

# Information Revelation and Market Crashes \*

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September 2004  
Revised: May 2005

**Abstract:** We show the possibility of market crash in rational expectations equilibrium due to information-regime switching. As the equilibrium price of an asset decreases in a smooth way due to adverse demand pressure of noise traders, it reaches a critical value at which the information of uninformed competitive traders changes from partial to full, and causes a discontinuous drop of the price. This happens as the information signal about the future asset payoff remains unchanged – it indicates low expected payoff. The rational expectations equilibrium price switches a regime from where it is uninformative to a regime where it is informative. At the critical price, uninformed traders “finally” realize that the information signal, which the informed traders knew all along, is low. The crucial feature of the model is that the distribution of the demand of noise traders has bounded support.

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\*Preliminary Draft.

## 1. Introduction

An important aspect of financial markets is that there is multitude of information potentially relevant to asset valuation, but only a small fraction of investors actively gather information. Investors have asymmetric information and they look to market prices in order to extract information about future payoffs. This leads to dual role of prices. Prices affect investors' portfolio demand, first, through budget constraints, and, second, through expectations.

Rational expectations equilibrium model of Radner (1978) and Grossman (1976, 1978) is a Walrasian model of competitive asset markets with asymmetric information and the dual role of prices. The model has been used to analyze how a competitive market serves to communicate information between traders, in particular, how market prices aggregate and reveal information. Conditions under which equilibrium prices reveal fully or partially the relevant information are now well understood (see an overview by Jordan and Radner (1980)).

It is generally accepted that information transmission in the markets plays an important role in market crashes. Yet, explaining market crashes, or large movements of asset prices, has been a challenge to rational expectations equilibrium models. In the two most (in)famous stock market crashes — the 1929 crash, and the crash of October 19th, 1987 — there were no significant economic news during the periods immediately surrounding the crashes. In their analysis of the October 1987 stock market crash, Genotte and Leland (1990) proposed an explanation of market crashes as *discontinuity* in the relationship between underlying environment and equilibrium asset prices. They showed that an infinitesimal shift in a parameter, such as unobserved supply shock (or “noise”), can cause discontinuous change of equilibrium price if some traders follow hedging strategies that generate upward sloping asset demand. Barlevy and Veronesi (2003) pointed out that the demand of uninformed competitive traders, who look up to current price for information, can be upward sloping, too, and lead to discontinuity of rational equilibrium prices even in the absence of hedging strategies.

In the presence of unobserved shocks or noise, rational equilibrium prices are typically non-revealing. Noise impedes agents' ability to extract information from prices, and consequently prices reveal only a fraction of total information. Most of rational expectations equilibrium models in the literature exhibit equilibria that reveal a fraction of information according to the same statistical rule throughout the entire range of values of information and noise. That is, equilibria display one regime of information revelation.

In this paper we demonstrate the possibility of a rational expectations equilibrium with information-regime switching that generates market crash. Information-regime switching occurs when equilibrium exhibits different rules of information revelation over subsets of values of information and noise. Switching from one information regime to another can occur in a discontinuous way and produce a crash.

Our model combines some features of Radner (1979) and Grossman and Stiglitz (1980) models. Investors select portfolios so as to maximize expected utilities of payoffs, and they receive private information signals that affect their expectations of asset payoffs. They extract information from market prices. We present an example with one signal, one risky asset, and one risk-free asset. Investors who observe the signal are informed. Uninformed are those who do not observe the signal. Following the tradition of Grossman and Stiglitz, there are "noise" traders whose portfolio demands are based on liquidity needs. These liquidity needs or noise are described by a random variable with bounded support. The presence of noise impedes the uninformed investor's ability to extract the signal from prices, and prevents the uninteresting situation of fully revealing rational expectations equilibrium.

Rational expectations equilibrium in our example turns out to have two information regimes. For high values of noise and the most favorable signal, and for low values of noise and the least favorable signal rational expectations equilibrium is fully revealing. For all remaining values of noise and signal the equilibrium is non-revealing, that is, it does not reveal the signal. Rational expectations equi-

librium price displays discontinuity as function of noise when it switches regimes from non-revealing to revealing. Under one scenario, as the equilibrium asset price decreases in a smooth way due to adverse demand pressure of the noise traders, it reaches a critical value at which the information of uninformed investors changes from partial to full, and causes a discontinuous drop of price. This happens as the information signal about the asset payoff remains unchanged – it indicates low expected payoff. At the critical price, uninformed investors “finally” realize that the information signal, which the informed investors new all along, is low.

