Sharing aggregate risks under moral hazard

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Abstract

This paper discusses some of the problems associated with the efficient design of insurance schemes in the presence of aggregate shocks and moral hazard. I consider an economy composed of groups. In each group, individuals are ex ante identical, but subject to idiosyncratic, group-specific shocks. A group may be for example the labour force in a given sector, workers being subject to the risk of unemployment. Without moral hazard, optimality requires full insurance against idiosyncratic shocks within a group, the so-called mutuality principle, allowing to treat a group by a representative agent. Aggregate macroeconomic risks are shared between representative individuals groups along the analysis of Borch (1960) and Wilson (1968). The question investigated in this paper is what remains of this analysis when the presence of moral hazard conflicts with the full insurance of idiosyncratic shocks. In particular, how is the sharing of macro-risks across groups affected by the partial insurance against idiosyncratic risks ? The design of unemployment insurance schemes in different economic sectors, and the design of pension annuities in an unfunded social security system are two potential applications.

Keywords : moral hazard, insurance, mutuality principle

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1 Introduction

This paper discusses some of the problems associated with the efficient design of insurance schemes in the presence of aggregate shocks and moral hazard. Consider for instance insurance against unemployment risks in different sectors. Without moral hazard, optimal risk sharing requires two properties: first full insurance against idiosyncratic shocks within each sector, and second the pooling of macro-economic risks affecting employment in the different sectors (along the lines first described by Borch (1960) Wilson (1968) and Malinvaud (1972), (1973)). Casual observation suggests that neither property holds in practice. Moral hazard explains why idiosyncratic shocks should not be fully insured. The question investigated here is whether this may have a significant impact on the sharing of macroeconomic risks.

The design of pension schemes is an important example that motivates this analysis. In several european countries, among them France, Germany, and Italy, the main part of the pension system is a pay as you go system, meaning that the pension benefits to the retirees are paid by the current workers. In the last thirty years, there is some evidence that the well being of retirees has significantly raised relative to that of the rest of the population. Although this relative increase has been the result of active policies at the beginning of the period, it has not been anticipated to a large extent. Indeed it resulted from a specific aspect of the design of pension benefits in a period of increase in unemployment. The basic reason is that the pension annuities are typically indexed on individual wages.[2] As a result, annuities increase more than the average income of young individuals as unemployment increases. This raises the question of why pension annuities are adjusted with the income of the workers instead of the income of the whole "young" generation, employed and unemployed. This question can be seen as the problem of sharing optimally the risk of unemployment (and

[2]The description is simplified here. Two elements matter: the initial level of pensions at retirement, and the adjustment during the retirement period. Whereas the initial level is still indexed on wage, there is a move to an adjustment with consumer price index. This move is too recent to have been significant yet.

of wages) among the population. A difficulty is that the welfare system, and the associated moral hazard problem, might play a role in explaining the high level of unemployment. If true, the extent to which retirees should bear the cost of unemployment is unclear. A proper analysis of the design of pension annuities should account for the moral hazard problem within the young generation.

To analyze the impact of moral hazard on risk sharing against aggregate shocks, I consider an economy composed of large groups, two for simplicity. Within each group, individuals are *ex ante* identical and subject to idiosyncratic, group-specific shocks. The probability of these shocks are influenced by the "state" of the economy (the macro risk), and also possibly by the non-observable effort expanded by the individual (moral hazard). The setting is one in which individuals cannot freely combine different contracts in various quantities : contracts are exclusive. This describes the current situation in many countries where insurance against pervasive risks, unemployment and health for instance, is provided at the country level through compulsory schemes. These policies are likely to be affected by, and to have an impact on, macro-economic activity. To our knowledge, not much has been said on how contracts should vary with the state of the economy. Indeed most research on moral hazard analyzes contracts in a given environment, i.e. in a given state of the economy.

The paper takes a second best approach to the optimal sharing of macro economic risks when moral hazard precludes perfect sharing of idiosyncratic risks. It studies how the contracts, defined *ex ante* before the state of the economy is known, should fluctuate in function of that state in order to maximize a weighted sum of utilities. If effort is observable, first best optimality can be obtained. It implies the *mutualization* of individual risks, leading to a representative agent for each group. Macro-economic risks, on the other hand, are unavoidable, and must be allocated among the representative agents according to some "sharing rules" that account for their respective attitudes towards risk, as studied by Wilson (1968). If effort is not observable, full insurance of idiosyncratic shocks discourages individuals from expanding any effort and the first best cannot be reached. What are the consequences on the sharing of aggregate risks ?

The sharing rules provide a benchmark to assess the impact of moral hazard. The paper derives first some conditions that allows one to compare the share of aggregate resources received by each group with those predicted by the sharing rules. For example, in the problem of pension design described above, the approach amounts to determine how the income of retirees and young individuals, employed and unemployed, should fluctuate in function of the state of the economy, accounting for the impact of unemployment benefits on incentives to find a job[3]. Under some plausible conditions, the share received by the young generation is decreased due to moral hazard.

The monotonicity of sharing rules with respect to the level of aggregate resources is a crucial property. It says that, as the state varies, individuals' consumption and utility levels vary in the same direction, either all upward or all downward. It turns out that the monotonicity property may fail under moral hazard, meaning that some may benefit from a change in the state while others are hurt. This may clearly create a difficulty in the implementation of insurance contracts, which may partly explain why macro-economic risks are poorly pooled, and mainly at a compulsory state level.

The paper is organized as follows. Section 2 describes the model and Section 3 studies the benchmark case of optimal insurance without moral hazard. Section 4 describes the optimal conditions, and Section 5 compares the expected consumption received by each group with the sharing rules without moral hazard. Section 6 discusses the impact that moral hazard has on the shape of the contracts.

2 The model

We consider an economy with a single good. The set of individuals is partitioned into observable classes or groups. Ex ante individuals within a class are identical. For simplicity, there are only two groups (or types) of indi-

[3]Note that, the (*ex ante*) weights on young and old individuals' welfare determine the expected level of pension benefits and contributions to social security. The (political) issue of choosing the weights, or the expected benefits, is not addressed.

viduals, each one of equal size, normalized to 1. A h-individual denote an individual in group h, $h = 1, 2$. The following characteristics are assumed.

Idiosyncratic and aggregate risks. There are two kinds of risks, at the individual and macro-economic level. Within a group, individuals are *ex ante* identical. They are *ex post* different with respect to the realization of an idiosyncratic shock, which determines their status and monetary output (or endowment). For instance, an individual may be employed or unemployed, be ill or healthy. There is a moral hazard problem if the probability distribution of the idiosyncratic shock is influenced by a level of effort exerted by the individual and not observable. A status is denoted by θ, and a level of effort by e.

The state of the economy, denoted by s, influences the environment individuals are facing, namely the monetary outputs and the probability of idiosyncratic shocks given a level of effort. To be more precise, the macroeconomic state s determines

- the monetary output $\omega_h(\theta|s)$ of a h-individual whose status is θ,
- the positive probability $p_h(\theta, e|s)$ for an h-individual to be in status θ if he exerts effort level e.

Status and monetary outputs can be identified if the state s influences the probability of success only, but not the monetary values.[4] A simple framework is one where individuals face a binomial risk, success or failure. More generally the status takes a finite number of values. They can be ordered, $\theta_1 < ... < \theta_m$ so that for each state s output $\omega(\theta|s)$ increases with θ. The set of macro states may be finite or infinite.

