The Ownership of Ratings*

Antoine Faure-Grimaud† Eloïc Peyrache‡ Lucía Quesada§

February 28, 2005

Abstract

In 2003, Standard & Poor's, in a move to enter the market for corporate governance assessments, proposed a new type of contract providing the auditee with the ability to withhold the rating. In this paper, we investigate under what circumstances such simple ownership contracts can emerge as the optimal arrangement. We show that the transfer of ownership over ratings to firms can only be an equilibrium outcome if firms are sufficiently uncertain of their quality at the time of hiring the certification intermediary and if the decision to get a rating is not observable. It is also most likely to be the optimal arrangement when competition between different intermediaries is sufficiently fierce.

Key Words: Certification, Corporate Governance.

Jel Classification:

*We thank George Dallas and Gurinder Badial from the Standard and Poor's Governance Unit for stimulating conversations. All remaining errors are ours.

†Department of Accounting and Finance, FMG at the London School of Economics,Houghton Street London WC2A 2AE, United Kingdom and CEPR. E-mail:a.faure-grimaud@lse.ac.uk

‡HEC School of Management, Paris, Finance and Economics department, 1, rue de la Libération - 78351, Jouy en Josas Cedex, France. E-mail: peyrache@hec.fr

§University of Wisconsin - Madison, Department of Economics, 6424 Social Science Building, 1180 Observatory Drive - Madison, WI, 53706, USA. E-mail: lquesada@ssc.wisc.edu

1 Introduction

On January 31st, 2003, Standard & Poor's assigned its first Corporate Governance Score (CGS) to a US company, the Federal Mortgage Association, Fannie Mae. Standard & Poor's corporate governance scores reflect its assessment of company's governance practices on a scale from 1 to 10 (incidentally, Fannie Mae was given a score of 9), and are based on a review of four important areas: ownership structure and influence, financial stakeholder rights, financial transparency and board structure. Those scores are described by S&P as *independent assessments* resulting form *an interactive process that does not follow a "check the box" approach.* It is therefore likely that those scores reflect some judgement by S&P of the governance quality and as such contain information not widely available otherwise. Importantly, the score is made public at the company's discretion. Quoting from S&P's website: "*The resulting report can either be used as a confidential diagnostic by the company or published as a Corporate Governance Score (CGS) .... A CGS is funded by the company being analyzed and is currently provided free of charge to investors, insurers and other interested parties*". In the same vein, it is noticeable that S&P commits not to reveal whether a particular company has even approached them for a score "*Assessments can be provided to companies on a confidential basis*". In short, companies pay a fee to hire S&P, then a score is produced, and the company may decide to disclose it at no extra fee. In fact, the option not to reveal the score is often exercised. S&P launched this product in the US in 2002 but started providing scores outside the US in 2000. In conversations with S&P, it appeared that a "good majority" of firms do not reveal their scores.

Although this business model is relevant to academics and practitioners involved in the important debate on corporate governance, and more specifically on the issue of whether a "market" solution will produce the right kind of information to investors, Standard & Poor's contractual offer is also of some interest to contract theorists at large. On page 30 of his 1995 book, Oliver Hart argues that "*the owner of an asset has residual control rights over that asset: the right to decide all usages of the asset in any way not inconsistent with a prior contract, custom or law*". It thus appears to us that what S&P is selling to firms is the ownership of an informational asset, their corporate governance score. Our research question is to identify the circumstances under which such a simple contract can emerge as the optimal possible arrangement.

To answer that question, we take a mechanism design approach and consider successively different market structures of the rating market. Our main result is that the transfer of ownership over ratings to firms can indeed emerge as the optimal contractual arrangement but only if firms are sufficiently uncertain of their quality at the time of hiring the intermediary. Moreover such an ownership contract is most likely to arise when the rating market is competitive. In fact, for a set of distribution functions of firms' qualities, a fully competitive market is a necessary condition for this result to obtain. Therefore we derive the result that simple ownership contracts can be the outcome of a competitive process. A related result is that competition between intermediaries will lead to less information being revealed in equilibrium. This, in the perspective of corporate governance or more generally of the working of certification markets, leads us to conclude that competition may actually be quite harmful.

The existing literature does not suggest that ownership contracts of the type documented above could be optimal. In an important paper, Alessandro Lizzeri (1999) shows two results. First, a monopolist intermediary will commit to never reveal its rating and will optimally make a simple announcement to the effect that a given firm has hired its services. If market participants expect every firm to hire such an intermediary, the absence of this announcement is an out of equilibrium outcome. Lizzeri shows that the only possible out of equilibrium beliefs must be that market participants (investors, consumers...) then take that firm to be of the lowest possible quality. Firms are then indifferent between going to the intermediary, and being extracted all the surplus created by this decision, or being mistaken for the worst kind. In a second result, Lizzeri shows that competition may lead to full information revelation. In a related set-up but with risk-averse buyers or competitive sellers, Eloïc Peyrache and Lucía Quesada (2004) show that the equilibrium will entail partial disclosure of information. But they do not consider contracts that give to the firm the ownership of its rating. The divergence between our results comes from us allowing for general contracts whereby intermediaries can charge fees contingent on the quality observed, but also from our insistence on renegotiation-proof contracts.

The literature on information disclosure is not supportive either of the view that once the firm has the ownership of its rating, it may actually choose to conceal this information. Sanford Grossman (1980) and Grossman and Hart (1980) show that when a seller can certify the quality of the good he sells, the only equilibria will involve some unravelling and will result in all the information being disclosed. Paul Milgrom (1981) contains an example with a similar result where in addition, revealing information is now costly. Masahiro Okuno-Fujiwara, Andrew Postlewaite and Kotaro Suzumura (1990) present a general analysis of games of imperfect information where a player can in a first move reveal some information about his type. They derive conditions under which this player fully discloses his type. In our set-up, once the firm has the ownership of its rating, it faces a strategic situation analogous to those looked at in this literature. But in contrast to those papers, we show that the firm may find it valuable to hide its type. The divergence comes from the fact that in our model, the choice to obtain a rating is itself a strategic variable.

Our analysis is a mechanism design exercise that involves both screening and signaling elements. With rational market participants, not revealing a rating is informative and will lead market participants to update their beliefs about those firms. The intermediary then wants to screen firms but has to account for the fact that their reservation utilities depend on the form of contract that the intermediary actually offers. It thus bears some relation with the literature on mechanism design with type dependent reservation utilities (Jean-Charles Rochet and Lars Stole 2002). But it also shares some features with the general set-up analyzed by Ilya Segal and Mike Whinston (1999, 2003) as ours is a case of contracting with (informational) externalities. Our final section deals with competition in contracts in such a context. Finally, there is also a literature on rating agencies that studies the reputational incentives of those agencies to manipulate their ratings (e.g. Roland Benabou and Guy Laroque 1993, or Beatriz Mariano 2004 and Enrico Sette 2004 among others). We ignore this issue, and concentrate on the signaling implications of the decision to hire a certification intermediary. We believe this feature to be most important in certification markets like corporate governance scores where the decision to hire such intermediary is discretionary and not directly dependent on other related decisions. In contrast, obtaining a rating is a pre-requisite to the issuance of bonds and this conditionality minimizes the relevance of signaling issues.

The paper is organized as follows. A brief second section introduces the model. In Section 3 we take the behavior of the intermediaries as given and we look for the conditions under which the option of concealing the rating may be of some value. Knowing those conditions, in Section 4 we investigate whether concealing the rating may be part of an equilibrium.

2 The model

Consider a firm, a certification intermediary (more than one in the competitive case) and a number ($n \geq 2$) of competitive, risk neutral, investors.[1] A firm's governance comes in various qualities which result in a value of v for its investors. Absent any additional information, we take for granted that investors consider v to be distributed according to some density $f(v)$. We take in fact $f(v)$ to be the uniform density on $[0, 1]$.[2] Therefore, initially, investors regard any firm as average, and take its value to be $\frac{1}{2}$. Suppose that, at the time of hiring the intermediary, the firm only gets a signal $\mu \in M$ about v. We assume that μ is uniform on $[v - \theta, v + \theta]$ for some $\theta \in [0, \frac{1}{2})$, so $M = [-\theta, 1 + \theta]$.

The intermediary possess a certification technology which provides a revealing signal σ about the true value of v at a cost $c > 0$. We suppose that σ is uniform on $[v - \omega, v + \omega]$. Except in one occasion, we will concentrate on the case where the signal is perfectly revealing, i.e. $\omega = 0$. We consider this signal to be hard information. That is, it can either be concealed or truthfully revealed. This can be viewed as a short cut for reputation concerns of the intermediary.[3]

The intermediary and the firm can enter into a transaction by which in exchange of a fee, the intermediary performs an audit that results in a rating of its corporate governance. We do not put any restriction on the class of contracts that can be offered to the firm. Based on the information they obtain, investors update their beliefs regarding the quality of the corporate governance and, given that they are risk neutral, pay the expected quality.

