Devaluation without common knowledge

Céline Rochon
THEMA, Université de Cergy-Pontoise*

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Abstract

In an economy with a fixed exchange rate regime that suffers a random adverse shock, we study the strategies of imperfectly informed speculators that may trigger an endogenous devaluation before it occurs exogenously. The game played by the speculators has a unique symmetric Nash equilibrium which is a strongly rational expectation equilibrium in the set of all strategies with delay. Uncertainty about the extent to which the Central Bank is ready to defend the peg increases the delay in selling the domestic currency and extends the ex ante mean delay between the exogenous shock and the devaluation. We determine endogenously the rate of devaluation.

Keywords: currency crises, fixed exchange rate regime, speculation, uncertainty of reserves, endogenous devaluation

JEL Classification: D82, D83, D84, E58, F31, F32

*Céline Rochon, THEMA, Université de Cergy-Pontoise, 33 bd du Port, 95011 Cergy-Pontoise, France, celine.rochon@eco.u-cergy.fr, tel: 33 1 34 25 67 59, fax: 33 1 34 25 62 33

1 Introduction

In the first generation models of currency crises, the exchange rate is fixed as long as the Central Bank has sufficient reserves to sustain its value in the face of an outflow of capital and a drain on its reserves. Such an outflow can be induced by a change in the terms of trade or by a deficit of the government that is financed by money creation (as in the model of Krugman (1979)). When the reserves of the Central Bank are depleted, the fixed exchange rate is abandoned and the rate is determined by the market in a new regime. In the first generation models, agents have perfect information about the process. The switch from a fixed to a floating rate is in general accompanied by a downward jump of the demand for the domestic currency.[1] Under perfect foresight (as in Krugman (1979)), this jump is achieved through a jump in the nominal quantity of money. At the time of the switch, the "currency crisis" takes the form of a run on the currency at the fixed rate. There is no discrete devaluation. There is no capital loss by the agents who hold the domestic currency and no strategic complementarity in running from the domestic currency.[2]

The second generation models have focussed on the discrete devaluations that generate multiple equilibria: if there is a run on the currency and all selling orders cannot be executed at the fixed exchange rate (before the reserves of the Central Bank are depleted), there is an incentive to run if all others run. The strategic complementarity[3] between the selling orders generate multiple equilibria. These models have focussed on the situation created by holdings of a domestic currency that are in excess after the peg is abandoned. The self-fulfilling nature of currency attacks is accounted for in these models; main contributions include Obstfeld (1986,1996), Velasco (1996), Jeanne (1997), and Jeanne and Masson (2000), among others.

Morris and Shin (1998) and Krugman (1996) both challenge the existence of multiple equilibria in second generation models. This is achieved by Morris and Shin (1998) through a one-period model without common knowledge[4], and by Krugman (1996) through a particular specification of the devaluation expectations and of the evolution of fundamentals.

One can criticize one-period models for two reasons. First, a critical feature of currency attacks is that they occur over some period of time. The basic problem of the agents is not *whether* to attack a currency, but *when* to attack. In this waiting game, agents observe the market, (Chamley (2003)). Second, as in the first generation models, agents should anticipate the switch and avoid to be in the position of running to escape a capital loss.

Guimarães (2004) presents a dynamic model of currency crises with frictions, in which agents get opportunities to change position according to a Poisson process. He

[1] In the model of Krugman (1979), the inflation rate jumps up after the switch because the government deficit is financed by seignorage.

[2] Other first generation models include Flood and Garber (1984), and Flood, Garber and Kramer (1996). The latter paper studies the effects of sterilization, by the addition of a bond market. Botman and Jager (2002) build on Krugman (1979) and Flood and Garber (1984) to present a multi-country setting in which coordination and contagion issues can be analyzed.

[3] A rigorous treatment of strategic complementarity requires the description of an ordering over the space of actions. See Cooper (1999) and Vives (1990).

[4] We refer here to a discrete time model, which can be thought of as a sequence of one-period games without strategic interaction between periods.

shows the existence of a unique equilibrium with agents delaying their attack. A discrete devaluation occurs, due to the frictions which slow down the trading process. The amount of the devaluation increases with the domestic holdings at the time of the devaluation, according to a posited relation.

When agents have imperfect information, a situation may arise in which agents end up with excess balance and a discrete devaluation takes place. In this paper, agents do not have common knowledge about each other's information and an outflow of capital. The absence of common knowledge generates a discrete devaluation after some finite time. The model does not rely on frictions, as in Guimarães (2004). In the limit case of perfect information, there is no devaluation, as in Krugman (1979).

Broner (2003) analyzes the issue of currency crises with imperfect information in a model with two types of identical agents: the first type of agents have perfect information about the level of the Central Bank's reserves that terminates the peg. The second type of agents have identical but imperfect information and they observe the actions of the agents in the first group. The model exhibits a continuum of equilibria in which agents of the first type can run at different dates, realizing all their orders, while agents of the second type can sell only a fraction of their holdings at the fixed rate and suffer a capital loss on the residual.

In the present model, each agent views himself as no better informed than others. Agents have imperfect information both about the magnitude of the Central Bank's reserves, and about the information of others. The former assumption is validated by the behavior of various Central Banks towards the diffusion of information during the ERM crisis in 1992-1994. Indeed, during that period and well after, some information about the reserves were published, but with a lag. Statistics published were incomplete.

One may wonder why a Central Bank would not always pretend that the level of its reserves is high. Credibility reasons prevent such claims. In order to protect itself in hard times, the Central Bank better be careful about what it is it reveals in good times. Note that even if the Central Bank were to publish its reserves, agents may not know how much the Central Bank can borrow, or how much it is ready to spend to defend the currency. In addition, there always remain some noise— from the balance of payments or else exogenous capital movements—which hinders the observation of the impact of the speculators on the reserves.

The model builds on that of Abreu and Brunnermeier (2003). They analyze the end of bubbles in a model where agents do not have common knowledge about the onset of the bubble. Although the focus of their analysis is a financial market, there is actually no formal analysis of the mechanism that determines the equilibrium price. The relation between that model and an actual financial market is difficult to see. For example, they assume that the amount of sales has no impact on the bubble price as long as these do not reach an assumed fixed threshold, and the bubble crashes when that level is reached.

In the present paper, the assumptions for iterative dominance have a nice economic foundation and the analysis is considerably simpler than in Abreu and Brunnermeier (2003): i) in the regime of fixed exchange rate, the asset price is fixed by the Central Bank and trades have indeed no impact on that price; ii) the upper bound on the level of total purchases that triggers a change of regime is naturally imposed by the total amount of reserves of the Central Bank; iii) the jump in the exchange rate which occurs at the end of the first regime can be determined endogenously in a macroeconomic model.

The model is presented in the next section. The arrival of a negative shock occurs according to a Poisson process in continuous time. After a shock, a fixed outflow of reserves takes place. The regime of fixed exchange rate is abandoned if the accumulated exogenous outflow plus the sellings by agents exceed the initial reserves of the Central Bank.