Preferences. Preferences of a h-individual are represented by a Von Neumann-Morgenstern utility function, u_h, over income levels c, separable in effort : $u_h(c) - ke$. The function u_h is defined over $[\underline{c}_h, +\infty[$ where $\underline{c}_h$ is a lower bound on wealth, possibly $-\infty$. It is concave, strictly increasing,

[4]Namely take θ and $\omega(\theta|s)$ to be equal. Such identification is always possible by setting $p(\omega|e, s)$ to be the probability of receiving ω. This would however imply to deal with supports for θ varying with the state or equivalently to allow some probabilities to be null. It is more convenient to work with positive probabilities, which explains why we have chosen to separate status from outputs.

and twice differentiable.

We shall assume that individuals in one group at least are risk averse.

Information. The *ex ante* distribution of macro-economic states, π and the distributions p_h are common knowledge.

Timing. The timing of the situation is the following one.

1. an insurance scheme is designed,
2. the macro-economic state is revealed,
3. each individual chooses a level of effort, and
4. individuals' status are observed, and contracts are implemented.

An insurance scheme specifies for each individual an insurance contract. I focus on insurance schemes that treat individuals within a group equally, meaning that they are all proposed an identical insurance contract at step 1. The timing makes possible for insurance contracts to account for both types of risk that affect an individual: a contract specifies the income level that an individual will receive at step 4 contingent on both his status and the macro-economic state, i.e. on θ and s.

Formally, an *insurance contract* for h is a contingent income plan : $\tilde{c}_h = (c_h(\theta|s))$, in which $c_h(\theta|s)$ denotes the income level of an h-individual whose status is θ if the macro state is s. The level of effort chosen at step 3 may depend on whether it is contractible or not. If it is, it is imposed, identical for all individuals in class h. If it is not, each individual chooses the optimal level, knowing the contingent income plan given s (as formally described in section**??**mor). At that time, all individuals in h are identical. Hence, whatever situation, individuals in h will exert an identical level of effort, that we denote by $e_h(s)$ and $\tilde{e}_h = (e_h(s))$.

An *insurance scheme* is described by an insurance contract for each group: $(\tilde{c}_h)_{h=1,2}$. It is feasible if, in each state, aggregate income is not larger than aggregate resources, accounting for the chosen level of efforts. To focus on the sharing of macroeconomic risks across groups, it is assumed that the frequency of an individual status within a group is exactly equal to its probability, i.e. $p_h(\theta, e_h(s)|s)$ for status θ given s and the chosen level of effort $e_h(s)$. Therefore, given a contract and the levels of effort exerted by individuals, there is no uncertainty at the aggregate level once the state of the economy is known. More precisely, given s, the θ-contingent plan $(c_h(\theta|s))$ and the level of effort $e_h(s)$, the aggregate output of group h, equal to

$$\Omega_h(s) = \sum_\theta p_h(\theta, e_h(s)|s)\omega_h(\theta|s). \tag{1}$$

is risk-less as well as its aggregate income:

$$C_h(s) = \sum_\theta p_h(\theta, e_h(s)|s)c_h(\theta|s). \tag{2}$$

With this notation, the *feasibility* of the scheme $(\tilde{c}_h)_{h=1,2}$ given effort levels $(\tilde{e}_h)_{h=1,2}$ writes as

$$\sum_h C_h(s) = \sum_h \Omega_h(s) \text{ in each state } s. \tag{3}$$

Optimality. Positive weights are assigned to each group, λ_h to h. A feasible insurance scheme is said to be optimal if it maximizes the weighted sum of the *ex ante* utilities of the groups over all feasible schemes. Thus, the welfare criterium associated to a scheme $(\tilde{c}_h)_{h=1,2}$ and effort levels $(\tilde{e}_h)_{h=1,2}$ is equal to

$$\sum_h \lambda_h [\sum_s \pi(s) U_h(\tilde{c}_h, \tilde{e}_h|s)]. \tag{4}$$

where $U_h(\tilde{c}_h, e_h|s)$ is the expected utility derived by a h-agent conditional on state s:

$$U_h(\tilde{c}_h, e_h|s) = \sum_\theta p_h(\theta, e_h(s)|s)u_h(c_h(\theta|s)) - k(s)e_h(s). \tag{5}$$

Exchanging the sums in the objective function (4), the optimization problem writes as

$$\text{maximize } \sum_s \pi(s)[\sum_h \lambda_h U_h(\tilde{c}_h, e_h|s)] \tag{6}$$

over $(\tilde{c}_h)_{h=1,2}$, and $(e_h)_{h=1,2}$ satisfying for each s the feasibility constraints (3), and the incentives constraints if effort is not contractible.

Since the constraints are independent across states s and the objective function (6) is separable, the optimization problem amounts to maximize in each state the weighted sum $\sum_h \lambda_h U_h(\tilde{c}_h, e_h)$ under the probability distribution for the idiosyncratic shocks and the constraints prevailing in that state. Evaluating how macro-economic risks are shared within the groups amounts to study how the solution to this program varies with s.

Remark. As a consequence, the distribution of the states π does not influence the shape of the contract : two distributions with the same support will have exactly the same optimal contract. The distribution however influences utility levels. Hence if one wants to select a contract that Pareto improves upon a given situation, the distribution matters through the choice of the weights.

3 The benchmark case without moral hazard

3.1 Sharing rules

This section recalls the basic features of optimal risk sharing in the absence of moral hazard. Without effort level, the probability distributions of individual states only depend on the state of nature, $p_h(\theta|s)$ for the probability of status θ given s. Thus, the wealth generated by each group h and the overall one are exogenous. In state s, they are respectively given by

$$\Omega_h(s) = \sum_\theta p_h(\theta|s)\Omega_h(\theta|s) \text{ and } \Omega(s) = \sum_h \Omega_h(s).$$

To be optimal, the allocation must satisfy the *mutuality principle*, which asserts that individuals' income levels are identical across states with identical aggregate wealth. Therefore, individuals bear risks only if it is unavoidable. This implies in particular that they are fully insured against idiosyncratic shocks. Furthermore, income levels depend on the macro-economic state only through the aggregate wealth realized in that state : they are described by *sharing rules* in the terminology of Wilson. Since individuals within a group are treated equally, their sharing rules are identical, S_h for a h-individual.

The sharing rules S_h are characterized as follows. For each level of aggregate wealth Ω, let $(S_1(\Omega), S_2(\Omega))$ be the unique pair (c_1, c_2) that solves

$$\lambda_1 u_1'(c_1) = \lambda_2 u_2'(c_2), \text{ and } c_1 + c_2 = \Omega. \tag{7}$$

Optimality is described formally as:

Without moral hazard, the optimal insurance contract fully insures individuals against idiosyncratic risks and allocates income according to the sharing rules (S_1, S_2) *: in each state* s*,*

$$c_h(\theta|s) = S_h(\Omega(s)) \textit{ for each } h.$$

If individuals in one group are risk neutral, the risk-averse individuals are fully insured against all risks, namely their income level is constant across states. If individuals are all risk averse, each sharing rule is strictly increasing. The precise shape of the sharing rules S_h depend on the utility functions of the two groups (and the weights), as will be illustrated below.

