Why are Buyouts Leveraged? The Financial Structure of Private Equity Funds

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Abstract

This paper presents a model of the financial structure of private equity firms. In the model, a private equity firm can raise financing either ex ante, before deals are discovered, or ex post, to fund specific deals. The general partner knows the quality of each deal but cannot credibly communicate this information to investors. The equilibrium of this model is consistent with a number of characteristics of the private equity industry: First, it explains why private equity investments are financed with a combination of ex ante and ex post financing. Second, the optimal securities issued by private equity firms in the model mirror practice, since ex ante investors' claims are similar to equity, ex post investors' claims are similar to debt, and general partners receive a percentage of the profits. Third, the model suggests that investments by private equity firms will exhibit extreme cyclicality, since there is overinvestment in good states of the world and underinvestment in bad states. Fourth, investments made in recessions will outperform on average investments made in booms. Finally, the model provides a rationale for why most private equity investments are pooled within funds.

Practitioner: "Things are really tough because the banks are only lending 4 times cashflow, when they used to lend 6 times cashflow. We can't make our deals profitable anymore."

Academic: "Why do you care if banks will not lend you as much as they used to? If you are unable to lever up as much as before, your limited partners will receive lower expected returns on any given deal, but the risk to them will have gone down proportionately."

Practitioner: "Ah yes, the Modigliani-Miller theorem. I learned about that in business school. We don't think that way at our firm. Our philosophy is to lever our deals as much as we can, to give the highest returns to our limited partners."

1. Introduction

The market for buying out businesses by private equity firms is enormous, totaling $xx billion over the 1990-2002 period. These purchases range from the now legendary Beatrice and RJR Nabisco acquisitions by KKR in the 1980s, to the current market in which private equity partnerships buy both large firms like Burger King to small businesses such as funeral homes. While buyouts originally were focused in the United States, they have become increasingly common in Europe; the Wall Street Journal recently estimated that 40% of M&A activity in Germany in 2004 is from private equity firms.(WSJ, Sept. 28, 2004, p. C1) These buyouts are generally highly leveraged; indeed, when most people refer to buyouts, they invariably include the adjective 'leveraged' in their description.

Buyouts, as well as other private equity investments, are generally made by funds that share a common organizational structure. Typically, these funds raise equity at the time they are formed, make investments that are levered whenever possible using the assets of the portfolio firm but not the fund as collateral, and have a finite life (see Sahlman (1990), or Fenn, Liang and Prowse (1997) for more discussion). The funds are usually organized as limited partnerships, with the limited partners (LPs) providing the capital and the general partners (GPs) making investment decisions and receiving a substantial share of the profits (most often 20%). While the literature has spent much effort understanding some aspects of the private equity market, it is very surprising that there is no clear answers to the basic questions of how funds are structured financially, and what the impact of this structure is. Why are most private equity investments made by funds that are financed by equity and have a finite life? Why are their investments financed by debt backed by the assets of the investment and not the fund? What should we expect to observe about the relation between bank lending practices, and the prices and returns of private equity investments? Why are booms and busts in the private equity industry so extreme?

According to the Modigliani-Miller theorem, capital structure decisions, including both fund structure and the financing of individual deals, is relevant only to the extent to which taxes, transactions costs, or real investment decisions are affected. Certainly the deductibility of interest payments is part of the reason why leverage is valuable at the portfolio firm level but not at the fund level since portfolio firms pay corporate taxes and funds can pass through profits to their partners tax free (see Kaplan (1989)). Yet, it seems unlikely that taxes are a complete answer: there is no evidence that buyouts are less levered when firms have tax shields limiting their corporate income taxes, and the same tax advantages are present in the targeted firms prior to the buyout, when firms typically have relatively modest leverage. Another commonly-cited explanation for leverage at the portfolio-firm level are the implicit incentives associated with leverage, in particular the fact that the commitment to pay interest limits management's discretion to waste the firm's excess cash flows (Jensen (1988)). Yet, managers of firms that are bought by private equity partnerships are monitored heavily and often replaced (Lerner 1995). It seems likely that direct monitoring by a knowledgeable practitioner personally receiving 20% of the profits would likely lead to better controls on managers than the more ad hoc constraints imposed by leverage. Furthermore, neither tax nor incentive benefits explain why the equity capital invested in portfolio firms is raised through a fund rather than deal by deal.

In this paper, we propose a new explanation for the financial structure of private equity firms. We present a model that explains a number of features of private equity markets, including the fact that private equity investments are generally done through funds that pool investments across the fund, the typical financial structure of raising equity at the fund level and supplementing it whenever possible with debt at the deal level, the payoffs to GPs of a "carry-like" structure in which they receive a fraction of the profits but one that is junior to that received by the LPs, the extreme "boom and bust" nature of investments by private equity firms, and the observed empirical regularity of investments made during busts outperforming investments made during booms on average.

The model is relatively stratightforward, relying mainly on one market friction that serves as the underlying source of deviations from the Modigliani-Miller benchmark. The friction we model is the notion that GPs making investment decisions have better information about the quality of their potential investments than their LPs or any potential lenders. This assumption seems plausible, given that GPs are specialists in evaluating companies who have substantial incentives to discover any relevant information.

