Managerial Ability and Capital Structure*

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ABSTRACT

This paper resolves an important puzzle in corporate finance — why do firms maintain such low levels of debt, given the apparently modest costs of bankruptcy? — by identifying a previously overlooked friction: the risk associated with human capital. In a setting in which employees are averse to their own human capital risk, while equity holders are not, we derive the optimal compensation contract and show that it implies that, with no other frictions, all firms will be unlevered. We then consider the effect of corporate taxes, and derive optimal debt levels that are consistent with the levels observed. An important characteristic of this friction is that employees bear the costs of bankruptcy, not firms. Because these costs are impossible to measure directly, existing empirical studies that attempt to measure the costs of bankruptcy grossly underestimate them.

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1 Introduction

Ever since Modigliani and Miller (1958) first proved that capital structure is irrelevant in a frictionless economy, financial economists have puzzled over the precise frictions that make the capital structure decision so important in reality. As yet, the most important friction that has been identified was identified by Modigliani and Miller themselves: corporate taxes. Because dividends are subject to corporate taxation while interest payments are not, firms can potentially realize significant tax savings by maintaining high levels of debt. However, in reality firms maintain only modest levels of debt. As Miller (1988) himself pointed out in a 30 year retrospective on his own work:

“In sum, many finance specialists, myself included, remain unconvinced that the high-leverage route to corporate tax savings was either technically unfeasible or prohibitively expensive in terms of bankruptcy or agency costs.” (p. 113)

Miller goes on to argue that corporate debt levels resulted from sub-optimal decision making, and points to two innovations that were happening at the time of the retrospective – the growth in junk bond markets and an explosion in the number of LBOs – as evidence of managers changing behavior and moving towards more optimal debt levels. However, subsequent developments have not borne out Miller’s prediction. In a recent study Graham (2000) finds (p. 1903) that “...even extreme estimates of distress costs do not justify observed debt policies.” Why then, do firms appear to have too little debt?

Clearly, an opposing friction must exist. However, economists have struggled to identify the precise source of this friction. High levels of debt increase the probability of bankruptcy. If there are deadweight costs associated with bankruptcy, these costs will be a disincentive to issue debt. However, in a seminal paper on the subject of debt weight bankruptcy costs, Haugen and Senbet (1978) argue that the importance of bankruptcy costs is necessarily limited to the costs of negotiating around these costs. Because debt holders bear the costs, they have incentives to recapitalize the firm outside of bankruptcy and thus avoid the costs altogether. Hence deadweight bankruptcy costs cannot exceed the cost of renegotiating, which rules them out as an effective counterweight to the large benefit of the tax shield.

Even though financial economists have not identified the precise nature of the costs associated with high debt levels, given the debt levels observed in the economy, substantial costs must exist. Hence it has become standard amongst corporate researchers simply to assume that some cost of debt exists, and move on to formulate theories of optimal capital structure. This has led to a number of theoretical predictions of optimal capital structure that have proved difficult to verify empirically. Indeed, in response to the apparent disconnect
between the empirical evidence and the theoretical predictions, Welch (2004) has concluded that capital structure decisions “remain largely a mystery.” (p. 106)

To the ancients, solar eclipses appeared to be random. The mystery was only removed once the underlying mechanism driving eclipses was identified. Hence, to make sense of the apparent randomness of corporate capital structure decisions one must first correctly identify the first order friction that acts as a counterbalance to corporate taxes and limits the use of corporate debt. One characteristic of the literature on bankruptcy costs is the apparent disconnect between the costs that researchers study and the ones identified in the popular press. During a corporate bankruptcy a major focus of the popular press is the human cost of bankruptcy, yet these costs have received minimal attention in the research literature.\(^1\) It is not difficult to understand why. In an efficient labor market there should be no human costs associated with bankruptcy. If employees are being paid their competitive wage, it should be a relatively costless matter to find a new job at the same wage. That is, for substantial human costs of bankruptcy to exist, most employees must be entrenched — they must incur costs associated with either not being able to find an alternative job, or taking another job at a substantially lower pay. At first blush such entrenchment is difficult to reconcile with optimizing behavior — even if labor markets are inefficient, why do shareholders ignore this inefficiency and instead overpay their employees? It would appear to be relatively costless to lower wages to their competitive levels, especially at times when the firm is facing the prospect of bankruptcy.\(^2\)

In this paper we argue that this initial intuition is wrong. In an economy with perfectly competitive capital and labor markets, one should expect large human costs of bankruptcy, and it is precisely these costs that limit the use of corporate debt. We begin with Harris and Holmström (1982) insight on the form of optimal employment contracts in perfect capital and labor markets. In a setting without bankruptcy they show that the optimal employment contract guarantees job security (employees are never fired), and pays employees a fixed wage that never goes down but rises in response to good news about employee ability. The intuition behind their result is that, while employees are averse to their own human capital risk, it is idiosyncratic, so equity holders can costlessly diversify it away by investing in many firms. Optimal risk sharing then implies that the shareholders will bear all this risk by offering employees a fixed wage contract. The problem with this contract is that employees cannot

\(^1\)In the next section we will review the literature on entrenchment, however none of those papers address the question of why it is optimal for the firm to have entrenched employees.

\(^2\)One possible explanation is the existence of firm specific human capital (see Neal (1995)). Yet in an efficient labor market it is not clear that employees are necessarily paid for their investments in human capital. Furthermore, even if they are, in a competitive economy like the United States it is hard to argue that most employees’ skills are not easily transferable, or that wages could not be lowered during financial distress.
be forced to work under it. That is, employees who turn out to be better than expected will threaten to quit unless they get a pay raise. This imperfection leads to the optimal employment contract Harris and Holmström (1982) identified.

The setting Harris and Holmström (1982) considered had no other frictions. In particular, firms carried no debt and equity holders did not have limited liability. To credibly commit to the terms of the contract, equity holders guaranteed the fixed wage by implicitly promising to make the wage payments even when the firm could not — that is, the equity holders had unlimited personal liability. In principle, there is no reason why the optimal equity contract requires limited liability. However, such contracts would be very difficult to trade in anonymous markets. Without the ability to trade, equity holders would no longer be able to diversify costlessly, and so the underlying assumption that they are not averse to human capital risk would be difficult to support. Hence, imposing the restriction that equity has limited liability is important.