The optimal risk sharing contract can be described as involving two types of insurance schemes: a full insurance contract for individual shocks within each group, and a contract sharing optimally the macro-economic risks across groups. The second contract can be reached by trading options on aggregate wealth. The Arrow-Debreu contingent price of one unit of consumption deliverable in state s is equal to the probability of the state $\pi(s)$ multiplied by the identical value of the weighted marginal utilities as given by (7).[5]

3.2 Contractible effort

The previous analysis extends easily to the situation in which individuals can exert an effort provided it is contractible. Given effort levels, the groups face the same problem as in the previous section: sharing the aggregate

[5]This is similar to Breeden and Litzenberger (1978) in a stock exchange setup. They show that if all individual endowments are issued as securities on a stock market, an optimal allocation can be reached by trading options on aggregate wealth. Demange and Laroque (1999) extends this result to situations in which individual risks are not all exchanged on the market.

consumption optimally between the groups. Thanks to the separability of preferences in effort and consumption, this immediately gives the following characterization[6]:

Proposition 1 *Assume that effort is observable. An optimal insurance scheme is characterized by*

- *optimal risk insurance For each group h, in each state s*

$$c_h(\theta|s) = S_h(\Omega(s)) \textit{ for each } \theta$$

- *optimal level of effort In each state s, for each h:*

$$e_h(s) \textit{ maximizes } e \to u'_h(S_h(\Omega(s)) \sum_\theta p_h(\theta, e|s)\omega_h(\theta) - ke$$

Therefore the mutuality principle still holds : individual's consumption levels depend only on aggregate wealth. Furthermore the same sharing rules apply whether there is no effort or effort is contractible. In states with identical aggregate resources, individual's consumption levels are identical as well, but not necessarily their level of effort. Take for example two groups with identical preferences, identical weights. Consider two states s and s' that are obtained by exchanging the distribution probability p and the outputs between the two groups. By symmetry, aggregate wealth is identical in the two states, hence individuals' consumption levels. However the levels of effort in general differ, and as a consequence the utility levels. Loosely speaking the risk in the cost of effort is not ensured, due to the linearity of preferences in effort.

Note that, because individuals are fully insured against their status risk, without enforcement of effort, they would choose the minimal level, which may be sub-optimal,

[6] Denote by $\alpha(s)$ the multiplier of the feasibility constraint in state s. The optimal consumptions and levels of effort maximize the lagrangean

$$\sum \lambda_h [\sum_\theta p_h(\theta, e_h|s) u_h(c_h(\theta|s)) - k(s)e_h(s)] + \alpha(s)[\sum_\theta p_h(\theta, e_h|s)(\omega_h(\theta) - c_h(\theta|s))].$$

Pensions As an illustrative example, let group 1 be composed of the young generation and group 2 the old one. Young individuals are employed with probability $p(e|s)$, generating a gross revenue $\omega_1(s)$ (index θ is unnecessary here and has been dropped) and are unemployed with probability $1-p(e|s)$ generating no revenue. Old individuals do not work, bear no risk. Therefore the expected aggregate resources generated in state s are equal to $\Omega(s) = (1-p(e|s)\omega(s))$.

By the mutuality principle, young workers are completely insured against unemployment, and income levels of both young and old are functions of $\Omega(s)$. This result suggests that pension annuities should not be indexed on individual wages, but rather on aggregate wages. In particular benefits should decrease as unemployment raises, and conversely. Such a result supports a *notional* pay as you go system, as has been recently implemented in Sweden, in which annuities are indexed on growth.

Under specific utility functions the sharing rules are easily described. Assume first that individuals have constant risk aversion, with an index of risk tolerance τ_h in group h. Let $\overline{\Omega} = E[\Omega(s)]$ be the *ex ante* expected revenue per worker over macro-economic states, accounting for the effort levels. Then each young, worker or unemployed, and each old individual get respectively

$$S_1(\Omega(s)) = \bar{c}_1 + \frac{\tau_1}{\tau_1+\tau_2}(\Omega(s)-\overline{\Omega}), S_2(\Omega(s)) = \bar{c}_2 + \frac{\tau_2}{\tau_1+\tau_2}(\Omega(s)-\overline{\Omega})$$

where $\bar{c}_1$ and $\bar{c}_2$ are two scalars that depend on the weights λ_h only and sum to the expected wealth $\overline{\Omega}$. So the fluctuations in aggregate wealth are allocated to the agents in proportion of their risk tolerance.

With an identical constant relative risk aversion, all income, in particular unemployment and pension benefits are fully indexed on aggregate wealth:

$$S_1(\Omega(s)) = \delta\Omega(s), S_2(\Omega(s)) = (1-\delta)\Omega(s)$$

where δ depends on the weights λ_h only.

4 Optimal scheme under moral hazard

This section assumes moral hazard, meaning that individuals' effort are not observable nor verifiable.

At the time an individual chooses a level of effort, he knows the state of the economy and the contract he is facing. Thus, individual h, knowing s and facing a contract $(c_h(\theta|s))$ chooses effort optimally, that is $e_h(s)$ that maximizes $\sum_\theta p_h(\theta, e|s)u_h(c_h(\theta|s)) - k(s)e$ over the possible levels of effort. Let $\hat{U}_h(\tilde{c}_h|s)$ be the obtained utility level:

$$\hat{U}_h(\tilde{c}_h|s) = \max_{e\in[0,e_{max}]} \left\{ \sum_\theta p_h(\theta, e|s)u_h(c_h(\theta|s)) - ke \right\}. \tag{8}$$

The optimization problem in state s maximizes the weighted sum of the expected utilities

$$\lambda_1 \hat{U}_1(\tilde{c}_1|s) + \lambda_2 \hat{U}_2(\tilde{c}_2|s)$$

under the feasibility constraint (3) :

$$\sum_h \sum_\theta p(\theta, e_h|s)[c_h - \omega_h](\theta|s) \leq 0,$$

where each e_h is optimal given $\tilde{c}_h$. It is convenient to decompose this problem into two sub problems, one intra-group, and the other inter-group.

Define the net transfer to group h as the expected surplus of its income over outputs at the optimum, accounting for the level of effort:

$$R_h(s) = \sum_\theta p(\theta, e_h|s)[c_h - \omega_h](\theta|s). \tag{9}$$

Surely, at the optimum, the insurance scheme within a group is optimal given the net transfer to that group, that is the contract $(\tilde{c}_h)$ maximizes $\hat{U}_h(\tilde{c}_h|s)$ under the constraint (9) where e_h is the optimal level given the contract and $R_h(s)$ is taken as given. This means that the optimization problem can be solved by considering

- the *intra-group* problem that chooses for each group an optimal insurance scheme, *given* a net transfer R_h to that group :

$$\mathcal{P}_h(R) : V_h(R_h|s) = \max_{\tilde{c}_h} \hat{U}_h(\tilde{c}_h|s)$$
$$\text{under the constraint } \sum_\theta p(\theta, e_h|s)[c_h - \omega_h](\theta|s) \leq R_h \tag{10}$$

in which e_h is the level chosen by h given contract $\tilde{c}_h$.