The crucial feature of the model is that the distribution of the demand of noise traders has bounded support. Normal distribution, which is frequently assumed in the literature, would not lead to discontinuous information-regime switching. The magnitude of price discontinuity depends on the number of uniformed investors in the market.

There have been several other papers concerned with the possibility of large price movements without substantial news. Closely related to our model are Genotte and Leland (1990) and Barlevy and Veronesi (2003). Romer (1993) and Hart and Tauman (2004) focus on imperfections in information processing among investors. Lee (1998) shows that transaction costs can impede information transmission and lead to excess volatility of prices.

## 2. Example

There are two dates, 0 and 1, and two states  $s = 1, 2$  at date 1. There are two assets with payoffs  $x_1 = (1, 1)$  and  $x_2 = (2, 1)$ . Asset prices are 1 for asset 1 and  $p$  for asset 2.

Agent  $i$ , for  $i = 1, 2$ , has expected utility function of date-1 consumption given by

$$\pi^i(1) \cdot \ln(c_1^i) + \pi^i(2) \cdot \ln(c_2^i), \quad (1)$$

where probabilities  $\pi^i(1)$  and  $\pi^i(2)$  reflect agent’s information in a way that will be specified later. There is no consumption at date 0.

Date-1 endowments are  $\omega^1 = (1, 3)$  and  $\omega^2 = (3, 1)$ , so that there is no aggregate

risk. There are no endowments at date 0.

Information is modeled by a signal that affects agents' probabilities of states. Signal  $\sigma$  can take one of two possible values:  $H$ , or  $L$ . Prior probability beliefs about states and signals are

$$\pi(s = 1, \sigma = H) = 0.3, \quad \pi(s = 2, \sigma = H) = 0.2, \quad (2)$$

$$\pi(s = 1, \sigma = L) = 0.2, \quad \pi(s = 2, \sigma = L) = 0.3. \quad (3)$$

If an agent observes signal H, her probability of state 1 is  $\pi(s = 1|\sigma = H) = 0.6$ . If she observes signal L, the probability of state 1 is 0.4. If she does not observe the signal, the probability is unconditional  $\pi(s = 1) = 0.5$ . Signal  $H$  indicates high expected payoff of asset 2, while signal  $L$  indicates low expected payoff of asset 2.

Agent 1 observes the signal – she is the *informed* agent. Agent 2 does not observe the signal – she is *uninformed*.

The third agent is a *noise* (or *liquidity*) trader. Her demands for assets 1 and 2 are functions of asset price  $p$  and a parameter  $y$ , and are given by

$$z_1(p, y) = -\frac{p}{(p-1)(2-p)}y, \quad z_2(p, y) = \frac{1}{(p-1)(2-p)}y, \quad (4)$$

for  $1 < p < 2$ . Parameter  $y$  can take any value in the interval  $[-1, 1]$  and is unobserved by either agent 1 or 2. We call  $y$  a *noise*, or *liquidity shock*.

The somewhat complicated form of portfolio demand of the noise trader is dictated by tractability – it allows for equilibria in linear form. The following may help the reader to understand the nature of these demands: The payoff of portfolio (4) in state 1 is  $\frac{y}{p-1}$ . Since the state-price of state 1 at asset prices  $(1, p)$  is  $p-1$ , date-0 value of state-1 payoff  $\frac{y}{p-1}$  is  $y$ . Further, portfolio(4) is self-financing, i.e.,  $z_1 + pz_2 = 0$ . Thus, the noise trader conducts a self-financing trade that generates state-1 payoff whose present value is  $y$ .

We assume that parameter  $y$  is uniformly distributed on the interval  $[-1, 1]$ , independent of  $\sigma$  and  $s$ .

