Devaluation without common knowledge

Céline Rochon

THEMA, Université de Cergy-Pontoise*

November 3, 2004

Abstract

In an economy with a fixed exchange rate regime that suffers an adverse shock, we study the delay strategies of speculators that may trigger an endogenous devaluation before it occurs exogenously. The game played by the speculators has a unique symmetric Nash equilibrium which is a strongly rational expectation equilibrium. A higher uncertainty about the initial level of reserves of the Central Bank increases the delay and extends the *ex ante* mean delay between the exogenous shock and the devaluation. We determine endogenously the rate of devaluation, bringing together the first and second generation models of currency crises.

Keywords: currency crises, fixed exchange rate regime, speculation, uncertainty of reserves, endogenous devaluation

JEL Classification: D82, D83, D84, E58, F31, F32

*Céline Rochon, THEMA, Université de Cergy-Pontoise, 33 bd du Port, 95011 Cergy-Pontoise, France, celine.rochon@eco.u-cergy.fr, tel: 33 1 34 25 67 59, fax: 33 1 34 25 62 33

1 Introduction

In the first generation models of currency crises, the exchange rate is fixed as long as the Central Bank has sufficient reserves to sustain its value in the face of an outflow of capital and a drain on its reserves. Such an outflow can be induced by a change in the terms of trade or by a deficit of the government that is financed by money creation (as in the model of Krugman (1979)). When the reserves of the Central Bank are depleted, the fixed exchange rate is abandoned and the rate is determined by the market in a new regime. In the first generation models, agents have perfect information about the process. The switch from a fixed to a floating rate is in general accompanied by a downward jump of the demand for the domestic currency.[1] Under perfect foresight (as in Krugman (1979)), this jump is achieved through a jump in the nominal quantity of money. At the time of the switch, the "currency crisis" takes the form of a run on the currency at the fixed rate. There is no discrete devaluation. There is no capital loss by the agents who hold the domestic currency and no strategic complementarity in running from the domestic currency.[2]

The second generation models have focussed on the discrete devaluations that generate multiple equilibria: if there is a run on the currency and all selling orders cannot be executed at the fixed exchange rate (before the reserves of the Central Bank are depleted), there is an incentive to run if all others run. The strategic complementarity between the selling orders generate multiple equilibria. These models have focussed on the situation created by holdings of a domestic currency that are in excess after the peg is abandoned. The self-fulfilling nature of currency attacks is accounted for in these models; main contributions include Obstfeld (1986,1996), Velasco (1996), Jeanne (1997), and Jeanne and Masson (2000), among others.

Morris and Shin (1998) and Krugman (1996) both challenge the existence of multiple equilibria in second generation models. This is achieved by Morris and Shin (1998)

[1] In the model of Krugman (1979), the inflation rate jumps up after the switch because the government deficit is financed by seignorage.

[2] Other first generation models include Flood and Garber (1984), and Flood, Garber and Kramer (1996). The latter paper studies the effects of sterilization, by the addition of a bond market. Botman and Jager (2002) build on Krugman (1979) and Flood and Garber (1984) to present a multi-country setting in which coordination and contagion issues can be analyzed.

through a one-period model without common knowledge[3], and by Krugman (1996) through a particular specification of the devaluation expectations and of the evolution of fundamentals.

One can criticize one-period models for two reasons. First, a critical feature of currency attacks is that they occur over some period of time. The basic problem of the agents is not *whether* to attack a currency, but *when* to attack. In this waiting game, agents observe the market, (Chamley (2003, 2004)). Second, as in the first generation models, agents should anticipate the switch and avoid to be in the position of running to escape a capital loss.

When agents have imperfect information, a situation may arise in which agents end up with excess balance and a discrete devaluation takes place. In this paper, agents do not have common knowledge about each other's information and an outflow of capital. The absence of common knowledge generates a discrete devaluation after some finite time.

Broner (2003) analyzes the issue of currency crises with imperfect information in a model with two types of identical agents: the first type of agents have perfect information about the level of the Central Bank's reserves that terminates the peg. The second type of agents have identical but imperfect information and they observe the actions of the agents in the first group. The model exhibits a continuum of equilibria in which agents of the first type can run at different dates, realizing all their orders, while agents of the second type can sell only a fraction of their holdings at the fixed rate and suffer a capital loss on the residual.

In the present model, each agent views himself as no better informed than others. Agents have imperfect information both about the magnitude of the Central Bank's reserves, and about the information of others. The former assumption is validated by the behavior of various central banks towards the diffusion of information during the ERM crisis in 1992-1994.[4]

[3]We refer here to a discrete time model, which can be thought of as a sequence of one-period games without strategic interaction between periods.

[4]Indeed, during that period and well after, the central banks of England and France, to give but a few examples, published some information about their reserves, but with a lag. Statistics published were incomplete.

The model builds on that of Abreu and Brunnermeier (2003). They analyze the end of bubbles in a model where agents do not have common knowledge about the onset of the bubble. Although the focus of their analysis is a financial market, there is actually no formal analysis of the mechanism that determines the equilibrium price. The relation between that model and an actual financial market is difficult to see. For example, they assume that the amount of sales has no impact on the bubble price as long as these do not reach an assumed fixed threshold, and the bubble crashes when that level is reached. Such a bound is necessary for iterative dominance.

In the present paper, the assumptions for iterative dominance have a nice economic foundation and the analysis is considerably simpler than in Abreu and Brunnermeier (2003): i) in the regime of fixed exchange rate, the asset price is fixed by the Central Bank and trades have indeed no impact on that price; ii) the upper bound on the level of total purchases that triggers a change of regime is naturally imposed by the total amount of reserves of the Central Bank; iii) the jump in the exchange rate which occurs at the end of the first regime can be determined endogenously in a macroeconomic model.

The model is presented in the next section. The arrival of a negative shock occurs according to a Poisson process in continuous time. After a shock, a fixed outflow of reserves takes place. The regime of fixed exchange rate is abandoned if the accumulated exogenous outflow plus the sellings by agents exceed the initial reserves of the Central Bank.

We begin by assuming an exogenous rate of devaluation. Agents have imperfect information about the shock. The mass of agents (a continuum) who are informed about the shock increases linearly with time. Hence each informed agent knows that a shock has occurred but not how long before he became informed, and he does not know how many other agents are informed. But each informed agent knows that the exogenous outflow will trigger a devaluation in some finite time. At each instant, an informed agent is comparing the higher return on the domestic currency with the risk of a devaluation.

