Can Boundedly Rational Agents Make Optimal Decisions? A Natural Experiment.[1]

Jonathan B. Berk
University of California, Berkeley and NBER

and

Eric Hughson
University of Colorado, Boulder

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[1]The authors would like to thank George Chen for research assistance. A copy of this paper is available on the WWW home page http://haas.berkeley.edu/∼berk/. Our respective e-mail addresses are berk@haas.berkeley.edu and eric.hughson@colorado.edu.

Abstract

The television game show *The Price Is Right* is used as a laboratory to test consistency of suboptimal behavior in an environment with substantial economic incentives. On the show, contestants compete sequentially in two closely related games. We document that contestants who use transparently suboptimal strategies in the objectively easier game use the optimal strategy almost all of the time in the game that is much more difficult to solve. Further, there is no consistency in the mistakes that are made in the two games. One cannot predict, conditional on play in one game, whether play in the other game will be optimal. The results have implications for the consistency of behaviorally based economic theory that relies on evidence derived in a laboratory setting.

1 Introduction

Economists who advocate using fully rational economic models in the presence of overwhelming evidence that most human beings are boundedly rational often cite two reasons for their position. First, they point out that often only a few people need to act in a fully rational way to set prices. The actions of the other people "wash out," either because they randomly average to zero, or because of the presence of arbitrageurs taking advantage of suboptimal behavior. The second reason is more subtle. Rather than deny that boundedly rational agents have a role in setting prices, supporters of the rational paradigm argue that the fact that people are incapable of solving for an optimal rule does not imply that people do not use optimal rules.

Human beings often use optimal decision rules without solving for them explicitly. For example, a major league pitcher can throw a curve ball without understanding the physics involved. One might well argue, however, that cases in which such decision rules are used are usually characterized by situations in which the rule is learned through experience. Although many economic applications fit this description, others do not. In applications in which boundedly rational participants have limited experience, how likely it that the predictions of an economic model that assumes full rationality will be useful? The objective of this paper is to provide an answer to this question by undertaking a natural experiment on the television game show *The Price Is Right*.

The answer to this question has implications for economic theory. Although it is straightforward to find examples where a group of agents behaving suboptimally will affect the prediction of a fully rational model, actually predicting exactly *how* their actions affect the equilibrium can be quite difficult. Predicting suboptimal behavior depends on whether consistent and predictable patterns of suboptimal behavior exist and can be detected. Much evidence of this has been found in the laboratory. Researchers have designed experiments that demonstrate consistent suboptimal behavior in the sense that if the experiment is repeated with different subjects, similar results obtain. Yet this does not shed much light on the question of immediate relevance to economists — if these same subjects were instead faced with a real world decision requiring the kind of skills that they failed to demonstrate in the laboratory, would their behavior be consistent? The answer to this question has become increasingly important because recently, many researchers have begun developing non–rationally based economic models in which the behavioral assumptions are justified based on the insights from these laboratory experiments.[1]

To derive useful economic insights from the mistakes people make in a laboratory setting, researchers need to demonstrate that the behavior observed in the laboratory is representative of what is observed in the real world. Unfortunately, there are a number of important differences between a laboratory and the real world. First, it is very difficult to match the incentives present in any real economic decision in the laboratory. Second, the people themselves differ. University undergraduates, the typical sample used, might behave quite differently in a laboratory than a typical person faced with an economic decision. Finally, the actual decision faced in the laboratory is never exactly the same as the decision faced in a real world setting. Of these differences, the last one is clearly the most important. If the predictions from a laboratory setting fail to hold up in the real world because of either one of the first two differences, one could imagine changing the laboratory setting to address this. However, one cannot hope to test each and every economically relevant decision people face in the laboratory. So if one cannot take an observed behavior in one context and then use it to predict behavior in a range of similar contexts, the relevance of psychological evidence for economic behavior is questionable.

The Price Is Right provides a unique environment to evaluate the importance of the last difference. One reason is that the first concern is neutralized — the incentives contestants face on the show are large.[2] Another reason is that we are able to observe the behavior of the same contestants in related but distinct problems. Furthermore, although the contestants are self selected to be people who are willing to spend six hours of their time attending the show, they most likely represent a broader cross section of society than the university undergraduates that are usually used as subjects in experiments. Consequently, the show's design provides an opportunity to directly test the hypothesis that people's inability to compute and use optimal strategies in one environment implies that they will not use optimal strategies in a closely related environment.

We utilize the first game of the show to identify individuals who use clearly suboptimal decision rules. We then test to see whether on a related, but not identical, game of the same show, these contestants continue to use suboptimal decision rules. We find that there is no relation between contestants' behavior in the two games. In

[1]See, for example, Daniel, Hirshleifer and Subrahmanyam (1998), Gervais and Odean (2001), Barberis, Huang and Santos (2000).

[2]In the games we study the expected value of winning is of the order of \$10,000.

the earlier game, about half the contestants use transparently suboptimal strategies, while in the later game, almost every contestant uses the optimal rule. Furthermore, the few contestants who do depart from the optimal strategy in the later game are no more or less likely to have used a suboptimal strategy in the earlier game. In stark contrast to the earlier game, there are so few mistakes on the later game that every implication of the fully rational model we test is confirmed by the data. What makes this result particularly surprising is that the *second* game is significantly more complex (and difficult to solve) than the earlier game. Contestants who do not use the optimal decision rule when it is transparent (and when significant losses ensue as a result), are somehow able to use an optimal decision rule that is much more complicated to derive and where, paradoxically, the loss for not using the rule is smaller.

The paper is organized as follows. In section 2, we describe the game show environment and detail the two games that contestants play: "Contestants' Row," and "The Wheel Spin." In section 3, we derive theoretical results. Empirical tests are performed in section 4. Section 5 discusses the results, and section 6 concludes the paper.

2 Description of the Game Show

We concentrate on two games played between contestants during the show. The first game is the initial round of the show called "Contestants' Row," in which four contestants sequentially guess the price of a product displayed on stage. The winner is the contestant that bids closest *without going over*. The prize for winning is the item up for bids as well as the opportunity to play additional games on the show. We concentrate on the fourth bidder's bidding behavior. If this contestant uses an optimal bidding strategy, then he or she will pick one of only four bids — $1 or $1 above any one of the three previous bids. Any other bid is suboptimal because this contestant can increase her chance of winning by lowering her bid until it becomes one of these four bids.

This round of the show has been studied by economists before. Both Bennett and Hickman (1993) and Berk, Hughson and Vandezande (1996) document that about half of the contestants who bid fourth do not use the optimal decision rule. Furthermore, not using the rule had real costs. Berk, Hughson and Vandezande (1996) show that had these contestants used the optimal rule, their likelihood of winning would have risen almost 50%, from 30.6% to 43.2%. In expected value terms this difference is on the order of $1000.

The second game we study is the "Wheel Spin." On *The Price Is Right*, the "Wheel Spin" is played after "Contestants' Row" has been played three times and three different winners have been determined. These three winners spin the wheel, and a winner is determined. Then, Contestants' Row is played three more times and the next three winners play the Wheel Spin game. The two winners of the Wheel Spin become the two contestants who compete in the final and largest payoff game of the show, the "Showcase Showdown."

On the Wheel Spin, the three contestants compete sequentially. The order is determined by the value of the prizes that each spinner has won up to that point in the show. The contestant who has won the least spins first, the one who has won the second–most spins next, and the one who has won the most spins last. The wheel has 20 numbers on it — the numbers from 5 to 100 in intervals of five. Each contestant may spin the wheel *up to* two times. After a contestant spins the wheel once, he or she can either spin again or stop. The contestant's score is the sum of both spins if she spins again, or just the first spin if she stops. Once one contestant's turn is finished, the next contestant spins. The winner is the contestant who score comes closest to 100 *without going over*. If two contestants tie, then each spins once more, and the contestant with the highest spin wins. If all three contestants tie, then all three spin once, and the highest spin wins. The expected value of participating in the Showcase Showdown (winning the wheel spin) is approximately $10,000, which is similar to the expected value of winning Contestants' Row, the initial round of the show.