Our first objective in this paper is to extend the Harris and Holmström (1982) results to a setting that includes both limited liability equity and debt. We show that the optimal employment contract in this setting requires only a minor modification to the Harris and Holmström (1982) contract. As in Harris and Holmström (1982), unless the firm is in financial distress, wages never fall, and rise whenever employees turn out to be better than expected. However, at the point that the firm cannot make interest payments, the employee takes a temporary pay cut. If the financial health of the firm improves, wages return to their contracted level. If it deteriorates further, and the firm cannot make interest payments even with the temporary wage concessions, it is forced into bankruptcy. In bankruptcy the firm can abrogate its contracts — employees can be terminated and new, more productive, employees can be hired to replace them. As a result, entrenched employees are forced to take a wage cut and earn their current market wage, either with the current firm or with a new firm.³

The form of this optimal employment contract has important implications for optimal capital structure. Note that eventually most employees are likely to become entrenched — because their pay is always increased, eventually it will likely be increased by too much. Because such employees are destroying value (the value of the firm would go up were they replaced), investors in the firm actually benefit from a bankruptcy filing because they can effectively fire such employees or lower their wages to competitive levels. Thus, ex post the effect of the optimal contract is to create a benefit to investors of filing for bankruptcy. Thus the Haugen and Senbet critique does not apply in this case — neither debt holders nor equity holders have an incentive to avoid bankruptcy.

³cite legal evidence on this
The implications of the optimal labor contract for the level of debt occur *ex ante*. The amount of risk sharing between investors and employees depends on the level of debt — under the optimal labor contract, higher debt levels imply less risk sharing. Thus, in the absence of taxes, the optimal level of debt is zero — with no frictions other than an incomplete market in human capital, all firms will be unlevered. In this setting, adding debt reduces profits because employees demand a higher wage *ex ante* to compensate them for the risk of bankruptcy. We then introduce corporate taxes into the model, and thereby derive a theory of optimal capital structure. The most important implication of our model is that it can resolve the apparent paradox present in the data: that firms maintain only modest levels of debt relative to the measured levels of the costs of bankruptcy. In a world with competitive labor and capital markets in which all firms optimally choose their capital structures, the costs of bankruptcy are borne by the firm’s employees, not the firm’s equity holders. It is the employees who trade off the benefits of higher wages against the costs of less human capital insurance. Because these costs are highly dependent on preferences, they are particularly difficult to measure. One would have to measure the *ex ante* utility associated with a change in the bankruptcy probability. Hence it is not surprising that people have not found direct evidence of these costs.

Although the direct costs of bankruptcy are difficult to measure, one might be able to find evidence of them indirectly. Clearly, industries that are more labor intensive will have higher costs of bankruptcy. Hence our model predicts that labor intensive firms will have lower levels of debt than capital intensive firms. Capital intensive firms tend to be larger (especially if accounting numbers are used as a measure of firm size), so a cross-sectional relation between debt levels and firm value will exist — large firms will be more highly levered. These predictions are supported by the existing empirical evidence — Titman and Wessels (1988), Rajan and Zingales (1995) and Fama and French (2002) all document a positive cross-sectional relation between leverage and firm size.

The model delivers a number of other cross-sectional implications. Firms with lower cash flow variation have higher levels of debt, as do older firms. Capital intensive industries should have higher wages, implying a positive relation between firm size and wages. This relation has been documented empirically, and it regarded as a puzzle by labor economists (see Brown and Medoff (1989)). Perhaps most importantly, we show that, for reasonable parameter values, the tradeoff between corporate taxes and optimal risk sharing alone can produce debt levels consistent with the evidence.

A surprising result in our setting is that, even in the presence of the frictions we study, there are times when the Modigliani-Miller proposition holds — that is, if the manager is being paid at his competitive wage, then the value of the firm does not change when the debt-
to-equity ratio changes. If the debt-to-equity ratio is adjusted at other times, a reduction in leverage reduces the value of equity and the firm — that is outside equity appears to be expensive, consistent with pecking order theory. On the other hand an increase in leverage increases the value of equity and the firm. Finally, managers and equity holders will only agree to a change in debt levels at discrete points in time. At most times, the firm’s debt-to-equity ratio will simply reflect current changes in the market value of the firm, consistent with the evidence in Welch (2002).

The rest of the paper is organized as follows ...

2 Review of the Literature

Like us, Chang (1992) models the optimal contract between investors and employees. However, in that paper he does not explicitly model the ability of employees, nor does he consider the role of labor markets. In his model employees are risk averse and thus should be given a constant wage. However, in some states of the world, value-enhancing restructurings should be undertaken, which are costly for the employees. It is assumed that the employees’ contract cannot be made contingent on such restructuring events and, as a consequence, employees always try to avoid restructuring. Investors therefore finance the firm with both equity and debt. If the firm defaults on the debt then investors are in charge and can force a restructuring. In a related paper, Chang (1993) focuses on the interaction between payout policy, capital structure and compensation contracts. Managers value control, so they must be motivated to pay out capital to the investors. The employee’s compensation is therefore linked to the payout to equityholders. However, the optimal payout level may change over time. Such a change is only feasible if control is transferred from the management to investors. This transfer is achieved by issuing the right amount of debt ex ante so that bankruptcy occurs in those states when new information about the optimal payout level is likely to be available.

More recently, Cadenillas, Cvitanić, and Zapatero (2004) analyze a firm with a risk averse employee who is subject to moral hazard. Equityholders compensate the employee with unlevered and levered equity, taking into account that the employee applies costly effort and selects the level of volatility, both of which affect expected returns. Several papers have analyzed the interaction between capital structure choice and the firm’s employees’ compensation and their incentives.

In an early contribution, Baldwin (1983) models a firm that undertakes a capital investment. Ex post, employees can appropriate the return to capital, since capital costs have been sunk. In response, the firm can impose costs on employees of adopting this strategy by
decreasing the productivity of the marginal units of capital so that if unions increase wages ex post, some plants must be shut down. Alternatively, the firm could issue debt. Then, if higher wages are demanded, bankruptcy occurs which is assumed to be costly for workers. Our focus is very different. We assume that both labor markets and capital markets are competitive, so that \textit{ex ante} the employee captures all the economic rents and makes the capital structure choice that maximizes his utility.

Stulz (1990) analyzes a firm where shareholders cannot observe either the firm’s cash flows or the employee’s investment decisions. Management always wants to invest as much as possible. Because shareholders know this, they will not always fully satisfy the employee’s demand for capital. Therefore the employee cannot take all positive NPV projects when the firm’s cash flows are low and its investment opportunities are good, and will overinvest when the firm’s cash flows are high and its investment opportunities are poor. It is shown that it is optimal for investors to design a capital structure consisting of debt and equity to reduce the costs of over- and underinvestment.