- the *inter-group* problem of allocating net transfers across groups : allocating $R_h(s)$ so as to maximize

$$\lambda_1 V_1(R_1|s) + \lambda_2 V_2(R_2|s)$$

under the feasibility constraint $\sum R_h = 0$.

This leads us to solve the intra-group problem for various values of transfers. It is worth noting that problem $\mathcal{P}(R)$ is the dual of a standard principal-agent problem with a risk neutral principal.[7] We shall take assumptions so that the so-called 'first order' approach to moral hazard problems is valid.[8] We shall take assumptions such that these problems are easil

Recall that status are ordered $\theta_1 < ... < \theta_m$ with resources increasing with with θ.

More precisely, the level of effort takes values in the interval $[0, e_{max}]$, and the probabilities $p(\theta, e|s)$ as a function of level e are differentiable. Furthermore:

- the cumulative probability $\sum_{\theta \leq t} p(\theta|e, s)$ is convex in effort e

- the ratio $p_e/p(\theta|e, s)$ is nondecreasing in θ.

The first assumption ensures that given a contract that is increasing in θ, an effort that satisfies the first order condition associated with (8) is indeed optimal. The second will give that an optimal contract is indeed increasing.

With two status levels, "failure" and "success", the assumptions are satisfied when the probability of success is increasing and concave in the effort level. The two outcomes case extends to more outcomes under the "spanning" condition, according to which

$$p(\theta|e) = \lambda(e)p(\theta) + (1 - \lambda(e))q(\theta)) \tag{11}$$

[7] In the "standard" problem the principal maximizes the surplus, $\sum_\theta p(\theta)(\omega - c)(\theta)$, under a minimum participation utility level for the agent.

[8] On this, see Grossman-Hart (1983), Rogerson (1985), and Jewitt (1988).

for some function λ that is increasing, concave, with $\lambda(0) = 0$, and $\lambda(01 = 0$ and probabilities p and q. The assumptions are satisfied if the ratio $p(\theta)/q(\theta)$ increases with θ.

The first order conditions on e are

$$\sum_{\theta} p_e(\theta, e)u(c(\theta)) - k = \begin{cases} \leq 0 & \text{if } e = 0 \\ = 0 & \text{if } 0 < e < e_{max} \\ \geq 0 & \text{for } e = e_{max} \end{cases} \tag{12}$$

4.1 Optimal contract within a group

This section considers the "intra-group" problem faced by a group in a given state when its budget constraint varies. To simplify notation, state s is omitted.

Lemma 1. *At an optimal solution of the intra-group problem faced by a group given trnafer R there are $\alpha > 0$, and $\beta \geq 0$ such that for each θ:*

$$\frac{1}{\alpha} = \frac{1}{u'(c(\theta))} - \frac{\beta}{\alpha}\frac{p_e}{p}(\theta, e) \tag{13}$$

The marginal value $V'(R)$ is equal to

$$1/E_{\theta}[\frac{1}{u'(c(\theta))}|e]. \tag{14}$$

Furthermore (i) either the optimal level of effort e is strictly positive, and

$$\beta(-\sum_{\theta} p_{ee}(\theta, e)u(c(\theta)) \leq \alpha \sum_{\theta} p_e(\theta, e)(\omega - c)(\theta) \text{ with } = \text{ if } e < e_{max}$$

(ii) or $e = 0$, $\beta = 0$, and $c(\theta) = c$ for each θ, which occurs if

$$u'(c)[\sum_{\theta} p_e(\theta, 0)\omega(\theta)] \leq k \tag{15}$$

Conditions (13) are the first order conditions on consumption given a level of effort where the multiplier α is associated to the budget constraint. They are the same as in a principal-agent problem, as expected from duality. According to (14), the marginal utility for an extra revenue R is equal to the inverse of the expectation of the inverse marginal utility, in which the expectation is taken according to the probability distribution derived from the effort level. One may somewhat understand this as follows. Consider a marginal change in the net transfer dR to the group. If the first order conditions are fulfilled, the marginal change in utility derived by this change is independent of how dR is allocated across the different status.[9] In other words, $V'(R)$ can be computed in many different ways. For example, it can be computed by allocating dR so as to equalize the marginal change in utility across various status θ: $u'(c(\theta))\delta c(\theta)$ is equal to some δu whatever θ. Such an allocation does not affect the level of effort, hence probabilities do not change. Thus the budget equation (10) is satisfied if

$$dR = \sum_\theta p(\theta, e)\delta c(\theta) = \delta u \sum_\theta p(\theta, e)/u'(c(\theta)).$$

This gives that the marginal change in utility is

$$\delta u = dR/E[\frac{1}{u'(c(\theta))}|e]$$

which explains formula (14).

By allocating $dR > 0$ instead uniformly across the various status, the "marginal cost" of moral hazard is derived as follows. A uniform consumption increase, $dc(\theta) = \delta$, typically reduces incentives.[10] Hence the increase δ may differ from dR. By the envelope theorem, the marginal change of utility is equal to $\delta E[u']$ so that (14) gives $\delta E[u'] = dR/E[1/u'])$. The extra amount of consumption that can be given to each θ-agent is smaller than dR by Jensen inequality (applied to the inverse function $1/x$). More precisely the decrease in the level of effort induces a loss of resources equal to $dR(1 - 1/(E[u']E[1/u']))$.

[9] The intuition is that if an allocation increases utility more than another one, both being feasible with the transfer $R + dR$, taking into account the impact on the effort level, then the difference would yield a feasible allocation for transfer R preferred to the standing allocation.

[10] Assuming e to be interior, the derivation of (12) gives $\delta \sum_\theta p_e u'(c) + \sum_\theta p_{ee} u(c) de = 0$. Thanks to the assumptions, the contract is increasing so that the term $\sum_\theta p_e u'(c)$ is negative (increasing likelihood ratio) as well as $\sum p_{ee} u(c)$ (second order condition).

Remark. As far as we know, the expression (14) has not been given in the literature. In a repeated moral hazard problem, Rogerson (1985) shows that the inverse of the marginal utility plays a role. More precisely, a risk neutral principal always benefits from equalizing the agent's inverse marginal utility in a first period to the discounted expected inverse marginal utility of the following period. Lemma 1 helps us to understand this result: the agent's marginal utility for income is equalized across periods.

4.2 Optimal contracts

Proposition 2 *Under moral hazard, an optimal insurance scheme satisfies the following conditions. In each state s*

- *(optimal risk sharing across groups) :*

$$\lambda_h \frac{\partial V_h}{\partial R_h} = \frac{\lambda_h}{E[\frac{1}{u'_h}|s]} \text{ is equalized across groups} \tag{16}$$

- *(optimal contract within a group) : for each h, $(c_h(\theta|s))$ is optimal for group h given the probability in state s and the net transfer $R_h(s)$.*

Condition (16) extends Borch condition (7) to situations with moral hazard. It says that the ratio of the groups' marginal utility for income should be kept constant across the states of the economy, where the marginal utility accounts of moral hazard.

4.3 Weak mutuality principle

The first natural question is what remains of the mutuality principle. One easily proves the following weak version.