The strategic choice contestants face on the Wheel Spin is whether or not to spin again. The advantage of spinning again is that the contestant can potentially increase his chance of winning by bringing his score closer to 100. The disadvantage is that he could go over 100 and eliminate himself from the game.

There are a number of additional prizes that can be awarded on the wheel spin. If any player gets exactly 100 either during their regular play or during a tie breaking spin, they get an additional $1000 and the chance to spin again. If on this second spin the player again gets 100 then he gets an additional $\$10,000$, or if he gets 5 or 15 he gets an additional $\$5,000$. For most of the paper we will ignore these extra prizes and consider the chance to compete in the "Showcase Showdown" as the only payoff of the wheel spin. The effect of including these prizes will be considered in Section 5.

Tenorio *et al* (1997) also study the Wheel Spin. The focus of that paper is on explaining contestant departures from optimality using Quantal Response Equilibrium. Our focus is quite different. It is to determine whether these departures are consistent with contestants' performance in an earlier game on the same show — Contestants' Row. Their study does however document the surprising ability of the rational model to predict actual winning probabilities on the Wheel Spin. Although their paper concentrates on the departures from optimality, the fact that these departures are far less frequent than what earlier studies observed on Contestants' Row suggests that contestants who previously used a transparently suboptimal strategy were now using an optimal one. It is not conclusive however, because a prerequisite to participating in the Wheel Spin is winning Contestants' Row. Since contestants who use optimal bidding strategies on Contestants' Row are much more likely to win, the sample of contestants spinning the wheel is biased in favor of optimality. One would therefore expect to see better average performance on the Wheel Spin than on Contestants' Row.

3 Theory

An important facet of the show is that the optimal strategy on Contestants' Row is much easier to derive than the optimal strategy on the Wheel Spin. This is a critical criterion for our study, since our main result is that contestants who suffer significant costs by not using the easily derivable optimal strategy on Contestants' Row, use a much more difficult to derive optimal strategy on the Wheel Spin. This brings up the issue of quantifying the level of complexity of the two strategies. We will use a crude measure — the amount of space required to formally derive the strategy. The optimal strategy for the fourth contestant on Contestants' Row is derived in four lines in Berk, Hughson and Vandezande (1996). The optimal strategy for the first and second contestant in the Wheel Spin is derived in this section and the accompanying appendix.[3] The second contestant's strategy takes six pages to derive, and the first contestant's strategy requires an additional three pages. Although this measure is admittedly crude, given the huge disparity in the length of these two derivations, we

[3]Tenorio *et al* (1997) state the optimal strategy, but do not derive it. They rely on numerical methods to obtain the stopping rule and the predicted winning probabilities. Our derivation proves that the optimal stopping rule they claim is not correct and so the winning probabilities we derive are not the same as the ones claimed in Tenorio *et al* (1997).

believe that it is accurate enough to establish that the optimal strategy of either of the first two contestants on the Wheel Spin is significantly more complex than the fourth contestant's strategy on Contestants' Row.

We refer to the result of a contestant's completed turn (that is, the single spin or the sum of the two spins if he chooses to spin again) as the contestants *score*. The i^{th} contestant's score is denoted b_i. If a contestant spins twice and the resulting score exceeds 100, then we will denote this by setting $b_i = 0$ for that contestant.

We name the contestants by the order of their spin, so, for example, the first contestant is the contestant who spins first. The only strategic choice contestants make in this game is whether or not to spin a second time. Contestants who spin later have an informational advantage because they observe earlier contestants' scores when deciding whether to spin again. Our objective is to determine each contestant's optimal strategy. Let $s_i(\cdot)$ be the i^{th} contestant's optimal stopping rule — if contestant i's first spin is *equal to* or greater than $s_i(\cdot)$, the first contestant will not spin again, otherwise he will. The following proposition derives the third contestant's optimal rule and probability of winning conditional on the earlier contestant's scores.

Proposition 1 *Let the result of the first and second contestants scores be b_1 and b_2 respectively with $b \equiv \max(b_1, b_2)$. We will restrict attention to the case when $b > 0$, otherwise the third contestant spins once and wins by default. If $b_1 \neq b_2$, the third contestant's optimal stopping rule is*

$$s_3(b) = \begin{cases} b+5 & b \leq 50 \\ b & b > 50 \end{cases} \tag{1}$$

and his conditional probability of winning is given by

$$P[\textit{Constant 3 wins}|b \textit{ and } b_1 \neq b_2] = \begin{cases} 1 - \left(\frac{b}{100}\right)^2 + \frac{1}{40}\left(\frac{b-5}{100}\right) & b \leq 50 \\ \left(\frac{41}{40} - \frac{b}{100}\right)\left(\frac{19}{20} + \frac{b}{100}\right) & b > 50. \end{cases} \tag{2}$$

If $b_1 = b_2$,the third contestant's optimal stopping rule is

$$s_3(b) = \begin{cases} b+5 & b < 70 \\ b & b \geq 70 \end{cases} \tag{3}$$

and his conditional probability of winning is given by

$$P[\textit{Constant 3 wins}|b \textit{ and } b_1 = b_2] = \begin{cases} 1 - \left(\frac{b}{100}\right)^2 + \frac{1}{60}\left(\frac{b-5}{100}\right) & b < 70 \\ \left(\frac{61}{60} - \frac{b}{100}\right)\left(\frac{19}{20} + \frac{b_2}{100}\right) & b \geq 70. \end{cases} \tag{4}$$

The second contestant's strategy is more complicated. The following proposition derives her optimal strategy as well as her probability of winning given the first contestant's score.

Proposition 2 *Let the first contestant's score be b_1. Then the second contestant's optimal stopping rule is*

$$s_2(b_1) = \begin{cases} 55 & 0 \le b_1 \le 50 \\ b_1 + 5 & 50 < b_1 < 70 \\ b_1 & 70 \le b_1 \le 100 \end{cases}$$

and her conditional probability of winning is given by

$$P[Constant\ 2\ wins|b_1]$$
$$= \begin{cases} \frac{29289}{64000} & b_1 = 0 \\ \frac{549167500+875\,b_1-200\,{b_1}^2+30\,{b_1}^3-3\,{b_1}^4}{1200000000} & b_1 \le 50 \\ \frac{414748750+3850500\,b_1-1675\,{b_1}^2-355\,{b_1}^3-4\,{b_1}^4}{1200000000} & 50 < b_1 < 70 \\ \frac{394986250+3856125\,b_1+1100\,{b_1}^2-335\,{b_1}^3-4\,{b_1}^4}{1200000000} & 70 \le b_1 \le 100 \end{cases} \quad (5)$$

To calculate the second contestant's unconditional probability of winning, we must derive the distribution of the first contestant's score. The next lemma does this as a function of the first contestant's optimal stopping rule.