The papers discussed thus far derive new insights for optimal capital structure choices by accounting for the effect on the actions and on the compensation of management and employees. However, all of these papers assume that capital structure is chosen by the investors. In practice, however, capital structure choice is one of management’s most important responsibilities.

Zwiebel (1996) provides the first formal model of a employee’s capital structure choice when ownership is separated from control. In this paper, an entrenched employee determines the firm’s capital structure, recognizing that he can only be replaced if the firm is taken over or if the firm goes bankrupt. Since the employee derives extra utility from keeping his job, he wishes to avoid being fired. This can be done by issuing debt. By doing that, the employee commits not to undertake negative NPV projects, and thereby makes a hostile takeover unprofitable. In equilibrium, managers with low abilities issue debt, and therefore do not take on negative NPV projects. This allows them to avoid both hostile takeovers and bankruptcy.

Morellec (2004) extends the model by Zwiebel (1996), and derives a continuous-time model of an entrenched employee who may find it optimal to issue debt to avoid a hostile takeover. He allows for a tax advantage of debt, so that there exists an optimal debt level even in the absence of agency problems. The paper shows how the employee’s capital structure choice deviates from the firm value maximizing capital structure.

Subramanian (2002) also analyzes a firm where the employee makes capital structure and investment decisions, taking his personal bankruptcy costs and risk aversion into account. In each period, the employee’s income is derived by a bargaining process with the equityholders.
Our analysis differs in several important ways from the previous literature on managerial capital structure choice discussed above. First, the existing literature relies on exogenous contracting restrictions. Existing papers have in common an exogenously specified managerial characteristic that destroys shareholder value, and cannot be eliminated by appropriate compensation contracts. In contrast, we analyze employee entrenchment in a model which solves for the optimal compensation contract. The only contracting restriction is that managers cannot commit to continue employment when their outside options generate a higher salary. This restriction, in combination with the assumption that managers with the ability to create value are in short supply, suffices to derive our results. Second, existing models on managerial choice of capital structure imply that bankruptcy is inefficient ex post, whenever costs are associated with it. In our model, it is ex post efficient to incur bankruptcy costs, since bankruptcy is the institution that cancels the firm’s existing wage contracts and allows restructuring with employees.

Our results are largely consistent with existing empirical evidence on capital structure. Using data from several countries, Rajan and Zingales (1995) find that leverage increases with the relative size of fixed assets, tangible assets, non-debt tax shields such as depreciation, and firm size. Furthermore, leverage is found to decrease with a firm’s volatility, its probability of bankruptcy, with its profitability and with the uniqueness of the firm’s product. All these findings are consistent with the implications from our model of employee entrenchment.

Several other empirical papers also find that firms’ capital structure choice is consistent with managerial entrenchment. Berger et al. (1997) test three alternative hypotheses for managerial capital structure choice. Managers may choose to underlever due to risk aversion; they may choose to overlever to inflate the voting power of their equity stakes and reduce takeover dangers; or they could overlever to signal restructurings in order to reduce the profitability of hostile takeovers. They find strong support for the first hypothesis. Managers which appear to be more entrenched (long tenure, compensation has low sensitivity to performance, few outside directors, no large shareholder) have low leverage. They also find that, after acquisition attempts, arrival of major stockholder-director etc., managers increase leverage.

Berkovitch et al. (2000) document additional evidence that management compensation is significantly related to firms’ capital structure choices and Jung et al. (96) also present support for the agency model of capital structure choice.

Strömberg (2000) analyzes cash auction bankruptcies in Sweden. According to Swedish bankruptcy law, if the firm enters bankruptcy, it ceases to exist, control is transferred to a trustee whose task it is to sell the assets for as high a price as possible, either piecemeal or going concern. The paper shows that sale backs of the assets to the original owners are very
common. The fact that the previous owners are receiving a stake of the reorganized firm is interpreted as deviation from absolute priority in order to avoid fire sales. These findings can also be interpreted in a way consistent with our model, which implies that the equity value is positive in bankruptcy. In our model, once the existing contracts with managers and/or employees have become null and void due to bankruptcy, equity has positive value. In Sweden the benefits of bankruptcy are reinforced by the fact that the government continues to pay the old wage levels for some time after bankruptcy (if the firm is reorganized as a going concern), while the new firm pays lower wages.

3 Optimal Labor Contract

In this section, we derive the optimal contract for a risk-averse employee working for a risk-neutral firm. We extend the results of Harris and Holmström (1982) by both allowing for debt, and imposing personal bankruptcy if the manager’s wage drops below zero. We also derive our results in continuous, rather than discrete, time.

The firm requires two inputs to operate, capital in the amount $K$ and an employee who is paid a wage $c_t$ and adds in period $t$, (uncertain) value, $Kt + \phi_t$. The firm raises the capital required by issuing debt, $D$, and equity $K - D$. The debt is perpetual and will turn out to be riskless, so it has coupon of $r$, the risk free rate of interest. The firm must pay corporate taxes at rate $\tau$ on earnings after interest expense and hence the interest tax shield is $Dr\tau$. Thus the firm produces after tax cash flows of $Kt + \phi_t - c_t + Dr\tau$ in period $t$, $Dr$ of which is paid out as interest on debt and the rest is paid out as dividends. Let $\beta \equiv e^{-r}$.

We assume that capital markets are perfectly competitive. The only source of risk in the model is uncertainty in the employee’s output which we assume is idiosyncratic to the employee and thus the firm. Consequently investors can diversify this risk away so the return on all invested capital is the risk free rate, $r$. We assume that the capital investment is irreversible and that their is no depreciation. At the point of bankruptcy, which occurs at the stopping time $T$, we assume all contracts can be unilaterally abrogated, so that the firm is no longer bound by the employee’s labor contract. Hence the firm can hire a new employee who will put the capital to productive use and because we assume that there are no costs of bankruptcy, the firm is restored to its initial state and hence its initial value. A bankruptcy filing therefore creates value in our model. For simplicity we assume that the equity holders are able to hold onto their equity stake and hence capture this value which restores the their initial equity stake. In fact the assumption that equity holders remain in control reflects the reality of Chapter 11 bankruptcy protection in the U.S.\footnote{Equity holders often maintain control even in countries without Chapter 11 protection, see Strömberg}, but most of the
results in this paper remain valid even when debt holders capture some or all of this value. Because the market value of equity, \( v_0 \), on first hiring the employee is the present value of all future cash flows we have

\[
v_0 = E_0 \left[ \int_0^T \beta^t ((K - D)r + \phi_t - c_t + Dr\tau) \, dt \right] + \beta^T v_0 \tag{1}
\]

so

\[
v_0 = \frac{E_0 \left[ \int_0^T \beta^t ((K - D)r + \phi_t - c_t + Dr\tau) \, dt \right]}{1 - \beta^T} = K - D + \frac{E_0 \left[ \int_0^T \beta^t (\phi_t - c_t + Dr\tau) \, dt \right]}{1 - \beta^T}
\]