The symmetric Nash equilibrium is analyzed in Section 3. Under some parameter conditions, an agent delays selling the domestic currency after he becomes informed.

In Section 4, the rate of devaluation is endogenously determined by setting a value of the post-devaluation real quantity of money. That section fills the gap between the first generation models of currency crises with perfect information and the second generation models with discrete devaluations and imperfect information.

Note that each agent knows that the exogenous flow is sufficient to trigger a devaluation after some finite time. Hence a very long delay is a dominated strategy. By iteration on the dominated strategies, one shows in Section 5 that the symmetric Nash equilibrium is the only one to survive the elimination process. The equilibrium is therefore a strongly rational expectation equilibrium (Guesnerie (1992)).[5]

Section 6 analyzes the impact of the uncertainty about the initial level of reserves by the Central Bank on the delay. It is shown that more uncertainty increases the delay in selling the domestic currency and extends the *ex ante* mean delay between the exogenous shock and the devaluation. A last section concludes.

2 The model

Consider an economy with a fixed exchange rate regime compatible with the economy's fundamentals. At some time θ, the economy suffers an adverse shock which changes the fundamentals, and makes the exchange rate incompatible with the fundamentals. The value of θ is determined by an exponential distribution with parameter λ per unit of time. The waiting time before the occurrence of the shock is parameterized by λ: it is the probability of the occurrence of an adverse shock per unit of time conditional on no previous shock. The cumulative distribution function is $F(\theta) = 1 - e^{-\lambda\theta}$ and the density is $f(\theta) = \lambda e^{-\lambda\theta}$.

There is a continuum of agents (speculators) of mass normalized to one who hold one unit of wealth each. We assume that each speculator is risk neutral, and can buy a fixed quantity of foreign currency normalized to one at the price of one.[6] The domestic currency yields a return of r per unit of time. The foreign currency yields no return.

[5] The interest of this equilibrium refinement lies in the fact that it involves no strong conditions on the coordination of agents, as compared with the Nash equilibrium concept.

[6] We could generalize to the case where the agent can buy a variable and bounded quantity of foreign currency.

Before the adverse shock, the reserves of the Central Bank are exogenously fixed at $\overline{R}$. The adverse shock generates an outflow of reserves. This outflow can be explained by a change in trades or capital movements. It is not observed by the agents. However, the agents know the structure of this outflow. Let the reserves of the Central Bank at time $\theta + s$ be equal to $R(s)$ with $R(0) = \overline{R}$, $R' < 0$ and $R(T) = 0$ for some $T > 0$ sufficiently large. Once a crisis has begun, the situation of the Central Bank deteriorates. We assume that the reserves of the Central Bank, as observed by the agents, are linear over time:

$$R(s) = \overline{R}(1 - \frac{s}{T}).$$

After some finite interval of time, here time T, the exogenous outflow depletes completely the reserves of the Central Bank assuming no activity by speculators. In this case, a currency crisis occurs exogenously. In general, speculators are active and will sell the currency, thus depleting the reserves at a faster pace. A currency crisis occurs endogenously when the mass of agents who have bought the foreign currency reaches the reserves of the Central Bank. To repeat, a currency crisis is equivalent to the complete depletion of the Central Bank's reserves. It occurs either exogenously or endogenously. When it occurs, the Central Bank devalues the currency by an exogenous value β (the foreign exchange rate appreciates by β), and the game is over. In section 4, the rate of devaluation β will be determined endogenously.

Once a shock has occurred at time θ, speculators become gradually informed about the existence of the shock. Following Abreu and Brunnermeier (2003), we assume that the flow of newly informed speculators is uniform: the agents become informed at a constant rate. The mass of informed agents at time $\theta + s$ is $s\sigma$, for some parameter $\sigma > 0$. A speculator informed at time $t > \theta$ knows only that $\theta < t$: he knows that an exogenous shock has occurred; he does not know when. It follows that he does not know how many other agents are informed.

3 Strategies for symmetric Perfect Bayesian Nash equilibria

Agents become gradually informed about the occurrence of the shock. As time goes, more and more agents become informed and the risk of devaluation increases. The payoff of trading goes up. Hence the natural strategy of an agent is to delay and then trade.

Consider an agent who receives a signal about θ at time t. In this section, we look for a symmetric equilibrium where the agent buys the foreign currency at time $t+y$. We assume all other agents delay for an interval of time x after being informed.

Given θ, let z denote the time elapsed between θ and the devaluation. Assume all agents delay for x. Then $\sigma(z-x)$ represents the purchases between $\theta+x$ and $\theta+z$ and $\overline{R}z/T$ represents the exogenous losses of the Central Bank. The equation

$$\sigma(z-x)+\overline{R}\frac{z}{T}=\overline{R} \tag{1}$$

defines z as a function of x: the time elapsed between the onset of the deterioration and the devaluation depends on the strategy x of the agents.

Consider an agent informed at time t. If no devaluation has occurred after he delayed for y, while all other agents delay for x, the reserves of the Central Bank are still positive at time $t+y$. The onset of the deterioration, θ, cannot date back to a very long time (otherwise reserves would already be depleted). More specifically, the earliest time for the shock is such that reserves would be depleted immediately after time $t+y$. Given the strategy x, θ satisfies $t-l\leq\theta\leq t$ for some function

$$l=\phi(y;x)$$

of y and x, with

$$\sigma(l+y-x)+\overline{R}\frac{l+y}{T}=\overline{R}, \tag{2}$$

where $\sigma(l+y-x)$ represents the purchases between $\theta+x$ and $\theta+l+y$ and $\overline{R}(l+y)/T$ represents the exogenous losses of the Central Bank.[7]

[7] Whenever $y=x$, we find (1) once we identify z with $l+y$.

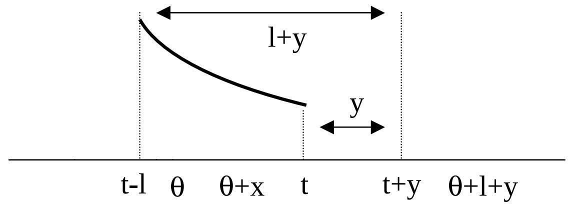

Figure 1

Equation (2) can be rewritten as follows:

$$l = \phi(y; x) = \frac{x + \frac{\overline{R}}{\sigma} - y(1 + \frac{\overline{R}}{\sigma T})}{1 + \frac{\overline{R}}{\sigma T}}. \tag{3}$$

Note that l, the length of the support of θ, is a decreasing function of the delay strategy y: the larger is the delay y, the smaller is l or else the closer is θ to t, the date at which the agent receives some information.

