Intro Logic: Week 3

Truth-Functionality

- The semantics of connectives of $\mathcal{L}_1$ ($\neg$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$) is very simple; it is captured in the associated truth tables.
- By contrast, many connectives of English function in a more intricate way.
  - E.g. 'because'. What would the truth table for this look like? (?, F, F, F)
  - Is 'Necessarily ...' truth functional? (?, F)
  - 'But'? 'unless'?
- A connective is *truth-functional* iff the truth-value of the compound sentence cannot be changed by replacing a direct subsentence with another sentence having the same truth value (in other words, iff it has only 'T' and 'F' in the main column of its truth table).

Material and Counterfactual Conditionals

- In English, the 'if ... then ...' structure features in both so-called *material* and *counterfactual* conditionals.
- Consider the sentence

  If Oswald didn't shoot Kennedy, somebody else did.

  This is a *material conditional*.
- Now consider the sentence

  If Oswald hadn't shot Kennedy, somebody else would have.

  This is a *counterfactual conditional*. Unlike the material conditional, it involves hypothetical scenarios.

- Plausibly, the material conditional can be formalised using '$\rightarrow$'. (In our world, both the antecedent and the consequent are false, but we consider the conditional to be true—this squares with the semantics for '$\rightarrow$'.)
- It is less clear that the counterfactual conditional can be formalised using '$\rightarrow$'. (In our world, both the antecedent ('Oswald didn't shoot Kennedy') and the consequent ('Someone else shot Kennedy') are false, but we might argue that the conditional is *false*—this does not square with the semantics for '$\rightarrow$'.)
- Formalising sentences involving counterfactuals is difficult; in the remainder of this course we'll focus mostly on material conditionals. (See Sider, *Logic for Philosophy* (OUP), for more on counterfactuals.)

Logical Form

Halbach (p. 59) gives the following recipe for getting a sentence of natural language (for us: English) into a form which we can then translate into $\mathcal{L}_1$:

1. *Check if the sentence can be reformulated in a natural way as a sentence built up form one or more sentences with a truth-functional connective. If this is not possible, then the sentence should be put in brackets and not analysed any further.*
2. *If the sentence can be reformulated in a natural way as a sentence built up from one or more sentences with a truth-functional connective, do so.*
3. *If that truth-functional connective is not one of the standard connectives ('not', 'or', 'and', 'if ... then ...', 'iff'), reformulate the sentence using the standard connectives.*
4. *Enclose the whole sentence in brackets, unless it is a negated sentence, that is, a sentence starting with 'it is not the case that'.*
5. *Apply the procedure, starting back at (1), to the next subsentence(s) (that is, to the sentence(s) without the standard connective from step (3)).*

Practically, it's not worth worrying about this too much (certainly don't memorise it!). Better to learn by doing examples:

(A) Rob and Tim will laugh, if the tutor can't pronounce Siobhan's name.

(B) If the ignition is turned on but there is no petrol in the tank, the engine will not start and I'll not be able to arrive in time.

(C) I am old only if I am old and I am not green.

(D) **Task:** Try other examples!

From Logical Form to Formal Language

We've now got to translate into $\mathcal{L}_1$. Here's Halbach's recipe:

1. *Replace standard connectives ('not', 'or', 'and', 'if ... then ...', 'iff') with their associated $\mathcal{L}_1$-connectives ($\neg$, $\vee$, $\wedge$, $\rightarrow$, $\leftrightarrow$).*
2. *Replace every English sentence with a sentence letter and delete the brackets surrounding the sentence letter. Use different sentence letters for distinct (that is, different) sentences and the same sentence letter for multiple occurrences of the same sentence.*
3. *Give a list (dictionary) of all sentence letters in the resulting $\mathcal{L}_1$-sentence together with the respective sentences they have replaced.*

Exercise: Apply this to (A)-(D) above.

Ambiguity

- Formalising in logic can be useful for identifying *scope ambiguities* in natural language sentences.
- Consider, for example, the sentence

  Brown is in Barcelona and Jones owns a Ford or Smith owns a Ford.

- **Task:** What are two different possible formulations of this sentence?

  - These two different formalisations (from different readings of the English sentence) are not logically equivalent.
  - **Exercise:** Show that these two formalisations are not logically equivalent (using truth tables).

- The *scope* of an occurrence of a connective in a sentence ϕ of $\mathcal{L}_1$ is the occurrence of the smallest subsentence of ϕ that contains this occurrence of the connective.
- **Task:** Identify the scope of the connectives in the two above formalisations.

Natural Language and Propositional Logic

(i) An $\mathcal{L}_1$-sentence obtained by translating a natural language sentence into $\mathcal{L}_1$ is called a *formalisation* of that natural language sentence.

(ii) A natural language sentence is a *propositional tautology* iff its formalisation in propositional logic (i.e. $\mathcal{L}_1$) is logically true (i.e. iff that formalisation is a tautology).

(iii) A natural language sentence is a *propositional contradiction* iff its formalisation in propositional logic is a contradiction.

(iv) A set of natural language sentences is *propositionally consistent* iff the set of all their formalisations in propositional logic is semantically consistent.

(v) An argument in natural language is *propositionally valid* iff and only if its formalisation in $\mathcal{L}_1$ is valid.

Work for Week 3

1. Halbach week 3, whole sheet
2. Peter Fritz week 3, exercises 3.2, 3.5

Links to both sets of exercises are available at logicmanual.philosophy.ox.ac.uk/

Solutions due at noon on Thursday week 3