The timing of the game can be summarized as follows. Nature chooses v and firms get an informative signal on their true value v. Not observing any of these variables, intermediaries post (simultaneously in the competitive case) contracts that can stipulate both an up-front fee p_0, a fee contingent on what rating is ultimately obtained $p(v)$ and a disclosure rule $d(v) \in [0, 1]$

[1] The risk neutrality assumption implies that there is no social value of information. The reader may then wonder why it is of any interest to study what information will be revealed in equilibrium in such a setting. It will be easy to incorporate in our model a loss function that would justify the need to provide accurate information to investors. We take the short cut that ultimately the more information is revealed by the rating intermediary, the better it is for society.

[2] None of the results contained in Section 3 depend on this assumption. Some of those in section 4 do, but we discuss there the extent to which this may be so (subsection 4.3) and the Appendix contains a more general treatment to assess their robustness.

[3] Our setting is thus different from the literature that studies the incentives of financial information providers in the context of cheap talk games, as in Mariano (2004) or Sette (2004). There, the rating agency can fully manipulate its report.

where $d(v)$ is the probability that a score of v is revealed. That is, $d = 0$ means that the rating is not revealed. Observing the contract offers, firms can approach an intermediary to provide a rating. The contract also specifies whether this information is to be made public or not. Once a firm hires an intermediary, the score is then revealed or not to the market. Finally, market participants update their beliefs on v, using Bayes' rule whenever possible. The firm's value is then equal to the updated expected value of v, given all the information provided.

3 The Value of the Option

We aim at understanding both the circumstances under which a simple ownership contract emerges as the optimal arrangement and the reasons why a non-negligible proportion of firms, once given the option, prefers to hide the score. We answer these two questions sequentially. In this section, we look for the conditions under which the option to withhold the rating is of some value for the firm. Indeed, if the option has no value, firms will never exert it and intermediaries cannot make profits from selling this option. Thus, a valuable option is a necessary (although not sufficient) condition for the optimality of simple ownership contracts.
We first derive the willingness to pay of a firm for any contract offered by the intermediary. Denote by ϕ the information that market participants have when no rating is revealed. The absence of a rating does not imply that market participants keep their prior beliefs, since they may learn something from the fact that no rating is revealed. For instance, if they observe that the firm hired an intermediary, they understand that the absence of a rating signifies that the firm's type is one for which $d(v) = 0$. If instead, the decision to hire the intermediary is kept secret, market participants may still infer from the absence of a rating that either the firm belongs to the set of values v for which $d(v) = 0$ or to the set of firms that do not hire the intermediary. In general, denote by $E[v/\phi]$ the updated value that market participants place on a firm without a rating.
Given an offer $(p_0, p(v), d(v))$, a firm of type v will go to the intermediary if and only if:

$$d(v)v + (1 - d(v))E[v/\phi_1] - p(v) - p_0 \geq E[v/\phi_0] \tag{1}$$

where ϕ_1 is the information if the firm asked for a rating but there was no revelation while ϕ_0 is the information when there is no revelation because the firm didn't ask for a score. In the case of secret contracting, $\phi_1 = \phi_0$ as market participants cannot tell whether the absence of a rating is due to the firm not hiring the intermediary or the intermediary hiding the rating of a rated firm. But in the case of public contracting, our previous discussion has highlighted that these two information sets may differ.

In the sequel, we will take the view that the firm and the intermediary(ies) will be able to fully realize gains from trade available to them under symmetric information. That is:

Assumption 1 *Parties cannot commit not to renegotiate.*

Whatever the contract signed initially, we require that there is no deviation at any stage of the game that would be preferred by both the intermediary and the firms. In particular, this assumption implies that the decision to reveal the rating or not must be *ex post efficient.* That is, consider a firm of type v that has hired the intermediary. Then, it is immediate to prove the following result.

Lemma 1 *A contract is renegotiation-proof if and only if it satisfies the following:*

$$d(v) = \begin{cases} 1 & \text{if} \quad v > E[v/\phi_1] \\ 0 & \text{if} \quad v < E[v/\phi_1]. \end{cases} \tag{2}$$

Proof.

The proof is obvious, once we remember that if the rating is not revealed, the firm is considered to be worth $E[v/\phi_1]$, while it is worth v if v is revealed. ■

In the core of the paper, threshold equilibria, as defined below, are shown to be the only relevant ones when considering optimal contracts. The intuition stems from the combination of equation (1) and Lemma 1, which implies that high types have higher willingness to pay for the rating.

Definition 1 *A threshold equilibrium is defined as an equilibrium in which all types v above a certain threshold $\hat{\mu}$ hire one intermediary, while all those below do not.*

We are able to narrow down our search for the circumstances under which hiding a rating may be part of an equilibrium by noticing that:

Proposition 1 *If intermediary i reveals that a given firm has hired its services (public contracting), then whatever the outcome of the rating, it is revealed:* $\forall\, i$, $\forall\, v$, $d_i(v) = 1$.

Proof.

Call $M_i \subset M$ the set of types who hire intermediary i. Call $V_i = \{v \in [0,1] : \exists \mu \in M_i : f(v|\mu) > 0\}$, the set of ratings that intermediary i is ever supposed to face and $\underline{v}_i := \inf V_i$ the lowest of those ratings. Suppose that there exists some subset $\bar{V}_i \subset V_i$ of ratings that are not revealed ($d_i(v) < 1$ for $v \in \bar{V}_i$). Under public contracting, we have that $E[v/\phi_1] = E[v/\mu \in M_i \text{ and } v \in \bar{V}_i]$. Now, define $\bar{v}_i := \sup \bar{V}_i$. If the intermediary faces $\bar{v}_i$, it is optimal to renegotiate the disclosure policy and set $d(\bar{v}_i) = 1$, since, by definition, $\bar{v}_i > E[v/\phi_1]$. This argument unravels and all types are revealed. ■

The proof of Proposition 1 does not assume any particular form of the contract offered by intermediary i, nor does it say anything about the market structure. It only uses the simple intuition that once a firm is seen to have hired an intermediary but that no rating is revealed, it must be bad news. Note that a setting where all firms ask for a rating can easily be reinterpreted as a particular form of public contracting, since in equilibrium investors know that the firm has hired an intermediary. Closely related is then the following result.

Corollary 1 *If all firms ask for a rating, then the option of withholding the score is of no value.*

An important implication is then that there cannot be an equilibrium where firms hire intermediaries but do not reveal ratings unless some of those hiring contracts are kept secret. We now explore this possibility, while successively considering settings where the firm is perfectly and imperfectly informed of its own type.

3.1 Fully Informed Firm

In this subsection, we study the case where $\theta = 0$ and we limit ourselves to secret contracting as we know that this is a necessary feature for the option to hide a rating to be of some value. Because the firm knows exactly its value v, the willingness to pay of any type who hires intermediary i is then:

$$d_i(v)v + (1 - d_i(v))E[v/\phi] - E[v/\phi] = d_i(v)(v - E[v/\phi])$$

as the hiring decision is secret. Moreover, we know from Lemma 1 how $d_i(v)$ must be designed in any renegotiation-proof equilibrium. As firms are informed about v they know exactly their maximum willingness to pay. In particular, a firm for which $v < E[v/\phi]$ in a given putative equilibrium is willing to pay 0 to hire the intermediary. From this, it follows that

Proposition 2 *If the firm is perfectly informed about its type, the option of hiding the rating has no value in any threshold equilibrium.*

Proof.

Since the willingness to pay of any type $v < E[v/\phi]$ is 0, two things can happen. First, if the intermediary actually charges a price of 0 to those types then all types hire the intermediary and this case is equivalent to public contracting. Second, if the intermediary charges a positive price to those types, they are better off not hiring the intermediary (the same information will be revealed to the market but it will save the fee). So, only firms with types $v \geq E[v/\phi]$ and for which $d_i(v) = 1$ hire the intermediary.

■

Whenever the firm is fully informed of its type at the time of hiring the intermediary, there is no difference between public and secret contracting. In both cases, a firm without a rating can only be a firm that has not hired the intermediary.

Proposition 2 only makes use of the fact that the equilibrium is of the threshold form. It is crucial to our analysis. Indeed, consider alternative equilibrium candidate where only low types hire some intermediary. Then, hiding information could be of some value, because the firm could be pooled with those high types that do not get rated. In section 4, we show that such candidates cannot be part of an equilibrium and that threshold structures are uniquely optimal.