Consider states that differ only by their monetary outputs and not by the probability of idiosyncratic shocks (given a level of effort). Let state s' be obtained from s by increasing all monetary outputs of group 1 by the same amount δ whatever the status θ and by decreasing those of group 2 by δ. Then, at the optimal contract, both groups get the same contingent contract in states s ans s'

$$c_h(\theta|s) = c_h(\theta|s') \text{ for each } \theta, h.$$

A direct proof is straightforward. Since the probabilities are identical, neither the set of the feasible allocations does not change (through a translation of the transfers) not the welfare criterium. The conditions that are required to compare two states are very strong, so the above mutuality principle is weak indeed. Another result one could hope for is that a uniform increase of 1's outputs, δ_1, greater than a uniform decrease of 2's outputs. δ_2 would lead to increase all contingent - consumptions levels This is not true. Indeed one can go from s to s' in two steps, first by increasing all monetary outputs of group 1 by δ_1 and by decreasing those of group 2 by δ_1, second by increasing the monetary outputs of group 2 by the positive amount $\delta_1 - \delta_2$. In the first step, thanks to the weak mutuality principle, contracts do not change. In the second step, only group 2's monetary outputs are increased. As shown by example 6.1 below (in which 2 is not subject to idiosyncratic shock), such an increase may be followed by a decrease in 2's consumption, due to a decrease in the level of effort exerted by group 1.

5 Shares leval

We start by studying how moral hazard affects the shares received by each group. Consider an outside observer, who has information only on the aggregate level of income of each group. In which direction does moral hazard distort the shares ? Formally, we want to compare the shares $C_h(s) = E_\theta[c_h(\theta|s)]$ with those associated to the same value for aggregate wealth $\Omega(s)$ if there was no moral hazard, namely with $S_h(\Omega(s))$. Next proposition answers this question under some conditions.

We first start with a special case in which moral hazard has no impact. Let individuals have a constant relative risk aversion equal to one : $u(c) = \ln c$.The group's marginal utility for income satisfies

$$V'(R) = 1/E[\frac{1}{u'(c)}] = 1/E[c] = u'(E[c]).$$

Thus, the group's marginal utility is equal to the marginal utility of one individual who receives the expected income as without moral hazard. This straightforwardly implies that conditions (16) coincide with the Borch conditions applied to expected consumption levels. This gives the corollary to proposition 2.

Corollary *Assume that group h is either not subject to moral hazard or has a log utility function, for $h = 1, 2$. Then at the optimal insurance scheme,*

$$C_h(s) = E_\theta[c_h(\theta|s)] = S_h(\Omega(s)),\ \textit{for } h = 1, 2 \textit{ each } s$$

Therefore, aggregate wealth is shared among the two groups as without moral hazard. The absence of distortion is due to the fact that the inverse of the marginal utility is linear for a log function. It suggests that more generally the concavity, or convexity, of the inverse of the marginal utility determines how shares are distorted in the presence of moral hazard. This is indeed true, as stated in the following Proposition.

Proposition 3 *Assume that $1/u_1'$ is convex or group 1 is not subject to moral hazard and that $1/u_2'$ is concave or group 2 is not subject to moral hazard. At the optimal scheme*

$$C_1(s) \leq S_1(\Omega(s)) \textit{ and } C_2(s) \geq S_2(\Omega(s)),$$

with a strict inequality if $1/u_1'$ is strictly convex and 1's effort is not minimal or $1/u_2'$ is strictly concave and 2's effort is not minimal.

Therefore group 1's share of aggregate wealth is not larger, and group 2's share is not less, than without moral hazard. An immediate implication of proposition 3 is the following one. Under the stated assumptions, consider two states in which aggregate wealth is identical. If group 1 provides no effort in one state, its share is surely not less than in the other state.

In the pension example, the retirees are not subject to moral hazard. Therefore proposition 3 applies whenever the inverse $1/u_1'$ is either convex or concave. The convexity of the inverse of marginal utility seems to be the more plausible assumption, at least for standard utility functions. Indeed, it is satisfied for constant risk aversion function. For a constant relative risk-aversion function, $1/u_1' = c^\gamma$ is convex if $\gamma\ > 1$, i.e. if individuals are relatively more risk averse than with a log function, as is usually assumed. Under this assumption, the expected share received by the young generation should be lowered to account for moral hazard.

5.1 A risk neutral group

This section assumes individuals in one group to be risk neutral, and in the other risk averse. Without moral hazard, consumption of the risk averse individuals are fully insured against all shocks, whether on their status or on macro-economic states: From Proposition 2, they get the constant income level S_1 defined by

$$\lambda_1 u_1'(S_1) = \lambda_2.$$

At the minimal consumption is reduced to the sure S_1 (the level of effort is however adjusted in function of the state). In the presence of moral hazard, it may be optimal not to insure the risk averse individuals against idiosyncratic shocks in some states so as to incite them to expand effort. The question is whether they are nevertheless insured against macro-economic shocks. To answer this question, consider the optimality conditions assuming individuals in group 2 to be risk neutral ($u_2(c) = c$, without bounds on their consumption levels). In a state s in which group 1's individuals are incited to exert effort, their consumption levels varie across status (since, for a positive β, $1/u_1'$ varies from (13)). Now, optimal sharing across groups as given by (16), implies, that in any state s:

$$E[\frac{1}{u_1'}|s] = \lambda_1/\lambda_2, \tag{17}$$

in other words the expectation of the inverse of marginal utilities is constant across states. Thus there are few chances for utility levels over consumption to be constant. If the distributions of $1/u_1'$ in various states can be compared in the sense of second order stochastic dominance, more can be said. Note that when no effort is exerted, the distribution is certain, hence surely dominates any other distribution.

We have already seen that the convexity of $1/u'_1$ plays a role. A related function that is worth considering is $(1/u'_1)(u_1^{-1})$, which is the derivative of u^{-1}. Since $u^{-1})(v)$ is the consumption level that ensures a uitlity level equal to v, $(1/u'_1()u_1^{-1})(v)dv$ is the additional consumption that must be given so as to increase utility from v to $v + dv$. Note that $1/u'_1(u_1^{-1})$ is surely convex if $1/u'_1$ is, but that the converse is false. For example, for a function with constant relative risk aversion γ, $1/u'_1$ is concave for $\gamma < 1$ but $1/u'_1(u_1^{-1})$ is convex if $\gamma > 1/2$.

Proposition 4. *Assume group 2 to be risk neutral. Let S_1 be the constant consumption level that group 1 gets if he is not subject to moral hazard ($u'_1(S_1) = \lambda_2/\lambda_1$). Let the distribution of $1/u'$ given state s be strictly dominated by that in state s'.*

If $1/u'_1$ is convex , then $E[C_1(s)] \leq E[C_1(s')] \leq S_1$,
the first inequality being strict if $1/u'_1$ is strictly convex.
If $(1/u'_1)(u_1^{-1})$ is convex then $E[u_1(\tilde{c}_1)|s] \leq E[u_1(\tilde{c}_1)|s'] \leq u(S_1)$,
the first inequality being strict if $(1/u'_1)(u_1^{-1})$ is strictly convex.

Thus, at the optimal scheme, group 1 is not insured against aggregate risk. In the case of a log function for example, ($1/u'_1$ is linear), the expected 1's consumption level is constant over all states, but in general, expected utility *Elnc* will vary with the state.