Lemma 1 *Assume that contestant 1's optimal stopping rule is to stop when his first spin is s_1 or greater. Then the probability that contestant 1's final score is b_1 is given by*

$$P[b_1] = \begin{cases} \frac{s_1(s_1-5)}{20000} & b_1 = 0 \\ \frac{b_1-5}{2000} & 0 < b_1 < s_1 \\ \frac{95+s_1}{2000} & s_1 \le b_1 \le 100 \end{cases} \quad (6)$$

Proof: If $b_1 < s_1$, contestant 1 must have spun twice, so the probability of getting b_1 is given by

$$\frac{b_1 - 5}{5}\frac{1}{20}\frac{1}{20} = \frac{b_1 - 5}{20000}.$$

If $b_1 \ge s_1$ then contestant 1 might have stopped on his first spin. So in this case the probability of getting b_1 is given by

$$\frac{1}{20} + \frac{s_1 - 5}{5}\frac{1}{20}\frac{1}{20} = \frac{s_1 + 95}{2000}.$$

Finally, the probability that the first contestant will go over is

$$1 - \sum_{i=2}^{\frac{s_1-5}{5}} \frac{5i-5}{2000} - \sum_{i=\frac{s_1}{5}}^{20} \frac{s_1+95}{2000} = \frac{s_1(s_1-5)}{2000}.$$

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The last proposition derives the first contestant's optimal stopping rule as well as her conditional probability of winning given her score.

Proposition 3 *The first contestant's optimal stopping rule is $s_1 = 70$. Furthermore, the probability of the first contestant winning given a score of b_1 is*

$$P[Contestant\ 1\ wins|b_1] = \begin{cases} 0 & b_1 = 0 \\ \frac{208750-42625\,b_1+16900\,{b_1}^2-75\,{b_1}^3+6\,{b_1}^4}{1200000000} & 5 < b_1 \le 50 \\ \frac{19375-4375\,b_1+925\,{b_1}^2-30\,{b_1}^3+3\,{b_1}^4}{300000000} & 50 < b_1 < 70 \\ \frac{263125-10625\,b_1+1750\,{b_1}^2-45\,{b_1}^3+3\,{b_1}^4}{300000000} & 70 \le b_1 \le 100 \end{cases} \tag{7}$$

Table 1 enumerates each contestant's probability of winning conditional on the first contestant's score. With these results is straightforward to calculate each contestant's unconditional probability of winning. The following corollary does this.

Corollary 1 *The probabilities that the first, second and third contestant will win are 30.815%, 32.96% and 36.225% respectively.*

Proof: The unconditional probability of the first contestant winning is

$$P[\text{Contestant 1 wins}] = \sum_{b_1} P[\text{Contestant 1 wins}|b_1]P[b_1|s_1 = 70].$$

Evaluating this expression using Lemma 1 and Proposition 3 gives

$$P[\text{Contestant 1 wins}] = 30.815\%.$$

Similarly,

$$\begin{aligned} P[\text{Contestant 2 wins}] &= \sum_{b_1} P[\text{Contestant 2 wins}|b_1]P[b_1|s_1 = 70] \\ &= 32.96\%. \end{aligned}$$

This implies that,

$$P[\text{Contestant 3 wins}] = 1 - P[\text{Contestant 2 wins}] - P[\text{Contestant 1 wins}] = 36.225\%.$$

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Table 1: **Contestants' Probability of Winning Conditional on the First Contestant's Score:** The first column shows the first contestant's score (b_1). The next three columns show, respectively, the probability that the first second and third contestant will win conditional on the score in the first column. The first row of the table is the case when the first contestant's score is over 100.

	Probability of Winning (%)		
b_1	1	2	3
Over	0	45.76	54.24
10	0.1215	45.76	54.12
15	0.2852	45.76	53.96
20	0.5397	45.74	53.72
25	0.9065	45.70	53.40
30	1.415	45.62	52.97
35	2.101	45.48	52.42
40	3.009	45.26	51.73
45	4.190	44.94	50.87
50	5.704	44.48	49.82
55	8.346	43.82	47.84
60	11.83	42.60	45.57
65	16.32	40.76	42.93
70	21.56	38.28	40.16
75	28.42	35.21	36.38
80	36.82	31.26	31.92
85	46.99	26.35	26.66
90	59.17	20.36	20.47
95	73.61	13.19	13.21
100	90.57	4.717	4.717

4 Empirical Results

We used the same sample of shows used in Berk, Hughson and Vandezande (1996). The sample consists of 112 broadcasts which were recorded on videotape and then manually transcribed. A few shows were interrupted by news stories, which left a total of 767 auctions on Contestants' Row. In 99 of these auctions all four contestants overbid, so there were 668 auctions that provided winners. Of the 668 winning bidders, we have data for 554 contestants who then also spun the wheel, 186 who spun the wheel first, and 184 each who spun the wheel second and third. The "missing" contestants (those that won Contestants' Row but did not spin the wheel) result from incomplete shows that were interrupted by breaking news stories.[4]

We begin by calculating the winning percentages on the Wheel Spin and comparing them to the theoretically predicted values — Table 2. From the table it is clear that the actual winning percentages are not significantly different to the predicted values. A similar result obtains when the winning frequency of each number is calculated (see Table 3). The predictions of the fully rational model cannot be rejected by the data. These results stand in sharp contrast to what was observed on Contestants' Row. There, Berk, Hughson and Vandezande (1996) show that a number of the theoretical predictions of the fully rational model are rejected by the data.

Table 2: **Winning Percentages by Contestant on the Wheel Spin:** The columns show the winning percentages by contestant. In this table we combined our sample with the numbers reported by Tenorio *et al* (1997). This increases the total number of observations to 466. The last row is the p-value, the probability (in percent) under the Null (that all contestants use the optimal strategy) of observing a deviation from the theoretical value greater than what is observed.

	Winning Probability (%)		
	1	2	3
Actual	29.83	32.40	37.77
Predicted	30.82	32.96	36.22
p-value	68.32	84.07	45.73

[4]Part of the time period in which these shows were recored spanned the O.J. Simpson trial.

Table 3: **Frequency of Each Winning Number:** The first column shows the winning contestant's score. The next column shows the number of occurrences at each score. There are 184 total occurrences. The next two columns show, respectively, the actual occurrence frequency and the theoretical predicted occurrence frequency. The last column is the p-value, the probability (in percent) under the Null (that all contestants use the optimal strategy) of observing a deviation from the theoretical value greater than what is observed.

Winning Score	Number Actual	Frequency (%) Actual	Frequency (%) Predicted	p-value (%)
≤ 55	9	4.89	4.83	79.47
60	8	4.34	2.19	4.19
65	3	1.63	2.93	42.02
70	7	3.80	5.13	53.65
75	15	8.15	6.98	43.17
80	14	7.61	9.22	54.41
85	25	13.59	11.89	40.39
90	26	14.13	15.05	82.37
95	38	20.65	18.75	44.49
100	39	21.20	23.03	62.31

The fact that the winning percentages are not statistically different from their theoretical values suggest that contestants might be using optimal stopping rules. In the next subsection we will verify this fact.

4.1 Optimal behavior in the Wheel Spin

Table 4 summarizes the spinning behavior of *Price is Right* contestants. Almost all players follow the optimal stopping rule. In the sample of 554 observations, there are only 21 errors. Not surprisingly, because the third spinner has the easiest decision, he almost never errs.[5] The second spinner made four mistakes, on a much harder problem.

Table 4: **Optimal Behavior by Position on the Wheel Spin:** The columns show the total number of observations (Total), the number of contestants who use the optimal rule (Optimal), the number that use a suboptimal rule (Suboptimal) and the percentage of the total who use the optimal rule (Pct. Optimal).

	Total	Optimal	Suboptimal	Pct. Optimal
All spinners	186	170	16	91.4
First spinner	184	180	4	97.8
Second Spinner	184	183	1	99.5

The first spinner, who has the hardest problem, still made only 16 mistakes. Given the first spin, the first spinner's optimal strategy was, empirically, to stop 65 times and spin 121. In fact, the first spinner never spun when she should have stopped, but did stop 16 out of the 121 times she should have spun, or 13.2%. All the errors consisted of stopping either on 60 (7 times of 12 total spins of 60) or 65 (9 times of 10 total spins of 65). Thus the only systematic mistake we can identify is that a subset of contestants use 60 or 65 instead of 70 as the optimal stopping rule. However, the cost of adopting either of these suboptimal rules is trivial. By choosing a policy of stopping on 65 (60) instead of 70, the first contestant's unconditional probability

[5]The single error was a case where the spinner stopped when tied with the first spinner at 45. In that case, spinning yields a probability of winning of 55%, whereas stopping yields a winning percentage of 50%.

of winning drops from 30.82% to 30.74% (30.40%) so the unconditional cost is only 0.08% (0.42%).