Before analyzing rational expectations equilibrium, we briefly discuss a simple method of calculating equilibrium asset prices in our model. An equilibrium for

given agents' probability vectors  $\pi^1, \pi^2$  is a price vector  $(1, p)$  such that the total portfolio demand of the three agents equals zero, when agents 1 and 2 select their optimal portfolios so as to maximize expected utility (1) subject to budget constraints

$$h_1^i + ph_2^i = 0, \quad (5)$$

$$c_s^i = \omega_s^i + x_{1s}h_1^i + x_{2s}h_2^i, \quad s = 1, 2. \quad (6)$$

Since asset markets are complete, we can find equilibrium asset prices by solving first for equilibrium state prices in markets for state-contingent claims. Asset prices can then be obtained applying the standard principle of valuation by state prices, i.e., as sums over states of asset payoffs multiplied by state prices.

Since agents have the standard Cobb-Douglas utilities, it is easy to solve for equilibrium state prices. Some details of the derivation are provided in Appendix A. Equilibrium price of asset 2, for given probabilities  $\pi^1, \pi^2$ , and liquidity shock  $y$ , is

$$p(y) = \frac{y}{2[\pi^1(1) - \pi^2(1)] + 4} + \frac{4 + 5\pi^1(1) - \pi^2(1)}{2[\pi^1(1) - \pi^2(1)] + 4}. \quad (7)$$

Note that  $p(y)$  is a linear function of noise  $y$ .

### 3. Rational Expectations Equilibrium

*Price forecast function* is a function  $\Phi : \{L, H\} \times [-1, 1] \rightarrow \mathfrak{R}_+$  that maps signal-noise pairs to prices of asset 2. Agent 2, who does not observe the signal, uses forecast function to infer the value of signal (and noise). If the price of asset 2 is  $p$ , she updates her prior belief by conditioning on  $\{\Phi = p\}$ . When choosing optimal portfolio at price vector  $(1, p)$ , she maximizes expected utility (1) with conditional probabilities  $\pi(s|\Phi = p)$  subject to budget constraint (6). Agent 1, who observes the signal, maximizes expected utility with probabilities conditional on the observed signal.

*Rational expectation equilibrium* is price forecast function  $\hat{\Phi}$  such that  $\hat{\Phi}(\sigma, y)$  is the equilibrium asset price for every realization of signal  $\sigma$  and noise  $y$ .

We first observe that full information equilibrium cannot be a rational expectations equilibrium on the entire domain of signal-noise pairs. *Full information equilibrium*  $p^f$  obtains when both agents observe the signal and adjust their probabilities accordingly. It follows from (7) that

$$p^f(H, y) = \frac{1}{4}y + 1.6, \quad (8)$$

$$p^f(L, y) = \frac{1}{4}y + 1.4. \quad (9)$$

This is shown Figure 1. For every price  $p \in [1.35, 1.65]$ , forecast function  $p^f$  does not reveal the signal. For example, price 1.5 could result either from signal  $H$  and  $y = -0.4$ , or from signal  $L$  and  $y' = 0.4$ . Full information equilibrium cannot be a rational expectations equilibrium because it is non-revealing. For the use later, we note that  $p^f$  reveals the signal at some prices. For every  $p \in [1.15, 1.35)$ ,  $p^f$  reveals signal  $L$ ; for every  $p \in (1.65, 1.85]$ ,  $p^f$  reveals signal  $H$ . In other words, forecast function  $p^f$  is a rational expectations equilibrium on two subsets of signal-noise pairs:  $\{L\} \times [-1, -0.2)$  and  $\{H\} \times (0.2, 1]$ .

We turn our attention now to the possibility of rational expectations equilibrium that does not reveal the signal to the uninformed agent. Let  $\Phi_\alpha$  be a price forecast function that assigns to any  $(\sigma, y)$  an equilibrium price of asset 2 when agent 2's probabilities of states are  $\pi^2 = (\alpha, 1 - \alpha)$  for some  $\alpha \in [0, 1]$ . Using (7), we have

$$\Phi_\alpha(H, y) = \frac{y}{5.2 - 2\alpha} + \frac{7 - \alpha}{5.2 - 2\alpha}, \quad (10)$$

$$\Phi_\alpha(L, y) = \frac{y}{4.8 - 2\alpha} + \frac{6 - \alpha}{4.8 - 2\alpha}. \quad (11)$$

