

Intro Logic: Week 6

Syntactic Proof Systems

- In previous weeks, we've seen how to show the validity of arguments in \mathcal{L}_1 or \mathcal{L}_2 .
- We might, however, seek a *purely syntactic* proof system, which enables us to show when a certain argument is valid.
- We focus on the *natural deduction* proof system (others are e.g. *axiomatic proof systems*, or *semantic tableaux*; these we do not consider).
- If there is a natural deduction proof of a sentence ϕ from a set of sentences Γ , we write $\Gamma \vdash \phi$.
- We want: $\Gamma \vdash \phi$ if and only if $\Gamma \models \phi$.
 - ‘If $\Gamma \vdash \phi$ then $\Gamma \models \phi$ ’ \longrightarrow ‘Soundness’.
 - ‘If $\Gamma \models \phi$ then $\Gamma \vdash \phi$ ’ \longrightarrow ‘Completeness’.
- The natural deduction proof system that interest us has this property (CP/MP/PP: see next term!)

Natural Deduction

- In natural deduction, we infer one sentence from other sentences, in accordance with a given collection of rules (see the end of this sheet).
- When we do so, we write the inferred sentence under the sentences from which it is inferred.
- The idea is that, if $\Gamma \vdash \phi$, we should be able to construct a proof, consisting of a series of lines, which begins with at most the sentences of Γ , and ends with ϕ .
- If there are any further assumptions (sentences from which the proof begins), they must have been *discharged* in accordance with the rules.

Work for Week 6

1. Halbach week 6, whole sheet.
2. No Fritz, although you might consult Carr's 'Natural Deduction Pack', if you're struggling with natural deductions and would like to look at some worked examples.

Links to both sets of exercises are available at logicmanual.philosophy.ox.ac.uk/

Solutions due at noon on Thursday week 6