The model is very much a dynamic extension of the famous Myers and Majluf (1984) model, in which informed firms raising capital from uninformed investors will always have an incentive to overstate the quality of potential investments so they cannot credibly communicate their information. The equilibrium of the Myers and Majluf model has debt as an optimal security because the asymmetric information leads to underpricing, and debt is less information-sensitive than equity, so it is associated with the lowest level of underpricing. This equilibrium sometimes is characterized by overinvestment, which occurs when the average project is positive NPV because both bad and good projects get financed. Alternatively, when the average project is negative NPV, since neither bad nor good projects can get financed there is underinvestment.

In our model, the GP faces several potential investment objects over time which require financing. This introduces a new financing decision for the GP relative to the static case. He can now decide whether to raise capital on a deal by deal basis (ex post financing), or raise a fund of capital to be used for several future projects (ex ante financing), or a combination of the two.

With ex post financing, the solution is the same as in the static Myers and Majluf model. Debt will be the optimal security, and GPs will choose to undertake all investments they can, even if they are value-decreasing. Whether deals will be financed at all depends on the state of the economy – in good times, where the average project is positive NPV, there is overinvestment, and in bad times there is underinvestment.

We show that ex ante financing can help to alleviate these problems. By tying the compensation of the GP to the collective performance of a fund, he has less of an incentive to invest in bad deals, since bad deals will contaminate his stake in the good deals. Thus, a fund structure often dominates deal-by-deal capital raising. Furthermore, debt is typically not the optimal security for a fund. Since the capital is raised before the GP has learned the quality of deals, there is no such thing as a "good" GP who tries to minimize underpricing by issuing debt. Indeed, issuing debt will maximize the risk shifting tendencies of a GP since it leaves him with a call option on the fund. We show that instead it is optimal to issue a security giving investors a debt contract plus a levered equity stake, leaving the GP with a "carry" at the fund level that resembles contracts observed in practice.

The downside with pure ex ante capital raising is that it leaves the GP with substantial freedom, since once the fund is raised he does not have to go back to the capital markets, and so can fund deals even in bad times. If the GP has not encountered enough good projects and is approaching the end of the investment horizon, or if economic conditions shift so that not many good deals are expected to arrive in the future, a GP with untapped funds has the incentive to "go for broke" and take bad deals. We show that it is therefore typically optimal to use a mix of ex ante and ex post capital. Giving the GP funds ex ante preserves his incentives to avoid bad deals in good times, but the ex post component has the effect of making the GP unable to finance bad deals in bad times. In addition the structure of the securities mirrors common practice; ex post deal funding is senior to the firm's claim, the LP's claim is senior to the GP's, and the GP's claim is a fraction of the profits. This financing structure turns out to be optimal in the sense that it is the one that maximizes the value of investments by minimizing the expected value of negative NPV investments undertaken and good investments ignored.

Even with this optimal financing structure however, investment nonetheless deviates from its first-best level. In particular, during good states of the world, firms are prone to overinvestment, meaning that some negative net present value investments will be undertaken. In addition, during bad states of the world there will be underinvestment, i.e., valuable projects that cannot be financed.

This investment pattern is an explanation for the common observation that the private equity investment process is extremely procyclical (see Gompers and Lerner (1999)). During recessions, there not only will not be as many valuable investment opportunities, but those that do exist will have difficulty being financed. Similarly, during boom times, not only will there be more good projects than in bad times, but bad projects will be financed in addition to the good ones. The implication of this pattern is that the informational imperfections we model are likely to exacerbate normal business cycle patterns of investment. It also suggests that there is some validity to the common complaint from GPs that during tough times it is difficult to get financing for even very good projects but during good times many poor projects get financed.

An important empirical implication of this result is that returns to investments made during booms will be lower on average than the returns to investments made during poor times. This finding is consistent with anecdotal evidence about poor investments made during the internet and biotech bubbles, as well as some of the most successful deals being initiated during busts. More formally, Kaplan and Schoar (2004) find evidence documenting that this pattern of returns is more general, with investments made during bad times underperforming investments made in good times.

The next section presents the model and its implications. There is a discussion and conclusion following the model.

2. Model

There is a general partner (GP) who raises money from investors to invest in candidate LBO firms.

There are two periods. Each period, a candidate firm arrives. We assume it costs I to invest in a firm. Firms are of two kinds: good (G) and bad (B). A good firm has cash flow $Z > 0$ for sure and a bad firm has cash flow 0 with probability $1 - p$ and cash flow Z with probability p where

$$Z > I > pZ$$

so that good firms are positive net present value investments and bad firms are negative NPV.

Each period a good firm arrives with probability α and a bad firm with probability $1 - \alpha$ (equivalently, we can assume that there are always bad firms available, and a good firm arrives with probability α).

To capture the concept of good and bad states of the economy, we assume the arrival probability α take values α_H and α_L with equal probabilty each period, where $\alpha_H > \alpha_L$. We assume α is observable but not verifiable, so it cannot be contracted on directly.

2.1. Capital Raising

We look at two forms of capital raising:

Ex post capital raising is done after the GP encounters a firm i. The security investors get is denoted $d_i(z_i)$ and is backed only by that firm's cash flows. We call the investors supplying ex post capital banks.

Ex ante capital raising is done before the GP encounters a firm. The security investors get is denoted $w(z)$ and is backed by the cashflow z from a portfolio of future firms. We call the investors supplying ex ante capital limited partners (LP's).

All securities have to satisfy the following monotonicity and limited liability conditions:

Monotonicity $w(z), z - w(z), d_i(z), z - d_i(z)$ are non-decreasing in z

Limited liability $z - w(z), z - d_i(z)$ are non-negative

These are standard assumptions. We also assume that the interest rate is zero and that all agents are risk-neutral.