We begin by assuming an exogenous rate of devaluation. Agents have imperfect information about the shock. The mass of agents (a continuum) who are informed about the shock increases linearly with time. Hence each informed agent knows that a shock has occurred but not how long before he became informed, and he does not know how many other agents are informed. But each informed agent knows that the exogenous outflow will trigger a devaluation in some finite time. At each instant, an informed agent is comparing the higher return on the domestic currency with the risk of a devaluation.

We first analyze the symmetric Nash equilibrium in Section 3. Under some parameter conditions, an agent delays selling the domestic currency after he becomes informed. Note that each agent knows that the exogenous flow is sufficient to trigger a devaluation after some finite time. Hence a very long delay is a dominated strategy. By iteration on the dominated strategies, we then show that the symmetric Nash equilibrium is the only one to survive the elimination process. The equilibrium is a strongly rational expectation equilibrium (Guesnerie (1992))[5] in the set of all strategies with delay. Note that any trade is admissible within the fixed constraints. Indeed, agents can take any position before the devaluation (the relative price of the two financial assets, the domestic and the foreign currencies, are the same). But it turns out that in the unique equilibrium, an agent sells the domestic currency after some delay. Before this delay, the agent strictly prefers to hold the domestic currency (conditional on no devaluation). After the delay, he strictly prefers to hold the foreign currency. Indeed, once the shock has occurred, the instantaneous rate of return of the foreign currency increases over time.

In Section 4, the rate of devaluation is endogenously determined by setting a value of the post-devaluation real quantity of money. This value could be made to depend on an anticipated policy of the Central Bank. As the rate of information increases, the model with endogenous devaluation rate and imperfect information approaches the model of currency crises with perfect information.

In section 5, we analyze the impact on the delay of the uncertainty about the level of reserves under which the Central Bank will stop defending the peg.[6] We show that the uncertainty increases the delay in selling the domestic currency and extends the *ex ante* mean delay between the exogenous shock and the devaluation.

A last section concludes.

[5] The interest of this equilibrium refinement lies in the fact that it involves no strong conditions on the coordination of agents, as compared with the Nash equilibrium.

[6] To give but one example, during the 1996-1997 currency crisis in Colombia, the Colombian Central Bank was publishing its reserves, but no one knew how much it would spend to defend its currency.

2 The model

Consider an economy with a fixed exchange rate regime compatible with the economy's fundamentals. At some time θ, the economy suffers an adverse shock which changes the fundamentals, and makes the exchange rate incompatible with the fundamentals. The value of θ is determined by an exponential distribution with parameter λ per unit of time. The waiting time before the occurrence of the shock is parameterized by λ: it is the probability of the occurrence of an adverse shock per unit of time conditional on no previous shock. The cumulative distribution function is $F(\theta) = 1 - e^{-\lambda\theta}$ and the density is $f(\theta) = \lambda e^{-\lambda\theta}$.

There is a continuum of agents (speculators) of mass normalized to one who hold one unit of wealth each. We assume that each speculator is risk neutral, and can buy a fixed quantity of foreign currency normalized to one at the price of one.[7] The domestic currency yields a return of r per unit of time. The foreign currency yields no return.

Before the adverse shock, the reserves of the Central Bank are exogenously fixed at $\overline{R}$. The adverse shock generates an outflow of reserves. This outflow can be explained by a change in trades or capital movements. It is not observed by the agents. However, the agents know the structure of this outflow. Let the reserves of the Central Bank at time $\theta + s$ be equal to $R(s)$ with $R(0) = \overline{R}$, $R' < 0$ and $R(T) = 0$ for some $T > 0$ sufficiently large. Once a crisis has begun, the situation of the Central Bank deteriorates. We assume that the reserves of the Central Bank, as observed by the agents, are linear over time:

$$R(s) = \overline{R}(1 - \frac{s}{T}).$$

After some finite interval of time, here time T, the exogenous outflow depletes completely the reserves of the Central Bank assuming no activity by speculators. In this case, a currency crisis occurs exogenously. In general, speculators are active and will sell the currency, thus depleting the reserves at a faster pace. A currency crisis occurs endogenously when the mass of agents who have bought the foreign currency reaches the reserves of the Central Bank. To repeat, a currency crisis is equivalent to the complete depletion of the Central Bank's reserves. It occurs either exogenously or endogenously. When it occurs, the Central Bank devalues the currency by an exogenous value β (the foreign exchange rate appreciates by β), and the game is over. In section 4, the rate of devaluation β will be determined endogenously.

Once a shock has occurred at time θ, speculators become gradually informed about the existence of the shock. Following Abreu and Brunnermeier (2003), we assume that the flow of newly informed speculators is uniform: the agents become informed at a constant rate. The mass of informed agents at time $\theta + s$ is $s\sigma$, for some parameter $\sigma > 0$. A speculator informed at time $t > \theta$ knows only that $\theta < t$: he knows that an exogenous shock has occurred; he does not know when. It follows that he does not know how many other agents are informed.

Let t denote the time when an agent becomes informed about the shock. Let $\mathcal{F}(s)$ be the foreign currency holding of an agent at time $t + s$. His portfolio in domestic and foreign currency at time $t + s$ is defined by $(1 - \mathcal{F}(s), \mathcal{F}(s))$.

[7] We could generalize to the case where the agent can buy a variable and bounded quantity of foreign currency. We will show that delaying more or less than the equilibrium time is strictly iteratively dominated.

A *strategy with delay* corresponds to holding only the domestic currency before time t, and the specification of a measurable function $\mathcal{F}(s)$ for $s \geq 0$ ($\mathcal{F}$ is defined on the positive real numbers equipped with the Lebesgue measure). The function $\mathcal{F}(s)$ specifies the holding of foreign currency contingent on no devaluation before time $t+s$.

In this model, a *delay strategy* y corresponds to holding only the domestic currency before time $t+y$, and if no devaluation has occurred at time $t+y$, to hold only the foreign currency thereafter.

One should emphasize that the set of strategies with delay is much wider than the set of delay strategies, which could also be called trigger strategies.

2.1 Intuition

The problem with one speculator

Consider a single speculator informed at time t. His problem can be described as follows: If he keeps the domestic currency, he earns r per unit of time. If he switches to the foreign currency, he earns no "dividend" on the currency (per unit of time, it is the probability of a devaluation multiplied by the capital gain of the devaluation). In general, the agent compares these two returns.

When our agent becomes informed, does he act immediately (assuming there are no other speculator)? In general, no:

He thinks: *I know that the shock has occurred before I got informed, but I think it will take some time before the reserves of the Central Bank are exhausted. In the mean time, I can earn a return r on the domestic currency.*

Our agent should not wait too long: he knows the reserves are exhausted a time T after a shock. The longer he waits, the higher the probability of a sudden devaluation. If he waits, say $T-\epsilon$, and by that time he is lucky that no devaluation has occurred, he knows that the devaluation will occur within ϵ and that this probability is very large.