We show in Appendix B that, if agent 2 uses  $\Phi_\alpha$  as price forecast function, then for every price  $p$  such that  $\Phi_\alpha$  does not reveal the signal at  $p$ , the updated probability of state 1 is

$$\pi(s = 1 | \Phi_\alpha = p) = 0.6 \frac{5.2 - 2\alpha}{10 - 4\alpha} + 0.4 \frac{4.8 - 2\alpha}{10 - 4\alpha}. \quad (12)$$

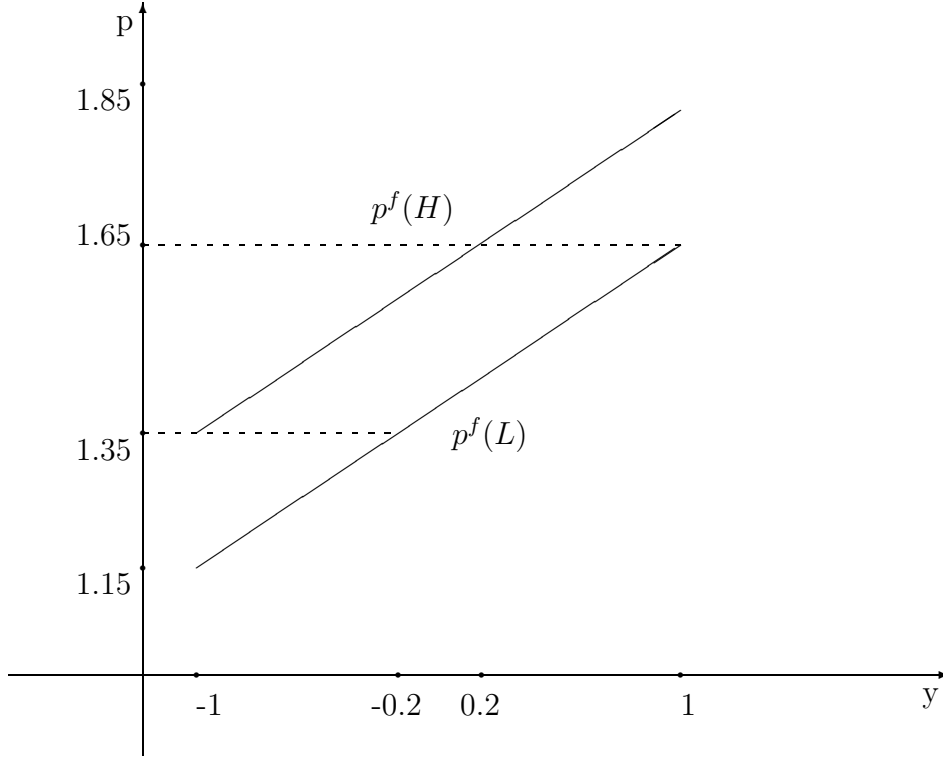


Figure 1: Full Information Equilibrium

For  $\Phi_\alpha$  to be a rational expectations equilibrium it is necessary that  $\alpha$  be equal to the probability in (12). This gives equation

$$\alpha = \frac{5.04 - 2\alpha}{10 - 4\alpha}. \quad (13)$$

The solution to (13) is  $\alpha^* = 0.51$ , and it gives the forecast function

$$\Phi_{\alpha^*}(H, y) = \frac{y}{4.18} + 1.55, \quad (14)$$

$$\Phi_{\alpha^*}(L, y) = \frac{y}{3.78} + 1.45 \quad (15)$$

shown in Figure 2. Price forecast function  $\Phi_{\alpha^*}$  is a rational equilibrium on two subsets of signal-noise pairs:  $\{L\} \times [-0.53, 1]$ , and  $\{H\} \times [-1, 0.67]$ . On these two



sets,  $\Phi_{\alpha^*}$  has exactly the same range of values – the interval  $[1.31, 1.71]$ . Consequently,  $\Phi_{\alpha^*}$  does not reveal the signal at any  $p \in [1.31, 1.71]$ . However, it does reveal signal L at any  $p \in [1.19, 1.31)$ , and signal H at any  $p \in (1.71, 1.79]$ . Thus,  $\Phi_{\alpha^*}$  is not a rational expectations equilibrium on the entire domain of noise-signal pairs.

