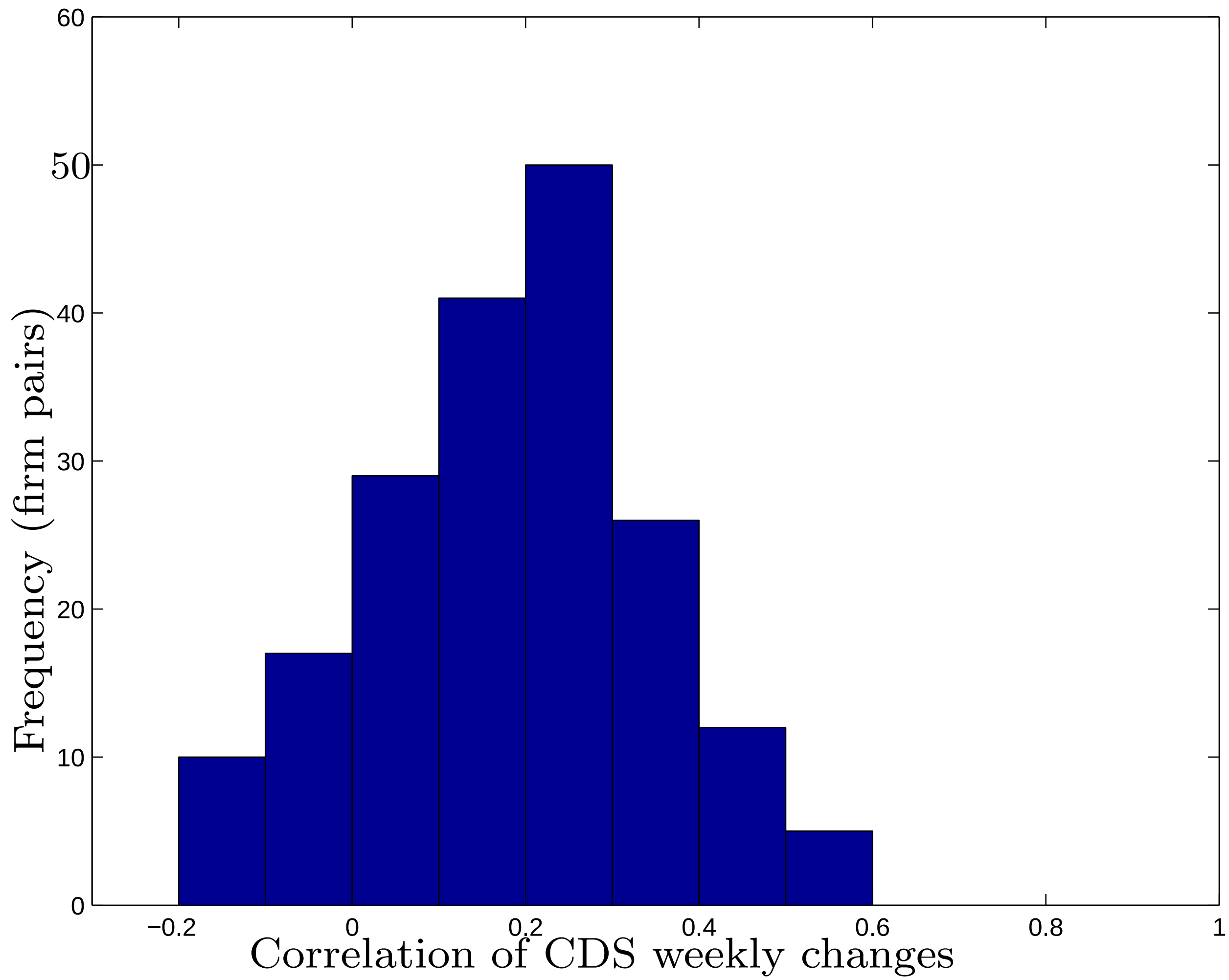


Figure 11: Distribution of Correlations of weekly % changes in Credit Default Swap Rates, Oil-Gas sector. (CIBC data)

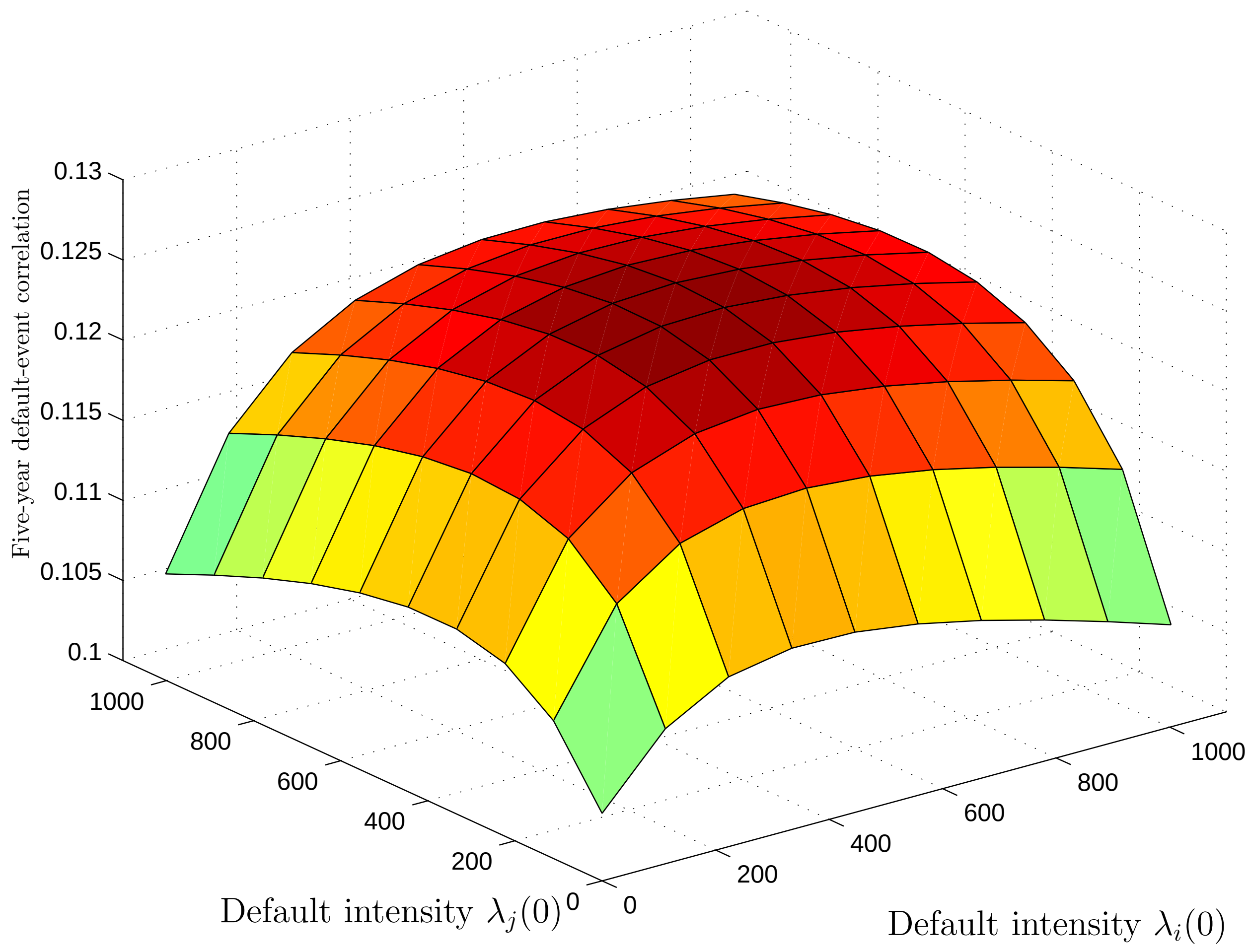


Figure 12: Five-year default-event correlations implied by doubly-stochastic default intensity estimates of Duffie and Wang (2004).