To summarize: our agent will do a Bayesian analysis, using the model, to compute the instantaneous probability of a devaluation at any time after he becomes informed. Let $\pi(x)$ be this probability if he delays x. We will show later that π is an increasing function.

The expected return from holding the domestic currency is r. The expected return from holding the foreign currency is $\beta\pi(x)$. Hence the optimal strategy of the agent is a delay x^* such that $r = \beta\pi(x^*)$. Before x^*, the domestic currency yields a higher return and the agent strictly prefers to hold the domestic currency (conditional on no devaluation). After x^*, the foreign currency yields a higher return and the agent strictly prefers to hold the foreign currency.

The problem with a continuum of speculators

The problem is essentially the same when there is a continuum of speculators.

Our speculator should not delay too long: since other speculators are active, he should delay less than in the above case.

Does he delay at all? Yes.

The "worst" case for him occurs when other speculators act as soon as they become informed. But in this case, when our speculator becomes informed, he knows the reserves have been partially depleted at a faster pace than when he was alone. But he is still betting, under suitable parameters of the model, that at the instant he becomes informed, the instantaneous probability of a devaluation is sufficiently low that holding the domestic currency for a while is optimal.

These arguments are used later to show that the model has a unique strongly rational expectations equilibrium in the set of strategies with delay. In that SREE, all agents have the same delay.

To analyze the workings of the model, we first make the assumption of a symmetric delay strategy in the next section. We will consider the much wider set of strategies with delay in section 3.2, and prove the uniqueness of the equilibrium in that set of strategies.

3 Strategies and equilibrium

Agents become gradually informed about the occurrence of the shock. As time goes, more and more agents become informed and the risk of devaluation increases. The payoff of trading goes up. The strategy of an agent is thus to delay and then trade.

The delay strategy of any agent depends on the time at which he becomes informed.

Given θ, let τ denote the time elapsed between θ and the devaluation. Assume first all agents delay for x. Then $\sigma(\tau - x)$ represents the purchases between $\theta + x$ and $\theta + \tau$ and $\overline{R}\tau/T$ represents the exogenous losses of the Central Bank. The equation

$$\sigma(\tau - x) + \overline{R}\frac{\tau}{T} = \overline{R} \tag{1}$$

defines τ as a function of x: the time elapsed between the onset of the deterioration and the devaluation depends on the strategy x of the agents.

Consider now an agent who receives a signal about θ at time t and assume all other agents delay for an interval of time x after being informed. We look for a symmetric equilibrium where our agent buys the foreign currency at time $t + y$.

If no devaluation has occurred after our agent delayed for y, while all other agents delay for x, the reserves of the Central Bank are still positive at time $t + y$. The onset of the deterioration, θ, cannot date back to a very long time (otherwise the reserves would already be depleted). More specifically, the earliest time for the shock is such that the reserves would be depleted immediately after time $t + y$. Given the strategy x, θ satisfies $t - l \leq \theta \leq t$ for some function $l = \phi(y; x)$ of y and x, with

$$\sigma(l + y - x) + \overline{R}\frac{l + y}{T} = \overline{R}, \tag{2}$$

where $\sigma(l + y - x)$ represents the purchases between $\theta + x$ and $\theta + l + y$ and $\overline{R}(l + y)/T$ represents the exogenous losses of the Central Bank.[8]

Equation (2) can be rewritten as follows:

$$l = \phi(y; x) = \frac{x + \frac{\overline{R}}{\sigma} - y(1 + \frac{\overline{R}}{\sigma T})}{1 + \frac{\overline{R}}{\sigma T}}. \tag{3}$$

[8] Whenever $y = x$, we find (1) once we identify τ with $l + y$.

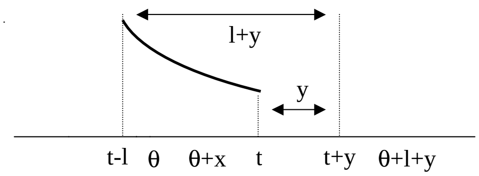

Figure 1

Note that l, the length of the support of θ, is a decreasing function of the delay strategy y: the larger is the delay y, the smaller is l or else the closer is θ to t, the date at which the agent receives some information.

For an agent informed at t which delays for y, θ belongs to the interval $[t-l, t]$; the minimal value taken by θ is $t-l$. If there is a devaluation at $t+y$, then θ is precisely equal to $t-l$.

Once the agent is informed at date t, he revises his belief about θ. After a delay of y, his subjective distribution about θ is the exponential distribution truncated on the support $[t-\phi(y;x), t]$. Its density is $f(\theta;y,x) = Ae^{-\lambda\theta}$ with $1/A = \int_{t-\phi}^{t} e^{-\lambda\theta} d\theta$. Hence,

$$f(\theta;y,x) = \frac{\lambda e^{-\lambda(\theta-t)}}{e^{\lambda\phi(y;x)} - 1}. \tag{4}$$

3.1 The reaction function

Given that all other agents delay for an interval of time x, the instantaneous probability of the devaluation for an agent informed at t which delays for y is the exponential distribution of θ truncated on the interval $[t-\phi(y;x), t]$ evaluated at $\theta = t - \phi(y;x)$:

$$\pi(y;x) = \frac{\lambda}{1 - e^{-\lambda\phi(y;x)}}. \tag{5}$$

Indeed, a devaluation occurs at $t+y$ if the reserves are depleted at $t+y$, i.e. if θ is at the earliest date of the support of its distribution for the agent, namely $t-\phi(y;x)$. A devaluation occurs according to a Poisson process with an instantaneous probability equal to the density of the distribution of θ at $t-\phi(y;x)$.

For a given strategy x, the function $\phi(y;x)$ is decreasing in y, and hence $\pi(y;x)$ is an increasing function of y: the larger is the delay, the larger is the instantaneous probability of devaluation.

Moreover, an agent knows that the largest delay between the time he gets informed and a devaluation occurs when he is the first to be informed. In this case, the delay is $y_0 = \frac{x+\frac{\overline{R}}{\sigma}}{1+\frac{\overline{R}}{\sigma T}}$. One verifies that $\pi(y;x)$ tends to infinity as y approaches y_0.

We now analyze the optimal delay. Assuming the agent delays for y, let $dy < 0$ denote a reduction in the delay and consider the impact of this modification on the gain in case of devaluation, $\beta\pi(y;x)$.

If $\beta\pi(y;x) > r$, then $(\beta\pi(y;x) - r)dy < 0$ and the agent should delay for an even shorter period of time. Indeed, the gain from the instantaneous probability of a devaluation is greater than the opportunity cost of the interest income on the domestic asset.

Similarly, if $\beta\pi(y;x) < r$, then $(\beta\pi(y;x) - r)dy > 0$ and hence the agent should delay for a longer period of time.

The *arbitrage condition* for buying the foreign currency after a delay of y is

$$\beta\pi(y;x) = r. \tag{6}$$

This condition defines a reaction function of an agent informed at t which delays y, when all other agents delay x.