For an agent informed at t which delays for y, θ belongs to the interval $[t - l, t]$; the minimal value taken by θ is $t - l$. If there is a devaluation at $t + y$, then θ is precisely equal to $t - l$.

Once the agent is informed at date t, he revises his belief about θ. After a delay of y, his subjective distribution about θ is the exponential distribution truncated on the support $[t - \phi(y; x), t]$. Its density is $f(\theta; y, x) = Ae^{-\lambda\theta}$ with $1/A = \int_{t-\phi}^{t} e^{-\lambda\theta} d\theta$. Hence,

$$f(\theta; y, x) = \frac{\lambda e^{-\lambda(\theta - t)}}{e^{\lambda\phi(y;x)} - 1}. \tag{4}$$

3.1 The reaction function

Given that all other agents delay for an interval of time x, the instantaneous probability of the devaluation for an agent informed at t which delays for y is the exponential distribution of θ truncated on the interval $[t - \phi(y; x), t]$ evaluated at $\theta = t - \phi(y; x)$:

$$\pi(y; x) = \frac{\lambda}{1 - e^{-\lambda\phi(y;x)}}. \tag{5}$$

Indeed, a devaluation occurs at $t+y$ if the reserves are depleted at $t+y$, i.e. if θ is at the earliest date of the support of its distribution for the agent, namely $t-\phi(y;x)$. A devaluation occurs according to a Poisson process with an instantaneous probability equal to the density of the distribution of θ at $t-\phi(y;x)$.

For a given strategy x, the function $\phi(y;x)$ is decreasing in y, and hence $\pi(y;x)$ is an increasing function of y: the larger is the delay, the larger is the instantaneous probability of devaluation.

Moreover, an agent knows that the largest delay between the time he gets informed and a devaluation occurs when he is the first to be informed. In this case, the delay is $y_0 = \frac{x+\frac{\overline{R}}{\sigma}}{1+\frac{\overline{R}}{\sigma T}}$. One verifies that $\pi(y;x)$ tends to infinity as y approaches y_0.

We now want to analyze the optimal delay. Assuming the agent delays for y, let $dy<0$ denote a reduction in the delay and consider the impact of this modification on the gain in case of devaluation, $\beta\pi(y;x)$.

If $\beta\pi(y;x) > r$, then $(\beta\pi(y;x)-r)dy < 0$ and the agent should delay for an even shorter period of time. Indeed, the gain from the instantaneous probability of a devaluation is greater than the opportunity cost of the interest income on the domestic asset.

Similarly, if $\beta\pi(y;x) < r$, then $(\beta\pi(y;x)-r)dy > 0$ and hence the agent should delay for a longer period of time.

The *arbitrage condition* for buying the foreign currency after a delay of y is

$$\beta\pi(y;x) = r. \tag{6}$$

This condition defines a reaction function of an agent informed at t which delays y, when all other agents delay x.

Using (5), we have

$$\beta\pi(y;x) = \frac{\beta\lambda}{1-e^{-\lambda\phi(y;x)}}. \tag{7}$$

Recall that for a given x, $\beta\pi(y;x)$ is increasing in y. From the expression of ϕ in (3), one can show that the graph of $\beta\pi(\cdot;x)$ is as shown in Figure 2 with $\beta\pi(0;x)>0$. From the figure, it is immediate that there is a unique value $y(x)>0$ such that $\beta\pi(y(x);x)=r$ if and only if $\beta\pi(0;x)<r$. If $\beta\pi(0;x)>r$, then the agent should buy the foreign currency without delay and hence $y(x)=0$ is his optimal strategy.

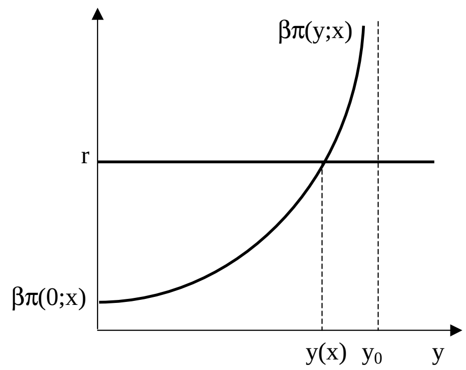

Figure 2

From the previous discussion, the *reaction function* of an agent informed at t which delays for y, given that all the other agents delay for x, is

$$y(x) = \frac{x + \frac{\overline{R}}{\sigma}}{1 + \frac{\overline{R}}{\sigma T}} - \frac{1}{\lambda} \log(\frac{r}{r - \beta\lambda}). \tag{8}$$

The slope of y as a function of x is smaller than one, and equal to $\frac{1}{1+\frac{\overline{R}}{\sigma T}}$. In order to forego trivial cases, we make the following assumption.

Assumption 1: $r > \beta\pi(0;0) = \frac{\beta\lambda}{1-e^{-A}} > \beta\lambda$, where $A = \frac{\lambda\frac{\overline{R}}{\sigma}}{1+\frac{\overline{R}}{\sigma T}}$.

Assumption 1 implies that $y(0) > 0$: the agent has a positive delay strategy.

The reaction function is illustrated in Figure 3. From Figure 3, it follows that there is a unique Nash equilibrium; it is the fixed point of the reaction function (8). The properties of the equilibrium strategy y^* are easily computed and are summarized in the next proposition.

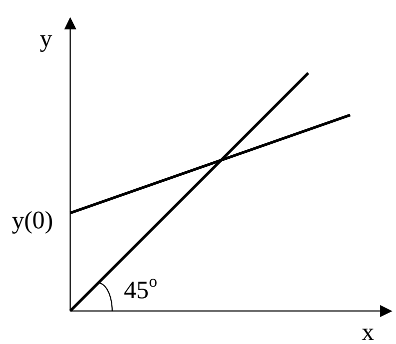

Figure 3

Proposition 3.1 *Under Assumption 1, the unique symmetric Nash equilibrium is given by*

$$y^* = T - (1 + \frac{\sigma T}{\overline{R}})\frac{1}{\lambda}\log(\frac{r}{r - \beta\lambda}).$$

The equilibrium strategy y^* is

- *increasing in T if $\frac{\overline{R}}{\sigma} > \frac{1}{\lambda}\log(\frac{r}{r-\beta\lambda})$,*
- *decreasing in σ,*
- *increasing in $\overline{R}$,*
- *increasing in λ if $\frac{\beta}{r-\beta\lambda} < \frac{1}{\lambda}\log(\frac{r}{r-\beta\lambda})$,*
- *decreasing in β and increasing in r.*

The condition $r > \beta\lambda$ is minimal in the sense that if r were not restricted to be above a given value, the coordination problem would become trivial: in an equilibrium, agents would buy the foreign currency as soon as they are informed.