Furthermore, we assume that there are an infinite supply of unserious fly-by-night operators that investors cannot distinguish from a serious GP. Fly-by-night operators only find useless firms with a pay-off of zero. However, if the security issued pays off less than the total cash flow whenever the cash flow is below the invested capital K, the fly-by-night operators can just sit on the money and earn rents. To screen them out of the market, we therefore require that:

Fly-by-night For invested capital K, $w(z), d(z) = z$ whenever $z \leq K$.

The timing of the model is as follows:

0: The GP can raise ex ante capital.

1: The first firm arrives and the GP finds out whether it is good or bad. The value of α is observed by everyone. The GP can raise ex post capital backed by firm 1.

2: The second firm arrives and the GP finds out whether it is good or bad. The new value of α is observed by everyone. The GP can raise ex post capital backed by firm 2.

3. Cash flows are realized and securities pay off.

We now compare three modes of financing: Pure ex post financing, pure ex ante financing, and a mix of ex ante and ex post financing.

3. Pure ex post capital raising

When capital is raised ex post and firm by firm, there is no link between the first and the second period and we can look at the one period problem. This is a standard signalling problem as in Nachman and Noe.

Since the GP has limited liability, he will have an incentive to seek financing for both good and bad firms. Thus, there can only exist a pooling equilibrium. The GP needs to raise I to invest in a firm. The security $d(z)$ is fully specified by the pay-off $d(Z)$ (since $d(0) = 0$). The break even condition for banks is

$$\alpha d(Z) + (1 - \alpha) pd(Z) \geq I$$

Thus, financing is feasible as long as

$$(\alpha + (1 - \alpha) p) Z \geq I$$

and in that case, the GP will invest in all firms. Otherwise, he cannot invest in any firms.

We make the following assumption:

Condition 3.1.

$$\frac{\alpha_L Z + (1 - \alpha_L) pZ + \alpha_H Z + (1 - \alpha_H) pZ}{2} < I$$

Condition 3.1 implies that ex post financing is not possible in the low state, since $\alpha_L Z + (1 - \alpha_L) pZ < I$. Thus, with pure ex post financing, there is always underinvestment in the low state since good deals cannot get financed.

Whether pure ex post financing is possible in the high state depends on whether $\alpha_H Z + (1 - \alpha_H) pZ > I$ holds. If it does, there is overinvestment in the high state since bad deals can get financed. We summarize the pure ex post case in the following proposition:

Proposition 1. *Pure ex post financing is never feasible in the low state. If*

$$\alpha_H Z + (1 - \alpha_H) pZ > I$$

it is feasible in the high state, where the GP issues security $d_i(Z)$ given by

$$d(0) = 0, d(Z) = \frac{I}{\alpha_H + (1 - \alpha_H) p}$$

and the GP captures all the surplus.

4. Pure Ex Ante Financing

We now study the case where the GP raises capital K in ex ante financing only to be used over the two periods for investment.

4.1. The capital constrained case: $K = 1I$

Suppose the GP raises $1I$ ex ante in period zero, and is not allowed to raise any ex post financing. This means he can invest in one deal over the two periods.[1]

The security $w(z)$ now has three potential pay-offs, $w(0) = 0$, $w(I) = I$ (which happens when the GP invests nothing, so from the fly-by-night condition he has to pay back everything), and $w(Z)$. Thus, the GP gets nothing unless he invests, which implies he will always invest in either period 1 or 2.

The GP's expected pay off is $Z - w(Z)$ from taking a good deal and $p(Z - w(Z))$ from taking a bad deal. It is then easy to see what his investment behavior will be. If a good firm arrives in period 1, he will take it. If a bad firm arrives, he will pass it up a bad project in the hope of encountering a good project in period 2. However, if he has not invested in period 1, he will take any deal that arrives in period 2.

Given this investment behavior, the GP maximizes his expected pay off such that investors break even:

$$\max_{w(Z)} E(\alpha)(Z - w(Z)) + (1 - E(\alpha))(E(\alpha) + (1 - E(\alpha))p)(Z - w(Z))$$

subject to the investor break even constraint:

$$E(\alpha)w(Z) + (1 - E(\alpha))(E(\alpha) + (1 - E(\alpha))p)w(Z) \geq I$$

The first term in the maximand is the case where the GP finds a good firm in period 1 and invest, the second term is his expected pay off from investing in the second period after having encountered (and passed up) a bad firm in period 1.

Solving for the optimal security and the set of parameters for which the investor can break even, we get the following result:

Proposition 2. *Pure ex ante financing with $K = I$ is feasible if*

$$(E(\alpha) + (1 - E(\alpha))E(\alpha))(Z - I) \geq (1 - E(\alpha))^2 (I - pZ)$$

[1] If the GP takes a deal in period 1, he would not be able to raise a new fund ex ante fund for period 2. This is because he would then have an incentive to take all deals in the second period, so the most he could promise LP's in expectation is

$$\frac{(\alpha_L + (1 - \alpha_L)p) + (\alpha_H + (1 - \alpha_H)p)}{2} Z$$

which, from Condition 3.1, is less than the needed investment amount I. Thus, LP's in this second fund would not break even.

The optimal security is given by

$$w(0) = 0, w(I) = I, w(Z) = \frac{I}{E(\alpha) + (1 - E(\alpha))(E(\alpha) + (1 - E(\alpha))p)}$$

and the GP captures all the surplus.

The shape of the contract is such that the GP gets a "carry", or a proportion of the pay offs above the supplied funds I.