Using (5), we have

$$\beta\pi(y;x) = \frac{\beta\lambda}{1 - e^{-\lambda\phi(y;x)}}. \tag{7}$$

Recall that for a given x, $\beta\pi(y;x)$ is increasing in y. From the expression of ϕ in (3), one can show that the graph of $\beta\pi(\cdot;x)$ is as shown in Figure 2 with $\beta\pi(0;x) > 0$. From the figure, it is immediate that there is a unique value $y(x) > 0$ such that $\beta\pi(y(x);x) = r$ if and only if $\beta\pi(0;x) < r$. If $\beta\pi(0;x) > r$, then the agent should buy the foreign currency without delay and hence $y(x) = 0$ is his optimal strategy.

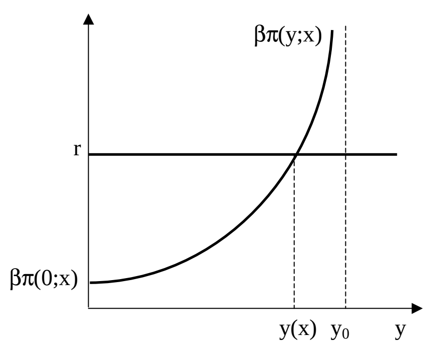

Figure 2

From the previous discussion, the *reaction function* of an agent informed at t which delays for y, given that all the other agents delay for x, is

$$y(x) = \frac{x + \frac{\overline{R}}{\sigma}}{1 + \frac{\overline{R}}{\sigma T}} - \frac{1}{\lambda}\log\left(\frac{r}{r - \beta\lambda}\right). \tag{8}$$

The slope of y as a function of x is smaller than one, and equal to $\frac{1}{1+\frac{\overline{R}}{\sigma T}}$.

In order to forego trivial cases, we make the following assumption.

Assumption 1: $r > \beta\pi(0;0) = \frac{\beta\lambda}{1-e^{-A}} > \beta\lambda$, where $A = \frac{\lambda\frac{\overline{R}}{\sigma}}{1+\frac{\overline{R}}{\sigma T}}$.

Assumption 1 implies that $y(0) > 0$: the agent has a positive delay strategy.

The reaction function is illustrated in Figure 3. From Figure 3, it follows that there is a unique Nash equilibrium; it is the fixed point of the reaction function (8). The slope of the reaction function shows that there is strategic complementarity within the set of delay strategies.

The properties of the equilibrium strategy y^* are easily computed and are summarized in the next proposition.

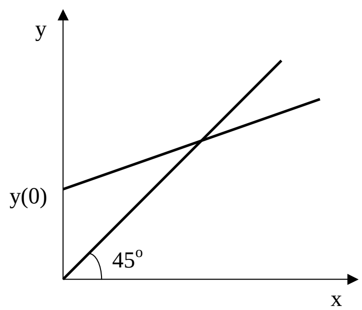

Figure 3

Proposition 3.1 *Under Assumption 1, the unique symmetric Nash equilibrium in the set of delay strategies is given by*

$$y^* = T - (1 + \frac{\sigma T}{\overline{R}})\frac{1}{\lambda}\log(\frac{r}{r - \beta\lambda}).$$

The equilibrium delay strategy y^* is increasing in T, in $\overline{R}$ and in r. It is decreasing in σ, in β and in λ.

The condition $r > \beta\pi(0,0)$ is minimal in the sense that if r were not restricted to be above a given value, the coordination problem would become trivial: in an equilibrium, agents would buy the foreign currency as soon as they are informed.

The equilibrium delay strategy is increasing in the interest rate and in the initial amount of reserves. The policies of the Central Bank are thus instrumental in the choice of the agent. As expected, the equilibrium delay strategy is negatively related to the gain in case of devaluation β and to the rate of information σ. The smaller the exogenous flow (the larger is T), the larger is the delay. Finally, if the probability λ of the occurrence of an adverse shock per unit of time, conditional on no previous shock, is large, then the delay is small.

Remark From (1), in equilibrium, the time τ between a shock θ and a devaluation is given by

$$\tau = T\frac{\overline{R} + \sigma y^*}{\overline{R} + \sigma T}.$$

As expected, $\tau < T$, and τ is an increasing function of T.

By definition, $1/\sigma$ is the time it takes for all agents to become informed of the shock. When the exogenous flow is small (T is large), all the agents are informed about the shock when a devaluation occurs: $\tau > 1/\sigma$. Using the definition of y^*, this condition is equivalent to

$$1 - \frac{\sigma}{\overline{R}\lambda}\log(\frac{r}{r - \beta\lambda}) > \frac{1}{\sigma T}.$$

Note that the LHS is always positive under Assumption 1.

A devaluation occurs when the exogenous flow is small, and it is not due to the fact that some speculators are not informed about the shock. Indeed, it may be that at the time of devaluation, all agents are informed, and that they all know that all agents are informed (i.e. $\tau > 2/\sigma$)[9]. A discrete devaluation is caused because speculators, who get informed sequentially, do not know their position in the information queue that was initiated by the shock.

3.2 Uniqueness of equilibrium

We now show that the equilibrium y^* in Proposition 3.1 is the unique stable equilibrium in the full set of strategies with delay. For this, we show that y^* is a strongly rational expectation equilibrium (SREE) (Guesnerie (1992)) — i.e. any strategy with delay $y \neq y^*$ is iteratively dominated and y^* is not iteratively dominated.[10] The equilibrium is "stable" in the sense that the sequence of reactions of the agents to any strategy of the others converges to the equilibrium.[11] In showing this result, we strengthen the symmetric Nash equilibrium with strategy y^*. Indeed, a critical issue in this paper is the coordination of agents without common knowledge. We here provide a refinement over Proposition 3.1, as it does not involve strong requirements on the coordination of agents, as compared to the Nash equilibrium.

Any trade is admissible within the fixed constraints. Agents can take any position before the devaluation (the relative price of the two financial assets, the domestic and the foreign currencies, are the same). Once the shock has occurred, the instantaneous rate of return of the foreign currency increases over time. For an agent informed about the occurrence of the shock, any long position in the domestic currency is a dominated strategy after some period of time. Any long position in the foreign currency for some time interval after the shock has occurred is also a dominated strategy.

The only restriction is that we assume the agents act only after being informed. Note that if an agent takes an action at time t without being informed, his strategy depends only on the time t. An action taken at some time t is successful only if other agents coordinate on the same date. Even if such a coordination could be achieved without some external device, it is not clear that there is a Nash equilibrium strategy where the action depends only on the time t.

Recall that if no shock has occurred, the stock of currency of speculators is strictly smaller than the reserves of the Central Bank, $\bar{R}$. An attack can be successful at time t only if a shock has occurred at some earlier time θ such that $\bar{R}(1-(t-\theta)/T) < 1$. Even if the probability of this event for an uninformed agent is small, all uninformed speculators could attack at the same time t and undo their position after a vanishingly short interval η if the attack fails. The opportunity cost of such a strategy can be made arbitrarily small when $\eta \to 0$. But if all agents follow such a strategy, any one of them

[9]It takes an interval of time $2/\sigma$ for all agents to know that all agents are informed: the last agent to be informed may be informed at time $\theta + 1/\sigma$, in which case, if he thinks he is the first to be informed, he will think that everyone is informed after another interval of time $1/\sigma$.