The equilibrium delay strategy is increasing in the interest rate and in the initial amount of reserves. The policies of the Central Bank are thus instrumental in the choice of the agent. As expected, the equilibrium delay strategy is negatively related to the gain in case of devaluation β and to the index of the mass of informed agents σ.

4 First generation models of currency crises

First generation models of currency crises are characterized by two exchange rate regimes, separated by the crisis. For simplicity, the demand for money depends only on the domestic inflation rate. In the first regime, the exchange rate is fixed and the government runs a deficit that is financed by the Central Bank. As the exchange rate is fixed, the price level is fixed (by purchasing power parity) and the demand for money, which depends on the inflation rate (0 in this regime) is constant. Hence the assets of the Central Bank remain constant, and the government bonds gradually crowd out the foreign reserves. This process must eventually stop. The exchange rate must eventually be abandoned. Following that event, there is a second regime in which the exchange rate floats and the level of foreign reserves in the Central Bank is constant. In that regime, the deficit which continues to be financed by money creation increases the money supply and the inflation rate is strictly positive and constant (and equal to the rate of appreciation of the foreign exchange). The jump in the inflation rate forces the real demand for money to jump down when the fixed rate regime is abandoned.

Krugman (1979) assumes that agents have perfect information. Under this assumption, the exchange rate cannot jump. The jump in the real quantity of money is therefore achieved by a jump in the nominal quantity of money: at the time of the switch, agents run to trade a stock of money equal to the difference in the nominal quantity demanded before and after the switch.

The model of Krugman (1979) remains unsatisfactory because of the perfect foresight assumption. Note that no sudden devaluation takes place in that model. This property does not fit the experiences of currency crises. We extend the model of the previous sections to address the issue analyzed by Krugman when agents have imperfect information about an exogenous shock that triggers a gradual depletion of the foreign currency reserves of the Central Bank. This model is equivalent to a model where the depletion of reserves is induced by a government deficit. We will show that a devaluation occurs in a currency crises where agents have incomplete information. When agents have near perfect information, the model will generate the same properties as in Krugman.[8]

Without loss of generality, we assume that the domestic quantity of money is equal to the liabilities of the Central Bank and that it is the sum of the speculators' holdings, K, and a demand for transactions, D.

As in the previous model, speculators hold domestic currency in a regime of fixed exchange rate because of the interest rate premium. After a devaluation, we assume no

[8]Broner (2003) also analyzes in a different setting a model of currency crisis with imperfect information. He assumes two types of agents: the first have perfect information and behave as in Krugman; the second observe only the actions of the first with a vanishingly small time lag. When the first type of agents run (always avoiding a capital loss because they have perfect information), the second type of agents are surprised to hold currencies which they can sell only with a capital loss. The model of Broner generates a continuum of equilibria among which one is selected by an ad hoc rule. In our model, there is a unique equilibrium which will be shown to be a SREE in the next section.

interest premium and speculators have no demand for the domestic currency. This is a stylized way to think of the model of Krugman where a portfolio equation is defined and in which the inflation rate plays the role of a tax on domestic currency. This "tax" increases in the second regime, leading to a decrease in the demand for domestic currency.

The demand for transactions is set such that

$$\frac{D}{P} = k,$$

where P is the exchange rate and k is a parameter. The value of P is equal to 1 under a fixed exchange rate and it is equal to the value determined by the market at the instant after the regime of fixed exchange rate is abandoned. The rate of devaluation β is now endogenous: $\beta = P - 1$.

Let K be the domestic currency holdings of the speculators at the time of devaluation. The quantity of money at that time is therefore $K + D$. Since speculators do not hold domestic currency after the devaluation, we have

$$\frac{D + K}{P} = k.$$

Hence

$$\beta = \frac{D + K}{k} - 1. \tag{9}$$

Let K_0 be the mass of speculators each holding initially one unit of domestic currency. The initial reserves of the Central Bank, $\overline{R}$, are larger than K_0. As in the previous sections, an exogenous shock occurs at some time θ after which there is an exogenous loss of reserves with a flow $\overline{R}/T$ per unit of time, where $\theta + T$ is the time at which an exogenous devaluation will occur.

The structure of information for speculators is the same as in the previous sections with the flow of newly informed agents per unit of time equal to σ.

At the time of devaluation $\theta + \tau$, the holdings of speculators are equal to

$$K = K_0 - \sigma(\tau - y^*),$$

where y^* is the equilibrium delay strategy. Using (1),

$$\tau = \frac{y^* + \frac{\overline{R}}{\sigma}}{1 + \frac{\overline{R}}{\sigma T}} = a + \nu y^*$$

where $a = \frac{\overline{R}}{\sigma}/[1 + \frac{\overline{R}}{\sigma T}]$ and $\nu = 1/[1 + \frac{\overline{R}}{\sigma T}] < 1$. Hence

$$K = K_0 - \sigma a + \sigma(1-\nu)y^*.$$

Substituting this value of K in (9), we get

$$\beta = \frac{K_0 - \sigma a + D - k}{k} + \frac{\sigma(1-\nu)y^*}{k}. \tag{10}$$

This expression defines the rate of devaluation, $\tilde{\beta}(y^*)$, as an increasing function of the equilibrium delay, y^*:

$$\tilde{\beta}(y^*) = \frac{1}{k}(K_0 - \frac{\overline{R}}{1 + \frac{\overline{R}}{\sigma T}} + D - k) + \frac{y^*}{k}\frac{\frac{\overline{R}}{T}}{1 + \frac{\overline{R}}{\sigma T}}. \tag{11}$$

This property is intuitive: if speculators delay longer, they hold more domestic currency at the time of devaluation and the rate of devaluation must be higher for the money market equilibrium after the devaluation.