The benefit of capital constrained ex ante financing is that investment is socially efficient in the first period. All good projects get taken, but bad projects are passed up. By giving the GP the right to do a second project if he passes up the first, he will engage in "winner picking" and will avoid bad projects. The cost of giving the GP this right is the inefficiencies in the second period: If the GP is out of funds, there is underinvestment, and if he has funds left, there is overinvestment. Whether this financing structure is better than the pure ex post case depends on the parameters. Comparing the feasible regions in Propositions 1 and 2, you get that pure ex ante capital raising is feasible for lower Z (or equivalently, higher I), and also more profitable for the GP, if

$$\alpha_L > \left(\frac{\alpha_H + \alpha_L}{2}\right)^2$$

When this condition does not hold, such as for low α_L and high α_H, ex post financing is better. The ex post contract is explicitly contingent on the realized value of α, so that no financing is done in bad states while all projects are done in the good state (when α_H is high enough to make the pooling contract feasible). When α_L goes to zero and α_H goes to one, this approaches the first best and is therefore better.

4.2. The unconstrained case: K =2I

Now suppose the GP raises $K = 2I$ of ex ante capital in period zero, and so can potentially invest in both periods.

The security $w(z)$ now has five potential pay offs: $\{w(0), w(Z), w(2I), w(Z + I), w(2Z)\}$. The fly-by-night condition implies that $w(0) = 0$ and $w(2I) = 2I$. For simplicity, we also assume that $Z \leq 2I$, which implies $w(Z) = Z$ from the monotonicity condition. Thus, it remains to specify $w(Z + I)$ and $w(2Z)$.

It is obvious that the GP will take all good projects he encounters over the two periods. Also, if no investment was made in period 1, he will take any project that comes in period 2 just as in the capital constrained case. The difference now is that the GP has enough money to take *all* projects in both periods, so the security has to be structured in such a way that he has no incentive to invest in bad projects

To characterize securities that satisfy this, we first investigate the GP's incentives in the second period. Suppose the GP has invested in a good project in period 1 and encounters a bad project in period two. Then, since the GP gets no pay off when the fund cash flows are below $2I$, the condition for passing up the bad project is

$$Z + I - w(Z + I) \geq p(2Z - w(2Z)) \tag{4.1}$$

This condition says that the GP's pay off cannot be too tilted towards high cash flows, or else he will have a "risk-shifting" incentive.

If the GP had invested in a bad project in period 1 and encounters another bad project, the condition for passing it up is

$$p(Z + I - w(Z + I)) \geq p^2(2Z - w(2Z))$$

which is the same as Condition 4.1.

Going back to period 1, suppose the GP encounters a bad project. Also, suppose that Condition 4.1 holds. Then, the condition for passing up the bad project is

$$\begin{aligned} & E(\alpha)(Z + I - w(Z + I)) + (1 - E(\alpha))\, p(Z + I - w(Z + I)) \\ \geq\ & pE(\alpha)(2Z - w(2Z)) + p(1 - E(\alpha))(Z + I - w(Z + I))) \end{aligned}$$

The left hand side is the expected pay off from not investing in period 1 and taking any project in period 2, the right hand side is the pay off from taking a bad project in period 1 and then only taking good projects in period 2. Noting that the second terms on both sides are equal, this is again equivalent to Condition 4.1.

To summarize, as long as Condition 4.1 holds, the GP will invest efficiently *except* when he encounters two bad projects in a row; Then, it is impossible to make him pass up the second project. If Condition 4.1 does not hold, all project will be taken, and the LP's will not break even. Therefore, the optimal contract is the one that maximizes the GP's pay off such that Condition 4.1 holds and the LP's break even. The break even condition for investors under this investment behavior is

$$E(\alpha)^2 w(2Z) + 2E(\alpha)(1 - E(\alpha))\, w(Z + I) + (1 - E(\alpha))^2 (pw(Z + I) + (1 - p)I) \geq 2I$$

The first term is the pay off when two good projects are encountered, the second when one good project is encountered, and the third term is the pay off when two bad projects are encountered so the GP takes the second one.

The feasible set and the optimal security design is characterized in the following proposition:

Proposition 3. *Pure ex ante financing with $K = 2I$ is feasible if*

$$\left(E(\alpha)^2 2 + 2E(\alpha)(1 - E(\alpha))\right)(Z - I) \geq (1 - E(\alpha))^2 (I - pZ)$$

Suppose $Z < 2I$ and $p < \frac{1}{2}$. Then, the optimal security is to give the LP debt with face value $2I$ plus a fraction k of profits above $2I$ and the GP captures all the surplus.

Suppose $p > \frac{1}{2}$. Then, the optimal security is to give the LP debt with face value $2I$ plus a convex function of profits above $2I$:

$$\begin{aligned} w(z) &= z \text{ for } z \leq 2I \\ w(Z+I) &= 2I + k(Z + I - 2I) \\ w(2Z) &> 2I + k(2Z - 2I)) \end{aligned}$$

and the GP may or may not capture all the surplus.

The proof is given in the Appendix. The security structure resembles the structure in private equity funds, where LP's get all cash flows below their invested amount and a proportion of the cash flows above that. It is essential to give the LP's this equity part to avoid the risk shifting tendencies of the GP so that he does not pick bad projects whenever he has invested in good projects or have the chance to do so in the future. The temptation to take bad projects is higher the higher the probability of success p is - therefore, when p is sufficiently high, the LP's stake will be even more loaded towards high cash flows. This is the security structure described at the end of the proposition. In this scenario, the GP may in fact need to share some of the surplus with the LP to be able to commit to the efficient investment behavior.