[10]Given that all agents have the same set of strategies $J \subset \mathbf{R}$, recall that a strategy y is iteratively dominated if there is a finite sequence of increasing sets $I_0 = \emptyset, ..., I_N$, with $y \in I_N$, such that strategies in I_k are strictly dominated when all agents play in the subset of strategies $J \setminus I_{k-1}$.

[11]Bernheim (1984), Pearce (1984).

has an incentive to act just before the others in order to avoid any rationing of the Central Bank's available reserves if the attack succeeds.

From these remarks, it is not obvious whether there is a Nash equilibrium in which uninformed agents take action. One may also conjecture that the introduction of a time lag between an order and its execution, or of a minimum length of time for holding an asset, would put a lower bound on the opportunity cost of attacking the currency and would eliminate any profitable strategy of action for an uninformed agent. These problems are bypassed to concentrate on the main issues.

Proposition 3.2 *The equilibrium delay strategy y^* in Proposition 3.1 is a strongly rational expectation equilibrium in the set of strategies with delay where agents take action after being informed of the shock.*

The proof can be found in Appendix A.

4 First generation models of currency crises

First generation models of currency crises are characterized by two exchange rate regimes, separated by the crisis. For simplicity, the demand for money depends only on the domestic inflation rate. In the first regime, the exchange rate is fixed and the government runs a deficit that is financed by the Central Bank. As the exchange rate is fixed, the price level is fixed (by purchasing power parity) and the demand for money, which depends on the inflation rate (0 in this regime) is constant. Hence the assets of the Central Bank remain constant, and the government bonds gradually crowd out the foreign reserves. This process must eventually stop. The exchange rate must eventually be abandoned. Following that event, there is a second regime in which the exchange rate floats and the level of foreign reserves in the Central Bank is constant. In that regime, the deficit which continues to be financed by money creation increases the money supply and the inflation rate is strictly positive and constant (and equal to the rate of appreciation of the foreign exchange). The jump in the inflation rate forces the real demand for money to jump down when the fixed rate regime is abandoned.

Krugman (1979) assumes that agents have perfect information. Under this assumption, the exchange rate cannot jump. The jump in the real quantity of money is therefore achieved by a jump in the nominal quantity of money: at the time of the switch, agents run to trade a stock of money equal to the difference in the nominal quantity demanded before and after the switch.

The model of Krugman (1979) remains unsatisfactory because of the perfect foresight assumption. Note that no sudden devaluation takes place in that model. This property does not fit the experiences of currency crises. We extend the model of the previous sections to address the issue analyzed by Krugman when agents have imperfect information about an exogenous shock that triggers a gradual depletion of the foreign currency reserves of the Central Bank. This model is equivalent to a model where the depletion of reserves is induced by a government deficit. We will show that a devaluation occurs in a currency crises where agents have incomplete information. When agents have near perfect information, the model will generate the same properties as in Krugman.[12]

Without loss of generality, we assume that the domestic quantity of money is equal to the liabilities of the Central Bank and that it is the sum of the speculators' holdings, K, and a demand for transactions, D.

As in the previous model, speculators hold domestic currency in a regime of fixed exchange rate because of the interest rate premium. After a devaluation, we assume no interest premium and speculators have no demand for the domestic currency. This is a stylized way to think of the model of Krugman where a portfolio equation is defined and in which the inflation rate plays the role of a tax on domestic currency. This "tax" increases in the second regime, leading to a decrease in the demand for domestic currency.

The demand for transactions is set such that

$$\frac{D}{P} = k,$$

where P is the exchange rate and k is a parameter. The value of P is equal to 1 under a fixed exchange rate and it is equal to the value determined by the market at the instant after the regime of fixed exchange rate is abandoned. The rate of devaluation β is now endogenous: $\beta = P - 1$.

Let K be the domestic currency holdings of the speculators at the time of devaluation. The quantity of money at that time is therefore $K + D$. Since speculators do not hold domestic currency after the devaluation, we have

$$\frac{D + K}{P} = k.$$

Hence

$$\beta = \frac{D + K}{k} - 1 = \frac{K}{k}. \tag{9}$$

Let K_0 be the mass of speculators each holding initially one unit of domestic currency. The initial reserves of the Central Bank, $\overline{R}$, are larger than K_0. As in the previous sections, an exogenous shock occurs at some time θ after which there is an exogenous loss of reserves with a flow $\overline{R}/T$ per unit of time, where $\theta + T$ is the time at which an exogenous devaluation will occur.

The structure of information for speculators is the same as in the previous sections with the flow of newly informed agents per unit of time equal to σ.

At the time of devaluation $\theta + \tau$, the holdings of speculators are equal to

$$K = K_0 - \sigma(\tau - y^*),$$

where y^* is the equilibrium delay strategy. Using (1), $\tau = a + \nu y^*$ where $a = \frac{\overline{R}}{\sigma}/[1 + \frac{\overline{R}}{\sigma T}]$ and $\nu = 1/[1 + \frac{\overline{R}}{\sigma T}] < 1$. Hence $K = K_0 - \sigma a + \sigma(1 - \nu)y^*$. Substituting this value

[12] Broner (2003) also analyzes in a different setting a model of currency crisis with imperfect information. He assumes two types of agents: the first have perfect information and behave as in Krugman; the second observe only the actions of the first with a vanishingly small time lag. When the first type of agents run (always avoiding a capital loss because they have perfect information), the second type of agents are surprised to hold currencies which they can sell only with a capital loss. The model of Broner generates a continuum of equilibria among which one is selected by an ad hoc rule. In our model, there is a unique equilibrium which is a SREE.

of K in (9), we get an expression which defines the rate of devaluation, $\tilde{\beta}(y^*)$, as an increasing function of the equilibrium delay, y^*:

$$\tilde{\beta}(y^*) = \frac{1}{k}(K_0 - \frac{\overline{R}}{1+\frac{\overline{R}}{\sigma T}}) + \frac{y^*}{k}\frac{\frac{\overline{R}}{T}}{1+\frac{\overline{R}}{\sigma T}}. \tag{10}$$

This property is intuitive: if speculators delay longer, they hold more domestic currency at the time of devaluation and the rate of devaluation must be higher for the money market equilibrium after the devaluation.

Consider now a devaluation rate β. From Proposition 3.1, the equilibrium delay strategy is

$$\tilde{y}^*(\beta) = T - (1+\frac{\sigma T}{\overline{R}})\frac{1}{\lambda}\log(\frac{r}{r-\beta\lambda}). \tag{11}$$

As already mentioned in that proposition, the delay in the equilibrium strategy is a decreasing function of the rate of devaluation β. When $\beta = 0$, then $\tilde{y}^*(\beta) = T$ and when $\beta = \frac{r}{\lambda}[e^\alpha - 1]/e^\alpha$ with $\alpha = \lambda \overline{R}T/[\overline{R}+\sigma T]$, then $\tilde{y}^*(\beta) = 0$.