Consider now a devaluation rate β. From Proposition 3.1, the equilibrium delay strategy is

$$\tilde{y}^*(\beta) = T - (1 + \frac{\sigma T}{\overline{R}})\frac{1}{\lambda}\log(\frac{r}{r - \beta\lambda}). \tag{12}$$

As already mentioned in that proposition, the delay in the equilibrium strategy is a decreasing function of the rate of devaluation β. When $\beta = 0$, then $\tilde{y}^*(\beta) = T$ and when $\beta = \frac{r}{\lambda}[e^\alpha - 1]/e^\alpha$ with $\alpha = \lambda\overline{R}T/[\overline{R} + \sigma T]$, then $\tilde{y}^*(\beta) = 0$.

The graphs of the functions $\tilde{\beta}(y^*)$ and $\tilde{y}^*(\beta)$ defined in (11) and (12) are represented in Figure 4.

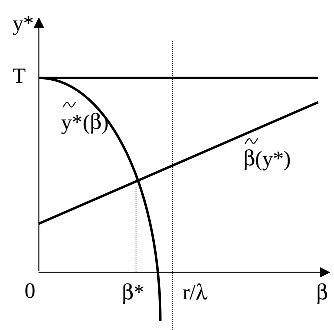

Figure 4

The parameters of the model are chosen such that in (11) for $\tilde{\beta}(y^*) = 0$, $y^* \in (0, T)$. Note that $y^* = 0$ for $k = K_0 + D - (\overline{R}/1 + \frac{\overline{R}}{\sigma T})$ and $y^* = T$ for $k = K_0 + D$. Hence, we must choose the parameters such that

$$k \in (K_0 + D - \frac{\overline{R}}{1 + \frac{\overline{R}}{\sigma T}}, K_0 + D).$$

The two schedules have a unique intersection that determines endogenously the rate of devaluation, β^*.

Recall that σ represents the rate of information or the speed at which the agents get informed. When σ increases, the agents become informed more quickly about the occurrence of the shock. Let us analyze the variations of the rate of devaluation and the equilibrium delay strategy with σ. We have

$$\frac{d\tilde{\beta}}{d\sigma} = -\frac{1}{k}\frac{\overline{R}^2}{(\sigma T)^2}\frac{1}{(1 + \frac{\overline{R}}{\sigma T})^2}(T - y^*), \tag{13}$$

which is negative for any $y^* \in (0, T)$, and

$$\frac{d\tilde{y}^*}{d\sigma} = -\frac{1}{\lambda}\frac{T}{\overline{R}}\log(\frac{r}{r - \beta\lambda}) < 0, \tag{14}$$

as found in Proposition 3.1. Hence, an increase in the rate of information decreases the gain in case of devaluation as long as the delay strategy is smaller than T (the time at which the devaluation occurs exogenously), and it decreases the delay.

Proposition 4.1 *The equilibrium value of the endogenous rate of devaluation, β^*, decreases when the rate of information σ increases.*

The limit value of the rate of devaluation as the rate of information tends to infinity is given by

$$lim_{\sigma \to \infty}\tilde{\beta}(y^*) = \frac{1}{k}(K_0 - \overline{R} + D - k) + \frac{\overline{R}}{kT}y^*.$$

As σ tends to infinity, the intersection between the functions $\tilde{\beta}(y^*)$ and $\tilde{y}^*(\beta)$, which determines the equilibrium value of the endogenous rate of devaluation, tends to the value of y^* for which $\tilde{\beta}(y^*) = 0$, namely

$$y^* = T(1 - \frac{K_0 + D - k}{\overline{R}}) > 0, \tag{15}$$

as $k \in (K_0 + D - \overline{R}, K_0 + D)$.

Proposition 4.2 *When the rate of information σ tends to infinity, the equilibrium value of the endogenous rate of devaluation β^* tends to zero. The equilibrium value of the delay tends to y^* defined by $\frac{y^*}{T} = 1 - \frac{K_0+D-k}{\bar{R}} > 0$.*

When σ tends to infinity, all agents are informed at nearly the same time. This limit case corresponds to the model of Krugman (1979) of the first generation. Speculators delay y^*, until the level of the Central Bank's reserves is just equal to their own balances, which they trade in at the same time with no capital loss. There is no jump of the exchange rate hence no devaluation when the peg is abandoned.

5 Iterated elimination of dominated strategies

A critical issue in this paper is the coordination of agents without common knowledge. In this context, the strength of the symmetric Nash equilibrium with strategy y^* that was analyzed previously is enhanced if the equilibrium is a strongly rational expectation equilibrium (SREE) (Guesnerie (1992)), i.e. if any strategy $y \neq y^*$ is iteratively dominated (and y^* is not iteratively dominated). Given that all agents have the same set of strategies $J \subset \mathbf{R}$, recall that a strategy y is iteratively dominated if there is a finite sequence of increasing sets $I_0 = \emptyset$,...,I_N, with $y \in I_N$, such that strategies in I_k are strictly dominated when all agents play in the subset of strategies $J \setminus I_{k-1}$.

We show in the next proposition that the equilibrium y^* is indeed a strongly rational expectation equilibrium. We will restrict the analysis to the set of delay strategies, where the agents act only after being informed.

If an agent invests at time t without being informed, his strategy depends only on the time t. An investment at some time t is successful only if other agents coordinate on the same date. Even if such a coordination could be achieved without some external device, it is not clear that there is a Nash equilibrium strategy where investment depends only on the time t. Recall that if no shock has occurred, the stock of currency of speculators is strictly smaller than the reserves of the central bank, $\bar{R}$. An attack can be successful at time t only if a shock has occurred at some earlier time θ such that $\bar{R}(1-(t-\theta)/T) < 1$. Even if the probability of this event for an uninformed agent is small, all uninformed speculators could attack at the same time t and undo their position after a vanishingly short interval η if the attack fails. The opportunity cost of such a strategy can be made arbitrarily small when $\eta \to 0$. But if all agents follow such a strategy, any one of them has an incentive to invest just before the others in order to avoid any rationing of the central bank's available reserves if the attack succeeds.

From these remarks, it is not obvious whether there is a Nash equilibrium with investment for uninformed agents. One may also conjecture that the introduction of a time lag between an order and its execution, or of a minimum length of time for holding an asset, would put a lower bound on the opportunity cost of attacking the currency and would eliminate any profitable strategy of investment for an uninformed agent. These problems are not essential here and are bypassed to concentrate on the main issues.