The initial value of equity must equal the value of the capital supplied, \( v_0 = K - D \), so we have

\[
E_0 \left[ \int_0^T \beta^t (\phi_t - c_t + Dr\tau) \, dt \right] = 0. \tag{2}
\]

Firms compete to hire finitely many managers of a given ability in a competitive labor market. As a result the firm cannot pay the employee less than his market wage (because otherwise he would quit and work for another firm). Because the supply of qualified managers is limited, at the market wage the manager must capture all the economic rents, so

\[
E_\tau \left[ \int_0^{\tau} \beta^{t-\tau} (\phi_t - c_t + Dr\tau) \, dt \right] \leq 0, \quad \forall \tau \in [0, T]. \tag{3}
\]

To avoid bankruptcy, the firm must be able to meet its interest obligation each period. Thus, because the dividend received by shareholders can never be negative, the employee’s wages cannot exceed the total cash generated by the firm less the amount required to service the debt, i.e.

\[
c_t \leq \phi_t + r [K - D(1 - \tau)]. \tag{4}
\]

We begin by assuming that the (stochastic) point at which the firm declares bankruptcy is independent of the labor contract. We then derive the optimal contract and show that under the optimal contract, this assumption is satisfied. Under this assumption, the optimal
contract solves the problem

\[
\max E_0 \left[ \int_0^T \beta^t u(c_t) \, dt \right] \tag{5}
\]

s.t. \[
E_0 \left[ \int_0^T \beta^t (\phi_t - c_t + Dr\tau) \, dt \right] = 0, \tag{6}
\]

\[
P_\tau \left[ \int_0^\tau \beta^{t-\tau} (\phi_t - c_t + Dr\tau) \, dt \right] \leq 0, \quad \forall \tau \in [0, T], \tag{7}
\]

\[
c_t - \phi_t - r [K - D(1 - \tau)] \leq 0, \quad \forall t \in [0, T]. \tag{8}
\]

The first two constraints are as in Harris and Holmström (1982). The last is new, and reflects the constraint that employee cannot earn a wage the firm cannot pay. Assuming the precision of \( \phi_t \) remains constant,\(^5\) the form of the optimal contract is similar to that in Harris and Holmström (1982):

**Proposition 1** The optimal contract is given at all dates prior to bankruptcy by

\[
c_t(\phi^t) = \min \left\{ \phi_t + r [K - D(1 - \tau)] , \max_{0 \leq s \leq t} \{ c^* (\phi_s) \} \right\}, \tag{9}
\]

\[
= \min \left\{ \phi_t + r [K - D(1 - \tau)] , c^*(\overline{\phi}_t) \right\}, \tag{10}
\]

where

\[
\phi^t \equiv \{ \phi_s; 0 \leq s \leq t \},
\]

\[
\overline{\phi}_t \equiv \max_{0 \leq s \leq t} \phi_t,
\]

and \( c^*(\overline{\phi}) \) is the (unique) increasing function of \( \overline{\phi} \) that sets the equity value of a new firm equal to \( K - D \) whenever \( \phi = \overline{\phi} \).

**Proof:** It is sufficient to verify that the contract given in the proposition maximizes the Lagrangian for program (5)–(8), and satisfies the complementary slackness conditions. The Lagrangian can be written (after first multiplying the constraints (7) and (8) by the unconditional probability of the respective \( \phi^t \), multiplying (8) by powers of \( \beta \), and then collecting terms) as follows:

\[
\max_{c_t} E_0 \int_0^T \beta^t \left[ u(c_t) + \lambda_{\phi_t} (\phi_t - c_t + Dr\tau) + \mu_t (c_t - \phi_t - r [K - D(1 - \tau)]) \right] \, dt. \tag{11}
\]

\(^5\)This assumption is for expositional simplicity.
where

\[ \lambda^t \equiv \int_0^t d\lambda_s(\phi^s), \tag{12} \]

and \( d\lambda_s(\phi^s) \) is the Lagrange multiplier corresponding to Equation (7). The first order conditions take the form

\[ u'(c_t) = \lambda^t - \mu_t. \tag{13} \]

Assume \( c_t \) is given by Equation (9), and define Lagrange multipliers

\[
\begin{align*}
\lambda^t &= u'(c^*(\phi_t)), \\
d\lambda_t &= d \left( u'(c^*(\phi_t)) \right), \\
\mu_t &= u'(c^*(\phi_t)) - u'(c_t).
\end{align*}
\tag{14-16}
\]

These immediately satisfy the first order condition, Equation (13). Since \( \phi_t \) is always increasing, Equation (15) immediately tells us that

\[
\begin{cases}
\leq 0 & \text{when } \phi_t = \phi_t, \\
0 & \text{when } \phi_t \neq \phi_t.
\end{cases}
\tag{17}
\]

Also, since at all times \( c_t \leq c^*(\phi_t) \), Equation (16) immediately tells us that

\[
\begin{cases}
\leq 0 & \text{when } c_t = c^*(\phi_t), \\
0 & \text{when } c_t \neq c^*(\phi_t).
\end{cases}
\tag{18}
\]

The contract defined by Equation (9), together with these Lagrange multipliers, thus maximizes the Lagrangian (by concavity, this is a consequence of its satisfying the first order conditions), and (together with the Lagrange multipliers) satisfies the complementary slackness conditions. By weak duality, the contract is also the solution to the original program, Equations (5)–(8).

Note that under the optimal contract the employee takes a temporary pay cut so that the firm can meet its interest obligations. If the employee gives up all his wages and the firm still cannot make interest payments, it is forced into bankruptcy. At that point the firm’s cash flow is \( Kr + \phi + Dr\tau \). The firm must declare bankruptcy when this cash flow is less than the interest owed, \( Dr \), that is when

\[ Kr + \phi + Dr\tau < Dr \]
or when \( \phi \) is less than \( \tilde{\phi} \), where

\[
\tilde{\phi} \equiv -(K - D(1 - \tau))r
\]

which is independent of the labor contract offered to the employee, so the optimal contract satisfies our initial assumption.