D. Testing for Doubly Stochastic Defaults

- The doubly stochastic property rules out contagion and others sources of correlation that are not captured by correlation in default intensity processes.
- Work with Sanjiv Das and Nikunj Kapadia provides a test of the doubly stochastic property.
- Goodness-of-fit tests indicate a rejection of the doubly-stochastic property if the measured default intensities are correct, but there is evidence that rejection may be due to missing macro-economic default covariates.
- Tests show no significant evidence of default clustering in excess of that implied by intensity correlation.

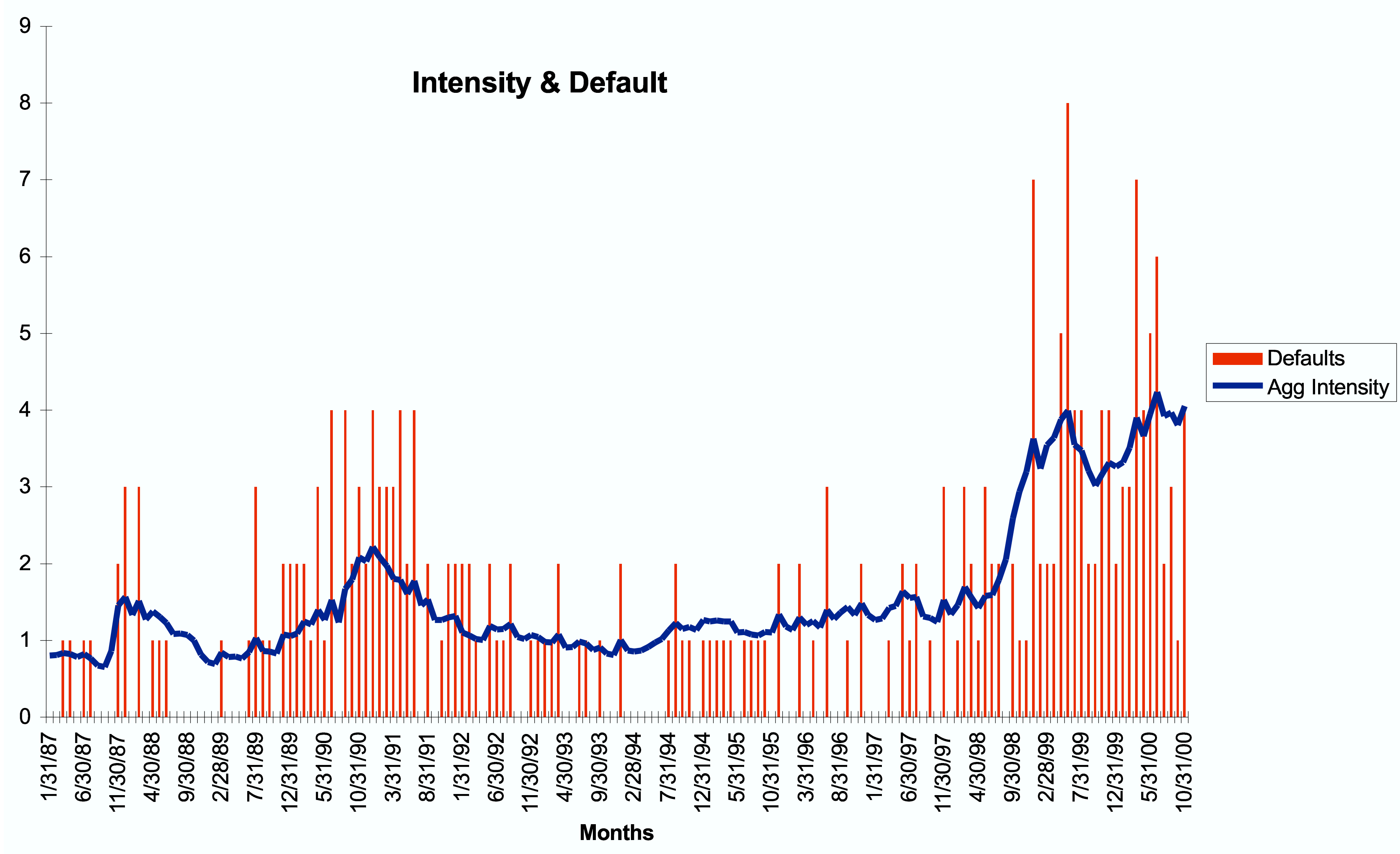


Figure 13: Aggregate default intensity and default incidence, 1987 to 2001. (Moody's data, from joint work with Sanjiv Das and Nikunj Kapadia).