Table 5 tabulates the different errors made and the occurrence frequency of error. It also provides a measure of the *conditional* cost of the error. That is, the table shows how much the winning probability would have increased, conditional on the result of the first spin, had the error not been made and the optimal rule followed instead.[6] The cost of the errors made here are substantially smaller than the cost incurred on Contestants' Row.[7] The only exception is one mistake a second contestant made.

Table 5: **Analysis of Errors on the Wheel Spin:** This table shows the frequency of all errors made on the Wheel Spin. The first column describes the type of error made, the second column show the frequency with which this error occurred in the sample of 554 observations. The final column shows the cost of the error. That is, the column shows by how much a contestant's probability of winning (in percent) would have increased had he used the optimal stopping rule instead.

Description of the Error	Frequ.	Cost (%)
Contestant 1 stopped on 60	7	6.84
Contestant 1 stopped on 65	9	2.35
Contestant 2 did not stop on 65 ($b_1 = 45$)	1	15.68
Contestant 2 stopped on 65 ($b_1 = 65$)	1	3.66
Contestant 2 stopped on 60 ($b_1 = 60$)	1	8.62
Contestant 2 stopped on 50 ($b_1 = 0$)	1	5.59
Contestant 3 stopped on 45 ($\max(b_1, b_2) = 45$)	1	5.00

The fact that most contestants use the optimal rule on the Wheel Spin is suggestive that contestants who use the suboptimal rule on Contestants' Row, use the optimal rule in the Wheel Spin. It is not conclusive, however, because contestants on Contestants' Row who use the optimal bidding strategy are more likely to win. Since

[6] Although adopting a *policy* of stopping on 65 (60) instead of 70 makes only a trivial difference to the unconditional probability of the first contestant winning, the cost of stopping in the subgame when the first spin is 65 (60) is non-trivial.

[7] Berk, Hughson and Vandezande (1996) report that the average winning probability of the fourth bidder would have increased by 13.36% had she used the optimal strategy instead of a suboptimal strategy.

winning on Contestants' Row is a prerequisite to competing in the Wheel Spin, there is a selection bias in the sample of contestants on the Wheel Spin in favor of optimal bidders. In the next subsection we will show, however, that this selection bias is not responsible for our results.

4.2 Optimal bidding on Contestants' Row

We define as optimal any bidding strategy that maximizes the probability of winning Contestants' Row. As in Berk, Hughson and Vandezande (1996), we define as optimal any bid made by the fourth bidder that is either a dollar above one of the previous three bids, or is less than \$100.[8]

We are concerned with optimal bidding behavior over the course of the entire show, which for some contestants lasts as long as six bidding rounds on Contestants' Row. Thus the possibility exists that the same player might use both an optimal bidding strategy on one round and a suboptimal one on another round on Contestants' Row. To classify these contestants, we define a contestant as an optimal bidder if she either (1) always bid optimally when given a chance to do so as the fourth bidder or (2) learned to bid optimally after having first bid suboptimally earlier in the show and then optimally.[9] We say that a player bids suboptimally who either (1) always bid suboptimally when given a chance to do so or (2) bid suboptimally after they have bid optimally earlier in the show. Last, there are some players, those who never have the chance to bid as the fourth bidder, about whom we have no information.[10]

Table 6 summarizes the bidding behavior of *Price is Right* contestants. Observe that, as predicted, there is a sample selection bias — bidders who win Contestants' Row are more likely to have bid optimally (56.8% of winners bid optimally vs. 53% of all bidders and 43% of non winners). Of the 767 bidding rounds where we know the winner, we have information about the fourth bidder's strategy in 764. Of those, the fourth bidder bid optimally in 389, or 50.9%[11] An optimal fourth bid won 168

[8]In Berk, Hughson and Vandezande (1996), we observe that changing the cutoff rule to, for example, \$5 more than one of the other three bids does not materially affect the results.

[9]Berk, Hughson and Vandezande (1996) show that the probability that a contestant will bid optimally increases substantially once another contestant on the same show bids optimally.

[10]Different definitions of optimality do not appear to materially affect the results. For example, the results did not change when bidders who bid first optimally and later suboptimally were classified as "no information."

[11]The percentage is lower than the 53% calculated for all bidders because suboptimal bidders are more likely to lose and hence bid again.

Table 6: **Optimal Behavior:** This table shows the frequency of optimal *bidding* on Contestants' Row and optimal *spinning* on the Wheel Spin. The columns are the total number of observations and the strategy used on Contestant Row by the fourth bidder: Optimal, Suboptimal, and No Information. The last column shows the number of bidders who used the optimal bidding strategy as a fraction of bidders who we identified as either optimal or suboptimal. The rows show the results for different partitions of the sample on both games. The number of spinners does not equal the number of winners of Contestants' Row because of the existence of missing observations caused by interrupted shows.

		Strategy Used on Contestants' Row			
	Total	Optimal	Subopt.	No Info.	Opt. %
Contestants' Row:					
All bidders	1003	258	229	516	53.0
Winners	668	197	150	321	56.8
Non winners	335	61	79	295	43.6
Wheel Spin:					
All Spinners	554	160	117	277	57.8
Spinners who spin correctly	533	153	114	266	57.3
Spinners who err	21	7	3	11	70.0
1^{st} or 2^{nd} spinners	370	107	78	185	57.8
Correct 1^{st} or 2^{nd} spinners	350	100	75	175	57.1
Erring 1^{st} or 2^{nd} spinners	20	7	3	10	70.0

of 389 times, or 43.2%; a suboptimal fourth bid won only 99 of 375 times, or only 26.4%.

Table 6 can answer two related questions. First, are optimal bidders on Contestants' Row more likely to spin correctly? Second, are optimal spinners on the Wheel Spin more likely to have bid correctly on Contestants' Row? One might argue that the third spinner's problem is transparent, so the table also contains results on the behavior of just the first and second spinners.

The table confirms that the selection bias is not driving our results. The reason that most contestants on the Wheel Spin use the optimal stopping rule is that contestants who previously used a suboptimal strategy on Contestants' Row, use the optimal stopping rule on the Wheel Spin. Of the 117 contestants who bid suboptimally on Contestants' Row, 114 of them used the optimal stopping rule on the Wheel Spin.

When we concentrate on the first and second spinners only, we find that optimal bidders spin correctly 100 of 107 times, or 93.5 %. But suboptimal bidders did even better, spinning optimally 75 of 78 times, or 96.2% of the time. Optimal spinners bid correctly 57.1% of the time, insignificantly different from the 57.8% likelihood that all spinners bid optimally. Finally, suboptimal spinners bid optimally 7 of 10 times, or 70% of the time (although there were only ten observations). In sum, we find *no* evidence that optimal bidders spin better than suboptimal bidders. The reason is, of course, that bidders spin correctly almost all of the time.[12]

5 Discussion

We have thus far ignored the effect of bonus payments on the wheel, the extra cash prizes awarded when a contestant hits 100 exactly. The expected value of getting a score of 100 is the $1000 paid on that spin as well as the additional possible winnings from the bonus spin. (Recall that when a contestant gets 100 exactly she is allowed to spin again.) If the wheel lands on 100 again she wins an additional $10,000, while if it lands on 15 or 5 she wins an additional $5,000. The only way these bonus payment can influence the analysis in Section 3 is if they change a contestant's optimal stopping rule.