We are now ready to derive the first implication of this labor contract on the capital structure choice of the firm:

**Proposition 2** *In the absence of corporate taxes, the optimal level of debt is zero.*

**Proof:** Consider two debt levels satisfying \( 0 \leq D_1 < D_2 \). First note that when \( \tau = 0 \), the firm’s cash flow is independent of the level of debt. Let \( T_1 \) and \( T_2 \) be the point of bankruptcy with debt levels \( D_1 \) and \( D_2 \) respectively. Note that \( T_1 > T_2 \). Because the firm’s cash flow is independent of the level of debt, the wage policy that pays the optimal wage when \( D = D_2 \) until \( T_2 \) and nothing thereafter is feasible when the debt level is \( D_1 \). However it is not optimal because the identical contract that pays strictly positive wages between \( T_2 \) and \( T_1 \) dominates it. Thus the employee is strictly better off with debt level \( D_1 \) than with debt level \( D_2 \). This is true for any \( 0 \leq D_1 < D_2 \), so the optimal debt level is zero.

Our next result shows that the Modigliani-Miller Proposition holds on when \( \phi = \tilde{\phi} \).

**Proposition 3** *The value of the firm, as well as the per share value of equity does not depend on the debt-to-equity ratio when \( \phi = \tilde{\phi} \).*

**Proof:** Debt is riskless and so always trades for \( D \). When \( \phi = \tilde{\phi} \) the value of equity is \( K - D \), so the value of the firm is \( D + (K - D) = K \) which is independent of the level of debt.

Of course, Proposition 3 presupposes that any change in the level of debt is an optimal response to market conditions so that all firms will follow the same policy and change their debt structures as well. When \( \phi \neq \tilde{\phi} \) an increase in the level of debt increases the value of the firm:

**Proposition 4** *If the optimal wage contract is fixed, then an increase (decrease) in the level of debt increases (decreases) firm value and the per share value of equity when \( \phi \neq \tilde{\phi} \).*
**Proof:** Assume the firm increases the level of debt by simultaneously repurchasing equity at the market price, leaving the amount of capital constant at $K$. Because the debt is riskless, its price will be unaffected by the purchase. However, the new level of debt has two effects. It increases all future cash flows to equity holders by the amount of the tax shield and shortens the time to bankruptcy. Both effects increase the value of equity. Hence the value of the firm and the value of equity rises.

The immediate implication of Propositions 4 is that whenever $\phi \neq \bar{\phi}$ outside equity will appear expensive — tapping outside equity markets results in a drop in the stock price. The only time that the firm can avoid this price drop is to issue when $\phi = \bar{\phi}$, that is, when the stock price is high. Thus, just as in Jensen and Meckling (1976) managers only tap outside equity markets when the stock price is high, but the underlying economic mechanism is different. Unlike in that model, in our world managers do not sell overpriced equity to unsuspecting investors, but rather, sell fairly priced equity to fully informed investors.

4 Implementing the Optimal Contract

In this section, we solve explicitly for the optimal contract, and explore its implications for optimal debt level. We derive an expression for the employee’s indirect utility function as a function of the level of debt and then optimize this function to find the optimal debt-to-equity ratio. We begin by first solving the firm’s optimization problem which will give us an explicit expression for the optimal wage as a function of the level of debt.

4.1 Optimal Wage

Define

$$\delta_t \equiv \phi_t - c_t + Dr\tau,$$

so the expected discounted value of the excess cash flows added by the employee is

$$V(\phi, \bar{\phi}, t) \equiv E_t \left[ \int_t^T e^{-r(\tau-t)} \delta_\tau \, d\tau \mid \phi_t = \phi, \bar{\phi}_t = \bar{\phi} \right].$$

and the value of equity is $K - D + V(\phi, \bar{\phi}, t)$. Whenever the employee is paid his competitive wage,

$$V(\phi, \phi, t) = 0.$$

13
At other times \( V(\phi, \bar{\phi}, t) < 0 \), that is, the value of equity is either equal to value when the employee is hired, or it is less. Note that this means the value of the firm is can never exceed the value if the human capital is replaced which is opposite to what \( q \) theory predicts about physical capital. There the value of the firm is never lower than the replacement value of physical capital.

Our object is to derive the cross-sectional implications of our model. To do that we will derive a closed form expression for firm value and employee utility. In this setting this requires making restrictive assumptions. The first one is that we will assume that \( \phi_t \) follows a random walk,

\[
d\phi_t = \sigma dZ.
\] (22)

This assumption greatly simplifies the analysis because the variance remains constant, so there is no time dependency in the problem, and \( V \) does not depend explicitly on \( t \). By Ito’s Lemma, when \( \phi_t < \bar{\phi}_t \),

\[
dV = V_\phi d\phi + \frac{1}{2} V_{\phi\phi} \sigma^2 dt.
\] (23)

First consider the state when the firm can make interest payments. In equilibrium, shareholders must earn a fair rate of return on their investment, i.e.

\[
E(dV) = (rV - \delta_t) dt.
\]

Combining these, we obtain a p.d.e. for \( V(\phi, \bar{\phi}) \):

\[
\frac{1}{2} \sigma^2 V_{\phi\phi} - rV + \delta_t = 0
\] (24)

The general solution to this equation is

\[
V(\phi, \bar{\phi}) = A(\phi)e^{\sqrt{2r}\phi/\sigma} + B(\phi)e^{-\sqrt{2r}\phi/\sigma} - \frac{c^*(\phi, D)}{r}.
\] (25)

When \( \phi = \bar{\phi} \), \( V \) equals zero (by (21)). In addition, as \( \phi \) approaches \( \bar{\phi} \), we have the additional boundary condition

\[
V_\phi(\phi, \bar{\phi}) = 0.
\] (26)

When the firm cannot meet its interest obligations so the firm is in financial distress, the employee takes a temporary pay cut to just make sure that the interest obligations are met.