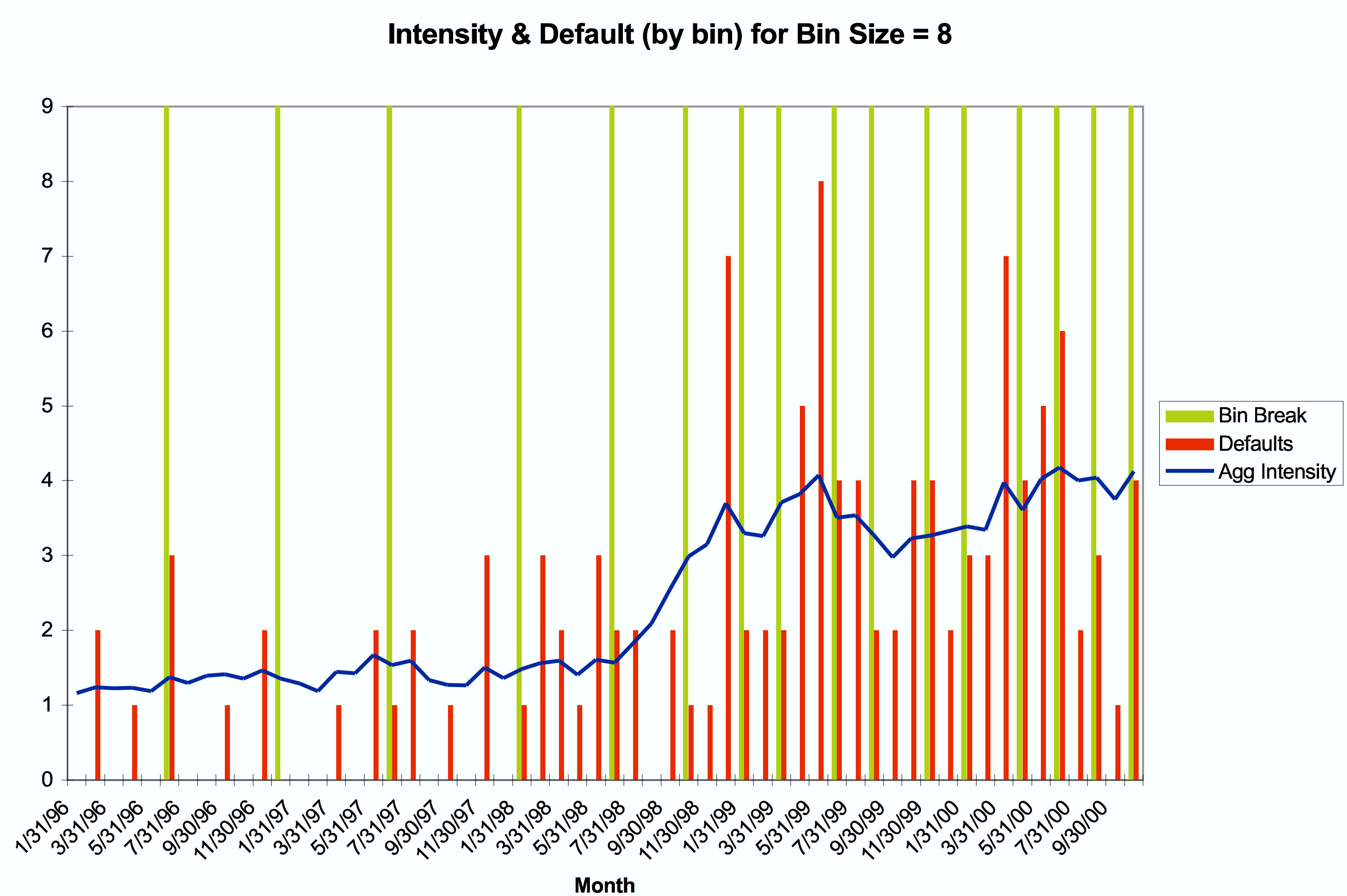


Figure 14: Breaking cumulative aggregate default intensity into time bins of size 8. Theoretical mean defaults per bin: 8. Actual mean 8.13. (Joint work with Sanjiv Das and Nikunj Kapadia).

Testable property of doubly stochastic

- Let $\overline{\lambda}(t) = \sum_{\{i:\tau(i)>t\}} \lambda_i(t)$ be the sum of the intensities of all alive firms at t.
- Let $K(t) = \#\{i : \tau(i) \leq t\}$ be the number of defaults by time t.
- Consider the time change $U(t)$ defined by

$$\frac{dU(t)}{dt} = \overline{\lambda}(t).$$

- For each $s \in [0, \infty)$, let $J(s) = K(U^{-1}(s))$ be the number of defaults by new time s.
- The doubly-stochastic property implies that J is a Poisson process with rate parameter 1.

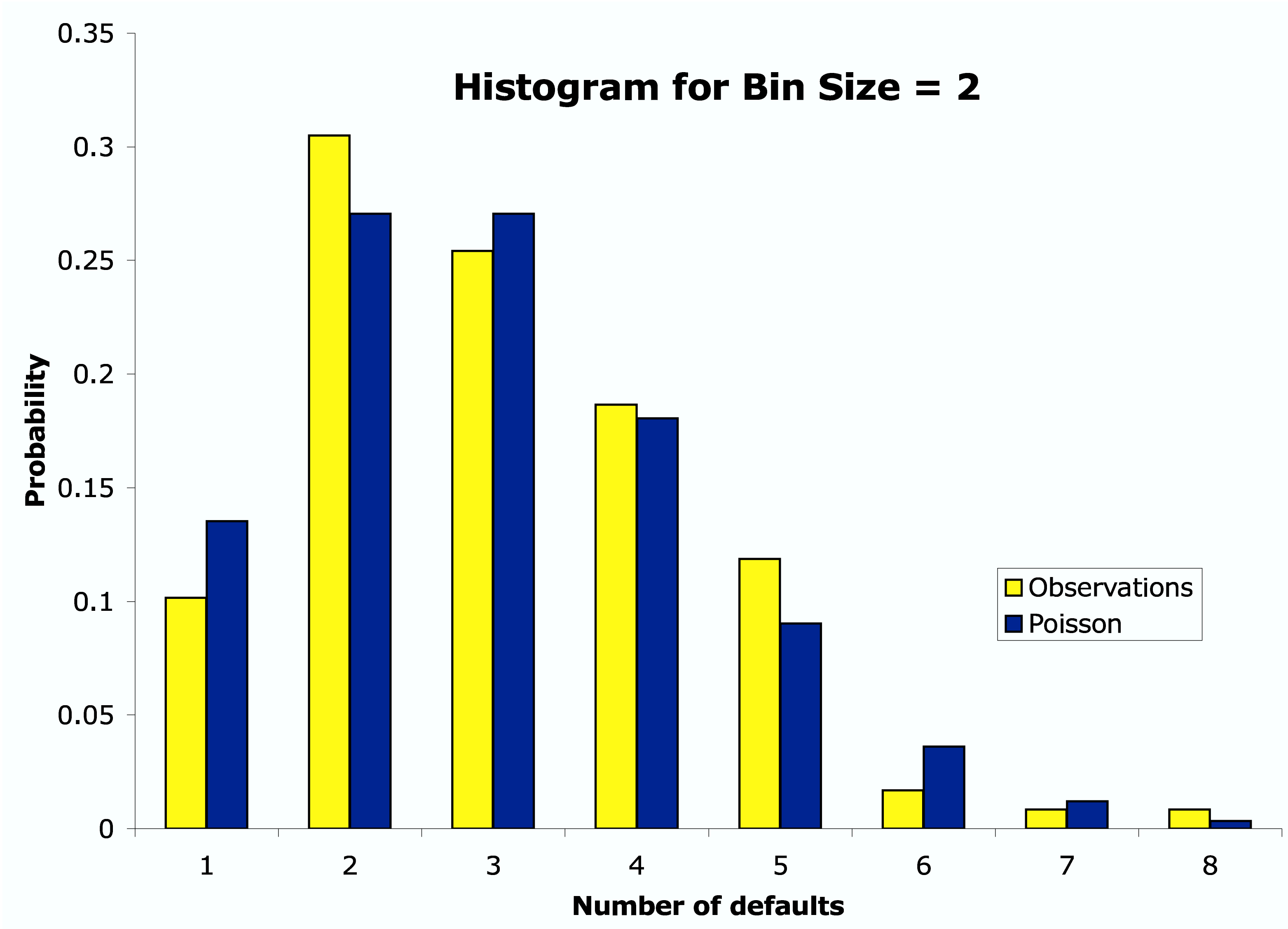


Figure 15: Empirical and Poisson distributions of defaults per size-2 bin. (Joint work with Sanjiv Das and Nikunj Kapadia).