The following proposition shows that the unconstrained ex ante solution is more efficient than both capital constrained ex ante fund raising and ex post fund raising.

Proposition 4. *Pure ex ante fund raising with $K = 2I$ is more efficient than pure ex ante fund raising with $K = I$.*

Pure ex ante fund raising with $K = 2I$ is more efficient than pure ex post fund raising if $(\alpha_H + (1 - \alpha_H)p)Z < I$, or if $(\alpha_H + (1 - \alpha_H)p)Z > I$ and

$$\frac{\alpha_L}{\left(\frac{\alpha_L + \alpha_H}{2}\right)^2} > \frac{I - pZ}{(1 - p)Z}$$

Proof: The only inefficiency in the $K = 2I$ case is that bad project in period 2 are taken after a bad project in period 1 was found. This inefficiency also exists in the $K = I$ case, but in that case good projects in period 2 are passed up if a good project was found in period 1. Therefore, the $K = 2I$ case is more efficient. Comparing the $2I$ case with the ex post case, the $2I$ case is more efficient whenever

$$\left(\alpha_H + p\left(1-\alpha_H\right)\right) Z < I$$

since ex post is not feasible then (and ex ante sometimes is). Ex post will be more efficient than $2I$ if

$$\alpha_H Z + \left(1-\alpha_H\right) pZ \geq I$$

and if

$$\left(1 - E\left(\alpha\right)\right)^2 \left(I - pZ\right) \geq 2\frac{1-\alpha_H}{2}\left(I - pZ\right) + 2\frac{\alpha_L}{2}\left(Z - I\right)$$

In the second expression, the left hand side is the efficiency loss for the $2I$ case, i.e. the loss from investing in a bad project the second period, which will happen if no investment was made in the first period. The right hand side is the efficiency loss for ex post raising, which is that some bad projects will get financing in the high state, and some good projects will not be financed in the low state. Rearranging this expression we get

$$\begin{aligned}
\left(1 - \frac{\alpha_H + \alpha_L}{2}\right)^2 \left(I - pZ\right) &\geq \left(1-\alpha_H\right)\left(I - pZ\right) + \alpha_L\left(Z - I\right) \\
\left(\frac{\alpha_H + \alpha_L}{2}\right)^2 \left(I - pZ\right) &\geq \alpha_L\left(1-p\right) Z \\
\frac{I - pZ}{\left(1-p\right) Z} &\geq \frac{\alpha_L}{E\left(\alpha\right)^2}
\end{aligned}$$

End Proof.

The first part of the Proposition shows the benefit of the fund structure - by tying the pay off of several projects together, the GP can now take all good projects, but will still avoid bad projects if there is any chance of finding a good project. The only inefficiency is that if the GP has not found any good projects either in period 1 or 2, he will take the bad project in period 2.

The comparison with ex post financing is less clear cut. Ex post financing has the disadvantage that the GP will always try to take any project he encounters. However, there is also a benefit - since the contract is set up ex post, it is automatically contingent on the realized value of α. Thus, you avoid financing completely in low states. If low states are very unlikely to have good projects (α_L close to zero) and high states have almost only good projects (α_H close to one) the inefficiency with ex post fund raising is small. With ex ante fund raising, the inefficiency is still significant since bad projects are taken in the low state.

5. Ex ante and ex post

We now examine the situation with both ex post and ex ante capital raising. The problem with unrestricted ex ante fund raising is the overinvestment tendencies in period 2 if no project is taken in period 1. Thus, there may be a benefit to make the GP *somewhat* capital constrained in the second period. In particular, we want to rule out investment in the low state in period 2. Since the ex ante contract cannot be made contingent on the state, we can implement this by using some ex post capital raising.

We assume the GP raises $K_1 + K_2$ in ex ante capital, where he is only allowed to use at most K_1 in the first period firm and at most K_2 in the second period firm. Since the problem is overinvestment in period 2, we set $K_1 = I$ (we verify later that this is optimal).

For financing a second period firm, the GP must issue an ex post security $d(z_2)$ in exchange for the needed capital $I - K_2$. Thus, the fund cash flow is $z = z_1 + (z_2 - d(z_2))$, and the fund security $w(z)$ is backed by this cash flow.

We look for the most efficient equilibrium and show that it can dominate all other forms of capital raising. The most efficient implementable equilibrium is one where the GP invests only in good firms in period 1, only in good firms in period 2 if there was an investment in period 1, and only in the high state if there was no investment in period 1. Note that this is more efficient than the $2I$ case since we avoid investment in the low state in period 2 after no investment has been done in period 1. It is also more efficient than ex post capital raising, since ex post capital raising has the added inefficiencies that no good projects are taken in low states and all bad projects are taken in high states. It is impossible to implement an equilibrium where the GP only invests in good firms over both periods, since if there is no investment in period 1, he will always have an incentive to invest in period 2 whether he finds a good or a bad firm.

Since we look for an equilibrium where the GP only invests in good firms in period 1, the ex post capital raising in period 1 has to be separating - banks know that only GP's with good firms ask for financing. Furthermore, since we want an equilibrium where GP's who have invested in the first period avoid all bad firms in period 2, the equilibrium is also separating in period 2 for GP's that invested in period 1. Also, note that in a separating equilibrium, whether you are in a good or a bad state of the economy does not impact incentives or capital raising. However, if no investment was made in period 1, the equilibrium is by necessity pooling since GP's will always have an incentive to invest. In this case, the capital raising will be affected by the state of the economy.