---

Financial distress occurs when all the revenues of the firm equal the interest owed:

$$Kr + \phi_t - c_t + Dr\tau = Dr$$

or when

$$\phi = \phi^* \equiv c - (K - D(1 - \tau))r.$$  \hspace{1cm} (27)

and

$$\delta = -(K - D)r.$$  \hspace{1cm} (28)

$\delta$ remains constant at this level while the firm is in financial distress — said another way, the firm pays zero dividends when it is in distress. Shareholders must still earn a fair rate of return on their investment while in financial distress, i.e.

$$E(dV^f) = (rV^f + (K - D)r) \, dt.$$  

so the o.d.e. in this region is

$$\frac{1}{2} \sigma^2 V^f_{\phi\psi} - rV^f - (K - D)r = 0.$$  \hspace{1cm} (29)

The general solution to this equation is

$$V^f(\phi, \bar{\phi}) = F(\bar{\phi})e^{\sqrt{2r} \phi/\sigma} + G(\bar{\phi})e^{-\sqrt{2r} \phi/\sigma} - (K - D).$$  \hspace{1cm} (30)

At the point the firm enters financial distress, $\phi^*$, the values and derivatives must be matched:

$$V(\phi^*, \bar{\phi}) = V^f(\phi^*, \bar{\phi})$$  \hspace{1cm} (31)

$$V_{\phi}(\phi^*, \bar{\phi}) = V^f_{\phi}(\phi^*, \bar{\phi})$$  \hspace{1cm} (32)

At the point of bankruptcy, $\bar{\phi}$, the firm fires the employee and replaces him with an employee who puts the capital to full productive use so

$$V^f(\bar{\phi}, \bar{\phi}) = 0.$$  \hspace{1cm} (33)
Using these boundary conditions to solve for the coefficients and the optimal wage gives:

\[
A(\bar{\phi}) = \left(4(D - K)r^{3/2} + \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma} - \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma}}} e^{-\frac{\sqrt{T\sigma}}{\sigma}} + 4\sqrt{r}(c - D\tau r - \bar{\phi})e^{-\frac{\sqrt{T\sigma}}{\sigma}} \right) r^{3/2}
\]

\[
B(\bar{\phi}) = \left(4(D - K)r^{3/2} - \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma} + \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma}}} e^{-\frac{\sqrt{T\sigma}}{\sigma}} \right) r^{3/2}
\]

\[
F(\bar{\phi}) = \left(4(D - K)r^{3/2} + \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma} - \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma}}} e^{-\frac{\sqrt{T\sigma}}{\sigma}} \right) r^{3/2}
\]

\[
G(\bar{\phi}) = \left(4(D - K)r^{3/2} - \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma} + \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma}}} e^{-\frac{\sqrt{T\sigma}}{\sigma}} \right) r^{3/2}
\]

and the wage is

\[
c = c^*(\bar{\phi}, D)
\]

where

\[
c^*(\bar{\phi}, D) \equiv \{c|\Delta(\phi, D, c) = 0, 0 \leq c < \bar{\phi} + D\tau \}
\]

and

\[
\Delta(\phi, D, c) \equiv \left(2\sqrt{2}(D - K)r^{3/2} + \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma} - \sqrt{2}e^{-\frac{\sqrt{T\sigma}}{\sigma}}} e^{-\frac{\sqrt{T\sigma}}{\sigma}} \right) r^{3/2} - \sigma + \sqrt{2}(c - D\tau r - \phi) + e^{2\sqrt{T\sigma}(D(1 + r)\sigma + \phi)} \left(\sigma - \sqrt{2}(D\tau + c)\right).
\]

Figure 1 plots the value of equity under the optimal wage contract as a function of the employee’s ability for the parameter values listed in Table 1 and what will turn to be the optimal level of debt. The value of equity equals the value of physical capital at inception and at bankruptcy, that is, at any point a new employee is hired and paid their market wage. At all other points the value of equity is less than the value of physical capital. Equity holders still get a fair market return because when the employee is hired, she is hired a wage below her ability — $c = 0.7$ in this case and her initial ability is $\bar{\phi} = 1$. This difference, plus the tax shield, generates a positive cash flow to equity holders that compensates for the drop in the value of equity and guarantees equity holders the competitive market expected return.
Figure 1: **Value of Equity**: The plot shows the value of equity as a function of employee ability ($\phi$) between $\phi = -0.96$ and $\bar{\phi} = 1$. The parameters values are listed in Table 1 with an initial debt-to-equity ratio (when $\phi = \bar{\phi}$) of 1.06, which is optimal.

4.2 **Employee’s Utility**

Given the wage schedule derived in the previous section, we can calculate the employee’s expected utility. Write

$$J(\phi, \bar{\phi}) \equiv E \left[ \int_0^\infty e^{-rt}u(c_t) \, dt \bigg| \phi_0 = \phi, c_0 = c^*(\bar{\phi}, D) \right].$$

To solve for $J$ explicitly we must make an assumption on preferences. Again we make a restrictive assumption that allows us to derive a closed from expression for $J$, we assume that

$$u(c) = -e^{-\gamma c} \quad (35)$$

When the firm is not in financial distress and $\phi$ is below $\bar{\phi}$, wages are constant so the Bellman equation for $J$ is

$$\frac{1}{2}\sigma^2 J_{\phi\phi} - rJ + u(c) = 0. \quad (36)$$
The general solution to this o.d.e. is

\[ J(\phi, \tilde{\phi}) = A(\tilde{\phi})e^{\sqrt{2r}\phi/\sigma} + B(\tilde{\phi})e^{-\sqrt{2r}\phi/\sigma} - \frac{e^{-\gamma c}}{r}. \]  (37)

where \( c = c^*(\tilde{\phi}, D) \). When the firm is in financial distress, the employee makes up the interest payment out of his pocket. Using (28), the firm goes into distress when \( \phi \) drops to \( \phi^* \). While in distress the managers compensation is \( c - (\phi^* - \phi) = \phi + r(K - D(1 - \tau)) = \phi - \phi \). So the Bellman equation in this region is

\[
\frac{1}{2}\sigma^2 J_{\phi\phi} - r J_{\phi} + u(\phi - \phi) = 0. \]  (38)

The general solution to this o.d.e. is

\[ J'(\phi, c) = C(\tilde{\phi})e^{\sqrt{2r}\phi/\sigma} + F(\tilde{\phi})e^{-\sqrt{2r}\phi/\sigma} - \frac{e^{-\gamma(\phi - \phi)}}{r - \frac{\gamma^2}{2}}. \]  (39)

Anytime \( \phi \leq \phi \) the employee loses his job, and cannot find another job at a positive wage so he chooses not to work and gets zero forever (his reservation wage in this model):\(^7\)

\[ J(0, 0) = \int_0^\infty e^{-rt}U(0)dt = -1/r \]

The first boundary condition is therefore

\[ J'(\phi, \tilde{\phi}) = -1/r \]  (40)

At the point of financial distress, \( \phi^* \), the values and slopes must match:

\[ J(\phi^*, \tilde{\phi}) = J'(\phi^*, \tilde{\phi}) \]  (41)

\[ J_\phi(\phi^*, \tilde{\phi}) = J'_\phi(\phi^*, \tilde{\phi}) \]  (42)

Using these three boundary conditions we can solve for the functions \( B(\tilde{\phi}), C(\tilde{\phi}) \) and \( F(\tilde{\phi}) \)