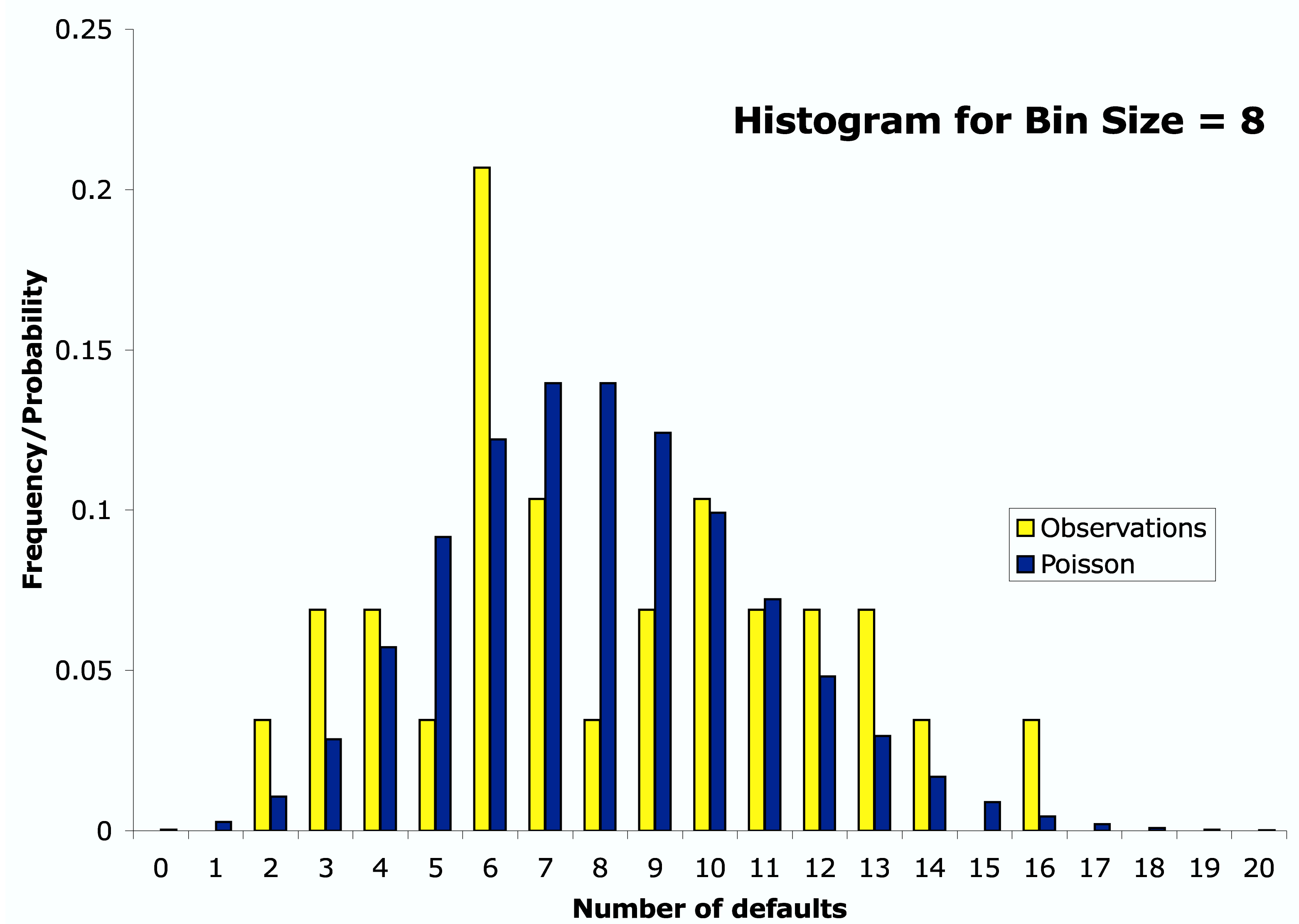


Figure 16: Empirical and Poisson distributions of defaults per size-8 bin. (Joint work with Sanjiv Das and Nikunj Kapadia).

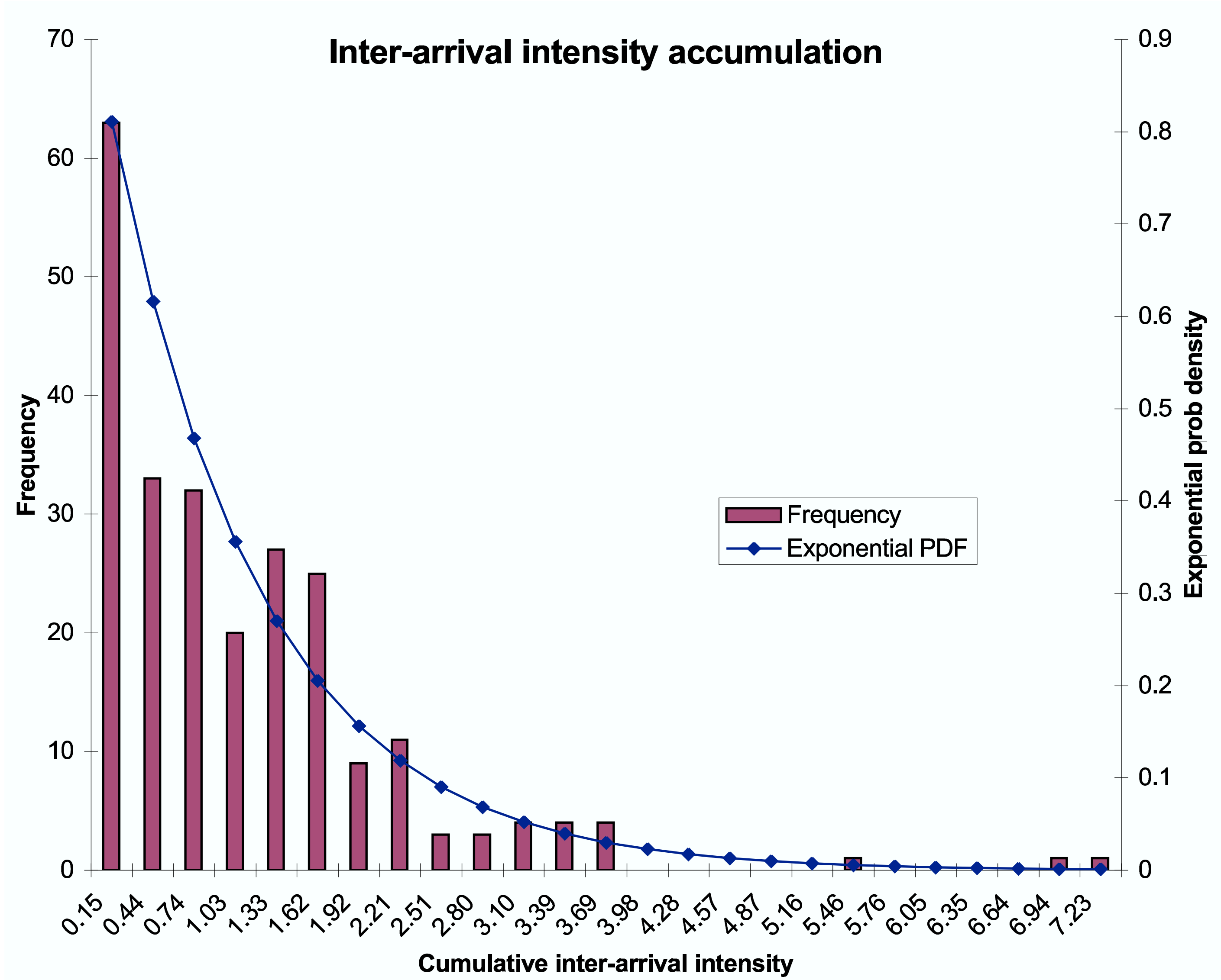


Figure 17: Empirical and exponential distributions of cumulative intensities between defaults (Joint work with Sanjiv Das and Nikunj Kapadia).