It turns out that whether the option is of some value or not is also independent of the precision of the intermediaries' technology. To see that, suppose that the intermediaries' rating is a noisy estimate of v. We show:

Proposition 3 *When $\omega > 0$, hiding the score has no value under public contracting or if the firm is fully informed.*

Proof.

The proof is immediate: It is enough to replace each time v by $E[v/\sigma]$ and all our previous results hold.

■

This result runs somehow against the intuition that this option may be valuable when a rating can be wrong: firms would get some protection against misleading low ratings. But this intuition ignores an important point: when the firm is fully informed on its type or contract under public contracting, and when necessarily the equilibrium has a threshold structure, a low rating cannot be bad news: market participants know that the firm's true value must be above a certain threshold. They would then disregard any rating that would suggest otherwise, on the account that the rating technology is noisy and that such a rating must be a "mistake".

3.2 Firm's Imperfect Knowledge and the Value of Secret Contracting

We have shown so far that the option of hiding the rating has no value neither when contracting is public nor when the firm is perfectly informed about its type. This is valid whatever the probability distributions of v and μ and whatever the market structure, as long as the equilibrium has a threshold structure.

Let us now consider the setting where the decision to hire the intermediary is still unobservable for the investors but firms are endowed with an imprecise signal regarding their own quality, i.e. $\theta > 0$. Why would this situation give value to the option of hiding the score? Consider a threshold equilibrium in which types below $\hat{\mu}$ do not hire any intermediary, while those above $\hat{\mu}$ do hire one. When the quality of the firm's information is low (θ is large), two types of mistakes can be observed:

1. firms with high signals and low true values may hire an intermediary;
2. firms with low signals and high true values may not hire any intermediary.

Imagine a firm above $\hat{\mu}$ who receives a significantly low rating. If the score is revealed, the firm will be known as a low type. If the firm is given the possibility to hide its score, it will be pooled with all those types below $\hat{\mu}$ who did not hire any intermediary. Given the second type of mistakes, the expected value of this group may be quite high and, the decision to withhold the rating profitable. This provides additional intuition for secret contracting to be a necessary condition. Indeed, whenever contracts are public, we have shown that $E[v|\phi_1] \neq E[v|\phi_0]$. More precisely, the space of firms' signals when no score is revealed is partitioned into two subsets. Whenever hiring an intermediary and deciding to withhold its score, a firm is then not pooled with firms whose signal was too low (below $\hat{\mu}$) to ask for a rating but whose true value might be high (if θ is high). Under renegotiation, standard unraveling process then applies and all firms who ask for a rating disclose their score.

The intuition then suggests that the option of hiding the score could be of some value when there is a non-negligible proportion of types not asking for a score and the quality of firms' ex ante signal is substantially low. We show by means of an example that the set of parameters for which the option has some value is not empty.

Example 1 *Consider the uniform case,* $v \sim U[0,1]$ *and* $\mu \sim U[v-\theta, v+\theta]$*. Suppose that* $\hat{\mu} = 1/4$ *and that* $\theta \in [0, 1/4]$*.*
We know that in any renegotiation-proof equilibrium the rating is not revealed if and only if $v < E[v/\phi]$*. The minimum score that one firm can get is* $\hat{\mu} - \theta = 1/4 - \theta$*. Therefore, the option has value as long as* $E[v/\phi] > 1/4 - \theta$*.*
It is a matter of some algebra to show that for $\theta > \frac{2\sqrt{3}-3}{4}$*, the previous inequality is verified.*

The previous example shows that, depending on the quality of the firm's signal, the option of hiding the score may be of some value. However, it does not prove that it can emerge as an equilibrium outcome. The next section deals with this issue.

4 Hiding the Rating as an Equilibrium Outcome

Whether or not giving the option to conceal the score emerges as an equilibrium outcome, depends on how the certification price is determined in equilibrium, since this defines the set of types who hire an intermediary. Therefore, it is important to distinguish different market structures. We look at two possible cases: a monopoly intermediary and two Bertrand competitors.

4.1 Monopoly Intermediary

A monopoly intermediary, by using a pricing scheme contingent on the firm's true value can extract all the surplus from any type who asks for a rating. That does not mean, however, that the intermediary always wants to attract all types, because the rating service is costly.
In order to be able to apply the results of the previous section, we need to show that, whatever the quality of the firm's information and the publicity of the contract, any equilibrium has a threshold structure. Call $M_1 \in M$ the set of types who hire the intermediary. A type $\mu \in M$ is willing to pay a maximum price of

$$p(\mu) = E[d(v)v + (1-d(v))E[v/\phi_1] - E[v/\phi_0]/\mu]$$

for a rating. According to Lemma 1, this reduces to

$$p(\mu) = \Pr(v > E[v/\phi_1]/\mu)E[v/v > E[v/\phi_1],\, \mu] + \Pr(v < E[v/\phi_1]/\mu)E[v/\phi_1] - E[v/\phi_0].$$

The profit of the intermediary is then

$$\Pi = \max_{M_1} \Pr(\mu \in M_1)\left(E[p(\mu)/\mu \in M_1] - c\right)$$

Lemma 2 *All equilibria belong to the class of threshold equilibria.*

Proof.

Suppose to the contrary that $\bar{\mu} \equiv \sup M_1 < \sup M$ and let us show that this cannot be part of a profit maximizing strategy. Consider instead the possibility that the intermediary offers to all $\mu \in [\bar{\mu}, \sup M]$ the contract offered to $\bar{\mu}$. The willingness to pay of all those types, $p(\mu)$ is no less than $p(\bar{\mu})$. Moreover, such a policy would reduce $E[v/\phi_0]$ as now better types are rated with strictly positive probability. In the case of public contracting, such a policy would also (weakly) increase $E[v/\phi_1]$ as better types hire the intermediary but do not reveal a rating with some probability. In the case of secret contracting, the intermediary wants as well $E[v/\phi_1] = E[v/\phi_0] = E[v/\phi]$ as small as possible and providing ratings of better types reduces $E[v/\phi]$. Therefore it must be that $\sup M_1 = \sup M$.

It cannot be either that there exist $\mu_1 < \mu_2 < \mu_3$ so that $\mu_1 \in M_1$ and $\mu_3 \in M_1$ while $\mu_2 \notin M_1$. Suppose to the contrary that this would be the case and take μ_3 to be the highest type above which all firms hire the intermediary. Again, μ_2's willingness to pay is no less than μ_1's. In the case of secret contracting, any subset of types $\mu \in [\mu_1, \mu_3]$ that is added in M_1 also allows the intermediary to (weakly) reduce $E[v/\phi]$. In the case of public contracting, the intermediary can make offers to some subset of types $\mu \in [\mu_1, \mu_2]$ but $\mu \notin M_1$, that are accepted, while at the same time making offers to some subset of types $\mu \leq \mu_1$ for whom $\mu \in M_1$ that are now rejected to guarantee that $E[v/\phi_1]$ does not decrease. This would weakly reduce $E[v/\phi_0]$ and therefore increase the monopolist's profits. ■

Better types offer better profit opportunities for the intermediary. Moreover contracting with better types allows the intermediary to reduce the average quality of the pool of firms that do not hire its services. The threshold nature of any equilibria allows us to use the results of Section 3 to conclude that if either firms are perfectly informed about their types or contracting is public, the option of concealing the score has no value.

We now proceed to show that in the case of uniform distributions, the monopolist does not find it profitable to offer the option of no disclosure. The Appendix contains a more general investigation.

Proposition 4 *Suppose v and μ are uniformely distributed on resp $[0,1]$ and $[v-\theta, v+\theta]$. For all $\theta \geq 0$, the unique renegotiation-proof contract is for the intermediary to offer $p_0 = 0$ and $p(v) = v$ and $d(v) = 1$, $\forall v$. Such equilibrium is supported by the out of equilibrium beliefs that* *a firm that does not have a rating must be worth zero. All types of firms accept this contract and whether contracting is secret or public is irrelevant. This results in a profit level of* $1/2 - c$ *for the intermediary.*[4]

Proof.