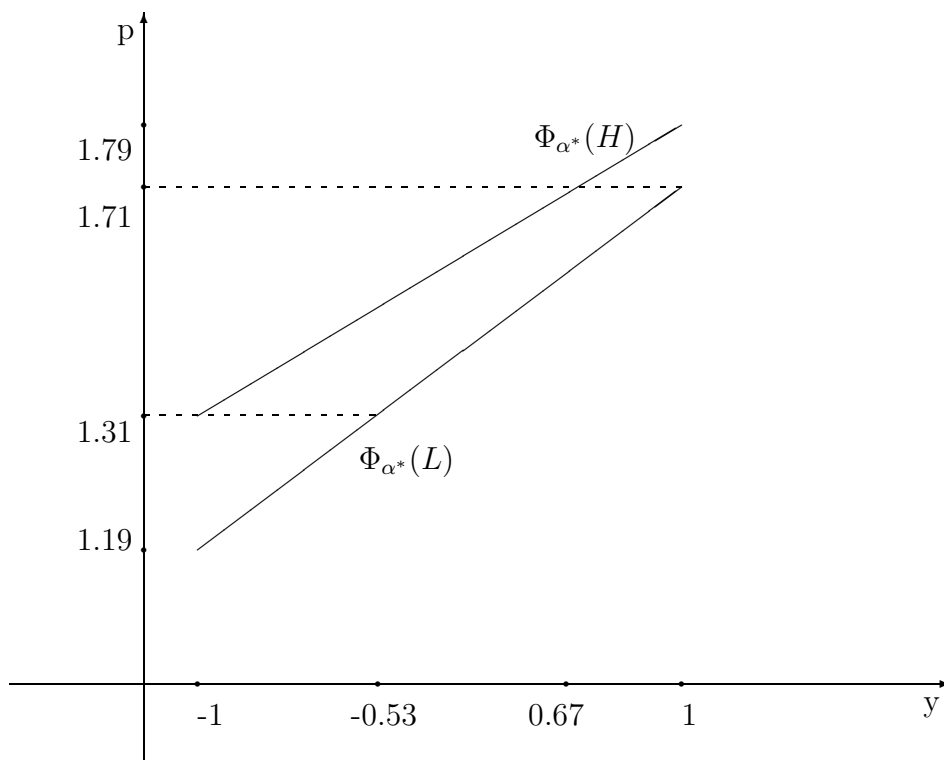


Figure 2: Function  $\Phi_{\alpha^*}$

It is interesting to note that forecast function  $\Phi_{\alpha^*}$  is almost, but not exactly, equal to the private information equilibrium. Private information equilibrium obtains when agent 2 probabilities of states are unconditional  $\pi(1) = \pi(2) = 0.5$ .

The rational expectation equilibrium combines the full information equilibrium with the non-revealing equilibrium.

**Proposition:** Price forecast function given by

$$\hat{\Phi}(H, y) = \begin{cases} \Phi_{\alpha^*}(H, y) & \text{if } y < 0.67, \\ p^f(H, y) & \text{if } y \geq 0.67 \end{cases} \quad (16)$$

and

$$\hat{\Phi}(L, y) = \begin{cases} p^f(L, y) & \text{if } y \leq -0.53, \\ \Phi_{\alpha^*}(L, y) & \text{if } y > -0.53 \end{cases}. \quad (17)$$

is a rational expectations equilibrium.

Figure 3 illustrates the rational expectations equilibrium, and justifies the assertion of the Proposition.