The comparison of the distributions is not easy: As can be seen from (13), a contract depends on a state both through the multiplier β of the incentives constraint and by the effort level. As the state varies, both can change, which makes comparisons difficult. Under the spanning condition (11) however with a linear function for λ, the distributions can always be compared. The reason is that optimal effort is either at the minimal or maximal value. In the first case, the sure level S_1 is obtained . If effort is maximal, only the value of β can change. As β increases, the distribution stochastically decreases in the sense of second order dominance.[11]

[11]To see this note that for θ for which $p_e > 0$, $1/u'(c(\theta))$ is larger than the mean λ and conversely for $\theta < \theta^*$ it is smaller. Thus, as β increases, the values larger than the mean all increase (since $p_e > 0$), and those smaller than the mean all decrease: this is a mean preserving spread.

This result can be translated into the framework of a risk neutral principal facing a risk averse agent in various situations, summarized in the "state".[12] If the principal keeps the expected utility level conditional on s of the risk-averse agent constant across states, the derived allocation is not optimal from the *ex ante* point of view. In other words, a principal who must give a minimal utility level to the agent before the state is known is better off by designing a contract in which the agent's utility level varies with the state.

6 Shares variations

The state of the economy can influence the environment in several ways, both through the distributions of probabilities individuals are facing and the outputs. This section studies how expected shares vary when monetary outputs vary.

6.1 Discontinuities

We have said nothing about whether the conditions given in proposition 2 that are necessary for optimality are sufficient. They are not. The reason is that the value of a state-contingent contract for a group is not concave, that is the function $\hat{U}_h(\tilde{c}_h|s)$ is not concave in $(\tilde{c}_h)$. As a consequence, the program $\mathcal{P}_h(R)$ is not necessarily convex, which result in discontinuities in its solution with respect to R. In fine, the optimal risk sharing and optimal transfers may present discontinuities with respect to the macro-economic states. We give here a simple example that illustrates this point.

It is quite easy to understand that the value function $\hat{U}_h(\tilde{c}_h|s)$ is not concave in $(\tilde{c}_h)$. By the envelope theorem, dropping index h and s, the marginal utility for consumption in state θ, $\frac{\partial \hat{U}(\tilde{c})}{\partial c(\theta)}$ is equal to $p(\theta|e)u(c(\theta)$ where e is

[12] Our analysis assumes that there are many agents within each group so that the law of large numbers applies. With a group composed of risk neutral individuals, however, whether there is a unique individual in that group or many does not affect the analysis. Thus we fall back on a model similar to principal agent model with individuals in group 2 as the agents.

the optimal level of effort. If θ is a "bad state", increasing consumption in that state may decrease the optimal level of effort and increase the probability of the state sufficiently so that marginal utility increases. Consider for example a simple model with two outcomes, bad, θ_b, and good θ_g. Assume also the probability linear in effort, so that the optimal level of effort will be minimum or maximum. Let p and q with $p > q$ be respectively the probabilities of the good state if the maximal and minimum levels are exerted. Starting with a contract for which e_{max} is optimal, increase consumption in case of failure $c(\theta_b)$, keeping $c(\theta_g)$ fixed. There is a threshold value $c^*(\theta_b)$ for which the optimal level of effort jumps to the minimum. At this point, the marginal utility $\frac{\partial \hat{U}(\tilde{c})}{\partial c(\theta_b)}$ jumps from $(1-p)u(c^*(\theta_b))$ to $(1-q)u(c^*(\theta_b))$. The jump is upward, which shows that $\hat{U}$ is not concave.

This may generate discontinuities in the contract with respect to the state, as illustrated by the following example, still assuming two outcomes and probability inear in effort. Only group 1 is subject to moral hazard. The state is parameterized by (ω_1, ω_2), where ω_1 is the output for an individual of group 1 in case of success, (0 in case of failure) and ω_2 the output for an individual of group 2. The out Both individuals have log function, and weights are taken to be equal.

Given a state (ω_1, ω_2), the value function $V_1(R)$ is easily computed as
$log(p\omega_1 + R) - a$ if $R \leq R^*$ (effort level is maximal)
$log(q\omega_1 + R)$ if $R \geq R^*$ (effort is minimal)
where R^* depends on the parameters and ω_1.

This gives the overall welfare as a function of the transfer R from group 2 to group 1 : $V_1(\omega_1, R) + log(\omega_2 - R)$. The function is concave on each interval where effort is constant, but has an upward jump at R^*. As a consequence it is not concave for all transfers, and the global maximum is obtained by comparing the two maxima on each interval. In Figure 1 the welfare function as a function of R is drawn in two states with identical value for ω_1 but two distinct values for ω_2. The dashed line represents the welfare for the lower value with a maximum obtained when effort is maximal, the plain line when it is minimal. The second graph depicts the set of states for

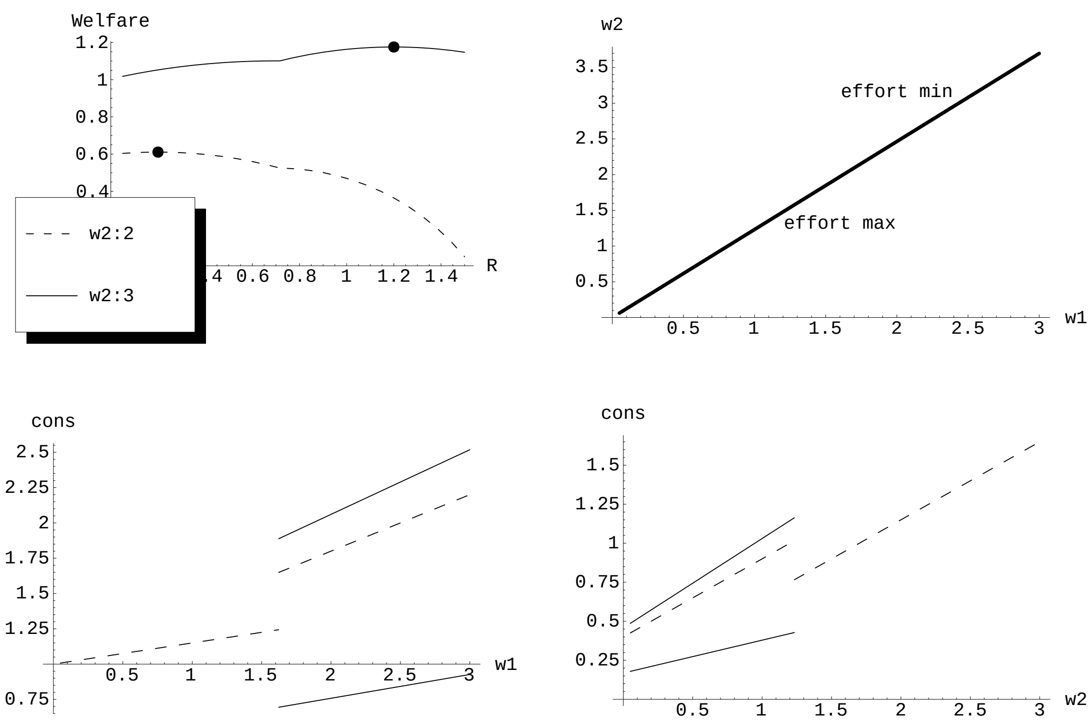

Figure 1: $k = 0.5$, $\pi_g = 0.8$, $\pi_b = 0.3$

1. Welfare as a function of the transfer from group 2 to group 1.
2. Locus of states for which effort is maximal or minimal.
3. Consumptions as a function of ω_1.
4. Consumptions as a function of ω_2.

which effort is maximal, below the line, or minimal, above the line.[13] If the value R^* is strictly between $R^1 = (\omega_2 - p\omega_1)/2$ and $R^0 = (\omega_2 - q\omega_1)/2$, welfare has two local maxima at R^1 and R^0, and a minimum at R^*. Simple computation gives that the global maximum is obtained by the transfer R^1 with a maximal level of effort if $\omega_2 \leq \omega_1 K$ for some constant K, and by R^0 with a minimal level of effort if the inequality is reversed.