\(^7\)This follows because at the point of bankruptcy, \( \phi < 0 \) and \( c^*(\phi, D) \leq \phi \) for any \( D \).
in terms of $A(\bar{\phi})$:

$$B(\bar{\phi}) = \frac{e^{-\frac{\sqrt{2r}}{\sigma} \gamma \sigma} \left( \sqrt{2r} \left( e^{\frac{2\sqrt{2r}}{\sigma}} - 1 \right) + \left( 1 - 2e^{c\left(\gamma + \frac{\sqrt{2r}}{\sigma}\right)} \right) \gamma \sigma \right)}{2e^{c\gamma \sigma - 2\sqrt{2r}} \sigma} r \left( \gamma^2 \sigma^2 - 2r \right) - e^{\frac{2\sqrt{2r}}{\sigma} \gamma \sigma} A(\bar{\phi})$$  \hspace{1cm} (43)

$$F(\bar{\phi}) = \frac{\gamma \sigma \left( 2\sqrt{2e^{-\frac{\sqrt{2r}}{\sigma} \gamma \sigma}} + e^{-\frac{\sqrt{2r} \gamma (\phi + c)}{\sigma}} \right) \left( 2\sqrt{\bar{r}} - \sqrt{2} \gamma \sigma \right)}{2\sqrt{2r} \left( 2r - \gamma^2 \sigma^2 \right)} - e^{\frac{2\sqrt{2r}}{\sigma} \gamma \sigma} A(\bar{\phi})$$  \hspace{1cm} (44)

$$C(\bar{\phi}) = \frac{e^{-\frac{\sqrt{2r}}{\sigma} \gamma \sigma} \left( -2\sqrt{r} + \sqrt{2} \gamma \sigma \right)}{2\sqrt{2e^{c(r)} \left( 2r - \gamma^2 \sigma^2 \right)}} + A(\bar{\phi})$$  \hspace{1cm} (45)

The derived utility is smooth on the boundary, that is, the full derivative with respect to $\phi$ just below $\bar{\phi}$ must be the same as the derivative just above $\bar{\phi}$.

$$\lim_{\phi \uparrow \bar{\phi}} \frac{\partial}{\partial \phi} J(\phi, \bar{\phi}) = \lim_{\phi \downarrow \bar{\phi}} \left( \frac{\partial}{\partial \phi} J(\phi, \bar{\phi}) + \frac{\partial}{\partial \bar{\phi}} J(\phi, \bar{\phi}) \frac{\partial \bar{\phi}}{\partial \phi} \right)$$  \hspace{1cm} (46)

The upper boundary condition is therefore:

$$\frac{\partial}{\partial \bar{\phi}} J(\phi, \bar{\phi}) = 0$$  \hspace{1cm} (47)

Taking derivatives and using (43) to substitute for $B(\bar{\phi})$ gives an ode in $A(\bar{\phi})$:

$$2e^{-c(\bar{\phi}, D)\gamma \gamma} + 2e^{\frac{\sqrt{2r}}{\sigma}} r A'(\bar{\phi})$$

$$= e^{-\frac{\sqrt{2r}(\bar{\phi} - 2\phi) + e^{c(\bar{\phi}, D)} \gamma \gamma}{\sigma}} \left( e^{\frac{\sqrt{2r}(c(\bar{\phi}, D) + \phi)}{\sigma}} + e^{\frac{\sqrt{2r}(e^{c(\bar{\phi}, D) - \phi})}{\sigma}} \right) \gamma + 2e^{c(\bar{\phi}, D) \gamma \gamma} r A'(\bar{\phi})$$

where we have explicitly written $c$ as $c(\bar{\phi}, D)$ to emphasize the wage’s dependency on $\bar{\phi}$. Integrating,

$$A(\bar{\phi}) = G + \int_{\bar{\phi}}^{\infty} e^{-c(u, D) \gamma \gamma} \left( 2e^{\frac{\sqrt{2r}(u - \phi)}{\sigma}} - e^{\frac{\sqrt{2r} c(\bar{\phi}, D)}{\sigma}} - e^{\frac{\sqrt{2r} c(u, D)}{\sigma}} \right) \frac{du}{2 \left( e^{\frac{\sqrt{2r}(u - \phi)}{\sigma}} - e^{\frac{\sqrt{2r} \phi}{\sigma}} \right) r}$$  \hspace{1cm} (48)

A sufficient condition for the convergence of this integral is $\sqrt{2r}/\sigma < \gamma$. To find $G$ we need to know the $\lim_{\bar{\phi} \to \infty} A(\bar{\phi})$. For a fixed $D$, large $\bar{\phi}$ implies that $c(\bar{\phi}, D)$ is large, so the firm
is permanently in financial distress (or goes bankrupt), that is the Bellman equation is

$$\frac{1}{2}\sigma^2 J_{\phi\phi}^p - rJ^p + u(\phi - \bar{\phi}) = 0.$$  \hfill (49)

The general solution to this o.d.e. is

$$J^p(\phi) = Ce^{\frac{\sqrt{2r}\phi}{\sigma}} + Fe^{-\frac{\sqrt{2r}\phi}{\sigma}} - e^{-\frac{\gamma(\phi - \bar{\phi})}{r - \frac{\gamma^2 \sigma^2}{2}}}.$$  \hfill (50)

When $\phi \to \infty$, $J^p$ cannot either be positive or infinity negative, so $C = 0$. Imposing the boundary condition at bankruptcy

$$J^p(\phi_b) = -1/r$$  \hfill (51)

and solving for $F$ gives

$$F = \frac{\gamma^2 \sigma^2 e^{\frac{\sqrt{2r}\phi}{\sigma}}}{r \left(2r - \gamma^2 \sigma^2\right)}.$$  \hfill (52)

Taking limits in (44) and (45) gives:

$$\lim_{\bar{\phi} \to \infty} F(\bar{\phi}) = \frac{\gamma^2 \sigma^2 e^{\frac{\sqrt{2r}\phi}{\sigma}}}{r \left(2r - \gamma^2 \sigma^2\right)} - e^{-\frac{\sqrt{2r}\phi}{\sigma}} A(\bar{\phi})$$

$$\lim_{\bar{\phi} \to \infty} C(\bar{\phi}) = A(\bar{\phi})$$  \hfill (53)

These coefficients must be the same in the limit, so $\lim_{\bar{\phi} \to \infty} A(\bar{\phi}) = 0$ which then implies that $G = 0$. So we have

$$A(\bar{\phi}) = \int_{\bar{\phi}}^\infty e^{-c^*(u,D)\gamma} \frac{2e^{\frac{\sqrt{2r}(u - \phi)}{\sigma}} - e^{-\frac{\sqrt{2r}c^*(u,D)}{\sigma}} - e^{-\frac{\sqrt{2r}c^*(u,D)}{\sigma}}}{2\left(e^{\frac{\sqrt{2r}(u - \phi)}{\sigma}} - e^{-\frac{\sqrt{2r}\phi}{\sigma}}\right)} r \, du$$  \hfill (54)