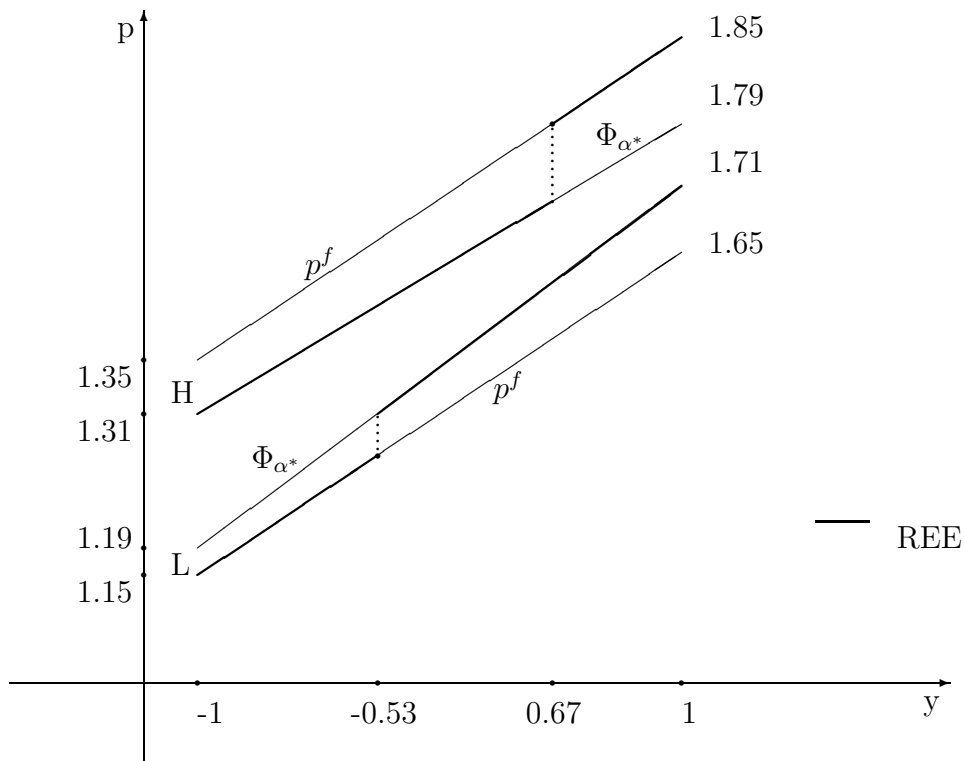


Figure 3: Rational Expectations Equilibrium

#### 4. Information-Regime Switching and Market Crash

Rational expectations equilibrium (16, 17) exhibits regime switching. On the subset of signal-noise pairs  $\{L\} \times [-1, -0.53]$ , equilibrium forecast function  $\hat{\Phi}$  is revealing. For every price  $p$  in the range of values of  $\hat{\Phi}$  over this set, forecast function  $\hat{\Phi}$  reveals signal L. Also, on the subset  $\{H\} \times [0.67, 1]$ , forecast function  $\hat{\Phi}$  is revealing. Here, for every price  $p$  in the range of values,  $\hat{\Phi}$  reveals signal H. On the subsets of signal-noise pairs  $\{L\} \times (-0.53, 1]$ , and  $\{H\} \times [-1, 0.67)$ , equilibrium forecast function  $\hat{\Phi}$  is non-revealing. For every price  $p$  in the range of values of  $\hat{\Phi}$  over these sets,  $\hat{\Phi}$  reveals two signal-noise pairs, one with L and another with H.

This rational expectations equilibrium has two points of discontinuity. They are  $(L, -0.53)$  and  $(H, 0.67)$ . In particular, the discontinuity at  $(L, -0.53)$  has some features of a market crash without news. When the signal is low and the liquidity shock is slightly above to the critical value of  $y = -0.53$ , the asset price is moderately low and the uninformed agent does not know that the signal is low. As the liquidity shock decreases, the asset price decreases in a smooth way. When the liquidity shock reaches  $y = -0.53$ , the uninformed agent realizes that the low asset price is incompatible with high signal and that the signal must be low. This causes an abrupt change of her expectations and leads to a drop in equilibrium price of the asset.

#### 5. Robustness.

What features of the example are important for rational expectations equilibrium with discontinuous regime switching? Are these features robust?

First, it is crucial that full information equilibrium is not revealing. Otherwise, it would be a rational expectations equilibrium with a single regime. The property that guarantees it, is

- $\min_y p^f(H, y) < \max_y p^f(L, y)$

It is a condition pertaining to relative significance of the noise and the signal for the asset price. It says that the impact of the signal does not fully dominate the

impact of noise.

Second, it is important that more favorable information about the payoff of the risky asset leads to higher equilibrium price (and lower expected return).