Proposition 5.1 *The equilibrium delay strategy y^* is a strongly rational expectation equilibrium in the set of delay strategies.*

Proof

- We prove first that any strategy $y > y^*$ is iteratively dominated.

  An agent informed at time t knows that a devaluation will occur no later than time $t+T$. If he delays for $y > T$, he misses for sure on the devaluation and earns the rate of return r at all time. We now prove that this strategy is dominated by the strategy to invest at some time prior to $t + T$.

  For an agent informed at time t, the support of the distribution of the time of a devaluation is contained in the interval $[t, t + T]$. Let $t + z$ be the essential upper-bound of this support. We can assume that $z > 0$. (If $z = 0$, immediate investment dominates any delay). By definition of z, for any $\delta < z$, the event that no devaluation has occurred at time $t + z - \delta$ has a strictly positive probability p for the agent informed at time t. Conditional on no devaluation before time $t + z - \delta$, the probability of a devaluation in the interval of time $[t + z - \delta, t + z]$ is equal to one. Under this event the return of investment at time $t + z - \delta$ evaluated at time $t + z$ is equal to $1 + \beta$.

  The difference between the payoff of this strategy and delaying more than T, evaluated at time $t + z$, is bounded below by the expression

$$D = p\,(1 + \beta - e^{r\delta}),$$

which is strictly positive for δ sufficiently small, as $p > 0$. A delay strictly greater than T is therefore strictly dominated. Hence no agent delays for more than T.

Assume agents delay no longer than x_k, with $x_1 = T$. The level of reserves at any time after θ is not greater than when all agents delay for the upper-bound x_k. Therefore, the delay between θ and a devaluation is bounded above by the value found in the symmetric case with all agents delaying x_k. Hence, if an agent informed at t delays y and no devaluation has occurred after that delay, then $t - \phi(y; x_k)$ is the lower bound of θ for any strategies of the other agents with delay no longer than x_k, and ϕ defined in equation (3). The support of θ is therefore in the interval $[t - \phi(y; x_k), t]$.

Recall that in the symmetric case where agents delay for x_k, a devaluation occurs at $t + y$ when θ is "at" (in the neighborhood of) the lower bound of the support $[t - \phi(y; x_k), t]$. In general, when θ is not restricted to this lower-bound, the instantaneous probability of a devaluation is not smaller than in the symmetric case. Therefore, the instantaneous rate of return of the foreign currency is bounded below by $\beta\pi(y; x_k)$ where $\pi(y; x_k)$ is the instantaneous probability of a devaluation in the symmetric case presented in section 3.

Let x_{k+1} be defined by $\beta\pi(x_{k+1}; x_k) = r$. We know that $\pi(y; x_k)$ is strictly increasing in y. Hence any delay strictly larger than x_{k+1} is strictly dominated.

The iteration process we have just described is illustrated in Figure 5. The sequence x_k is such that $x_{k+1} = y(x_k)$ is monotonically converging to y^*. Therefore, any strategy $y > y^*$ is iteratively dominated.

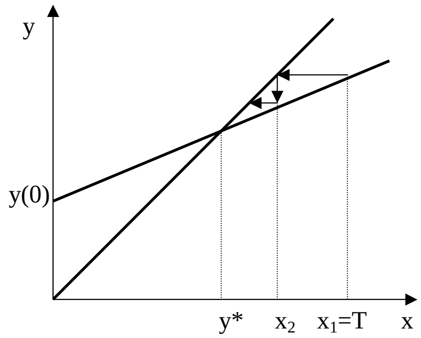

Figure 5

- We now show that any strategy $y < y^*$ is iteratively dominated.

  By an argument similar to the one used in the previous case, for any strategy of the other agents who delay at least x_k, with $x_1 = 0$, the instantaneous probability of a devaluation for an agent informed at t who delays y is bounded above by $\pi(y; x_k)$. Let x_{k+1} be defined by $\beta\pi(x_{k+1}; x_k) = r$. Since $\beta\pi(y; x_k)$ is strictly increasing in y, for any $y < x_{k+1}$, then $\beta\pi(y; x_k) < r$. Hence any delay shorter than x_{k+1} is strictly dominated.

  The iteration process is illustrated in Figure 6. The sequence x_k is such that $x_{k+1} = y(x_k)$ is monotonically converging to y^*. Therefore, any strategy $y < y^*$ is iteratively dominated.

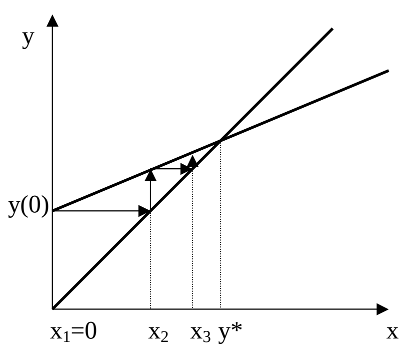

Figure 6

□

6 Uncertainty about the reserves

In this last section, we study the influence of uncertainty about the reserves on the instantaneous probability of a devaluation, in order to determine whether the central bank should or not reveal the level of its reserves.

Assume the initial amount of reserves, $\overline{R}$, can take one of two values, $\overline{R}_1 = \overline{R} - \epsilon$ and $\overline{R}_2 = \overline{R} + \epsilon$, with equal probability. The outflow of reserves is independent of the initial level of reserves and is the same as in the model with certainty. A devaluation will then occur exogenously either at date T_1 or date T_2, where

$$T_1 = \frac{(\overline{R} - \epsilon)}{\overline{R}} T, \qquad T_2 = \frac{(\overline{R} + \epsilon)}{\overline{R}} T,$$

with T as in Section 2. We assume that the reserves of the Central Bank are linear over time:

$$R_1(s) = (\overline{R} - \epsilon)(1 - \frac{s}{T_1}), \qquad R_2(s) = (\overline{R} + \epsilon)(1 - \frac{s}{T_2}).$$

Without loss of generality, we will denote by R_i the state in which the reserves are given by the function $R_i(s)$. Let S denote the event of being informed about the possible occurrence of a crisis, while the crisis (the devaluation) has not yet occurred.