See Appendix. ■

There are several effects at work behind this proposition. Suppose that instead the monopolist would offer the option. First to have any value, our previous results indicate that the monopolist should not rate all types of firms. So only some subset of firms whose signals are above some threshold would hire its services. The option would be most attractive to firms who on the basis of their signal hired the intermediary, but discover that their exact value is rather low. This is most likely to happen for a set of types close to the threshold level. This possible benefit has to be balanced with two associated costs of the option from the monopolist's point of view. As already mentionned, it would now be necessary that some subset of types do not contract with the intermediary: a loss in market share. Second, it would reduce the willingness to pay of all types that hired the intermediary and prefer to see their score revealed. Now for those firms, their willingness to pay is reduced because not having a rating results in a positive market value. In the case of uniform distributions, these two costs exceed the benefit. It is worth emphasizing that there is some cross-subsidization between types. Indeed, whenever $v \leq c$, the intermediary loses money on those types. It still pays for the intermediary to take them on, as not doing so will admittedly save $v - c$ but will reduce the willingness to pay for its services of all firms with higher values as it would increase $E[v/\phi]$. The situation is in fact quite similar to a discriminating monopolist who starts contracting with the consumers with the highest valuation to pay, and who is surely willing to do so with all consumers with a valuation exceeding his cost of delivering the good or service (as again, the monopolist extracts all the surplus). But there is an extra effect in our set up: as the monopolist walks down the demand curve, the demand curve is shifted upward. This second effect explains that the monopolist is willing to contract with types with a lower valuation than c. The fact that the monopolist prefers taking all types is specific to the uniform assumption (Lemma 5 in the appendix presents a more general class of distributions for which it will still be the case that

[4] To be precise, the equilibrium outcome is unique, but there is an indeterminacy in prices as long as $p_0 + p(v) = v$.

the monopolist takes on all types). This trade-off, between making a loss on low types and increasing the willingness to pay of all types above, remains general[5].

This last proposition is in sharp contrast with the result of Lizzeri (1999) that the optimal contract in the monopolist case is to offer $p_0 = 1/2$ and $d(v) = 0$ for any v. The intermediary only publicly reveals that the firm has hired the intermediary. This equilibrium is supported by out of equilibrium beliefs that a firm that does not hire the intermediary must be worth zero (and this out of equilibrium beliefs appear to be the only ones that can support an equilibrium for any distribution of types whose c.d.f. is logconcave). Although Lizzeri only considers non contingent contracts (i.e. p and d do not depend on v), and $c = 0$, these restrictions do not explain why our results differ: it is noticeable that the monopolist's profit when offering $p_0 = 1/2$ and $d(v) = 0$ is still $1/2 - c$. In other words, this contract is still optimal once contingent mechanisms are allowed. The difference comes from our insistence on renegotiation-proofness. Indeed, in equilibrium, a firm who hires the intermediary is worth 1/2 if no information is revealed. Therefore, a firm of type $v > 1/2$ that has hired the intermediary is willing to pay an extra fee (at most $v - 1/2$) for the intermediary to reveal its type instead of remaining valued as an average firm in the absence of such an announcement. If the firm and the intermediary can renegotiate the initial contract, they will find a way to certify this information to the market as doing so creates an additional surplus.

We now turn to the implementation of this optimal contract.

Proposition 5 *The optimal contract is implementable with a simple ownership contract where the intermediary owns the rating and where the firm and the intermediary renegotiate over its revelation.*

Indeed, the ownership of the rating confers to the intermediary the right to use this informational asset in any way it wants. In particular, the intermediary has the right to hide the rating. This would result in the firm being valued at $E[v/\phi]$ and therefore the intermediary can bargain to extract $v - E[v/\phi]$. In our case where the intermediary takes on all the types, the previous out of equilibrium beliefs imply $E[v/\phi] = 0$. Notice that in the more general case where it could happen that the monopolist does not contract with all the types, it would remain true that the optimal contract would be implemented by the intermediary keeping the ownership of the rating. Then, as long as the intermediary is in a monopolistic situation, one

[5] Proposition 5 below remains valid even when the monopolist prefers not to contract with all types.

should not expect to see a contractual arrangement of the type currently used by Standard and Poor's.

The bottom line of this section is then the following: when the intermediary is a monopolist, and firms' value is uniformly distributed, then all firms get rated and all ratings are revealed. To do so, the intermediary implements some cross-subsidization between firms since it makes a loss on low quality firms. Indeed, for such firms, the expected return from providing a rating is lower than the cost of exerting expertise.

The rating market could not work better from the viewpoint of information revelation, taking as given the distribution of firms' quality. Therefore, if one is sufficiently confident that market forces will lead to the intermediary getting ownership of the rating and to renegotiation taking place, then one should see no need for regulatory intervention. This conclusion should however be toned down: firms have no incentives to improve their governance, as their equilibrium payoff is independent of v.

4.2 Competing Intermediaries

We now turn our attention to the opposite case of Bertrand competition: two intermediaries who can produce a perfectly revealing score at a cost c compete on the rating market.

Unsurprisingly, an equilibrium where the intermediary keeps full ownership of the rating and captures all the surplus as described in Propositions 4 and 5 cannot survive in a competing framework. Indeed, the standard Bertrand-like reasoning applies and one of the intermediaries will always lower his price to attract all firms willing to obtain a score. This is summarized in the following lemma:

Lemma 3 *When two identical intermediaries compete in contracts, they price at marginal cost and make zero profit.*

Proof.

Assume that one intermediary offers a contract $\{p_0, p(v), d(v)\}$ that generates strictly positive profits. Then the other intermediary say j, could adopt the same disclosure structure $d(v)$ and slightly undercut prices. By doing so, intermediary j would attract all firms. Therefore, whatever the contractual offers, intermediaries' expected profits have to be zero in equilibrium.

Now let us show that firms profits per type also have to be zero. To the contrary suppose that, at equilibrium there exist a type $\tilde{v}_2$ to whom intermediary i offers a combination of an access price $p_{i\,0}$ and a price contingent on quality $p_i(\tilde{v}_2)$ so that $p_{i\,0} + p_i(\tilde{v}_2) > c$. Given that, in expectation, he is making zero profits, there must exist at least one type $\tilde{v}_1$ such that $p_{i\,0} + p_i(\tilde{v}_1) < c$. Then Intermediary i can offer a new schedule, $p_{i\,0} + p_i'(\tilde{v}_1) \geq c$. The worst that could happen is that type $\tilde{v}_1$ does not accept i's offer. But this increases i's profits since he is making losses on this type. Therefore this cannot be part of an equilibrium and necessarily $p_{i\,0} + p_i(v) = c \; \forall v$ and $\forall i$. ■

Being aware of the pricing strategy that emerges in equilibrium, it is now easy to show that any equilibrium must have a threshold structure.

Lemma 4 *Any equilibrium is a threshold equilibrium in which there exist some values $\hat{\mu}$ and $\hat{v}$ so that: a firm asks for a rating if and only if $\mu \geq \hat{\mu}$. Moreover, if the probability of not revealing the score is strictly positive, it must be that a firm reveals its score, v, if and only if $v \geq \hat{v}$.*

Proof.

Suppose that there is an equilibrium in which the firm does not go to any intermediary if and only if $\mu \in M_1 \subset M$. Define M_2^i the set of types who go to intermediary i, so $M = M_1 + M_2^1 + M_2^2$. Moreover, for those firms that have asked for a score to intermediary i, if $v \in V_1^i$ the score is revealed and if $v \in V_2^i$ the score is not revealed. Define

$$\phi = \{(\mu, v) : [\mu \in M_1 \text{ or } [\exists\, i : \mu \in M_2^i \text{ and } v \in V_2^i]\},$$

the set of signals and scores such that no score is revealed. So, if no score is revealed buyers are willing to pay $E_v[v|(\mu, v) \in \phi] = E_v[v|\phi]$.

Take a firm in M_2^i. For this to be an equilibrium, we need that wants to reveal the score if and only if $v \in V_1^i$. That is

$$\begin{aligned} E_v[v|\phi] &\leq v \quad \forall v \in V_1^i, \\ E_v[v|\phi] &\geq v \quad \forall v \in V_2^i. \end{aligned}$$

These two conditions together imply that either $V_2^i = \emptyset$ or there must be a type $\hat{v} = E_v[v|\phi]$, such that all $v < \hat{v}$ are in V_2^i and all $v > \hat{v}$ are in V_1^i. Therefore, the revelation rule has a threshold structure.