E. Collateralized Debt Obligations

- Useful for regulatory bank capital relief.
- Mitigate illiquidty.
- Current practice for pricing and rating is primitive.
- Current pricing framework does not integrate with models of credit-spread risk.

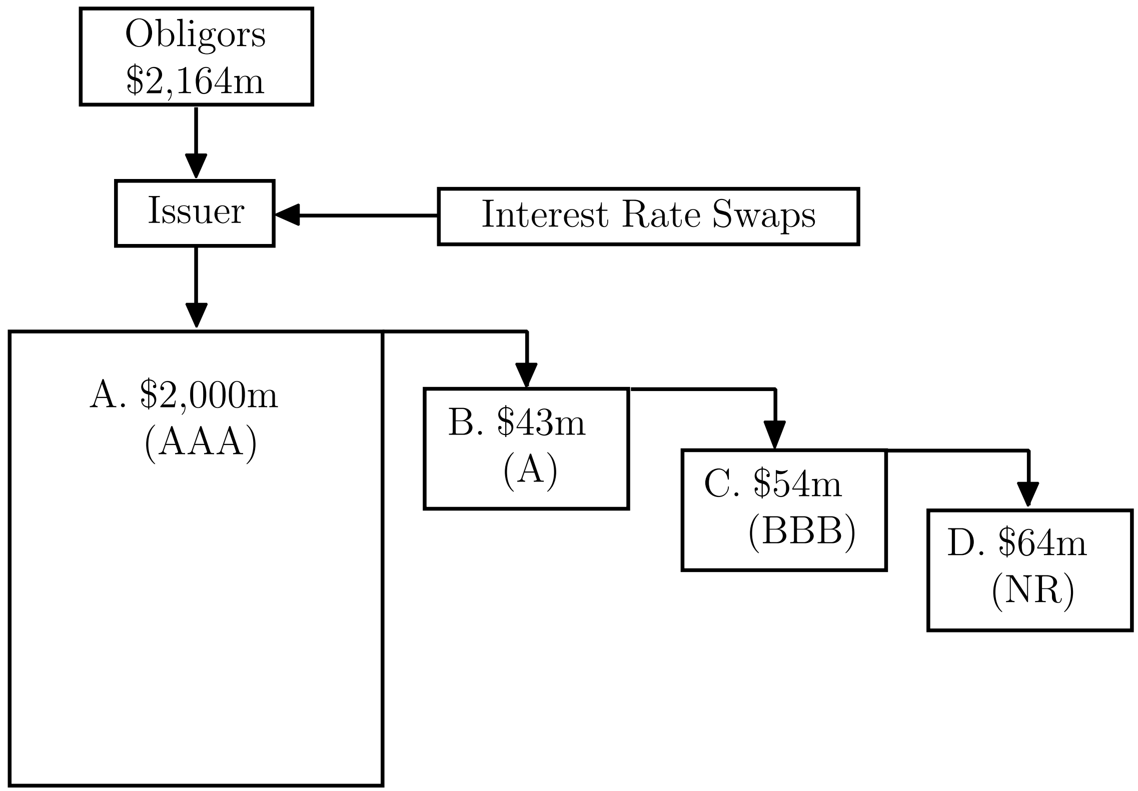
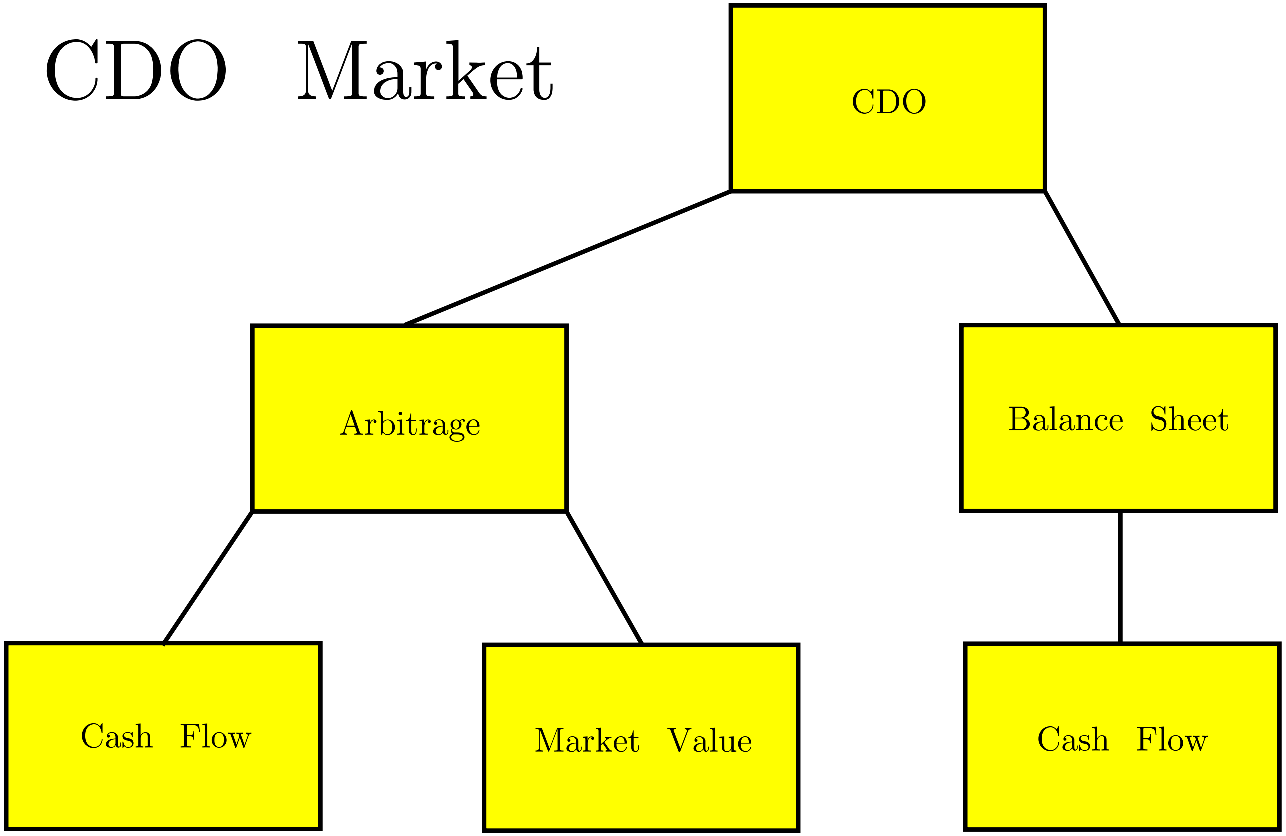


Figure 18: Nations Bank 1997 CLO Tranches

CDO Market



Source: Morgan Stanley

Table 2: CDO Spreads (basis points). Source: Citigroup, Feb., 2004.