The graphs of the functions $\tilde{\beta}(y^*)$ and $\tilde{y}^*(\beta)$ defined in (10) and (11) are represented in Figure 4.

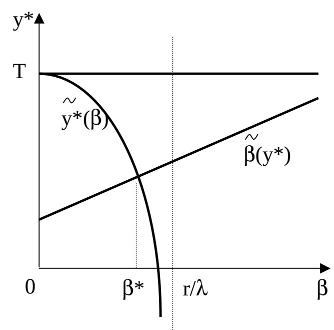

Figure 4

The parameters of the model are such that in (10) for $\tilde{\beta}(y^*) = 0$, $y^* \in (0, T)$. The two schedules have a unique intersection that determines endogenously the rate of devaluation, β^*.

As in section 3.1, we can see that when the exogenous flow is small (T is large), all the agents are informed when the devaluation occurs ($\tau > 1/\sigma$). Indeed, assume the exogenous flow is small. Then the vertical intercept of the function $\tilde{\beta}$ gets very large, which implies a very large equilibrium value y^*. We then have $y^* > 1/\sigma$ and thus $\tau > 1/\sigma$ as τ is proportional to y^*.

Recall that σ represents the rate of information or the speed at which the agents get informed. When σ increases, the agents become informed more quickly about the occurrence of the shock. Let us analyze the variations of the rate of devaluation and the equilibrium delay strategy with σ. We have

$$\frac{d\tilde{\beta}}{d\sigma} = -\frac{1}{k}\frac{\overline{R}^2}{(\sigma T)^2}\frac{1}{(1+\frac{\overline{R}}{\sigma T})^2}(T - y^*), \tag{12}$$

which is negative for any $y^* \in (0,T)$, and

$$\frac{d\tilde{y}^*}{d\sigma} = -\frac{1}{\lambda}\frac{T}{\overline{R}}\log(\frac{r}{r-\beta\lambda}) < 0, \tag{13}$$

as found in Proposition 3.1. Hence, an increase in the rate of information decreases the gain in case of devaluation as long as the delay strategy is smaller than T (the time at which the devaluation occurs exogenously), and it decreases the delay.

Proposition 4.1 *The equilibrium value of the endogenous rate of devaluation, β^*, decreases when the rate of information σ increases.*

The limit value of the rate of devaluation as the rate of information tends to infinity is given by

$$lim_{\sigma\to\infty}\tilde{\beta}(y^*) = \frac{1}{k}(K_0 - \overline{R}) + \frac{\overline{R}}{kT}y^*.$$

As σ tends to infinity, the intersection between the functions $\tilde{\beta}(y^*)$ and $\tilde{y}^*(\beta)$, which determines the equilibrium value of the endogenous rate of devaluation, tends to the value of y^* for which $\tilde{\beta}(y^*) = 0$, namely

$$y^* = T(1 - \frac{K_0}{\overline{R}}) > 0. \tag{14}$$

Proposition 4.2 *When the rate of information σ tends to infinity, the equilibrium value of the endogenous rate of devaluation β^* tends to zero. The equilibrium value of the delay tends to y^* defined by $\frac{y^*}{T} = 1 - \frac{K_0}{\overline{R}} > 0$.*

When σ tends to infinity, all agents are informed at nearly the same time. This limit case corresponds to the model of Krugman (1979) of the first generation. Speculators delay y^*, until the level of the Central Bank's reserves is just equal to their own balances, which they trade in at the same time with no capital loss. There is no jump of the exchange rate hence no devaluation when the peg is abandoned.

5 Uncertainty about the reserves

In this last section, we study the influence of uncertainty about the reserves on the instantaneous probability of a devaluation, in order to determine whether the Central Bank should reveal or not the extent to which it will defend the peg.

Let the initial amount of reserves be $\overline{R}$. Assume the Central Bank can decide to defend the peg using more than its reserves through some borrowing, or else decide to stop defending the peg before the reserves are exhausted. Assume the only thing the agents know is that the Central Bank will adopt one of the two policies, with equal probability.

The outflow of reserves is the same as in the model with certainty and the reserves of the Central Bank are linear over time, as defined previously:

$$R(s) = \overline{R}(1 - \frac{s}{T}).$$

Let $A_1 = \overline{R} - A$ and $A_2 = \overline{R} + A$. The parameter A represents the differential in the level of reserves with respect to the mean of the reserves, $\overline{R}$. A devaluation will then occur exogenously[13] either at date T_1 or date T_2, where

$$T_i = \frac{A_i}{\overline{R}} T$$

with T as in section 2.

Without loss of generality, we denote by R_i the state in which the reserves are exogenously depleted at T_i. Let S denote the event of being informed about the possible occurrence of a crisis, while the crisis (the devaluation) has not yet occurred.

Assuming an agent delays for y and given that all other agents delay for an interval of time x, the *instantaneous probability of the devaluation* is

$$\pi_u = \mu \frac{\lambda}{1 - e^{-\lambda \phi_1(y;x)}} + (1 - \mu) \frac{\lambda}{1 - e^{-\lambda \phi_2(y;x)}}, \tag{15}$$

where μ is the probability that the state is R_1 given that the devaluation has not occurred yet and ϕ_i defines the length of the support of θ if the devaluation occurs exogenously at date T_i.

A few computations, the details of which can be found in Appendix B, show that the uncertainty about the reserves extends the delay in selling the domestic currency.

Proposition 5.1 *The uncertainty about the reserves decreases the instantaneous probability of a devaluation π_u.*

Should the Central Bank reveal its policy regarding the reserves or not?

We have assumed until now that the priors are 1/2 for every agent ($P(R_1) = P(R_2) = 1/2$). We will assume from now on that the Central Bank wishes to maximize the *ex ante* expected delay between the shock and the devaluation.

When there is a distribution of possible levels of reserves used to defend the peg, define the *certainty equivalent* as the level of reserves equal to the mean of these level of reserves. The following result is shown in Appendix B.

Proposition 5.2

i. When there is more than one possible level of reserves used by the Central Bank to defend the peg, which are not observed, the mean delay between the shock and the devaluation is greater than the delay under the equivalent certainty case.

ii. When there is more than one possible level of reserves used by the Central Bank to defend the peg, which are assumed to be perfectly known, the mean delay between the shock and the devaluation is equal to the delay under the equivalent certainty case.

From Proposition 5.2, we can infer that if the objective of the Central Bank consists in maximizing the ex ante expected delay between the shock and the devaluation, it should not reveal any information about the extent to which it will defend the peg.

[13] A devaluation occurs exogenously when the exogenous outflow generated by the shock depletes completely the reserves of the Central Bank without any intervention of speculators.