[12]This is true even when one considers the first and second spinners only. In that case, spinning is optimal 94.6% of the time.

Ceteris paribus, the presence of these payments increases the incentive to spin again. Given a first spin, the expected payoff from possible bonus payments of spinning again is

$$\frac{1}{20}\left(\$1,000 + \frac{1}{20}\$10,000 + \frac{2}{20}\$5,000\right) = \$100$$

If the expected value of the Showcase Showdown is \$10,000, then this extra payment adds 1% to the expected payoff the Wheel Spin. If contestants only derive utility from the monetary rewards, they would be prepared to lower their probability of winning by up to 1% to spin again for the opportunity of getting a bonus prize. This number is dwarfed by the cost of spinning again when a first spin gives a result that the propositions identify as being optimal to stop on.

There is also a countervailing effect. By choosing to spin again the contestant not only reduces her expected cash reward derived from the chance to appear on the Showcase Showdown, but also her non–pecuniary reward from appearing on the Showcase Showdown itself. The Showcase Showdown is the final game of the show and the entire show builds up to this game. The winner is the "star" of the show for the day and her family members are allowed to join her on stage at the close of the show. Although the utility of this is hard to quantify, it is certainly clear that some fraction of the utility gained by appearing on *The Price Is Right* is the chance to appear on national television. The fraction of the total payoffs this comprises depends on each individual, but given the self selection bias involved, it is difficult to believe that for most contestants this benefit would comprise less than 1% of the expected payoff or be worth less than \$100. In light of this, the presence of the bonus payments do not alter contestants' optimal stopping rules.

It is tempting, given our results, to re–evaluate the complexity of the Wheel Spin. One could argue that for most results of the first spin, the decision is "obvious." For example, it might, in retrospect now appear "obvious" that a first contestant who say, gets 45, should spin again. As the formal theory in this paper demonstrates, the actual proof of this fact is complex, time consuming and not obvious at all. It is certainly harder to derive than the fourth player's strategy on Contestant's Row. Clearly, contestants do not undertake such an analysis — they just seem to know the optimal rule. Any reader of this article is herself a human being with a similar cognitive makeup to the contestants on *The Price Is Right.* Since these contestants never make this mistake, it is not surprising that a reader of this article would not either and like the contestants, regard this decision as "obvious." The interesting, but at this point, unanswered question is why does a decision that is objectively difficult appear easy?

It is hard to resist speculating about the reasons for our results. The most astonishing fact is the huge disparity in strategies across what *a priori* appear to be closely related settings. Why is it that contestants who do not use a transparently optimal strategy in one game, somehow use a much more complicated optimal strategy in a related game? We believe the answer lies in a puzzle that Berk, Hughson and Vandezande (1996) note.

It is very likely that the contestants on the *The Price Is Right* have watched the show on television prior to appearing as contestants. Since the optimal strategy of the fourth contestant on Contestants' Row is clear once you see it, why do people not learn the strategy from watching the show on television? Furthermore, although Berk, Hughson and Vandezande (1996) do note a marked increase in the likelihood that the fourth contestant bids optimally once another contestant *on the same show* uses the optimal strategy, 42% of fourth contestants *still* do not use the strategy.

We conjecture that the reason why so many contestants do not learn the optimal strategy is that the focus of the game — the amount of the prize — provides little information about the optimal rule. As on any game show, viewers' attention is drawn to the answer so they miss the opportunity to learn the optimal bidding rule. This is not the case on the Wheel Spin, however. There, the focus of the game, the winning number, reveals much about the optimal stopping rule. This is because after watching enough shows, a typical viewer will get some idea of what score is likely to win, that is, the frequencies in Table 3. As this table makes clear, viewers would learn that, unconditionally, the probability of the winning score being less than 70 is low. This information would be very useful in determining when to stop.

Although there is no way to test this hypothesis in on the *The Price Is Right* itself, there clearly is some subtle difference between these two games that leads to such radically different behavior. Such subtle differences have been documented to cause large changes in outcomes in a laboratory setting. For example, Rapoport, Stein, Parco, and Nicholas (2001) shows that although in the two–person centipede game, backward induction fails (see e.g. McKelvey and Palfrey (1992)), in the three person centipede game, the noncooperative subgame perfect Nash equilibrium emerges after repeated play between anonymous subjects.

6 Conclusion

The important lesson in this paper is that objectively difficult decisions can be correctly made even by boundedly rational agents. What this implies is that the task of predicting when boundedly rational agents act suboptimally is complex. One cannot just take evidence of suboptimality that is derived in a laboratory and assume that similar behavior will be observed in economic contexts such as financial markets. Yet, such practice is becoming increasingly common.

The last decade has been characterized by an enormous growth in interest in economic models that do not rely on assumptions of full rationality. Nowhere is this more true than in the field of financial economics where the area has *de facto* become its own subfield termed *Behavioral Finance*. Here the field has progressed well beyond the early models that just assume random departures from rationality (noise trader models). Today there are numerous examples of financial market equilibria that are based on explicit models of boundedly rational human behavior. Often, the motivation for the behavior that is modeled is laboratory evidence. Yet, these papers provide no evidence that suboptimal behavior that is observed in the laboratory is in any way predictive of behavior in financial markets.

Behavioral research in the last 40 years has made great strides in studying human behavior. Mainly as a result of experimental studies, a large body of work exists that documents consistent and predictable departures in human decision making from the rational model.[13] That is, *given the same experimental setup* people's behavior has been shown to be predictable based on the results of the earlier experiments — departures from rationality have been shown to be both predictable and consistent. Based on these results a number of researchers have begun to explicitly incorporate these departures into economic models. What this paper shows is that one must be careful when taking results about human behavior that have been derived in experiments into the real world.

[13] A review of this literature is beyond the scope of this paper. Interested readers can consult Rabin (1998).

Appendix

A Proof of Proposition 1

Proof: First consider that case when $b_1 \neq b_2$. Contestant 3 will spin again whenever her first spin is strictly less than b. If her first spin is equal to b, then if she chooses not to spin again she has a 50% chance of winning on the tie breaking spin, so she will choose to spin again only if $b \leq 50$. If $b > 50$, her probability of winning on the first spin is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{40} \tag{8}$$

otherwise it is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}. \tag{9}$$

Let f be the result of the first spin. To win on the second spin, the wheel must land between $b-f$ and $100-f$. If it equals $b-f$ the second contestant stands only $\frac{1}{2}$ a chance of winning, otherwise she wins outright. Thus, so long as $b \neq f$, her probability of winning on the second spin is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{40}. \tag{10}$$

If $b = f$, then the second spin cannot result in a tie, so her probability of winning in this case is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}. \tag{11}$$

So her unconditional probability of winning is her probability of winning on the first spin plus her probability of winning on the second spin times the probability of not stopping on the first. Taking the two cases separately:

- $b > 50$:{i.e., (8) + (10) Pr[f $< b$]}

$$\begin{aligned}&\left(\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{40}\right)+\left(\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{40}\right)\left(\frac{b-5}{100}\right)\\ &=\left(\frac{41}{40}-\frac{b}{100}\right)\left(\frac{19}{20}+\frac{b}{100}\right)\end{aligned} \tag{12}$$

- $b \leq 50$: {i.e., (9) + (10) Pr[f $<b$] + (11) Pr[f$=b$]}

$$\left(\left[\frac{100-b}{5}\right]\frac{1}{20}\right)+\left(\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{40}\right)\left(\frac{b-5}{100}\right)+\left(\left[\frac{100-b}{5}\right]\frac{1}{20}\right)\left(\frac{1}{20}\right)$$
$$=1-\left(\frac{b}{100}\right)^2+\frac{1}{40}\left(\frac{b-5}{100}\right) \tag{13}$$