Collateral	AAA (Sr)	AAA (Jr)	AA	A	BBB	BB
HY CBO	NA	50-55	120-130	185-200	350-400	750-850
IG CBO	NA	50-55	120-130	195-205	375-425	800-900
HY CLO	40	45-48	95-105	155-165	270-280	650-675
ABS CDO	45	43-48	115-125	170-180	315-330	650-700
TRUPS	55–58	55-58	120-125	165-170	280-300	550-650

Pricing Example

- Collateral: 100 ten-year straight coupon bonds.
- Default Modeling: jump-diffusion intensities with correlation.
- Recovery: independent and uniform $[0, 100]$.
- 3 tranches: senior bond, mezzanine bond, junior residual.
- Simple prioritization schemes: uniform and fast.

Correlated Multi-Issuer Intensities

- Risk-neutral default intensities $\lambda_1^*, \ldots, \lambda_{100}^*$.
- $\lambda_i^* = X_c + X_i$, common factor X_c, name-specific factor X_i.
- This allows us to hold the issuer-level default-time distribution fixed and vary correlation by adjusting the parameters of X_i and X_c.
- We vary jumpiness of intensities (spreads), holding spreads and vols constant.
- Base case has mean of 2 jumps per 10 years, mean jump in spread of 500 basis points.

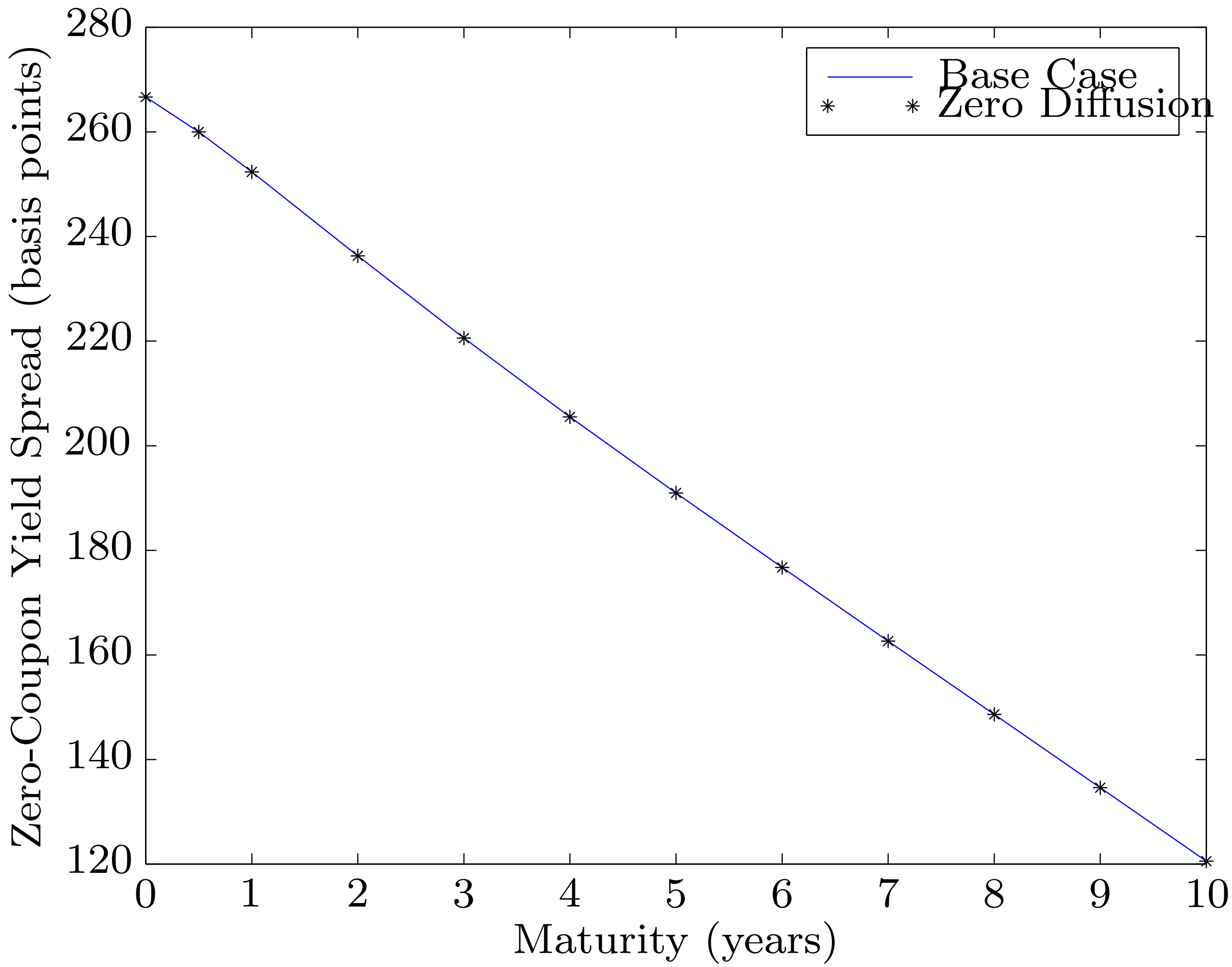
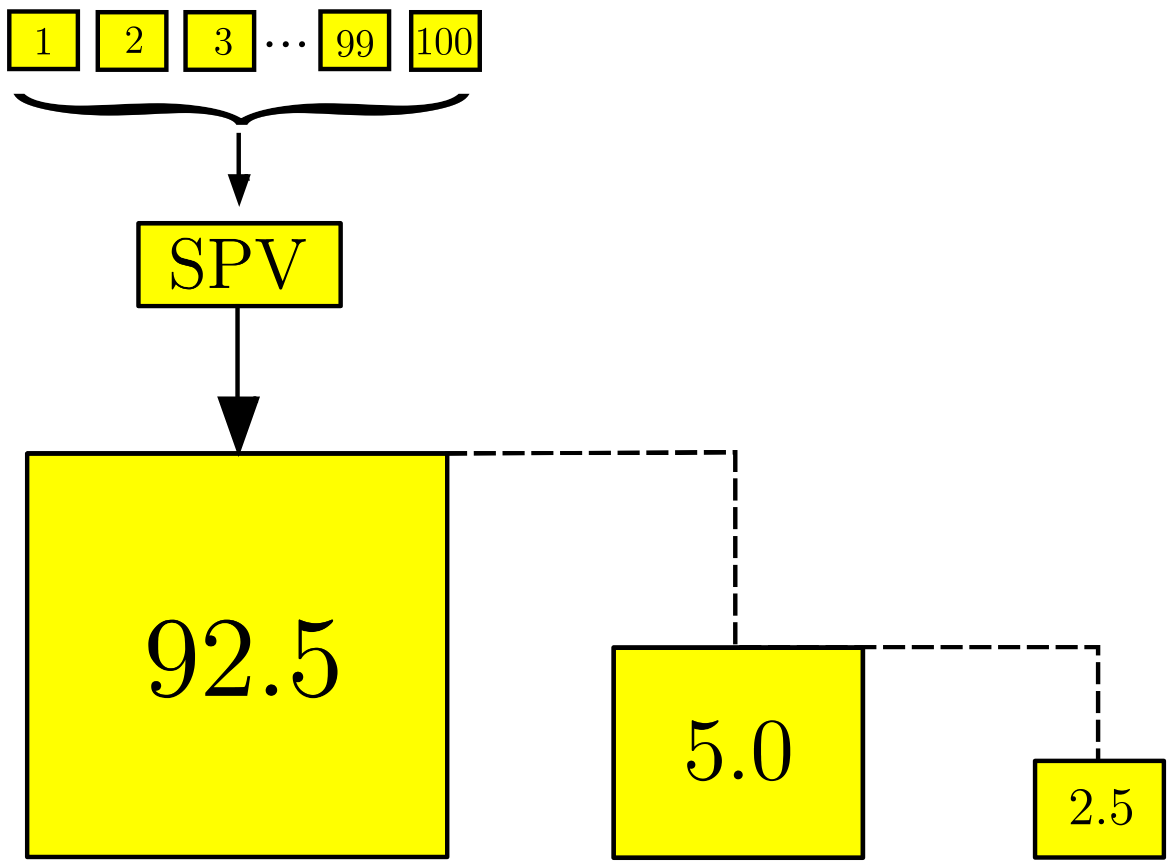
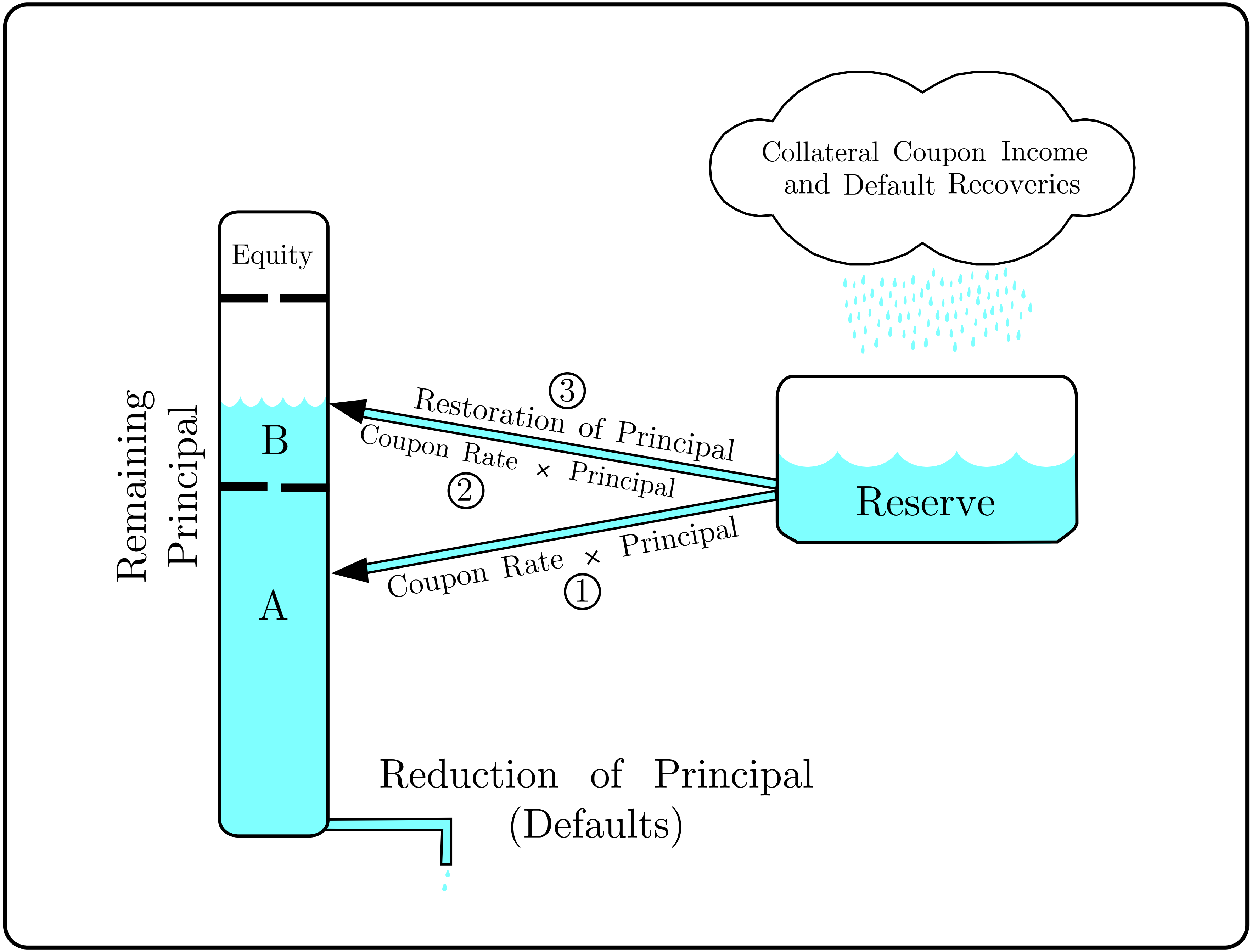