To verify that this equilibrium exists, we solve by backward induction, starting with investment behavior in the second period.

5.1. Period 2 fund raising

We now check conditions in period 2 such that the GP can only invest in the high state if no period 1 investment was made, but invests in all good projects regardless of the state if there was a period 1 investment.

- **Case 1: No investment in period 1**

When there has been no investment in period 1, all GP's will seek ex post financing, so the equilibrium is pooling. The internal capital K_2 has to be set high enough so that the GP can invest in the high state but low enough such that the GP cannot invest in the low state. The condition for this is:

$$\alpha_H Z + (1-\alpha_H)\, pZ \geq I - K_2 \geq \alpha_L Z + (1-\alpha_L)\, pZ \tag{5.1}$$

The pay out $d(z_2)$ for the bank providing ex post capital to break even in the high state is given by

$$(\alpha_H + (1-\alpha_H)\, p)\, d(Z) = I - K_2$$

so the security $d(z_2)$ is given by

$$d(Z) = \frac{I - K_2}{\alpha_H + (1-\alpha_H)\, p}$$

- **Case 2: Investment was made in period 1**

Since we assume there is a separating equilibrium in which banks assume that GP's that invested in period 1 only invest in good projects in period 2, the GP can issue claims $d(Z) = I - K_2$ to raise $I - K_2$. Thus, the GP will always invest in good firms. We need to check that the GP has no incentive to invest in bad firms.

If the GP invested in a good firm in period 1, he will pass up a bad firm if:

$$\begin{aligned} & Z + K_2 - w(Z+K_2) \qquad (5.2)\\ > \;& p\left[2Z - d(Z) - w(2Z - d(Z))\right] + (1-p)\left[Z - w(Z)\right] \\ = \;& p\left[2Z - (I-K_2) - w(2Z - (I-K_2))\right] + (1-p)\left[Z - w(Z)\right] \end{aligned}$$

The last term is the case where the bad firm does not pay off, and the fund defaults on its period 2 ex post debt.

We also assume for the purposes of this section that $Z < 2I$. This implies that

We also have to check the off-equilibrium behavior where the GP invested in a bad firm in period 1 to make sure there is no deviation in period 1. Thus, if the GP invested in a bad firm in period 1 he will pass up a bad firm in period 2 if:

$$
\begin{aligned}
& p(Z+K_2-w(Z+K_2))+(1-p)\,(K_2-w\,(K_2)) \\
> \;& p^2\,[2Z-(I-K_2)-w\,(2Z-(I-K_2))] \\
& +p\,(1-p)\,[Z-w\,(Z)] \\
& +(1-p)p\,[Z-(I-K_2)-w\,(Z-(I-K_2))]
\end{aligned}
$$

The two last terms are, respectively, the case where the first bad firm pays off and the second does not, and the case where the first bad firm does not pay off and the second does.

We also assume for the purposes of this section that $Z < 2I$. From the fly-by-night assumption, this implies that $w\,(Z-(I-K_2)) = Z-(I-K_2)$, because $I+K_2$, which is the invested amount, is bigger than $Z-(I-K_2)$. Noting that the fly by night condition also implies that $w\,(K_2) = K_2$, and dividing by p, we can rewrite the condition as

$$
\begin{aligned}
& Z+K_2-w(Z+K_2) \\
> \;& p\,(2Z-(I-K_2)-w\,(2Z-(I-K_2)))+(1-p)\,(Z-w\,(Z))
\end{aligned}
\tag{5.3}
$$

which is the same as condition 5.2.

5.2. Period 1 Investment

In period 1, it is always optimal to invest in a good project. We must check that the GP does not want to invest in a bad project to sustain the separating equilibrium.

The condition for not investing in a bad project in period 1 becomes:

$$
\begin{aligned}
& \frac{1}{2}\,(\alpha_H+p\,(1-\alpha_H))\left(Z+I-\frac{I-K_2}{\alpha_H+(1-\alpha_H)\,p}-w\left(Z+I-\frac{I-K_2}{\alpha_H+(1-\alpha_H)\,p}\right)\right) \\
> \;& pE(\alpha)\,(2Z-(I-K_2)-w\,(2Z-(I-K_2)))
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
& \frac{\alpha_H+p\,(1-\alpha_H)}{\alpha_H+\alpha_L}\left(Z+I-\frac{I-K_2}{\alpha_H+(1-\alpha_H)\,p}-w\left(Z+I-\frac{I-K_2}{\alpha_H+(1-\alpha_H)\,p}\right)\right) \\
> \;& p\,(2Z-(I-K_2)-w\,(2Z-(I-K_2)))
\end{aligned}
\tag{5.4}
$$

As long as this condition holds, the GP has no incentive to take a bad firm in period 1, so that the period 2 separating equilibrium is upheld. The optimal contract is then found by maximizing GP pay off subject to conditions 5.3 and 5.4 and the LP break even condition.

We now outline when this equilibrium is feasible for investors.