Now consider the case when $b_1 = b_2$. As before contestant 3 will spin again whenever her first spin is strictly less than b. However, in this case if her first spin is equal to b, then if she chooses not to spin again, since all three contestants will participate in the tie breaking spin, she has only a $\frac{1}{3}$ chance of winning, so she will choose to spin again only if $b < 70$. If $b \geq 70$, her probability of winning on the f is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{60} \tag{14}$$

otherwise it is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}. \tag{15}$$

If $b \neq f$ probability of winning on the second spin is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{60}. \tag{16}$$

If $b = f$, then the second spin cannot result in a tie, so her probability of winning in this case is

$$\left[\frac{100-b}{5}\right]\frac{1}{20}. \tag{17}$$

Taking the two cases separately, her unconditional probability of winning is:

- $b \geq 70$:{i.e., (14) + (16) Pr[f $<b$]}

$$\left(\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{60}\right)+\left(\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{60}\right)\left(\frac{b-5}{100}\right)$$
$$=\left(\frac{61}{60}-\frac{b}{100}\right)\left(\frac{19}{20}+\frac{b}{100}\right) \tag{18}$$

- $b < 70$: {i.e., (15) + (16) Pr[f $<b$] + (17) Pr[f$=b$]}

$$\left(\left[\frac{100-b}{5}\right]\frac{1}{20}\right)+\left(\left[\frac{100-b}{5}\right]\frac{1}{20}+\frac{1}{60}\right)\left(\frac{b-5}{100}\right)+\left(\left[\frac{100-b}{5}\right]\frac{1}{20}\right)\left(\frac{1}{20}\right)$$
$$=1-\left(\frac{b}{100}\right)^2+\frac{1}{60}\left(\frac{b-5}{100}\right) \tag{19}$$

which completes the proof. •

B Proof of Proposition 2

Proof: We first calculate the probability of the second contestant getting a particular b_2, given $b_1 > 0$. The probability of observing a b_2 such that $s_2(b_1) \leq b_2 \leq 100$ is equal to the probability of getting b_2 on the first spin plus the probability of getting b_2 after both spins times the probability of not stopping:

$$P[b_2|s_2(b_1) \leq b_2 \leq 100] = \frac{1}{20} + \left[\frac{s_2(b_1) - 5}{100}\right]\frac{1}{20}. \tag{20}$$

If $b_2 < s_2(b_1)$, then it must have taken 2 spins so the probability of observing this is

$$P[b_2|b_2 < s_2(b_1)] = \left[\frac{b_2 - 5}{100}\right]\frac{1}{20}. \tag{21}$$

When contestant 2 has beaten contestant 1's score, his probability of winning is just the complement of contestant 3's probability of winning. Thus, from Proposition 1 we have

$$P[\text{Contestant 2 wins}|b_2 \text{ and } b_1 < b_2] = \begin{cases} \left(\frac{b_2}{100}\right)^2 + \frac{1}{40}\left(\frac{b_2-5}{100}\right) & b \leq 50 \\ 1 - \left(\frac{41}{40} - \frac{b_2}{100}\right)\left(\frac{19}{20} + \frac{b_2}{100}\right) & b > 50. \end{cases}$$

and

$$P[\text{Contestant 2 wins}|b_2 \text{ and } b_1 = b_2] = \begin{cases} \frac{1}{2}\left[\left(\frac{b_2}{100}\right)^2 + \frac{1}{60}\left(\frac{b_2-5}{100}\right)\right] & b \leq 70 \\ \frac{1}{2}\left[1 - \left(\frac{61}{60} - \frac{b_2}{100}\right)\left(\frac{19}{20} + \frac{b_2}{100}\right)\right] & b > 70. \end{cases}$$

We will leave it to the reader to verify that $s_2(b_1) > 50$ when $b_1 \leq 50$. Contestant 2's probability of winning, given that $b_1 \leq 50 < s_2(b_1)$.

$$\begin{aligned}
&P[\text{Constant 2 wins}|b_1] \\
&\quad = \sum_{b_2} P[\text{Contestant 2 wins}|b_2, b_1 \text{ and } b_1 \leq b_2]P[b_2|s_2(b_1)] \\
&\quad = \sum_{i=\frac{b_1+5}{5}}^{10} \left(\left(\frac{5i}{100}\right)^2 - \frac{1}{40}\left(\frac{5i-5}{100}\right)\right)\left[\frac{5i-5}{100}\right]\frac{1}{20} \\
&\qquad + \sum_{i=11}^{\frac{s_2(b_1)-5}{5}} \left(1 - \left(\frac{41}{40} - \frac{5i}{100}\right)\left(\frac{19}{20} + \frac{5i}{100}\right)\right)\left[\frac{5i-5}{100}\right]\frac{1}{20} \\
&\qquad + \sum_{i=\frac{s_2(b_1)}{5}}^{20} \left(1 - \left(\frac{41}{40} - \frac{5i}{100}\right)\left(\frac{19}{20} + \frac{5i}{100}\right)\right)\left(\frac{1}{20} + \left[\frac{s-5}{100}\right]\frac{1}{20}\right)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\left(\left(\frac{b_1}{100}\right)^2-\frac{1}{60}\left(\frac{b_1-5}{100}\right)\right)\left[\frac{b_1-5}{100}\right]\frac{1}{20}\\
=\;&\frac{1}{1200000000}(394686250+875\,b_1-200\,{b_1}^2+30\,{b_1}^3-3\,{b_1}^4 \qquad (22)\\
&\qquad +3812500\,s_2(b_1)+5950\,s_2(b_1)^2-385\,s_2(b_1)^3-s_2(b_1)^4).
\end{aligned}
$$

If $70 > s_2(b_1) > b_1 > 50$, then

$$
\begin{aligned}
&P[\text{Constant 2 wins}|b_1]\\
=\;&\sum_{i=\frac{b_1+5}{5}}^{\frac{s_2(b_1)-5}{5}}\left(1-\left(\frac{41}{40}-\frac{5i}{100}\right)\left(\frac{19}{20}+\frac{5i}{100}\right)\right)\left[\frac{5i-5}{100}\right]\frac{1}{20}\\
&+\sum_{i=\frac{s_2(b_1)}{5}}^{20}\left(1-\left(\frac{41}{40}-\frac{5i}{100}\right)\left(\frac{19}{20}+\frac{5i}{100}\right)\right)\left(\frac{1}{20}+\left[\frac{s-5}{100}\right]\frac{1}{20}\right)\\
&+\frac{1}{2}\left(\left(\frac{b_1}{100}\right)^2-\frac{1}{60}\left(\frac{b_1-5}{100}\right)\right)\left[\frac{b_1-5}{100}\right]\frac{1}{20}\\
=\;&\frac{1}{1200000000}(395611250+2875\,b_1+1300\,{b_1}^2+50\,{b_1}^3-3\,{b_1}^4 \qquad (23)\\
&\qquad +3812500\,s_2(b_1)+5950\,s_2(b_1)^2-385\,s_2(b_1)^3-s_2(b_1)^4)
\end{aligned}
$$

Taking the derivative of either (22) or (23) with respect to $s_2(b_1)$ provides:

$$
\frac{3812500+11900\,s_2(b_1)-1155\,s_2(b_1)^2-4\,s_2(b_1)^3}{1200000000}. \tag{24}
$$

Setting this expression equal to zero and solving gives $s_2(b_1) = 56.98$. It is easy to verify that (22) and (23) are maximized for $s_2(b_1) = 55$, since the expressions are concave and $s_2(b_1) = 60$ provides a lower value. Thus, $s_2(b_1) = 55$ is the optimal stopping rule when $b_1 < 55$.