Figure 19: Zero-Coupon Yield Spreads, With and Without Diffusion

CBO Example





Darrell Duffie, Stanford University, June 2004

Coupon Rates and Tranches

- Risk-free rate 6%.
- Par coupon spread on collateral approximately 250 basis points.
- A Tranche: Face Value 92.5, par spread of 18 basis points.
- B Tranche: Face Value 5.0, par spread of 710 basis points.
- Equity residual: Base-case value of 2.5.

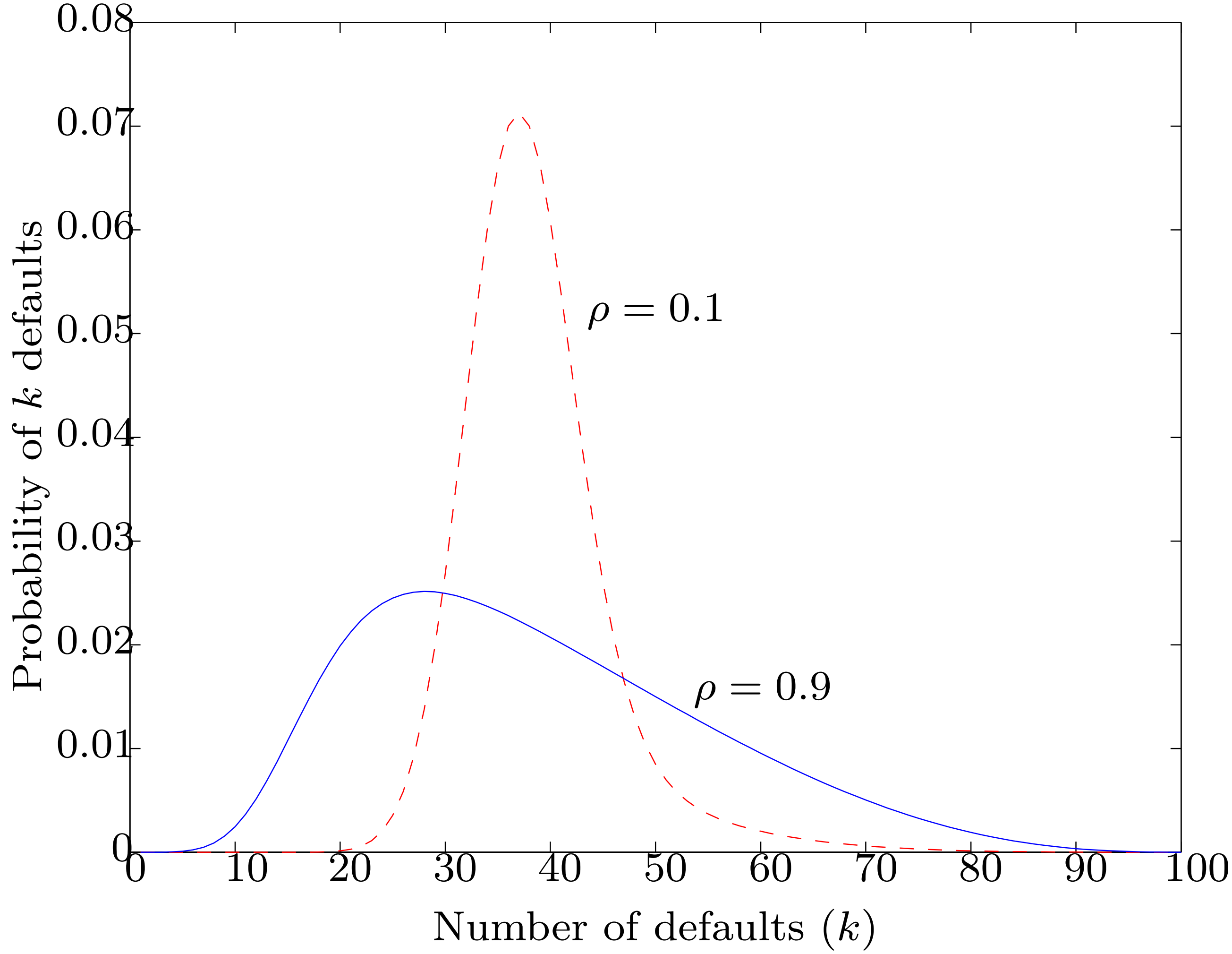


Figure 20: The probability of k defaults, for high and low correlation (base case).

Table 3: Conditional probabilities of default and diversity scores

		$\text{corr}(\lambda_i^*, \lambda_j^*) = 0.1$		$\text{corr}(\lambda_i^*, \lambda_j^*) = 0.5$		$\text{corr}(\lambda_i^*, \lambda_j^*) = 0.9$	
Set	p_i	$p_{i \mid j}$	divers.	$p_{i \mid j}$	divers.	$p_{i \mid j}$	divers.
1	0.386	0.393	58.5	0.420	21.8	0.449	13.2
2	0.386	0.393	59.1	0.420	22.2	0.447	13.5
3	0.386	0.392	63.3	0.414	25.2	0.437	15.8
4	0.386	0.393	56.7	0.423	20.5	0.454	12.4

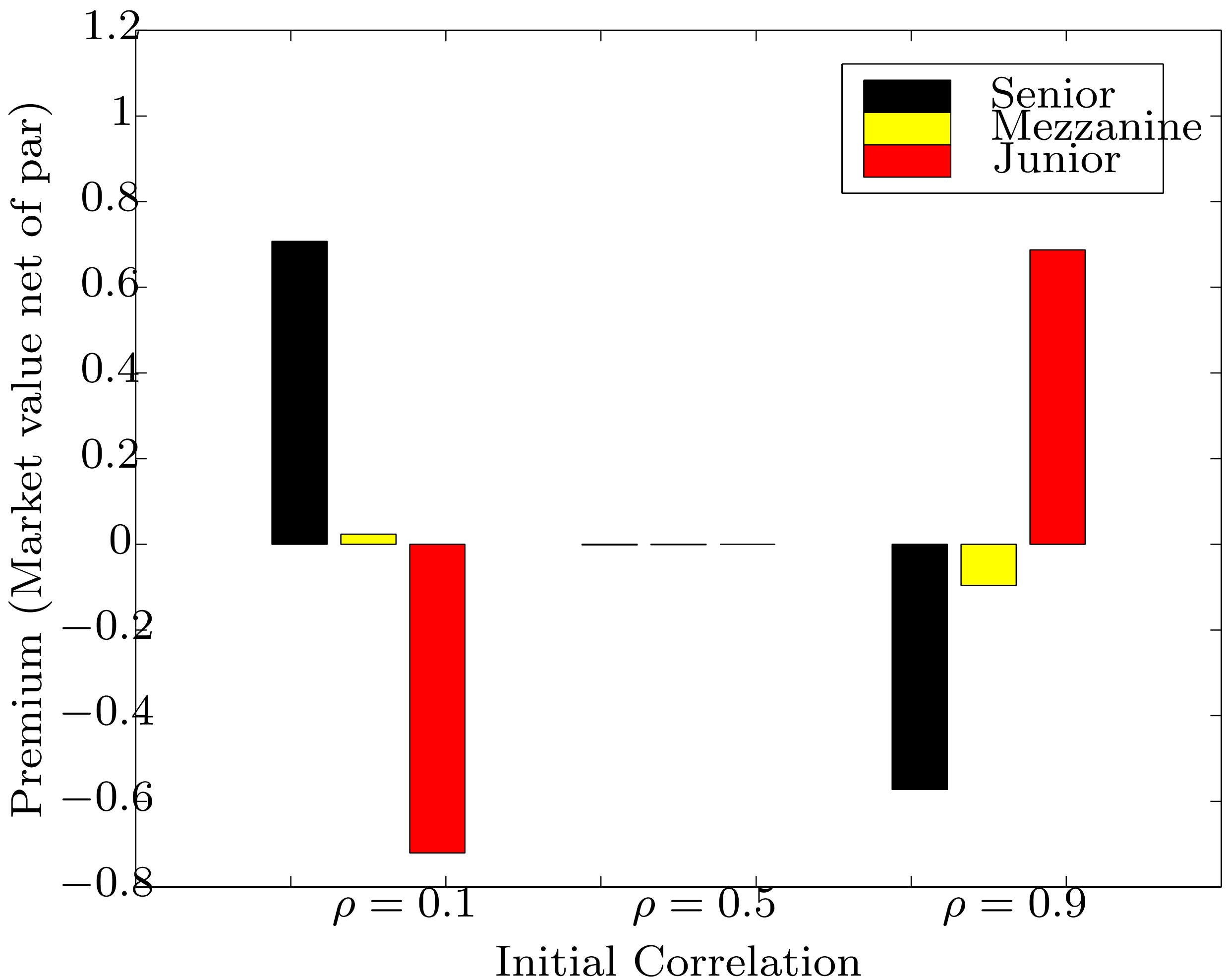


Figure 21: Impact on market values of correlation, uniform prioritization, Parameter Set.