The shape of the contracts are given for a fixed value of ω_2 as ω_1 increases and similarly for a fixed value of ω_1 as ω_2 increases. The important point is that group 2 may be hurt by an increase in its output. The reason is that such an increase triggers an increase in the transfer that shifts the effort level to its minimum, and as a consequence decrease overall wealth.

6.2 Group's risk aversion

Discontinuities are due to change in effort level. One may also determine how the shares vary as the state of the economy changes but the same level of effort is implemented. As resources vary, the budget constraint of the intra-group problems $(\mathcal{P}(\mathcal{R})$. vary:

$$C_h(s) \leq p_h\omega_h(s) + R.$$

Let $V_h(\omega_h, R_h)$ be the associated value function. The analysis of Wilson extends somewhat by using this indirect utility function. In particular the variation in income levels can be appraised through the "risk aversion" coefficient of V_h. Let us give the argument. If the overall aggregate resources are Ω, the shares satisfy

$$\lambda_1 \frac{\partial V_1}{\partial R_1}(\omega_1, R_1) = \lambda_2 \frac{\partial V_2}{\partial R_2}(\omega_2, R_2) \text{ with } C_1 + C_2 = \Omega. \tag{18}$$

[13] The maximum on $R \leq R^*$ is obtained at $R^1 = (\omega_2 - p\omega_1)/2$ if this transfer is smaller than R^*, and at R^* otherwise. For this value the marginal utility of the two groups, the inverse of expected consumption, are equalized: the expected consumptions of each group are equal. Similarly, the maximum on $R \geq R^*$ is obtained at $R^0 = (\omega_2 - q\omega_1)/2$ for which marginal utility of the two groups are equalized if $R^0 > R^*$, and at R^* otherwise.

By the envelope theorem, $\frac{\partial V_1}{\partial \omega_1(\theta)}(\omega_1, R_1) = p(\theta)\frac{\partial V_1}{\partial R_1}(\omega_1, R_1)$. Thus differentiation of (18) gives

$$\lambda_1 V_1''(R_1)dC_1 = \lambda_2 V_2''(R_2)dC_2$$

with $dC_1 + dC_2 = d\Omega$, where to simplify notation $V_h''(R_h)$ denotes the second derivative of V with respect to R. Thus

$$d(C_1) = \frac{\rho_2}{\rho_1 + \rho_2}d\Omega,\ d(C_2) = \frac{\rho_1}{\rho_1 + \rho_2}d\Omega$$

where

$$\rho_h = \frac{-V_h''}{V_h'}(R_h).$$

It remains to evaluate the group risk aversion when resources vary. Since marginal utility $V'(R)$ is given by the inverse of $E[1/u']$, along states with a constant level of effort, one gets

$$V_h''(C_h) = E[-\frac{u''}{u'^2(\theta)}\frac{dc_h}{dR}(\theta)]/E[\frac{1}{u'}]^2$$

in which the expectation is taken over the values of θ with respect to the probability conditional on the effort being e.

Proposition 5 *Consider states that are distinct only for the values of the resources, $\omega_h(\theta|s), h = 1, 2$. In the states for which group h provides the same level of effort, the risk aversion satisfies*

$$\frac{1}{\rho} = E[\frac{u_h^{'}}{-u"_h}] - bcov(\frac{-u_h^{'2}}{u_h''}, \frac{1}{u_h'})^2 \tag{19}$$

for some positive b.

In other words, the group's risk tolerance is equal to the expectation of the risk tolerance over the various status diminished by a correcting term. As expected, the correcting term is null for a log function. Otherwise moral hazard tends to decrease the risk tolerance of the group.

Example

With only two outcomes, the expression takes a simpler form that is easy to obtain. The incentives constraint and the feasibility constraint fully determine how the optimal contract within the group is adjusted in function of R. Indeed keeping the effort level constant implies that $u'_h(c_h)dc_h$ is constant hence equal to dR. This gives the following formula for the group risk aversion

$$-\frac{V''}{V'}(R) = E[-\frac{u''}{u'}.\frac{1}{u'^2}]/E[\frac{1}{u'}]^2$$

. Under individual's constant aversion r, the "group " risk aversion coefficient is equal to $r(1+\frac{var(1/u')}{E[1/u'']^2})$. It is clearly larger ρ.

. Under constant relative risk aversion γ simple computation gives that

$$-\frac{V''}{V'}(R)Ec = \gamma E[c^{2\gamma-1}]E[c]/(Ec^{\gamma})^2$$

To apply this to the pension example, let group 2 be not subject to moral hazard, and assume the same isoelastic function for both groups. The equalization of weighted marginal utility with respect to income (16) writes as

$$\lambda_1 C_2^{\gamma} = \lambda_2 E[c_1^{\gamma}],$$

and risk aversion index as

$$\rho_h = \gamma\frac{E[c_h^{2\gamma-1}]}{[Ec_h^{\gamma}]^2}.$$

Thus

$$\frac{\rho_1}{\rho_2} = [\frac{\lambda_2}{\lambda_1}]^{\frac{1}{\gamma}}\frac{E[c_1^{2\gamma-1}]}{[Ec_1^{\gamma}]^{2-1/\gamma}} = [\frac{\lambda_2}{\lambda_1}]^{\frac{1}{\gamma}}E\left[(\frac{c_1^{\gamma}}{E[c_1^{\gamma}]})^{\frac{2\gamma-1}{\gamma}}\right].$$

The share received by group 1 varies less under moral hazard if the last expectation term is larger than 1, that is if function $x^{\frac{2\gamma-1}{\gamma}}$ is convex, and more if it is concave. Thus, the young's share is less variable when either the coefficient of relative risk aversion γ is less than one half or more than 1. To understand why, note that at the same level of share risk aversion is increased by moral hazard. However, the level of the shares is also affected by moral hazard: this may have an opposite impact on risk aversion if the shares are increased. From proposition 3, this is the case if $\gamma < 1$. For $1/2 < \gamma < 1$ the effect due to increased share levels dominates. To sum up, moral hazard has the following impact on the share of group 1:

if $\gamma < 1/2$, it increases the share and lowers its variation with resources

if $1/2 < \gamma < 1$, it increases the share and its variation

if $\gamma > 1$, moral hazard decreases both the share of group 1 and its variation.

7 Conclusion

We have shown that moral hazard may significantly distort the sharing of aggregate risks. In particular, optimality may lead to contracts that have undesirable properties: individuals may suffer from an increase of their own outputs.