When $b_1 > 55$, it is clearly suboptimal to stop at 55. However, what is not clear is whether it is optimal to stop at b_1 or at $b_1 + 5$. When $55 \leq b_1 < 70$ we have

$$
\begin{aligned}
&P[\text{Constant 2 wins}|b_1 \text{ and } s_2(b_1)=b_1]\\
=\;&\sum_{i=\frac{b_1+5}{5}}^{20}\left(1-\left(\frac{41}{40}-\frac{5i}{100}\right)\left(\frac{19}{20}+\frac{5i}{100}\right)\right)\left(\frac{1}{20}+\left[\frac{b_1-5}{100}\right]\frac{1}{20}\right)\\
&+\frac{1}{2}\left(\left(\frac{b_1}{100}\right)^2-\frac{1}{60}\left(\frac{b_1-5}{100}\right)\right)\left(\frac{1}{20}+\left[\frac{b_1-5}{100}\right]\frac{1}{20}\right)\\
=\;&\frac{394036250+3860375\,b_1+1250\,{b_1}^2-335\,{b_1}^3-4\,{b_1}^4}{1200000000} \qquad (25)
\end{aligned}
$$

or

$$
\begin{aligned}
&P[\text{Constant 2 wins}|b_1 \text{ and } s_2(b_1) = b_1 + 5] \\
&= \sum_{i=\frac{b_1+5}{5}}^{20} \left(1 - \left(\frac{41}{40} - \frac{5i}{100}\right)\left(\frac{19}{20} + \frac{5i}{100}\right)\right)\left(\frac{1}{20} + \left[\frac{b_1}{100}\right]\frac{1}{20}\right) \\
&\quad + \frac{1}{2}\left(\left(\frac{b_1}{100}\right)^2 - \frac{1}{60}\left(\frac{b_1-5}{100}\right)\right)\left(\left[\frac{b_1-5}{100}\right]\frac{1}{20}\right) \\
&= \frac{414748750 + 3850500\, b_1 - 1675\, {b_1}^2 - 355\, {b_1}^3 - 4\, {b_1}^4}{1200000000}. \qquad (26)
\end{aligned}
$$

so,

$$
\begin{aligned}
&P[\text{Contestant 2 wins}|b_1 \text{ and } s_2(b_1) = b_1 + 5] - P[\text{Contestant 2 wins}|b_1 \text{ and } s_2(b_1) = b_1] \\
&= \sum_{i=\frac{b_1+5}{5}}^{20} \left(1 - \left(\frac{41}{40} - \frac{5i}{100}\right)\left(\frac{19}{20} + \frac{5i}{100}\right)\right)\left(\frac{1}{20}\frac{1}{20}\right) \\
&\quad - \frac{1}{2}\left(\left(\frac{b_1}{100}\right)^2 - \frac{1}{60}\left(\frac{b_1-5}{100}\right)\right)\left(\frac{1}{20}\right) \\
&= \frac{4142500 - 1975\, b_1 - 585\, {b_1}^2 - 4\, {b_1}^3}{240000000}.
\end{aligned}
$$

This expression is positive for all values in the range $55 \le b_1 < 70$. Thus $s_2(b_1) = b_1+5$ in this range.

When if $b_1 \ge 70$ we have

$$
\begin{aligned}
&P[\text{Constant 2 wins}|b_1 \text{ and } s_2(b_1) = b_1] \\
&= \sum_{i=\frac{b_1+5}{5}}^{20} \left(1 - \left(\frac{41}{40} - \frac{5i}{100}\right)\left(\frac{19}{20} + \frac{5i}{100}\right)\right)\left(\frac{1}{20} + \left[\frac{b_1-5}{100}\right]\frac{1}{20}\right) \\
&\quad + \frac{1}{2}\left(1 - \left(\frac{61}{60} - \frac{b_1}{100}\right)\left(\frac{19}{20} + \frac{b_1}{100}\right)\right)\left(\frac{1}{20} + \left[\frac{b_1-5}{100}\right]\frac{1}{20}\right) \\
&= \frac{394986250 + 3856125\, b_1 + 1100\, {b_1}^2 - 335\, {b_1}^3 - 4\, {b_1}^4}{1200000000} \qquad (27)
\end{aligned}
$$

or

$$
\begin{aligned}
&P[\text{Constant 2 wins}|b_1 \text{ and } s_2(b_1) = b_1 + 5] \\
&= \sum_{i=\frac{b_1+5}{5}}^{20} \left(1 - \left(\frac{41}{40} - \frac{5i}{100}\right)\left(\frac{19}{20} + \frac{5i}{100}\right)\right)\left(\frac{1}{20} + \left[\frac{b_1}{100}\right]\frac{1}{20}\right)
\end{aligned}
$$

$$
+\frac{1}{2}\left(1-\left(\frac{61}{60}-\frac{b_1}{100}\right)\left(\frac{19}{20}+\frac{b_1}{100}\right)\right)\left(\left\lceil\frac{b_1-5}{100}\right\rceil\frac{1}{20}\right)
$$
$$
= \frac{414698750+3861250\,b_1-1825\,{b_1}^2-355\,{b_1}^3-4\,{b_1}^4}{1200000000}. \tag{28}
$$

Using these two expressions gives

$$
\begin{aligned}
&P[\text{Contestant 2 wins}|b_1 \text{ and } s_2(b_1)=b_1+5]-P[\text{Contestant 2 wins}|b_1 \text{ and } s_2(b_1)=b_1]\\
&= \sum_{i=\frac{b_1+5}{5}}^{20}\left(1-\left(\frac{41}{40}-\frac{5i}{100}\right)\left(\frac{19}{20}+\frac{5i}{100}\right)\right)\left(\frac{1}{20}\frac{1}{20}\right)\\
&\quad -\frac{1}{2}\left(1-\left(\frac{61}{60}-\frac{b_1}{100}\right)\left(\frac{19}{20}+\frac{b_1}{100}\right)\right)\left(\frac{1}{20}\right)\\
&= \frac{3942500+1025\,b_1-585\,{b_1}^2-4\,{b_1}^3}{240000000}.
\end{aligned}
$$

This expression is negative when $b_1 \geq 70$, so in this range $s_2(b_1)=b_1$. Summarizing,

$$
s_2(b_1)=\begin{cases} 55 & 0\leq b_1\leq 50\\ b_1+5 & 50<b_1<70\\ b_1 & 70\leq b_1\leq 100\end{cases}.
$$

Substituting this rule into (22)-(28) and simplifying provides:

$$
\begin{aligned}
&P[\text{Constant 2 wins}|b_1]\\
&= \begin{cases}\frac{29289}{64000} & b_1=0\\ \frac{549167500+875\,b_1-200\,{b_1}^2+30\,{b_1}^3-3\,{b_1}^4}{1200000000} & b_1\leq 50\\ \frac{414748750+3850500\,b_1-1675\,{b_1}^2-355\,{b_1}^3-4\,{b_1}^4}{1200000000} & 50<b_1<70\\ \frac{394986250+3856125\,b_1+1100\,{b_1}^2-335\,{b_1}^3-4\,{b_1}^4}{1200000000} & 70\leq b_1\leq 100\end{cases}
\end{aligned} \tag{29}
$$

where the probability that contestant 2 wins, conditional on contestant 1 going over ($b_1=0$), is derived by setting $b_1=5$ in (22). •