Consider now the participation decision. In equilibrium we have that $\forall i$:

$$\begin{aligned} E_v[v|\phi] &\geq \Pr(v \in V_1^i|\mu) E_v[v|v \in V_1^i, \mu] + \Pr(v \in V_2^i|\mu) E_v[v|\phi] - c \quad \forall \mu \in M_1, \\ E_v[v|\phi] &\leq \Pr(v \in V_1^i|\mu) E_v[v|v \in V_1^i, \mu] + \Pr(v \in V_2^i|\mu) E_v[v|\phi] - c \quad \forall \mu \in M_2^i. \end{aligned}$$

Now $\Pr(v \in V_2^i|\mu) = 1 - \Pr(v \in V_1^i|\mu)$, so the above conditions can be rewritten as:

$$\begin{aligned} c &\geq \Pr(v \in V_1^i|\mu) \left(E_v[v|v \in V_1^i, \mu] - E_v[v|\phi]\right) \quad \forall \mu \in M_1, \\ c &\leq \Pr(v \in V_1^i|\mu) \left(E_v[v|v \in V_1^i, \mu] - E_v[v|\phi]\right) \quad \forall \mu \in M_2^i. \end{aligned}$$

Since the revelation rule has a threshold structure, $\Pr(v \in V_1^i|\mu)\left(E_v[v|v \in V_1^i, \mu] - E_v[v|\phi]\right)$ is an increasing function of μ and this means that there is a $\hat{\mu}^i$ defined as $\Pr(v \in V_1^i|\hat{\mu}^i)\left(E_v[v|v \in V_1^i, \hat{\mu}^i] - E_v[v|\phi]\right) = c$ such that all $\mu < \hat{\mu} := \min_i \hat{\mu}^i$ are in M_1 and $\forall \mu > \hat{\mu}$, there is an i such that $\mu \in M_2^i$. ■

Given that we must have a threshold equilibrium, our previous analysis already tells us that if the firm knows exactly its value ($\theta = 0$), the option is of no value.

Therefore, let us consider the case of secret contracting and $\theta > 0$. We know that the option of hiding the score may have some value. But for this is to be more than a theoretical possibility we need to exhibit an equilibrium where there exists some $\mu \geq \hat{\mu}$ for which the set of attainable values of v contains some range for which $v < E_v(v/\phi)$. This last is itself endogenous.

Consider a putative symmetric equilibrium where both intermediaries offer the same contract. Suppose that firms with $\mu \geq \hat{\mu}$ ask for certification while others don't. A firm that asks and obtains a rating v can either reveal it and get v or withhold it and get:

$$E_v(v/\phi) = E_v(v/\mu \leq \hat{\mu} \textit{ or } \mu \geq \hat{\mu} \text{ and } v \leq \hat{v})$$

where $\hat{v}$ is the type which ex post is just indifferent between revealing $\hat{v}$ or nothing, $\hat{v} = E_v(v/\phi)$.

To simplify notation in what follows, call $\underline{v} = \max\{\hat{v}, \hat{\mu} - \theta\}$ the lowest score that gets revealed and $\overline{v} = \min\{1, \hat{\mu} + \theta\}$ the highest score that a type $\hat{\mu}$ can obtain.

The intermediaries make zero profits in equilibrium, and the threshold type indifferent between hiring or not any intermediary is, $\hat{\mu}$, is defined by

$$\Pr(v > \underline{v}|\hat{\mu})E_v[v|v > \underline{v}, \hat{\mu}] + \Pr(v \leq \underline{v}|\hat{\mu})E_v(v/\phi) - c = E_v(v|\phi). \tag{3}$$

Using the results above, we also get that,

$$\begin{aligned} \Pr(\phi) &= \int_0^{\underline{v}} \int_{v-\theta}^{v+\theta} \frac{1}{2\theta} d\mu\, dv + \int_{\underline{v}}^{\overline{v}} \int_{v-\theta}^{\hat{\mu}} \frac{1}{2\theta} d\mu\, dv, \\ \hat{v} = E_v(v|\phi) &= \frac{1}{\Pr(\phi)} \left[\int_0^{\underline{v}} \int_{v-\theta}^{v+\theta} \frac{v}{2\theta} d\mu\, dv + \int_{\underline{v}}^{\overline{v}} \int_{v-\theta}^{\hat{\mu}} \frac{v}{2\theta} d\mu\, dv \right]. \end{aligned} \tag{4}$$

Rewriting this expression using that $E(v|v > \underline{v}, \hat{\mu}) = \frac{\overline{v}+\underline{v}}{2}$ and $\Pr(v > \underline{v}|\hat{\mu}) = (\overline{v} - \underline{v})\, f(v|\hat{\mu})$, we get that $(\hat{v}, \hat{\mu})$ are obtained by solving equations (3) and (4). We are now equipped to provide the following result.

Proposition 6 *When $\theta \leq c$, the only equilibrium entails full revelation for all types who ask for a score to any intermediary, i.e., the option of hiding the score has no value in equilibrium. A firm hires any one of the intermediary iff $\mu \geq \widehat{\mu}$ where*

$$E_v[v|\hat{\mu}] - c = E_v[v|\mu \leq \hat{\mu}]$$

Proof.

See appendix ■

To gain some intuition for these results, suppose first that $\theta = 0$. Then $\mu = v$ and the last equation implies that all firms with a value above $2c$ hire an intermediary. Indeed doing so costs them c while the absence of a rating leads the market to value them at $2c/2 = c$. Can it be now that when $\theta > 0$, the threshold above which a firm hires an intermediary remains equal at $2c$? If it was so, and θ is small enough, the benefit of getting a rating would be for this threshold type, $E_v[v|2c] - c = c$. What would be the expected value of a firm without a rating? Compared to the case $\theta = 0$, market participants must realize that firms may make mistakes in their decision to hire an intermediary. There are two possible mistakes: firms with low signals but high true values do not apply for a rating, while firms with high signals and low true values do. Both mistakes imply that the value of a firm without a rating is higher than c. Hence, it must be that the threshold type is at least equal to $2c$ and goes up with θ. This observation has two implications.

First, contrast this situation with the monopoly outcome for the same parameter values. There, all firms go to the intermediary and all information is revealed. We can thus conclude that competition between intermediaries reduces the amount of information that is revealed to market participants.

Second, the fact that the threshold goes up with the noise implies that this reduction in information production due to competition is all the more pronounced when firms are more uncertain about their true value. This is precisely when the value of certification is highest.

We have shown that if the quality of the firms' information is relatively high (θ small) the option of hiding the score has no value and therefore, in equilibrium all scores are revealed. However, this equilibrium does not exist anymore when the firm is poorly informed about its type (θ large). Indeed, in that case a firm is no longer guaranteed that the lowest possible score it can get is necessarily higher than the market particpants expectation in the absence of a rating. An intermediary may now want to deviate to offering secret contracts that include the option of no disclosure. By doing this, an intermediary insure the firm against bad scores. Indeed, secret contracting allows a firm who gets a low rating to hide behind a firm with a low signal (below $\widehat{\mu}$), but whose true value may be high because θ is large. It now remains to show that, for θ large, there is an equilibrium in which some scores are not revealed. We now show:

Proposition 7 *If $\theta > c$ there is a unique renegotiation-proof equilibrium in which both intermediaries charge a price equal to c, keep contracting secret and there is no disclosure for types below $\hat{v}$ and there is full disclosure for types above $\hat{v}$. This equilibrium can be implemented by transferring property rights to the firm.*

Proof.

See Appendix ■

The unravelling result breaks down when we introduce secret contracting and firms have noisy information about their types. It is interesting to note that this happens when the rating industry is more valuable: it is not very costly (c is low) and it considerably improves the information available (θ is large).

4.3 Discussion

We have shown in the previous section that, at least for the uniform distribution, the option of withholding the rating can be of some value in equilibrium only when the market is competitive. A fair question to ask is whether this is a general property or it depends on the assumed distribution function.

The intuition for why the competitive case can be extended to other distributions is fairly simple. Since the equilibrium price is equal to the marginal cost, c, there are always some types who prefer to stay out of the rating market. Therefore, with secret contracting and poor information held by the firms, some firms will prefer to be pooled with those types who stayed out rather than reveal their low true values.

For the case of a monopoly intermediary, however, things are less clear. Our result that the option has no value relies on the fact that the monopoly intermediary does not want to leave some types out of the rating market. This certainly depends on the uniform distribution. Under alternative distribution functions, the intermediary may decide to leave some types out and the option of hiding the score could become valuable.

Even though a general analysis is a difficult task, we can however provide an interesting additional result.

Proposition 8 *The option of withholding a rating has less value under a monopoly market structure than in the case of Bertrand competition.*

Proof.