Proposition 5. *The equilibrium is feasible if and only if it creates social surplus, and*

$$\frac{\alpha_H + p(1-\alpha_H)}{\alpha_H + \alpha_L} > p \tag{5.5}$$

Proof: If $\frac{\alpha_H+p(1-\alpha_H)}{\alpha_H+\alpha_L} < p$, it is impossible to make Condition 5.4 hold since the GP's pay off at cash flow $z = 2Z - (I - K_2)$ must be at least as big as at $z = Z + I - \frac{I-K_2}{\alpha_H+(1-\alpha_H)p}$ from the monotonicity requirement. If $p < \frac{\alpha_H+p(1-\alpha_H)}{\alpha_H+\alpha_L}$, we can set

$$\begin{aligned} w\left(2Z - (I - K_2)\right) &= 2Z - (I - K_2) - \varepsilon \\ w\left(Z + I - \frac{I - K_2}{\alpha_H + (1-\alpha_H)\,p}\right) &= Z + I - \frac{I - K_2}{\alpha_H + (1-\alpha_H)\,p} - \varepsilon \\ w\left(Z + K_2\right) &= Z + K_2 - \varepsilon \end{aligned}$$

and satisfy all conditions. Making ε small, the investor can break even whenever the equilibrium creates social surplus.

End Proof.

This shows that the mix of ex ante and ex post capital solution is more efficient than any other solution as long as Condition 5.5 is satisfied, since it creates more surplus. Note that the condition is easier to satisfy when α_H is high and α_L is low, which is exactly when pure ex post raising may dominate pure ex ante raising with $K = 2I$. Thus, some form of ex ante fund raising should always be used.

The fund security has the same characteristics as before - to satisfy Conditions 5.5 and 5.3 you need to give the LP a debt contract plus some part of the upside so that the GP does not engage in risk shifting.

A few things are worth noting about the nature of the equilibrium. Even though the solution is the most efficient that can be implemented, there are still investment distortions. There is overinvestment in the good state since some bad projects get taken, and there is underinvestment in bad states since some bad projects get passed up. Thus, we should expect deals made in good times to underperform and deals that are economically sound that not get financed in bad times.

Note also that the solution involves two types of investors, LPs and banks. In fact, it is essential that these be different agents. One could imagine that the GP might just as well go back to the LPs in period 2 and ask for the ex post capital. But after no investment has been made in period 1, the LPs have an incentive to veto the investment if it does not break even so they can get their capital back. However, if this was done, the incentives for the GP in period 1 would unravel, and he would take all projects.

6. Discussion and Conclusions

An enormous literature in corporate finance concerns the capital structures of firms and the manner in which firms decide to finance investments. Yet, much financing today is done through private capital markets, by private equity firms who receive funding from limited partners and use this money to finance investments in both new ventures and buyouts of existing companies. These firms follow a common financial structure: they are finite-lived limited partnerships who raise equity capital from limited partners before any investments are made (or even discovered) and then supplement this equity financing with debt at the individual deal level whenever possible. General partners have most decision rights, and receive a percentage of the profits (usually 20%), which is junior to all other securities. Yet, while this financial structure is responsible for a very large quantity of investment, we have no theory explaining why it should be so prevalent.

This paper presents a model of the financial structure of a private equity firm. The firm can finance its investments either ex ante, by pooling capital across future deals, or ex post, by financing deals once knows about them. The financial structure chosen is the one that maximizes the value of the fund, which will depend on financial structure because managers have better information about deal quality than potential investors. The value maximizing financial structure of the firm minimizes the losses both from expected bad investments that are undertaken and good investments that are ignored.

The model leads to a number of of predictions that are consistent with commonly observed features of the private equity industry. First, deals are financed by a combination of ex ante and ex post financing, i.e., private equity funds raise capital both initially at the fund level and subsequently at the deal level. Second, the nature of the optimal securities derived by the model appears to mimic actual securities used by private equity firms; ex post securities in individual deals is senior like debt, while payments to LPs is junior to the debt-like securities used in the deals but senior to the payments to the GPs. Third, the model predicts that observed investments in the private equity market will be extremely procyclical, with the already procyclical nature of investment opportunities augmented by the overinvestment in good times and underinvestment in bad times. Fourth, consistent with both casual observation (the internet and biotech bubbles) as well as more formal empirical evidence, this overinvestment and underinvestment predicts that average returns to investments made during booms will be worse than returns to investments made during recessions. Finally, the model provides a rationale for the observation that most private equity investments are done through funds that raise at least some capital ex ante and pool capital across deals.

In addition, the intuitions coming from our model are consistent with other common observations about the private equity industry. For example, there are circumstances where investors do provide financing for individual deals. Sellers often pay for partial financing of their firms, GPs syndicate investments across funds, and approach LPs for coinvestment opportunities. Each of these types of financing can be thought of in terms of our model in that they all occur in circumstances where the degree of information asymmetry is likely to be low. For example, when a seller helps to finance a deal, it typically supplements bank financing and is likely to occur when the seller has better information about his firm than the bank. When GPs syndicate deals, they share information usually agree on the prospects for an investment, so that information asymmetry is potentially minimized. When funds ask LPs to coinvest, our model suggests that they tend to ask more sophisticated LPs, who can evaluate the deal themselves and be assured that it is a good investment. Finally, in those circumstances where specific funds are raised to finance particular deals, there should be a good reason why the initiating GP did not do the entire investment by himself. One potential reason is that the fund could be constrained in the size of its investment by its charter; an example of such a situation is Exxel's acquisitions of Argencard and Norte (see Hoye and Lerner (1995), Ballve and Lerner (2001)).