C Proof of Proposition 3

Proof: The case when the first contestant's score exceeds 100 ($b_1=0$) is trivial. For the case when $b_1>0$,

$$
P[\text{Contestant 1 wins}|b_1] = \tag{30}
$$

$$
\begin{aligned}
= \quad & P[b_2 > 100 \text{ or } b_2 < b_1|b_1]P[b_3 > 100 \text{ or } b_3 < b_1|b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] \\
& + \frac{1}{2}P[b_2 > 100 \text{ or } b_2 < b_1|b_1]P[b_3 = b_1|b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] \\
& + \frac{1}{2}P[b_3 > 100 \text{ or } b_3 < b_1|b_1 \text{ and } b_2 = b_1]P[b_2 = b_1|b_1] \\
& + \frac{1}{3}P[b_3 = b_1|b_1 \text{ and } b_2 = b_1]P[b_2 = b_1|b_1]. \qquad (31)
\end{aligned}
$$

The probability that after two spins, $b_2 < b_1$ is

$$
P[b_2 < b_1|b_1] = \sum_{i=2}^{\frac{b_1-5}{5}} \left(\frac{1}{20}\right)\left(\frac{i-1}{20}\right) = \frac{(b_1-5)(b_1-10)}{20000}
$$

Let f_i be the result of the i^{th} contestant's first spin. Then the probability that the second contestant goes over 100 is the probability of not stopping on the first spin times the probability of going over on the second spin:

$$
\begin{aligned}
P[b_2 > 100|b_1] &= P[f_2 < s_2(b_1)|b_1]P[b_2 > 100|b_1 \text{ and } f_2 < 55] \\
&= \sum_{i=1}^{\frac{s_2(b_1)-5}{5}} \left(\frac{1}{20}\right)\left(\frac{i}{20}\right) \\
&= \begin{cases} \frac{11}{80} & b_1 < 55 \\ \frac{(b_1+5)b_1}{20000} & 55 \leq b_1 < 70 \\ \frac{(b_1-5)b_1}{20000} & 70 \leq b_1 \leq 100 \end{cases}
\end{aligned}
$$

where we have used the optimal stopping rule $s_2(b_1)$ derived in Proposition 2. Using these two results provides,

$$
\begin{aligned}
P[b_2 > 100 \text{ or } b_2 < b_1|b_1] &= P[b_2 > 100|b_1] + P[b_2 < b_1|b_1] \\
&= \begin{cases} \frac{2750+(b_1-5)(b_1-10)}{20000} & b_1 \leq 50 \\ \frac{(b_1+5)b_1+(b_1-5)(b_1-10)}{20000} & 50 < b_1 < 70 \\ \frac{2(b_1-5)^2}{20000} & 70 \leq b_1 \leq 100 \end{cases} \qquad (32)
\end{aligned}
$$

Similarly, the probability that the third contestant will not beat the first contestant given that the second contestant has not beaten the first contestant is:

$$
P[b_3 < b_1|b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] = \sum_{i=2}^{\frac{b_1-5}{5}} \left(\frac{1}{20}\right)\left(\frac{i-1}{20}\right) = \frac{(b_1-5)(b_1-10)}{20000}
$$

The probability that the third contestant goes over 100, conditional on the second contestant not beating the first contestant, is

$$
P[b_3 > 100|b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] = \begin{cases} \sum_{i=1}^{\frac{b_1}{5}} \left(\frac{1}{20}\right)\left(\frac{i}{20}\right) & b_1 \leq 50 \\ \sum_{i=1}^{\frac{b_1-5}{5}} \left(\frac{1}{20}\right)\left(\frac{i}{20}\right) & b_1 > 50 \end{cases}
$$

$$= \begin{cases} \frac{b_1(b_1+5)}{20000} & b_1 \le 50 \\ \frac{b_1(b_1-5)}{20000} & b_1 > 50, \end{cases}$$

which implies that

$$\begin{aligned} &P[b_3 > 100 \text{ or } b_3 < b_1 | b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] \\ &= P[b_3 > 100 | b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] + P[b_3 < b_1 | b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] \\ &= \begin{cases} \frac{(b_1+5)b_1+(b_1-5)(b_1-10)}{20000} & b_1 \le 50 \\ \frac{2(b_1-5)^2}{20000} & b_1 > 50 \end{cases} \end{aligned} \tag{33}$$

Finally we need to deal with the ties. Using the same logic that was used in the proof of Lemma 1,

$$P[b_3 = b_1 | b_1 \text{ and } (b_2 > 100 \text{ or } b_2 < b_1)] = \begin{cases} \frac{b_1-5}{2000} & b_1 \le 50 \\ \frac{b_1+95}{2000} & b_1 > 50 \end{cases} \tag{34}$$

$$P[b_2 = b_1 | b_1] = \begin{cases} \frac{b_1-5}{2000} & b_1 < 70 \\ \frac{b_1+95}{2000} & b_1 \ge 70 \end{cases} \tag{35}$$

$$P[b_3 = b_1 | b_1 \text{ and } b_2 = b_1] = \begin{cases} \frac{b_1-5}{2000} & b_1 < 70 \\ \frac{b_1+95}{2000} & b_1 \ge 70 \end{cases} \tag{36}$$

$$P[b_3 > 100 \text{ or } b_3 < b_1 | b_1 \text{ and } b_2 = b_1] = \begin{cases} \frac{(b_1+5)b_1+(b_1-5)(b_1-10)}{20000} & b_1 < 70 \\ \frac{2(b_1-5)^2}{20000} & b_1 \ge 70 \end{cases} \tag{37}$$

Substituting (32)-(37) into (31) and simplifying provides

$$P[\text{Contestant 1 wins} | b_1] = \begin{cases} \frac{208750-42625\,b_1+16900\,{b_1}^2-75\,{b_1}^3+6\,{b_1}^4}{1200000000} & b_1 \le 50 \\ \frac{19375-4375\,b_1+925\,{b_1}^2-30\,{b_1}^3+3\,{b_1}^4}{300000000} & 50 < b_1 < 70 \\ \frac{263125-10625\,b_1+1750\,{b_1}^2-45\,{b_1}^3+3\,{b_1}^4}{300000000} & b_1 \ge 70 \end{cases} \tag{38}$$

The optimal s_1 maximizes:

$$P[\text{Contestant 1 wins} | s_1] = \sum_{b_1} P[\text{Contestant 1 wins} | b_1] P[b_1 | s_1], \tag{39}$$

where $P[b_1|s_1]$ is given by Lemma 1. It is left to the reader to show that (39) is convex in s_1, so that the optimal stopping rule is the smallest x such that

$$P[\text{Contestant 1 wins} | s_1 = x + 5] - P[\text{Contestant 1 wins} | s_1 = x] < 0.$$

Evaluating this expression for $x = 65$ gives:

$$\begin{aligned}
&P[\text{Contestant 1 wins}|s_1 = 70] - P[\text{Contestant 1 wins}|s_1 = 65] \\
&= \sum_{i=14}^{20} \frac{263125 - 10625\,5i + 1750\,(5i)^2 - 45\,(5i)^3 + 3\,(5i)^4}{300000000} \left(\frac{5}{2000}\right) \\
&\quad - \frac{19375 - 4375\,(65) + 925\,(65)^2 - 30\,(65)^3 + 3\,(65)^4}{300000000} \left(\frac{100}{2000}\right) \\
&= \frac{4613}{6000000} > 0.
\end{aligned}$$

Repeating this calculation for $x = 70$ gives:

$$\begin{aligned}
&P[\text{Contestant 1 wins}|s_1 = 75] - P[\text{Contestant 1 wins}|s_1 = 70] \\
&= \sum_{i=15}^{20} \frac{263125 - 10625\,5i + 1750\,(5i)^2 - 45\,(5i)^3 + 3\,(5i)^4}{300000000} \left(\frac{5}{2000}\right) \\
&\quad - \frac{263125 - 10625\,(70) + 1750\,(70)^2 - 45\,(70)^3 + 3\,(70)^4}{300000000} \left(\frac{100}{2000}\right) \\
&= -\frac{459347}{192000000} < 0.
\end{aligned}$$

So the optimal stopping rule is $s_1 = 70$. •

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