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8 Proofs

Proof of Proposition 1. The proof follows similar arguments as for a principal agent model. Index s and h are dropped to simplify notation. Consider the indirect utility $\hat{U}$ of an individual who is faced with a θ-contingent plan $\tilde{c} = c(\theta)$ and chooses the optimal level of effort. Recall that we take the appropriate assumptions that ensure the first order condition (12) on effort level to be sufficient. By the envelope theorem, the derivative of $\hat{U}$ with respect to $c(\theta)$ is $p(\theta, e)u(c(\theta))$ computed at the optimal e.

Denote by α and β the multipliers associated with the feasibility constraint and the incentives constraint (12). This gives the lagrangean of the problem:

$$\hat{U} + \alpha(R + \sum_{\theta} p(\theta, e)[\omega - c](\theta) + \beta(\sum_{\theta} p_e(\theta, e)u(c(\theta)) - k).$$

An optimal allocation satisfies the first order conditions :

(consumption) each θ

$$u'(c(\theta)) - \alpha p(\theta, e) + \beta(\sum_{\theta} p_e(\theta, e)u'(c(\theta))) = 0$$

which can be written as

$$1 - \frac{\alpha}{u'(c(\theta))} + \beta\frac{p_e(\theta, e)}{p(\theta, e)} = 0,$$

i.e. (13).

(incentives)

$[\sum_\theta p_e(\theta,e)u(c(\theta)) - k] + \alpha[\sum_\theta p_e(\theta,e)(\omega - c)(\theta)] +$

$$\beta[\sum_\theta p_{ee}(\theta,e)u(c(\theta)] = 0 \text{ if } 0 < e < e_{max} \tag{20}$$

$$\leq 0 \text{ if } e = 0, \geq 0 \text{ if } e = e_{max}$$

Since $\sum_\theta p(\theta,e) = 1$, one has $\sum_\theta p_e(\theta,e) = 0$. Therefore multiplying (13) by $p(\theta,e)$ and taking the sum over θ gives that

$$\frac{1}{\alpha} = E_\theta[\frac{1}{u'(c(\theta))}].$$

By the envelope theorem α is equal to the marginal utility for revenue.

Two types of consumption plans within a group are obtained depending on whether the price of the incentives constraint, β, is nul or not.

Consider first the case where $\beta = 0$. Then by (13) consumption levels are equalized across individual states θ: $c(\theta)$ is constant equal to c. Therefore, $\sum_\theta p_e(\theta,e)u(c) = 0$, so that (IC) implies that effort is minimal. Also $\sum_\theta p_e(\theta,e)c = 0$. Thus condition (20) is met if

$$-k + \alpha[\sum_\theta p_e(\theta,0)(\omega(\theta)] \leq 0$$

Using that the value c satisfies $u'(c) = \alpha$, this gives

$$u'(c)[\sum_\theta p_e(\theta,0)\omega(\theta)] \leq k$$

which is condition (15)

Consider the second case where $\beta \neq 0$. Surely $e > 0$. Therefore the incentive constraint (IC) is binding: $\sum_\theta p_e(\theta,e)u(c) = k$, which implies that the level of effort is not minimal. Thus the first term in (20) is null, which gives that the value of β satisfies :

$$\beta \geq \alpha \frac{[\sum_\theta p_e(\theta,e)(\omega - c)(\theta)]}{-[\sum_\theta p_{ee}(\theta,e)u(c(\theta)]}$$

with an equality if $e < e_{max}$.

The fact that even if $e = e_{max}$ (IC) is satisfied as an equality is a standard result in incentives theory: if a contract is designed so as to incite the agent to provide the maximal level of effort, there is no gain in "inciting him more " by making the contract more risky. ■

Proof of Proposition 3. Let us rewrite the optimality condition (16):

$$\lambda_1/E_\theta[\frac{1}{u_1'(c_1(\tilde{\theta}|s))}] = \lambda_2/E_\theta[\frac{1}{u_2'(c_2(\tilde{\theta}|s))}].$$

Assume $1/u_1'$ to be convex. Then

$$E_\theta[\frac{1}{u_1'(c_1(\tilde{\theta},s))}] \geq \frac{1}{u_1'(E_\theta[c_1(\tilde{\theta}|s)]} = \frac{1}{u_1'(C_1(s))}.$$

Note also that if the group is not subject to moral hazard in state s, the just above inequality holds (as an equality), independently of the convexity of $1/u_1'$.

Using a similar argument, if $1/u_2'$ is concave or group 2 is not subject to moral hazard,

$$E_\theta[\frac{1}{u_2'(c_2(\tilde{\theta},s))}] \leq \frac{1}{u_2'(C_2(s))}.$$

Thus the optimality condition (16) gives $\lambda_1 u_1'(C_1(s)) \geq \lambda_2 u_2'(C_2(s))$. Now by definition

$$C_1(s) + C_2(s) = S_1(\Omega(s)) + S_2(\Omega(s)) = \Omega(s).$$

Furthermore, by Borch condition (7) :

$$\lambda_1 u_1'(S_1(\Omega(s))) = \lambda_2 u_2'(S_2(\Omega(s)).$$

Assume by contradiction $S_1(\Omega(s)) < C_1(s)$. Surely, $S_2(\Omega(s)) > c_2(s)$ and the successive inequalities hold :

$$\lambda_1 u_1'(S_1(\Omega(s))) > \lambda_1 u_1'(C_1(s)) \geq \lambda_2 u_2'(C_2(s)) > \lambda_2 u_2'(S_2(\Omega(s)),$$

which contradicts (7). ■

Proof of Proposition 5. To simplify notation, index h is dropped. Consider a change in the feasibility constraints of the group $d\omega$ and dR. This will change the optimal contract, and the multipliers, $d\alpha$ and $d\beta$. Write the first order conditions (13) as

$$u'(c(\theta))[1+\beta\frac{p_e}{p}(\theta)]=\alpha.$$

The marginal variation of contingent income ($dc(\theta)$), effort de and the multipliers $d\alpha$ and $d\beta$. satisfy

$$\frac{-u''}{u'}dc(\theta)=\frac{-d\alpha}{\alpha}+u'\frac{d\beta}{\beta}(\frac{1}{u'}-\frac{1}{\alpha})+\frac{u'}{\alpha}\beta d(\frac{p_e}{p})de \text{ each } \theta \tag{21}$$

These expressions give the variation of the contingent income and effort as function of $d\alpha$ and $d\beta$. In order to keep the level of effort constant, $de=0$, the marginal change in contingent consumption must satisfy

$$\sum_\theta p_e u'(c(\theta))dc(\theta)=0 \tag{22}$$

$$\sum_\theta p dc(\theta)=dW \tag{23}$$

and (21 becomes

$$\frac{-u''}{u'}dc(\theta)=\frac{-d\alpha}{\alpha}+u'\frac{d\beta}{\beta}(\frac{1}{u'}-\frac{1}{\alpha}) \text{ each } \theta \tag{24}$$

Plugging (24 into (22) gives the value of $d\beta$ as a function of $d\alpha$

$$d\beta=(\sum_\theta p_e\frac{u'^2}{-u''})d\alpha \tag{25}$$

Plugging this expression into (**??**) and rearranging yields

$$-\alpha E[dc]=d\alpha[E_\theta\frac{u'}{-u''}-\frac{(\sum_\theta p_e\frac{u'^2}{-u''})^2}{\sum_\theta\frac{u'^3}{-u''}\frac{p_e^2}{p}}]$$

■