6 Conclusion

We have considered an economy with a fixed exchange rate regime that has suffered an adverse shock at some random time. Speculators know that a devaluation will occur at some exogenous time in the future. A coordination problem appears as a devaluation may well be triggered by the actions of the speculators before it occurs exogenously. The strategies of the speculators are at the heart of this coordination problem.

In this game, the agents become gradually informed at a constant rate about the occurrence of the shock. A devaluation occurs exogenously or endogenously as soon as the reserves of the Central Bank are completely depleted.

We have shown that the game played by the speculators has a unique symmetric Nash equilibrium which is a strongly rational expectation equilibrium. We therefore depart from the second generation models characterized by multiple equilibria as we present a determinate equilibrium solution, as in Morris and Shin (1998). Moreover, we have determined endogenously the rate of devaluation.

The model could be extended to include additional shocks on the fundamentals. For example, the exogenous outflow that was set here by a random process could be stopped according to a second random process. One can imagine that a policy of the Central Bank that was able to delay a crisis under a permanent shock could avoid entirely a crisis when the shock is transitory.

There are various ways of introducing uncertainty in this model. We have focussed here on an uncertainty regarding the level of reserves used by the Central Bank to defend its currency. We could equally well have considered different rates of information, or else different masses of speculators.

The issue of transparency is crucial in international financial market policies. The results in this paper show that in the short term, during a crisis, transparency may have adverse effects for the policy maker. However, it is well acknowledged by international organizations that transparency is efficient in the long term as it may foster credibility. Clearly, an analysis linking long term and short term issues is desirable, and will be the subject of a subsequent paper.

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7 Appendix A : Proof of Proposition 3.2

- We prove first that any strategy $y > y^*$ is iteratively dominated.

  Recall that for an agent informed at time t, $\mathcal{F}(s)$ is his foreign currency holding at time $t+s$. His portfolio in domestic and foreign currency at time $t+s$ is defined by $(1-\mathcal{F}(s), \mathcal{F}(s))$.

  We will say that an agent does not delay longer than s' if $\mathcal{F}(s) = 1$ for any $s \geq s'$. Note that this definition does not restrict the path of $\mathcal{F}(s)$ for $s < s'$.

  The instantaneous expected rate of return of the portfolio specified by the strategy at time $t+s$ is $r + \mathcal{F}(s)[\beta\tilde{\pi}(s) - r]$, where $\tilde{\pi}(s)$ is the instantaneous probability of a devaluation and depends on the strategy of others.

If $\beta\tilde{\pi}(s) > r$, for some interval of time dt, any strategy $\mathcal{F}$ with $\mathcal{F} < 1$ for a set of positive measure in the interval of time dt is strictly inferior to the same strategy where $\mathcal{F}$ is replaced by 1 in that interval of time.

If $\beta\tilde{\pi}(s) < r$, for some interval of time dt, any strategy $\mathcal{F}$ with $\mathcal{F} > 0$ for a set of positive measure in the interval of time dt is strictly inferior to the same strategy where $\mathcal{F}$ is replaced by 0 in that interval of time.

An agent informed at time t knows that a devaluation will occur no later than time $t + T$.

We may assume that for any strategy, $\mathcal{F}(s) = 1$ for $s \geq T$ without any impact on the probability $\tilde{\pi}(s)$ as a devaluation occurs before time $t + T$, where t is the instant an agent is informed.

Assume now that the agents delay no longer than x_k, with $x_1 = T$. The level of reserves at any time after θ is not greater than when all agents delay for the upper-bound x_k. Therefore, the delay between θ and a devaluation is bounded above by the value found in the symmetric case with all agents delaying x_k. Hence, for an agent informed at t, if no devaluation has occurred by time $t + y$, then $t - \phi(y; x_k)$ is the lower bound of θ for any strategies of the other agents with delay no longer than x_k, and ϕ defined in equation (3). The support of θ is therefore in the interval $[t - \phi(y; x_k), t]$.

Recall that in the symmetric case where agents delay for x_k, a devaluation occurs at $t + y$ when θ is "at" (in the neighborhood of) the lower bound of the support $[t - \phi(y; x_k), t]$. In general, when θ is not restricted to this lower-bound, the instantaneous probability of a devaluation is not smaller than in the symmetric case. Therefore, $\tilde{\pi}(y) \geq \pi(y; x_k)$ where $\pi(y; x_k)$ is the instantaneous probability of a devaluation in the symmetric case presented in section 3.

Let x_{k+1} be defined by $\beta\pi(x_{k+1}; x_k) = r$. We know that $\pi(y; x_k)$ is strictly increasing in y. Hence for any $y > x_{k+1}$, and for any strategy of the other agents that do not delay longer than x_k, $\tilde{\pi}(y) \geq \pi(y; x_k) > \pi(x_{k+1}; x_k) = \frac{r}{\beta}$. Holding the domestic currency for any $y > x_{k+1}$ (i.e. $\mathcal{F}(y) < 1$ for a set of measure different from zero) is dominated by $\mathcal{F}(y) = 1$.

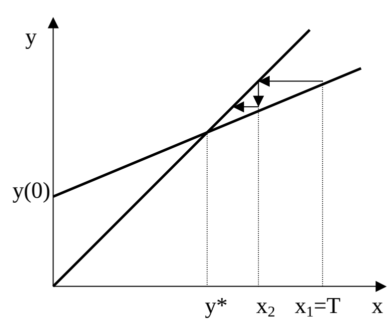

Figure 5

The sequence $\{x_k\}$ is generated by the reaction function y defined in section 3: $x_{k+1} = y(x_k)$. This sequence is monotonically converging to y^*, as illustrated in Figure 5. Therefore, any strategy with a delay beyond y^* is iteratively dominated.

- We now show that any strategy $y < y^*$ is iteratively dominated. The argument is similar to the one used in the previous case.

  Assume the agents delay at least x_k, with $x_1 = 0$. The delay between θ and a devaluation is bounded below by the value found in the symmetric case with all agents delaying x_k. For an agent informed at t, if no devaluation has occurred by time $t+y$, then $t-\phi(y;x_k)$ is the lower bound of θ for any strategies of the other agents with delay of at least x_k, and ϕ defined in equation (3). The support of θ is therefore in the interval $[t-\phi(y;x_k), t]$.

  Recall that in the symmetric case where agents delay for x_k, a devaluation occurs at $t+y$ when θ is "at" (in the neighborhood of) the lower bound of the support $[t-\phi(y;x_k), t]$. In general, when θ is not restricted to this lower-bound, $\tilde{\pi}(y) \leq \pi(y;x_k)$ where $\pi(y;x_k)$ is the instantaneous probability of a devaluation in the symmetric case presented in section 3.

  Let x_{k+1} be defined by $\beta\pi(x_{k+1};x_k) = r$. Since $\pi(y;x_k)$ is strictly increasing in y, for any $y < x_{k+1}$, then $\tilde{\pi}(y) \leq \pi(y;x_k) < \pi(x_{k+1};x_k) = \frac{r}{\beta}$. Hence holding the foreign currency for any $y < x_{k+1}$ (i.e. $\mathcal{F}(y) \geq 0$ for a set of measure different from zero) is dominated by $\mathcal{F}(y) = 0$.