The profit of the monopolist is given by:

$$\Pi(\hat{\mu}) = \int_{\hat{\mu}}^{\mu_{\max}} [p(\mu, \hat{\mu}) - c] f(\mu) d\mu,$$

with

$$p(\mu, \hat{\mu}) = \int_{\hat{v}(\hat{\mu})}^{1} (v - \hat{v}(\hat{\mu})) f(v/\mu) dv.$$

Deriving this function with respect to $\hat{\mu}$ we get

$$\frac{\partial \Pi}{\partial \hat{\mu}} = [p(\hat{\mu}, \hat{\mu}) - c] f(\hat{\mu}) + \int_{\hat{\mu}}^{\mu_{\max}} \frac{\partial p(\mu, \hat{\mu})}{\partial \hat{\mu}} f(\mu) d\mu.$$

We know that under Bertrand competition, the lowest type who asks for a rating is $\hat{\mu}^B$ such that

$$p(\hat{\mu}^B, \hat{\mu}^B) = c.$$

Evaluating the monopolist's first order condition at $\hat{\mu}^B$, we get

$$\frac{\partial \Pi}{\partial \hat{\mu}}(\hat{\mu}^B) = \int_{\hat{\mu}^B}^{\mu_{\max}} \frac{\partial p(\mu, \hat{\mu}^B)}{\partial \hat{\mu}} f(\mu) d\mu < 0$$

An intermediary, in a monopoly setting, then attracts more firms seeking for a rating and, therefore, lowers the value of the option of withholding the score. ■

That is, even though we cannot say that the option of hiding the score would never have value for a monopolist in the general case, the set of parameters for which some scores are not revealed in the monopoly case is smaller than the same set in a competitive environment.

References

[1] Albano, Gianluigi and Alessandro Lizzeri (2001), "Strategic Certification and the Provision of Quality", *International Economic Review*, Vol 42(1), pp 267-295.

[2] Benabou, Roland and Guy Laroque (1992), "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility", *The Quarterly Journal of Economics*, Vol. 107(3), pp 921-58.

[3] Grossman, Sanford and Oliver Hart (1980), "Disclosure Laws and Takeover Bids", *The Journal of Finance*, 35, pp 323-334.

[4] Hart, Oliver (1995), "Firms, Contracts and Financial Structure", *Clarendon Lectures in Economics*, Oxford University Press.

[5] Lizzeri, Alessandro (1999), "Information Revelation and Certification Intermediaries", *The Rand Journal of Economics,* Vol 30(2), pp 214-231.

[6] Milgrom, Paul (1981), "Good news and Bad news: Representation Theorems and Applications, *The Bell Journal of Economics*, 12, pp 380-391.

[7] Mariano, Beatriz (2004), "Competition, information Revelation and the Market for Ratings", *mimeo, FMG.*

[8] Okuno-Fujiwara, Masahiro, Andrew Postlewaite and Kotaro Suzumura (1990), "Strategic Information Revelation", *The Review of Economic Studies*, 57, pp 25-47.

[9] Peyrache, Eloïc and Lucía Quesada (2004), "Strategic Certification" *mimeo, HEC and University of Wisconsin, Madison.*

[10] Rochet, Jean-Charles and Lars Stole (2002) "Nonlinear Pricing with Random Participation", *Review of Economic Studies*, Vol. 69(1), pp 277-311

[11] Segal, Ilya and Mike Whinston (1999), "Contracting with Externalities", *The Quarterly Journal of Economics* 114(2), pp. 337-388.

[12] Segal, Ilya and Mike Whinston (2003), "Robust Predictions for Bilateral Contracting with Externalities", *Econometrica*, 71(3), pp. 757-791.

[13] Sette, Enrico (2004), "Reputation and Long term incentives of financial information Providers", *mimeo, FMG.*

[14] Standard and Poor's Corporate Governance Services (2005), Corporate Governance Evaluation and Scores, http://www.governance.standardandpoors.com/.

5 Appendix

5.1 Proof of Proposition 4

We need first to prove two intermediate results (Lemmata 5 and 6 below).

Lemma 5 *For any distribution of types $v \in [0,1]$ for which $E[v] = 1/2$ and so that $E[v/v \leq x] \geq \frac{x}{2}$ for all x, it is the case that the intermediary's profit cannot exceed $1/2 - c$.*

Proof.
We derive the optimal policy of the monopolist for any distribution of types. The profit function is

$$\max_{\hat{v}} \int_{\hat{v}}^{1} \left[v - E\left[v/v \leq \hat{v}\right] - c\right] dF(v)$$

Denote by $g(\hat{v}) = E\left[v/v \leq \hat{v}\right]$, $g'(\hat{v}) = \frac{f(\hat{v})}{F(\hat{v})}\left[\hat{v} - g(\hat{v})\right]$. The first order condition for an interior solution is

$$\begin{gathered} -\left(\hat{v} - g(\hat{v}) - c\right) f(\hat{v}) - (1 - F(\hat{v}))g'(\hat{v}) = 0 \\ \Leftrightarrow -\hat{v} + g(\hat{v}) + c - \frac{1 - F(\hat{v})}{F(\hat{v})}\left[\hat{v} - g(\hat{v})\right] = 0 \\ \Leftrightarrow (-\hat{v} + g(\hat{v}))\left(1 + \frac{1 - F(\hat{v})}{F(\hat{v})}\right) + c = 0 \\ \Leftrightarrow (-\hat{v} + g(\hat{v})) + cF(\hat{v}) = 0 \end{gathered}$$

The first term is negative for any $\hat{v} > 0$. In the case of uniform distribution, the left hand side of the last equation reduces to $\hat{v}(c - \frac{1}{2})$, which is always negative as $c < 1/2$.
For more general distribution functions, we can show the following: at any point $\hat{v}^*$ for which $\Pi'(\hat{v}^*) = 0$ it must be that $\Pi(\hat{v}^*) \leq \frac{1}{2} - c$. Indeed at such points, we would have $cF(\hat{v}^*) = \hat{v}^* - g(\hat{v}^*)$. The profit function satisfies for any $\hat{v}$,

$$\begin{aligned} \int_{\hat{v}}^{1} \left[v - g(\hat{v}) - c\right] dF(v) &= \int_{0}^{1} \left[v - g(\hat{v}) - c\right] dF(v) - \int_{0}^{\hat{v}} \left[v - g(\hat{v}) - c\right] dF(v) \\ &= \frac{1}{2} - g(\hat{v}) - c - \int_{0}^{\hat{v}} v dF(v) + F(\hat{v})\left(g(\hat{v}) + c\right) \\ &= \frac{1}{2} - g(\hat{v}) - c + F(\hat{v})c \end{aligned}$$

so that

$$\Pi(\hat{v}^*) \leq \frac{1}{2} - c \Leftrightarrow cF(\hat{v}^*) \leq g(\hat{v}^*)$$

but at an interior point, this inequality simplifies to

$$\hat{v}^* - g(\hat{v}^*) \leq g(\hat{v}^*),$$

so that, if for any $\hat{v}$, $g(\hat{v}) = E[v/v \leq \hat{v}] \geq \hat{v}/2$, this condition is satisfied. For instance, any distribution for which $F(v)$ is convex satisfies this inequality. ■

Lemma 6 *Assume that* $v \in [0,1]$ *according to some density* $f(.)$ *for which* $E[v] = 1/2$. *Assume that* $\mu \sim [\mu_{\min}, \mu_{\max}]$ *according to some* $f(\mu/v)$ *density function. Call* $V(\mu) = \{v \in [0,1] : f(v/\mu) > 0\}$ *and* $v_{\min}(\mu) = \inf V(\mu)$. *We denote* $\hat{v} \equiv E[v/\phi]$. *Then the intermediary's profit* $\Pi(\hat{\mu})$ *cannot exceed* $\frac{1}{2} - c$ *if:*

$$-\hat{v}(\hat{\mu}^*) + F(\hat{\mu}^*)c \leq 0,$$

where

$$\hat{v}(\hat{\mu}) = E(v/\phi) = \frac{\int_{\mu_{\min}}^{\hat{\mu}} \int_0^1 v f(v/\mu) f(\mu) dv d\mu + \int_{\hat{\mu}}^{\mu_{\max}} \int_0^{\hat{v}} v f(v/\mu) f(\mu) dv d\mu}{\Pr(\phi)},$$

Proof.