- $\Phi_\alpha(L, y) < \Phi_\alpha(H, y), \quad \forall y.$
- $\Phi_{\alpha'}(\sigma, y) < \Phi_\alpha(\sigma, y)$  for  $\alpha' < \alpha, \quad \forall y, \sigma = H, L.$

The first condition guarantees that there cannot be a non-revealing equilibrium over the entire domain of signal-noise pairs. The two conditions together give rise to the configuration of forecast functions as in Figure 3. The positive relation between information content and asset price is intuitively appealing, but it cannot be always guaranteed (see Admati (1985)).

Third, it is crucial that the distribution of liquidity shocks have bounded support. Uniform distribution of shocks is important for generating piecewise linear rational expectations equilibrium, but not for discontinuous regime switching. One can show that as long as the density function  $f_y$  of the liquidity shocks is uniformly bounded away from zero and bounded above, then there cannot be a piecewise linear equilibrium with continuous regime switching. We summarize these conditions as

- density  $f_y$  is non-zero on an interval  $[\underline{b}, \bar{b}]$  and satisfies  $0 < \epsilon \leq f_y(t) \leq B.$

Needless to say, normal distribution, which is frequently used in the literature, does not satisfy this condition.

## Appendix.

**A.** We use  $q$  for state price of state 1 and  $(1 - q)$  for state price of state 2. With this normalization, the valuation relation for asset 1 whose price is 1 is guaranteed, and the price of asset 2 is related to state prices via  $p = 1 + q$ . We solve for equilibrium value of  $q$  by writing demand functions for consumption in state 1 for all three agents and equating the total demand to the aggregate consumption endowment in state 1. By Walras Law, the market for consumption in state 2 will be cleared, too.

Agents 1 and 2 have the standard Cobb-Douglas utility functions. Their demand functions for consumption in state 1 are

$$c^1(1) = \pi^1(1) \frac{[q + 3(1 - q)]}{q}, \quad \text{and} \quad c^2(1) = \pi^2(1) \frac{[3q + (1 - q)]}{q}. \quad (18)$$

The noise trader's demand for consumption in state 1 is  $\frac{y}{q}$ . The market clearing condition is

$$c^1(1) + c^2(1) + \frac{y}{q} = 4. \quad (19)$$

It follows that the equilibrium state price of state 1 is

$$q(y) = \frac{y}{2[\pi^1(1) - \pi^2(1)] + 4} + \frac{3\pi^1(1) + \pi^2(1)}{2[\pi^1(1) - \pi^2(1)] + 4}. \quad (20)$$

**B.** Let  $p$  be such that  $\Phi_\alpha(L, y) = \Phi_\alpha(H, y') = p$  for some  $y, y'$ . We have

$$\pi(s = 1 | \Phi_\alpha = p) = \lim_{h \rightarrow 0} \pi(s = 1 | \Phi_\alpha \in [p - h, p + h]) \quad (21)$$

We introduce the following notation:

$$E_H^h = \{(\sigma, y) : \sigma = H, p - h \leq \Phi_\alpha(H, y) \leq p + h\}, \quad (22)$$

$$E_L^h = \{(\sigma, y) : \sigma = L, p - h \leq \Phi_\alpha(L, y) \leq p + h\}. \quad (23)$$

Then,

$$\pi(s = 1 | \Phi_\alpha \in [p - h, p + h]) = \pi(s = 1 | E_H^h \cup E_L^h) = \quad (24)$$

$$\pi(s = 1 | E_H^h) \frac{\pi(E_H^h)}{\pi(E_H^h) + \pi(E_L^h)} + \pi(s = 1 | E_L^h) \frac{\pi(E_L^h)}{\pi(E_H^h) + \pi(E_L^h)} \quad (25)$$

Since  $y$  is uniformly distributed and independent of  $\sigma$ , and  $\Phi_\alpha$  is linear of the form (11), we obtain

$$\pi(E_H^h) = 0.5(5.2 - 2\alpha)h, \quad \pi(E_L^h) = 0.5(4.8 - 2\alpha)h. \quad (26)$$

Further,

$$\pi(s = 1|E_H^h) = \pi(s = 1|\sigma = H) = 0.6 \quad (27)$$

$$\pi(s = 1|E_L^h) = \pi(s = 1|\sigma = L) = 0.4 \quad (28)$$

Equation (12) follows now from (21) and (25).

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