Assuming the agent delays for y and given that all other agents delay for an interval of time x, the *instantaneous probability of the devaluation* is

$$\pi_u = \mu \frac{\lambda}{1 - e^{-\lambda \phi_1(y;x)}} + (1 - \mu) \frac{\lambda}{1 - e^{-\lambda \phi_2(y;x)}}, \tag{16}$$

where μ is the probability that the state is R_1 given that the devaluation has not occurred yet:

$$\mu = P(R_1|S) = \frac{P(S|R_1)P(R_1)}{P(S|R_1)P(R_1) + P(S|R_2)P(R_2)} = \frac{P(S|R_1)}{P(S|R_1) + P(S|R_2)} \tag{17}$$

(as $P(R_1) = P(R_2) = 1/2$), and $l_1 = \phi_1(y;x)$ and $l_2 = \phi_2(y;x)$ satisfy

$$\sigma(l_1 + y - x) + (\overline{R} - \epsilon)\frac{l_1 + y}{T_1} + \epsilon = \overline{R}, \tag{18}$$

$$\sigma(l_2 + y - x) + (\overline{R} + \epsilon)\frac{l_2 + y}{T_2} - \epsilon = \overline{R}. \tag{19}$$

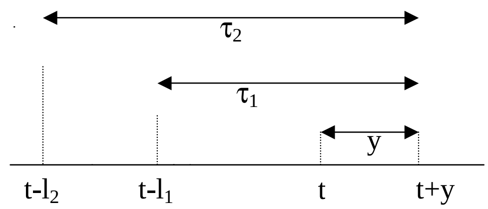

Figure 7

Equation (18) defines the length of the support of θ if the devaluation occurs exogenously at date T_1:

$$l_1 = \phi_1(y;x) = \frac{x + \frac{\overline{R}-\epsilon}{\sigma} - y(1 + \frac{\overline{R}-\epsilon}{\sigma T_1})}{1 + \frac{\overline{R}-\epsilon}{\sigma T_1}}, \tag{20}$$

while equation (19) defines the length of the support of θ in case the devaluation occurs exogenously at date T_2:

$$l_2 = \phi_2(y;x) = \frac{x + \frac{\overline{R}+\epsilon}{\sigma} - y(1 + \frac{\overline{R}+\epsilon}{\sigma T_2})}{1 + \frac{\overline{R}+\epsilon}{\sigma T_2}}. \tag{21}$$

Using the definitions of T_1 and T_2, we have

$$l_1 = \phi_1(y;x) = \frac{x + \frac{\overline{R}-\epsilon}{\sigma} - y(1 + \frac{\overline{R}}{\sigma T})}{1 + \frac{\overline{R}}{\sigma T}}, \tag{22}$$

$$l_2 = \phi_2(y;x) = \frac{x + \frac{\overline{R}+\epsilon}{\sigma} - y(1 + \frac{\overline{R}}{\sigma T})}{1 + \frac{\overline{R}}{\sigma T}}. \tag{23}$$

In (17), $P(S|R_i)$ is the instantaneous probability of the devaluation if the state is R_i, and it is computed as follows:

$$P(S|R_i) = e^{-\lambda(t-\phi_i(y;x))} \int_0^{\phi_i(y;x)} \lambda e^{-\lambda u} du = e^{-\lambda(t-\phi_i(y;x))}(1 - e^{-\lambda \phi_i(y;x)}).$$

Hence $\mu = P(R_1|S)$ defined in equation (17) reads

$$\mu = P(R_1|S) = \frac{e^{-\lambda(t-\phi_1(y;x))}(1 - e^{-\lambda\phi_1(y;x)})}{e^{-\lambda(t-\phi_1(y;x))}(1 - e^{-\lambda\phi_1(y;x)}) + e^{-\lambda(t-\phi_2(y;x))}(1 - e^{-\lambda\phi_2(y;x)})}$$

or else

$$\mu = \frac{e^{\lambda\phi_1} - 1}{e^{\lambda\phi_1} + e^{\lambda\phi_2} - 2}.$$

Equation (16), defining the instantaneous probability of a devaluation, can now be computed and it is given by[9]

$$\pi_u = \lambda \frac{e^{\lambda\phi_1} + e^{\lambda\phi_2}}{e^{\lambda\phi_1} + e^{\lambda\phi_2} - 2}. \tag{24}$$

Using $\phi_i(y; x) = \tau_i - y$ (Figure 7), and multiplying by $e^{\lambda y}/e^{\lambda y}$ gives

$$\pi_u = \lambda \frac{e^{\lambda\tau_1} + e^{\lambda\tau_2}}{e^{\lambda\tau_1} + e^{\lambda\tau_2} - 2e^{\lambda y}}. \tag{25}$$

By definitions (18) and (19), we have

$$\tau_1 = \frac{\overline{R} - \epsilon + \sigma x}{\sigma + \frac{\overline{R}}{T}} = a_1 + \nu x, \qquad \tau_2 = \frac{\overline{R} + \epsilon + \sigma x}{\sigma + \frac{\overline{R}}{T}} = a_2 + \nu x, \tag{26}$$

with $a_1 = a - \eta,\ a_2 = a + \eta$ where $a = \frac{\overline{R}}{\sigma + \frac{\overline{R}}{T}},\ \eta = \frac{\epsilon}{\sigma + \frac{\overline{R}}{T}}$, and $\nu = \frac{\sigma}{\sigma + \frac{\overline{R}}{T}}$.

With these definitions, the instantaneous probability of a devaluation (equation (25)) can be rewritten as follows:

$$\pi_u(y; x) = \lambda \frac{\frac{e^{\lambda a_1} + e^{\lambda a_2}}{2}}{\frac{e^{\lambda a_1} + e^{\lambda a_2}}{2} - e^{\lambda(y - \nu x)}}. \tag{27}$$

The function e^x being convex, we have

$$\frac{e^{\lambda a_1} + e^{\lambda a_2}}{2} > e^{\lambda a},$$

and the next result follows.

Proposition 6.1 *The uncertainty about the reserves decreases the instantaneous probability of a devaluation π_u. It extends the delay.*

Should the Central Bank reveal the level of reserves $\overline{R}_i$ or not?

We have assumed until now that the priors are 1/2 for every agent ($P(\overline{R}_1) = P(\overline{R}_2) = 1/2$).

Suppose the Central Bank wishes to maximize the *ex ante* expected delay between θ and the devaluation, namely $\tau = \frac{\tau_1 + \tau_2}{2}$.

Case 1 The Central Bank *does not* reveal the reserves.

[9]Note that for $\phi_1 = \phi_2 = \phi$, then $\pi_u = \pi$: the instantaneous probability of devaluation in case of uncertainty equals the instantaneous probability of devaluation with no uncertainty.