A firm who received an ex ante signal μ is ready to pay at most

$$p(\mu) = \Pr(v \geq E(v/\phi)/\mu)[E(v/v \geq E(v/\phi), \mu) - E(v/\phi)],$$

Whenever he attracts all types of sellers, the intermediary's profit is then

$$\begin{aligned} \Pi(\hat{\mu}) &= \int_{\hat{\mu}}^{\mu_{\max}} [p(\mu) - c] f(\mu) d\mu \\ &= \int_{\hat{\mu}}^{\mu_{\max}} [\Pr(v \geq E(v/\phi)/\mu)[E(v/v \geq E(v/\phi), \mu) - E(v/\phi)] - c] f(\mu) d\mu. \end{aligned}$$

By definition we can express $\hat{v}(\hat{\mu})$ as follows,

$$\hat{v}(\hat{\mu}) = E(v/\phi) = \frac{\int_{\mu_{\min}}^{\hat{\mu}} \int_0^1 v f(v/\mu) f(\mu) dv d\mu + \int_{\hat{\mu}}^{\mu_{\max}} \int_0^{\hat{v}} v f(v/\mu) f(\mu) dv d\mu}{\Pr(\phi)},$$

where

$$\Pr(\phi) = \int_{\mu_{\min}}^{\hat{\mu}} \int_0^1 f(v/\mu) f(\mu) dv d\mu + \int_{\hat{\mu}}^{\mu_{\max}} \int_0^{\hat{v}} f(v/\mu) f(\mu) dv d\mu.$$

This implies, in particular that

$$\frac{d\hat{v}}{d\hat{\mu}} = \frac{f(\hat{\mu})}{\Pr(\phi)} \int_{\hat{v}}^1 (v - \hat{v}) f(v/\hat{\mu}) dv.$$

The profit function then writes

$$\Pi(\hat{\mu}) = \int_{\hat{\mu}}^{\mu_{\max}} \Pr(v \geq \hat{v}(\hat{\mu})/\mu)[E(v/v \geq \hat{v}(\hat{\mu}), \mu) - \hat{v}(\hat{\mu})] f(\mu) d\mu - (1 - F(\hat{\mu}))c.$$

It is equal to $\frac{1}{2} - c$ whenever $\hat{\mu} = \mu_{\min}$ and to 0 whenever $\hat{\mu} = \mu_{\max}$. If it can take any higher value, it must be at an interior optimum. Maximizing profits with respect to $\hat{\mu}$, we obtain the first order (necessary) condition for an interior solution:

$$\begin{aligned} -\int_{\hat{v}}^1 (v - \hat{v}) f(v/\hat{\mu}) f(\hat{\mu}) dv - \frac{d\hat{v}}{d\hat{\mu}} \int_{\hat{\mu}}^{\mu_{\max}} \int_{\hat{v}}^1 f(v/\mu) f(\mu) dv d\mu + f(\hat{\mu})c &= 0, \\ -\int_{\hat{v}}^1 (v - \hat{v}) f(v/\hat{\mu}) dv - \frac{1 - \Pr(\phi)}{\Pr(\phi)} \int_{\hat{v}}^1 (v - \hat{v}) f(v/\hat{\mu}) dv + c &= 0, \\ \int_{\hat{v}}^1 (v - \hat{v}) f(v/\hat{\mu}) dv &= \Pr(\phi)c. \end{aligned}$$

Suppose that there exists a value of $\hat{\mu}$ satisfying the first order condition above. Let us define the condition under which, at this point, the profit is lower than $\frac{1}{2} - c$. That is

$$\Pi(\hat{\mu}^*) = \int_{\hat{\mu}^*}^{\mu_{\max}} \Pr(v \geq \hat{v}(\hat{\mu}^*)/\mu)[E(v/v \geq \hat{v}(\hat{\mu}^*), \mu) - \hat{v}(\hat{\mu}^*)]f(\mu)d\mu - (1 - F(\hat{\mu}^*))c \leq \frac{1}{2} - c. \tag{5}$$

Using the fact that

$$\Pr(\phi^*)\hat{v}^* + (1 - \Pr(\phi^*))E(v/v \geq \hat{v}^* \text{ and } \mu \geq \hat{\mu}^*) = \frac{1}{2},$$

equation (5) can be written

$$-\hat{v}(\hat{\mu}^*) + F(\hat{\mu}^*)c \leq 0,$$

or, plugging in the first order condition, we get

$$-\hat{v}(\hat{\mu}^*) + \frac{F(\hat{\mu}^*)}{\Pr(\phi^*)} \int_{\hat{v}(\hat{\mu}^*)}^{1} (v - \hat{v}(\hat{\mu}^*))f(v/\hat{\mu}^*)dv \leq 0. \tag{6}$$

■

Let us now show that the conditions derived in Lemmas 5 and 6 are satisfied in the case of the uniform distributions considered in the text. Simple Bayesian updating gives the density of μ,

$$f(\mu) = \int_0^1 f(\mu|v)f(v)dv = \begin{cases} \frac{\mu+\theta}{2\theta} & if \quad \mu \in [-\theta, \theta) \\ 1 & if \quad \mu \in [\theta, 1-\theta] \\ \frac{1+\theta-\mu}{2\theta} & if \quad \mu \in (1-\theta, 1+\theta] \end{cases}$$

Using Bayes' rule, we get

$$f(v|\mu) = \frac{f(\mu|v)f(v)}{f(\mu)} = \begin{cases} \frac{1}{1+\theta-\mu} & if \quad \mu > 1-\theta, & v \in [\mu-\theta, 1], \\ \frac{1}{2\theta} & if \quad \mu \in [\theta, 1-\theta], & v \in [\mu-\theta, \mu+\theta], \\ \frac{1}{\mu+\theta} & if \quad \mu < \theta, & v \in [0, \mu+\theta]. \end{cases}$$

Define now $x(\mu) = \int_0^1 vf(v/\mu)dv$. We have

$$x(\mu) = \int_0^1 vf(v/\mu)dv = \begin{cases} \frac{\mu+\theta}{2} & if \quad \mu \in [-\theta, \theta) \\ \mu & if \quad \mu \in [\theta, 1-\theta] \\ \frac{1-\theta+\mu}{2} & if \quad \mu \in (1-\theta, 1+\theta] \end{cases}$$

We can then deduce $h(\widetilde{x})$ the firms' ex ante distribution of valuations, where $\widetilde{x}$ denotes a particular realization of x: $h(\widetilde{x}) = \left(x^{-1}(\widetilde{x})\right)' f(x^{-1}(\widetilde{x}))$.

$$h(\widetilde{x}) = \begin{cases} \frac{2\widetilde{x}}{\theta} & if \quad \widetilde{x} \in [0, \theta) \\ 1 & if \quad \widetilde{x} \in [\theta, 1-\theta] \\ \frac{2(1-\widetilde{x})}{\theta} & if \quad \widetilde{x} \in (1-\theta, 1] \end{cases}$$

and

$$\Pr(\widetilde{x} \leq \hat{v}) = \begin{cases} \frac{\hat{v}^2}{\theta} & if \quad \hat{v} \in [0, \theta) \\ \hat{v} & if \quad \hat{v} \in [\theta, 1-\theta] \\ 1 - \frac{1}{\theta} + \frac{2\hat{v}}{\theta}\left(1 - \frac{\hat{v}}{2}\right) & if \quad \hat{v} \in (1-\theta, 1] \end{cases}$$

We start with checking that the condition of Lemma 6 is satisfied in this case. First suppose that $\hat{\mu} < \theta$. The necessary condition for the option to have value is then that $\hat{v} > 0$.

Noticing that

$$\hat{v}(\hat{\mu}) = E[v|\phi] \Leftrightarrow \hat{\mu} = \hat{v} - \theta + (6\theta)^{\frac{1}{3}}\, \hat{v}^{\frac{2}{3}}. \tag{7}$$

the condition given by equation (6), rewrites

$$-\frac{2\theta^{\frac{2}{3}}\hat{v}^{\frac{4}{3}} + 6^{\frac{2}{3}}\theta^{\frac{1}{3}}\hat{v}^{\frac{5}{3}}}{8\theta^{\frac{2}{3}}\hat{v}^{\frac{1}{3}} + 2^{\frac{5}{3}}3^{\frac{2}{3}}\theta^{\frac{1}{3}}\hat{v}^{\frac{2}{3}}} \leq 0,$$

which is easily satisfied for all values of θ whenever $\hat{v} > 0$.

Suppose now that $\theta < \hat{\mu} < 1 - \theta$. Again, we have

$$\hat{v}(\hat{\mu}) = E[v|\phi] \Leftrightarrow \hat{\mu} = \hat{v} - \theta + (6\theta)^{\frac{1}{3}}\, \hat{v}^{\frac{2}{3}}. \tag{8}$$

The necessary condition for the option to have value is given by

$$\hat{v} > \hat{\mu} - \theta \Leftrightarrow \hat{v} < \left(\frac{4}{3}\right)^{\frac{1}{2}} \theta \tag{9}$$

Given that $\frac{F(\hat{\mu}^*)}{\Pr(\phi)} < 1$, a sufficient condition for equation (6) to be satisfied is that

$$-\hat{v}(\hat{\mu}^*) + \int_{\hat{v}(\hat{\mu}^*)}^{1} (v - \hat{v}(\hat{\mu}^*)) f(v/\hat{\mu}^*) dv \leq 0. \tag{10}$$

Using (8) condition (10) rewrites

$$\begin{aligned} \hat{v} &\geq \int_{\hat{v}(\hat{\mu}^*)}^{\hat{\mu}+\theta} (v - \hat{v}(\hat{\mu}^*)) f(v/\hat{\mu}^*) dv \\ \Leftrightarrow \hat{v} &\leq \frac{16}{9}\theta \end{aligned}$$

This last equation is trivially satisfied whenever condition (9) is satisfied. Finally the case where $1 - \theta < \hat{\mu}$ is irrelevant in the monopoly case.