  The sequence $\{x_k\}$ is generated by the reaction function y in section 3: $x_{k+1} = y(x_k)$. This sequence is monotonically converging to y^* as illustrated in Figure 6. Therefore, any strategy $y < y^*$ is iteratively dominated.

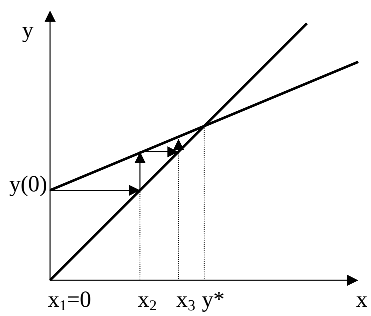

Figure 6

□

8 Appendix B : Uncertainty about the reserves

Given the notation introduced in section 5, the *instantaneous probability of the devaluation* for an agent that delays for y, given that all other agents delay for an interval of time x, is

$$\pi_u = \mu\frac{\lambda}{1-e^{-\lambda\phi_1(y;x)}} + (1-\mu)\frac{\lambda}{1-e^{-\lambda\phi_2(y;x)}}, \tag{16}$$

where μ is the probability that the state is R_1 given that the devaluation has not occurred yet:

$$\mu = P(R_1|S) = \frac{P(S|R_1)P(R_1)}{P(S|R_1)P(R_1) + P(S|R_2)P(R_2)} = \frac{P(S|R_1)}{P(S|R_1) + P(S|R_2)} \tag{17}$$

(as $P(R_1) = P(R_2) = 1/2$), and $l_i = \phi_i(y;x)$ defines the length of the support of θ if the devaluation occurs exogenously at date T_i:

$$l_i = \phi_i(y;x) = \frac{\sigma x + A_i}{\sigma + \frac{\overline{R}}{T}} - y. \tag{18}$$

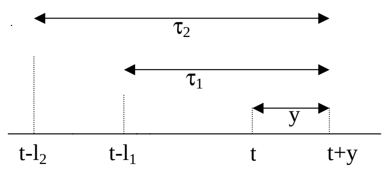

Figure 7

The instantaneous probability of the devaluation if the state is R_i, $P(S|R_i)$, is computed as follows:

$$P(S|R_i) = e^{-\lambda(t-\phi_i(y;x))} \int_0^{\phi_i(y;x)} \lambda e^{-\lambda u} du = e^{-\lambda(t-\phi_i(y;x))}(1 - e^{-\lambda\phi_i(y;x)}).$$

Hence

$$\mu = \frac{e^{-\lambda(t-\phi_1(y;x))}(1 - e^{-\lambda\phi_1(y;x)})}{e^{-\lambda(t-\phi_1(y;x))}(1 - e^{-\lambda\phi_1(y;x)}) + e^{-\lambda(t-\phi_2(y;x))}(1 - e^{-\lambda\phi_2(y;x)})} = \frac{e^{\lambda\phi_1} - 1}{e^{\lambda\phi_1} + e^{\lambda\phi_2} - 2}.$$

The instantaneous probability of a devaluation is thus given by[14]

$$\pi_u = \lambda \frac{e^{\lambda\phi_1} + e^{\lambda\phi_2}}{e^{\lambda\phi_1} + e^{\lambda\phi_2} - 2}. \tag{19}$$

By definition, $\phi_i(y;x) = \tau_i - y$ (Figure 7), where $\tau_i = a_i + \nu x$, with $a_1 = a - \eta$, $a_2 = a + \eta$, $a = \frac{\overline{R}}{\sigma + \frac{\overline{R}}{T}}$, $\eta = \frac{A}{\sigma + \frac{\overline{R}}{T}}$, and $\nu = \frac{\sigma}{\sigma + \frac{\overline{R}}{T}}$.

From these observations, the instantaneous probability of a devaluation takes the following form:

$$\pi_u(y;x) = \lambda \frac{\frac{e^{\lambda a_1} + e^{\lambda a_2}}{2}}{\frac{e^{\lambda a_1} + e^{\lambda a_2}}{2} - e^{\lambda(y - \nu x)}}. \tag{20}$$

The function e^x being convex, Proposition 5.1 follows.

Should the Central Bank reveal its policy regarding the reserves or not?[15]

We have assumed until now that the priors are 1/2 for every agent ($P(R_1) = P(R_2) = 1/2$). Suppose the Central Bank wishes to maximize the *ex ante* expected delay between θ and the devaluation, namely $\tau = \frac{\tau_1 + \tau_2}{2}$.

[14]Note that for $\phi_1 = \phi_2 = \phi$, then $\pi_u = \pi$: the instantaneous probability of devaluation in case of uncertainty equals the instantaneous probability of devaluation with no uncertainty.

[15]The Central Bank designs its policy without being informed of the shock. Any information about the shock would be a signal.

Case 1 The Central Bank *does not* reveal its policy.

In this case, $a_1 = a - \eta$ and $a_2 = a + \eta$, and the *ex ante* expected delay between θ and the devaluation is $\tau = \frac{\tau_1+\tau_2}{2} = a + \nu x$, where x is solution to $\pi_u(x;x) = c$ with $c = r/\beta$ a parameter that depends on the interest rate and the rate of devaluation, and $\pi_u(x;x)$ is the equilibrium value of the instantaneous probability of devaluation in (20):

$$\pi_u(x;x) = \lambda \frac{\zeta}{\zeta - e^{\lambda(1-\nu)x}}, \qquad \text{with} \qquad \zeta = \frac{e^{\lambda(a-\eta)} + e^{\lambda(a+\eta)}}{2}.$$

Case 2 The Central Bank reveals its policy.

In this case, either $a_1 = a_2 = a - \eta$ or $a_1 = a_2 = a + \eta$.

If $a_1 = a_2 = a+\eta$, then the delay between θ and the devaluation is $\tau_+ = a+\eta+\nu x_+$ where x_+ is solution to $\pi(x;x) = c$ with $\pi(x;x)$ the instantaneous probability of the devaluation under certainty (computed in Section 3.1):

$$\pi(x;x) = \lambda \frac{e^{\lambda(a+\eta)}}{e^{\lambda(a+\eta)} - e^{\lambda(1-\nu)x}}.$$

If $a_1 = a_2 = a-\eta$, then the delay between θ and the devaluation is $\tau_- = a-\eta+\nu x_-$ where x_- is solution to $\pi(x;x) = c$ with

$$\pi(x;x) = \lambda \frac{e^{\lambda(a-\eta)}}{e^{\lambda(a-\eta)} - e^{\lambda(1-\nu)x}}.$$

The *ex ante* expected delay between θ and the devaluation is

$$\tau = \frac{\tau_- + \tau_+}{2} = a + \nu(\frac{x_- + x_+}{2}). \tag{21}$$

These observations lead to Proposition 5.2.