In this case, $a_1 = a - \eta$ and $a_2 = a + \eta$, and the *ex ante* expected delay between θ and the devaluation is

$$\tau = \frac{\tau_1 + \tau_2}{2} = \frac{a - \eta + \nu x + a + \eta + \nu x}{2} = a + \nu x, \tag{28}$$

where x is solution to

$$\pi_u(x; x) = c \tag{29}$$

with $c = r/\beta$ a parameter that depends on the interest rate and the rate of devaluation, and $\pi_u(x; x)$ is the equilibrium value of the instantaneous probability of devaluation in (27):

$$\pi_u(x; x) = \lambda \frac{\zeta}{\zeta - e^{\lambda(1-\nu)x}}, \qquad \text{with} \qquad \zeta = \frac{e^{\lambda(a-\eta)} + e^{\lambda(a+\eta)}}{2}.$$

Case 2 The Central Bank reveals the reserves.

In this case, either $a_1 = a_2 = a - \eta$ or $a_1 = a_2 = a + \eta$.

If $a_1 = a_2 = a + \eta$, then the delay between θ and the devaluation is

$$\tau_+ = a + \eta + \nu x_+$$

where x_+ is solution to $\pi(x; x) = c$ with $\pi(x; x)$ the instantaneous probability of the devaluation under certainty (computed in Section 3.1):

$$\pi(x; x) = \lambda \frac{e^{\lambda(a+\eta)}}{e^{\lambda(a+\eta)} - e^{\lambda(1-\nu)x}}.$$

If $a_1 = a_2 = a - \eta$, then the delay between θ and the devaluation is

$$\tau_- = a - \eta + \nu x_-$$

where x_- is solution to $\pi(x; x) = c$ with

$$\pi(x; x) = \lambda \frac{e^{\lambda(a-\eta)}}{e^{\lambda(a-\eta)} - e^{\lambda(1-\nu)x}}.$$

The *ex ante* expected delay between θ and the devaluation is

$$\tau = \frac{\tau_- + \tau_+}{2} = a + \nu(\frac{x_- + x_+}{2}). \tag{30}$$

When there is a distribution of possible values for the initial reserves, define the *certainty equivalent* as the level of reserves equal to the mean of the reserves. The above observations lead to the following results.

Proposition 6.2 *When there is more than one possible value for the initial reserves, which are not observed, the mean delay between the shock and the devaluation is greater than the delay under the equivalent certainty case.*

Proposition 6.3 *When there is more than one possible value for the initial reserves which are assumed to be perfectly known, the mean delay between the shock and the devaluation is equal to the delay under the equivalent certainty case.*

Corollary 6.4 *If the objective of the Central Bank consists in maximizing the ex ante expected delay between the shock and the devaluation, it should not reveal any information about the initial reserves.*

7 Conclusion

We have considered an economy with a fixed exchange rate regime that has suffered an adverse shock at some random time. Speculators know that a devaluation will occur at some exogenous time in the future. A coordination problem appears as a devaluation may well be triggered by the actions of the speculators before it occurs exogenously. The delay strategies of the speculators are at the heart of this coordination problem.

In this game, the agents become gradually informed at a constant rate about the occurrence of the shock. A devaluation occurs exogenously or endogenously as soon as the reserves of the Central Bank are completely depleted.

We have first shown that the game played by the speculators has a unique symmetric Nash equilibrium which is a strongly rational expectation equilibrium. We therefore depart from the second generation models characterized by multiple equilibria as we present a determinate equilibrium solution, as in Morris and Shin (1998). Secondly, we have determined endogenously the rate of devaluation, bringing together the first and second generation models of currency crises.

In this paper, uncertainty about the initial level of reserves of the Central Bank increases the delay and extends the *ex ante* mean delay between the exogenous shock and the devaluation. Consequently, if the objective of the Central Bank consists in maximizing the *ex ante* expected delay between the shock and the devaluation, it should not reveal any information about the initial reserves.

The model could be extended to include additional shocks on the fundamentals. For example, the exogenous outflow that was set here by a random process could be stopped according to a second random process. One can imagine that a policy of the Central Bank that was able to delay a crisis under a permanent shock could avoid entirely a crisis when the shock is transitory.

The issue of transparency is crucial in international financial market policies. The results in this paper show that in the short term, during a crisis, transparency may have adverse effects for the policy maker. However, it is well acknowledged by international organizations that transparency is efficient in the long term as it may foster credibility. Clearly, an analysis linking long term and short term issues is desirable, and will be the subject of a subsequent paper.

References

Abreu, D., Brunnermeier, M., 2003. Bubbles and crashes. Econometrica 71, 173–204.

Botman, D., Jager, H., 2002. Coordination of speculation. Journal of International Economics 58, 159–175.

Broner, F., 2003. Discrete devaluations and multiple equilibria in a first generation model of currency crises. mimeo, University of Maryland.

Chamley, C., 2003. Dynamic speculative attacks. American Economic Review 93, 603–621.

Chamley, C., 2004. Rational herds: Economic models of social learning. Cambridge University Press.

Flood, R., Garber, P., 1984. Collapsing exchange rate regimes: some linear examples. Journal of International Economics 17, 1–13.

Flood, R., Garber, P., Kramer, C., 1996. Collapsing exchange rate regimes: another linear example. Journal of International Economics 41, 223–234.

Guesnerie, R., 1992. An exploration of the eductive justifications of the rational expectations hypothesis. American Economic Review 82, 1254–1278.

Jeanne, O., 1997. Are currency crises self-fulfilling? A test. Journal of International

Economics 43, 263–286.

Jeanne, O., Masson, P., 2000. Currency crises, sunspots and markov-switching regimes. Journal of International Economics, 50, 327–350.

Krugman, P., 1979. A model of balance-of-payments crises. Journal of Money, Credit and Banking 11, 311–325.

Krugman, P., 1996. Are currency crises self-fulfilling? NBER Macroeconomics Annual, MIT Press, Cambridge, MA.

Morris, S., Shin, H.S., 1998. Unique equilibrium in a model of self-fulfilling currency attacks. American Economic Review 88, 587–597.

Obstfeld, M., 1986. Rational and self-fulfilling balance-of-payments crises. American Economic Review 76, 72–81.

Obstfeld, M., 1996. Models of currency crises with self-fulfilling features. European Economic Review 40, 1037–1047.

Velasco, A., 1996. Credibility, flexibility and multiplicity. European Economic Review 40, 1023–1035.