Now that we know that in the case of uniform distribution, the monopolist will not offer the option to hide the score, we know check that the condition of Lemma 5 is satisfied for the distribution of firms' ex ante valuation, $h(.)$. We just need to check whether $E[v/v \leq \hat{v}] \geq \hat{v}/2$ for this distribution. It is quite easy to see that the cdf is convex and that therefore this is the case.

5.2 Proof of Proposition 6

Suppose that there is an equilibrium with full revelation for all types who ask for a score to any intermediary. This means that $V_2^i = \emptyset$, $\forall i$. Thus, observing a firm who got rated but did not reveal the score is out-of-equilibrium. Assuming that the out-of-equilibrium beliefs are such that rated firms who do not reveal their rating have $v = 0$ we just need to determine the type $\hat{\mu}$ who is indifferent between asking or not for a score.

All firms with $\mu \geq \hat{\mu}$ hire an intermediary, while those below do not. We have:

$$E_v[v/\mu \leq \hat{\mu}] = E_v[v/\mu = \hat{\mu}] - c. \tag{11}$$

Suppose first that $\theta \leq \hat{\mu} \leq 1 - \theta$ or, indirectly that $\Pr(\mu \leq \hat{\mu}) = \hat{\mu}$. We then check the conditions under which the equilibrium satisfies such restrictions. Using the implied conditional distributions (see the proof of Proposition 4), the previous indifference condition becomes

$$\frac{1}{\hat{\mu}}\left[\int_{-\theta}^{\theta}\int_{0}^{\mu+\theta} vf(v|\mu)f(\mu)dvd\mu + \int_{\theta}^{\hat{\mu}}\int_{\mu-\theta}^{\mu+\theta} vf(v|\mu)f(\mu)dvd\mu\right] = \int_{\hat{\mu}-\theta}^{\hat{\mu}+\theta} vf(v|\hat{\mu})dv - c,$$

or

$$c = \frac{\hat{\mu}}{2} - \frac{\theta^2}{6\hat{\mu}} \equiv h(\hat{\mu}, \theta)$$

which gives $\hat{\mu} = c + \sqrt{c^2 + \frac{1}{3}\theta^2}$.

We need to verify that $\hat{\mu} \in [\theta, 1 - \theta]$, which is true if

$$\frac{\theta}{3} \leq c \leq \frac{3(1-\theta)^2 - \theta^2}{6(1-\theta)}.$$

Note first that for any $\theta \in [0, \frac{1}{2}]$, $\frac{3(1-\theta)^2-\theta^2}{6(1-\theta)} \geq \frac{1}{6} \geq c$.

When $c < \frac{\theta}{3}$ we have that the solution satisfies $\hat{\mu} < \theta$. The indifference condition writes

$$\begin{aligned} \frac{4\theta}{(\hat{\mu}+\theta)^2}\int_{0}^{\hat{\mu}+\theta}\int_{v-\theta}^{\hat{\mu}} vf(v|\mu)f(\mu)dvd\mu &= \int_{0}^{\hat{\mu}+\theta} vf(v|\hat{\mu})dv - c \\ \frac{(\hat{\mu}+\theta)}{3} &= \frac{(\hat{\mu}+\theta)}{2} - c, \end{aligned}$$

which gives $\hat{\mu} = 6c - \theta$.

We need to prove that it is indeed optimal for the intermediaries to set $V_2 = \emptyset$. We know that this is optimal if and only if for all types who ask for a score the rating is above $\hat{v} = E_v[v/\phi]$. That is, we need to prove that $\max\{0, \hat{\mu} - \theta\} \geq \hat{v}$.

Take first the case $\frac{\theta}{3} > c$, or $\hat{\mu} = 6c - \theta$. From equation (4), we get

$$\hat{\mu} = \hat{v} - \theta + (6\,\theta)^{\frac{1}{3}}\hat{v}^{\frac{2}{3}}. \tag{12}$$

Plugging these two equations together, we get

$$6c = \hat{v} + (6\,\theta)^{\frac{1}{3}}\hat{v}^{\frac{2}{3}}$$

from which, obviously, $\hat{v} > 0$. Therefore $\hat{v} > 0 > \hat{\mu} - \theta$, and a positive fraction of firms ask for a rating but do not disclose it. Thus, $V_2 = \emptyset$ cannot be part of an equilibrium when $\theta > 3c$. Therefore, transferring all property rights to the firm is strictly profitable.

Consider now the case where $\frac{\theta}{3} < c < \frac{1}{6}$ or, $\hat{\mu} = c + \sqrt{c^2 + \frac{1}{3}\theta^2}$.

Using again equation (4), we get $\hat{\mu} = \hat{v} - \theta + (6\,\theta)^{\frac{1}{3}} \hat{v}^{\frac{2}{3}}$, or

$$c + \sqrt{c^2 + \frac{1}{3}\theta^2} = \hat{v} - \theta + (6\,\theta)^{\frac{1}{3}} \hat{v}^{\frac{2}{3}}. \tag{13}$$

The right-hand-side of (13) is an increasing function of $\hat{v}$. Therefore $\hat{v} > \hat{\mu} - \theta$ if and only if the left-hand side of (13) is greater than the right-hand-side evaluated at $\hat{v} = \hat{\mu} - \theta$. This gives

$$2\theta > (6\,\theta)^{\frac{1}{3}} \left[c + \sqrt{c^2 + \frac{1}{3}\theta^2} - \theta \right]^{\frac{2}{3}},$$

or

$$\theta < c.$$

it is an equilibrium to have $V_2 = \emptyset$ if and only if $\theta < c$.

5.3 Proof of Proposition 7

Suppose that $\theta > c$. If there is such an equilibrium, it must be that $(\hat{\mu}, \hat{v})$ solve the system of equations (3) and (4). We have to look at 2 cases: $\hat{\mu} < \theta$ and $\theta \leq \hat{\mu} \leq 1 - \theta$.

Consider first the case: $\hat{\mu} < \theta$. Solving (3) and (4) we get

$$\begin{aligned} \hat{v} &= \frac{c^2(2c + 18\theta) + 2c^{\frac{3}{2}}(c + 3\theta)\sqrt{c + 12\theta}}{18\theta^2}, \\ \hat{\mu} &= \hat{v} - \theta + (6\theta)^{\frac{1}{3}} \hat{v}^{\frac{2}{3}}. \end{aligned}$$

We just need to check that $\hat{\mu} < \theta$, which is true if $c < \alpha\,\theta$, with $\alpha = \frac{5}{2}\left(5 + 3\sqrt{5}\right)^{\frac{1}{3}} - 1 - 5^{\frac{2}{3}} \left(\frac{2}{5 + 3\sqrt{5}}\right)^{\frac{1}{3}} \simeq 0.459433$.

Since in this case $\hat{\mu} < \theta$, the probability that the score is not reveal is positive if and only if $\hat{v} > 0$, which is true as long as $c > 0$ and $\theta > 0$.

Consider now the second case: $\theta \leq \hat{\mu} \leq 1 - \theta$. Solving (3) and (4) we get

$$\begin{aligned} \hat{v} &= 2\frac{\sqrt{3}}{3}\,\theta^{\frac{1}{4}} c^{\frac{3}{4}}, \\ \hat{\mu} &= \hat{v} - \theta + (6\theta)^{\frac{1}{3}} \hat{v}^{\frac{2}{3}}. \end{aligned}$$

Now, we need to check first that $\theta \leq \hat{\mu}$, which is true if $c \geq \alpha\,\theta$ and $\hat{\mu} \leq 1 - \theta$, always true because $\theta \in [0, \frac{1}{2}]$ and $c \in [0, \frac{1}{6}]$.

In this case, the probability that the score is not revealed is positive if and only if $\hat{v} > \hat{\mu} - \theta$. Using the solution we have found, this inequality is verified if and only if $\theta > c$, which is satisfied by assumption. To show that this is an equilibrium, we need to check that no intermediary has incentives to deviate. First, no change of price alone can be profitable. If the intermediary increases the price, all types will go to the other intermediary and therefore profits cannot increase. If the intermediary decreases the price, he will attract all types, but make negative profits.

Second, the intermediary could change the price and the revelation policy. Changing the revelation policy to no disclosure for types above $\hat{v}$ would increase the willingness to apply for rating, but is not renegotiation-proof. Changing the revelation policy to full disclosure for types below $\hat{v}$ decreases the willingness to apply for rating and therefore the intermediary has to reduce the price and will make negative profits. Deviation to public contracting would have the same effect. Therefore, there is no profitable deviation for the intermediary.