To plot the employee’s utility we use the parameters listed in Table 1. Our intention here is not to calibrate the model — it is far too simple to capture all the complexities of actual capital structure decisions. However, to evaluate whether the effects we study are economically important, we attempt to pick parameters that are economically realistic. We use a risk aversion coefficient of 2, consistent with values derived from experiments and a tax rate of 30%, close to the U.S. corporate tax rate. Another important parameter is the fraction of revenue attributable to labor versus capital. We pick an initial $\phi_0 = \bar{\phi} = 1$ and $K = 50$. With $r = 3\%$, this implies that the revenue attributable to capital is $Kr = 1.5$. So at these
parameter values, the revenue attributable to labor is two thirds the revenue attributable to capital.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>K</td>
<td>50</td>
</tr>
<tr>
<td>Initial $\phi$</td>
<td>$\bar{\phi}$</td>
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<tr>
<td>Risk Aversion</td>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>$r$</td>
<td>3%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$\sigma$</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values

Figure 2 plots the derived utility function, $J$, as a function of the debt-to-equity ratio for the parameters in Table 1. Note the utility is maximized when the debt-to-equity ratio is 1.06, the ratio we used to generate Figure 1. What is important about this result is that it implies that the importance of human capital risk is of the same order as taxes, and that with this friction alone can act as a counterbalance to taxes and deliver realistic debt-to-equity ratios. In the next section we will explicitly derive the optimal level of debt by maximizing $J$.

Figure 2: Employee’s Derived Utility: The figure shows the employee’s utility, $J$, as a function of the debt-to-equity ratio for the parameters in Table 1.

4.3 The Optimal Level of Debt

To get the optimal level of debt, we maximize the employee’s derived utility function. The main complication is that $c^*(\bar{\phi}, D)$ is only defined implicitly by (34). First, write $J$ as an explicit function of $D$, $J(\phi, \bar{\phi}, D)$. At the point of inception when $\phi = \bar{\phi}$, compute the
gradient of $J$, that is the full derivative of $J$ with respect to $D$:

$$\frac{d}{dD} J(\bar{\phi}, \bar{\phi}, D) = \frac{\partial}{\partial D} J(\bar{\phi}, \bar{\phi}, D) + \frac{\partial}{\partial \bar{\phi}} J(\bar{\phi}, \bar{\phi}, D) \frac{\partial \bar{\phi}}{\partial D} = \frac{\partial}{\partial D} J(\bar{\phi}, \bar{\phi}, D).$$

where the second line follows from the upper boundary condition (47) on the derived utility function. So the optimal level of debt solves

$$\frac{\partial}{\partial D} J(\bar{\phi}, \bar{\phi}, D) = 0$$

Figure 3 plots the optimal debt-to-equity ratio at inception as a function of the tax rate. Note that the book level of debt is constant in the model, but because the level of debt is determined when $\phi = \bar{\phi}$ the market level of debt varies. Since $V \leq 0$, market leverage is always equal to or less than book leverage. As the plot shows, the presence of human capital risk alone is enough to generate realistic leverage ratios even in the presence of significant tax rates.

Figure 3: Optimal D/E

One thing to note is that the optimal level of leverage is a function of the amount of physical capital $K$. Ceteris paribus, more physical capital implies a lower probability of bankruptcy and thus a higher optimal level of debt. Figure 4 plots the optimal debt-to-equity ratio as a function of the fraction of revenues attributable to physical capital. The clear inference is that labor intensive firms should have lower levels of debt, something that
is, at least anecdotally, characteristic of the economy. Furthermore, since physical capital intensive firms tend to be large (especially if accounting numbers are used as a measure of firm size), this also delivers the empirical implication that larger firms have higher leverage, which is consistent with the empirical evidence.

Figure 4: Firm Size and Debt Levels: The plot shows the optimal debt-to-equity ratio as a function of the percentage of firm value attributable to physical capital, $K$. Each line corresponds to an economy with different levels of cash flow uncertainty, $\sigma$.

An interesting question is what the cross-sectional variation in the capital versus labor intensity of firms implies about wages. Ceteris paribus, labor intensive industries have a higher probability of bankruptcy so there is less scope for risk sharing and one would expect higher wages in these industries. On the other hand, firms in these industries endogenously respond by holding less debt, thus increasing the probability of bankruptcy. Figure 5 shows that the endogenous response is enough to reverse the initial effect — physical capital intensive firms and hence larger firms pay higher wages. This is a robust characteristic of the data and is regarded as a puzzle by labor economists (see Brown and Medoff (1989)).
Figure 5: **Physical Capital Intensive Firms Pay Higher Wages:** The plot shows the cross sectional distribution of wages, $c$, (at optimal debt levels) for different levels of physical capital. Each line corresponds to an economy with differing levels of cash flow uncertainty, $\sigma$. 

![Diagram](image-url)
We next turn to the effect of uncertainty on the debt-to-equity ratio. Intuitively, the effect is clear. More uncertainty implies more risk, so the endogenous response is to reduce debt levels, as Figure 6 demonstrates.

Figure 6: **Optimal D/E as a Function of Cash Flow Uncertainty:** The plot shows the optimal debt-to-equity ratio as a function of the level of cash flow uncertainty, $\sigma$ at three different levels of risk aversion, $\gamma$.

An important driver of underlying cash flow uncertainty is firm age — younger firms face higher levels of uncertainty. Therefore an immediate implication of this result is that younger firms should maintain lower leverage levels, another robust characteristic of the data.\(^8\) Growth firms also have higher uncertainty, which again reproduces the empirical finding that growth firms have less leverage.\(^9\)

The level of uncertainty varies across industries, so this is another driver of cross-sectional variation in wages. Figure 7 plots the optimal wages of firms as a function of the debt-to-equity ratio. It shows that at these parameter values, firms with higher leverage pay higher wages.

\(^8\) We need a reference here.
\(^9\) Put in refs
Figure 7: **Firms with Higher Leverage Pay Higher Wages:** The plot shows the cross sectional distribution of wages, $c$, and debt levels for firms with different levels of cash flow uncertainty. Each line corresponds to an economy with agents of differing levels of risk aversion, $\gamma$.

5 Conclusion

This paper is at a very early stage and so your comments are particularly welcome!
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