However, our model falls short in that it fails to explain a number of important features of private equity funds. First private equity funds tend to be finitely-lived; we provide no rationale for such a finite life. Second, while one might expect much of our analysis to apply equally to hedge funds, it is not clear that it does. Hedge funds are financed predominately by levered equity and we have no explanation for this phenomenon. Third, most venture deals are staged in a number of rounds. While a number of explanations for staging are in the literature (see Gompers and Lerner (1999)), ideally staging should come as an implication of a more general model of private equity firms' capital structures such as the one presented here. Finally, while we identify potential investment distortions arising even when funds use the optimal financial structure, we do not have a clear understanding of what practitioners and policy-makers could conceivably do to minimize these distortions. Knowing about any conceivable such policies clearly is a potentially valuable to contribution to the study of, as well as the practice of private equity.

7. Appendix

Proof of Proposition 3:

For the case $Z < 2I$, the text shows that the project is feasible as long as Condition 4.1 holds and investors break even. Note that for $w(Z+I) = Z+I-\varepsilon$ and $w(2Z) = 2Z-\varepsilon$, Condition 4.1 reduces to

$$\varepsilon \geq p\varepsilon$$

and so holds automatically. Making ε small, we can then give LP's close to all the cash flows without changing investment behavior, and the feasible set is then defined from the break even condition of the LP when he gets all the cash flows.

We now show that the same feasibility condition holds for the case $Z > 2I$, in which case we can have $w(Z) < Z$. Now, the condition for not investing in a bad project in period 2 after having invested in a bad project in period 1 becomes:

$$p\left(Z+I-w(Z+I)\right) \geq p^2\left(2Z-w(2Z)\right) + 2p\left(1-p\right)\left(Z-w(Z)\right)$$

or

$$Z+I-w(Z+I) \geq p\left(2Z-w(2Z)\right) + 2\left(1-p\right)\left(Z-w(Z)\right) \tag{7.1}$$

The condition for not investing in a bad after a good project becomes:

$$Z+I-w(Z+I) \geq p\left(2Z-w(2Z)\right) + \left(1-p\right)\left(Z-w(Z)\right) \tag{7.2}$$

Note that this is implied by Condition 7.1.

Suppose Condition 7.1 does not hold. Then, the condition for not investing in a bad project in period 1 becomes

$$\begin{aligned} & E(\alpha)\left(Z+I-w\left(Z+I\right)\right) + \left(1-E(\alpha)\right)p\left(Z+I-w\left(Z+I\right)\right) \\ \geq\ & E(\alpha)\left(p\left(2Z-w\left(2Z\right)\right) + \left(1-p\right)\left(Z-w\left(Z\right)\right)\right) + \left(1-E(\alpha)\right)\left(p^2\left(2Z-w\left(2Z\right)\right) + 2p(1-p)\left(Z-w\left(Z\right)\right)\right) \end{aligned}$$

or

$$Z+I-w\left(Z+I\right) \geq p\left(2Z-w\left(2Z\right)\right) + \left(1-p\right)\frac{E(\alpha)+\left(1-E(\alpha)\right)2p}{E(\alpha)+\left(1-E(\alpha)\right)p}\left(Z-w\left(Z\right)\right) \tag{7.3}$$

This implies Condition 7.2 but is weaker than Condition 7.1. However, Condition 7.1 does not need to be satisfied as long as Condition 7.3 holds since there will never be a situation where a bad project is taken in period 1. Thus, it is enough that Condition 7.3 is satisfied to get the same investment behavior as for the $Z < 2I$ case, and the same break even condition for the LP will result. Again, note that we can always satisfy Condition 7.3 in the same way as above by setting $w(Z) = Z, w(Z+I) = Z + I - \varepsilon$ and $w(2Z) = 2Z - \varepsilon$. Thus, by giving the investor all the cash flows, the same feasibility condition results.

The optimal security is found by maximizing GP pay off subject to the corresponding conditions for the $Z > 2I$ and $Z < 2I$ cases and the break even condition. The Proposition describes optimal securities for the $Z < 2I$ case (the $Z > 2I$ case is similar).

First, suppose that $p < \frac{1}{2}$. Suppose the security is $w(z) = z$ for $z \leq 2I$ (debt component) and $w(z) = 2I + k(z - 2I)$ (equity component) so the GP gets a carry $1 - k$ for cash flows above $2I$.with Plugging in values in Condition 5.3 gives

$$Z + I - (2I + k(Z + I - 2I)) \geq p\,(2Z - (2I + k(2Z - 2I)))$$

or

$$Z - I - k(Z - I) \geq 2p\,((Z - I) - k(Z - I))$$

This holds if and only if $p \leq \frac{1}{2}$. Thus, when $p \leq \frac{1}{2}$ we can always satisfy Condition 5.3 with this security structure. Also, we can always make the LP break even condition bind. Setting $k = 1$ gives the LP all the cash flows, so he can break even as long as we are in the feasible set. That we can make the break even condition bind follows since at $k = 0$, the LP does not break even. Thus, there is always some interior k for which the LP gets no surplus and the GP captures all the surplus. Hence, the security is optimal.

For $p > \frac{1}{2}$, we have to give the LP relatively more in the high state (increase $w(2Z)$ relative to $w(Z + I)$) to satisfy Condition 5.3. Now, Condition 5.3 is the binding constraint. The solution is found by solving for $w(2Z)$ in terms of $w(Z + I)$:

$$Z + I - w(Z + I) = p\,(2Z - w(2Z))$$

$$w(2Z) = 2Z - \frac{Z + I - w(Z + I)}{p}$$

Then, set $w(Z + I)$ as small as possible subject to the break even constraint. For some parameter values, the LP will get surplus even at the lowest feasible $w(Z + I) = 2I$.

End Proof.

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