Intro Logic: Week 1

Preliminaries

1. Introductions
   (a) Students
   (b) Me (james.read@hertford.ox.ac.uk)
2. Course materials: logicmanual.philosophy.ox.ac.uk/
3. What is logic?
   (a) Natural language – e.g. English, French, Arabic, etc.
   (b) Formal language – ‘strip down’ natural language; capture essential features
   (c) Logic is the study of such formal languages
4. Why study logic?
   (a) Easier to study the structure of arguments, where people go wrong, etc.
      i. Can project back features of formal languages to natural language—useful in debates, written exchanges, etc.
      ii. Given that it’s easier to work with, it’s become the *lingua franca* of a lot of contemporary analytic philosophy.
   (b) It’s been used to develop philosophical theses in itself—e.g. the early Wittgenstein noted that the logical languages that we will study contained ‘atomic propositions’—which he took to be mirrored in ‘atomic facts’ in the world.
   (c) Interesting to study for its own sake (as a branch of mathematics).
5. Finals courses that use logic.

If *vs.* only if

1. Consider the sentence

   I will do my logic work if the coin lands heads.

   This is the same as

   If the coin lands heads, I will do my logic work.

   - It means that the coin landing heads is a *sufficient* condition for my doing my logic work: if the coin lands heads, I will *definitely* do the work.
   - *But*, it doesn't rule out that if the coin doesn't land heads, I might still do my logic work. E.g. if the coin lands tails, I might still do the work. The sentence is silent on this situation.

2. Now consider the sentence

   I will do my logic work only if the coin lands heads.

   This is the same as

   Only if the coin lands heads will I do my logic work.

   - It means that the coin landing heads is a *necessary* condition for my doing my logic work: if I'm to do my logic work, the coin *must* land heads. It rules out the possibility that there are other ways that I might end up doing my logic work.
   - *But*, it doesn't rule out that there might be *further* conditions for my doing my logic work, even once the coin lands heads. E.g. it might also be the case that I need to not be sick, if I'm to do my logic work.

3. Finally, consider the sentence

   I will do my logic work if and only if the coin lands heads.

This is the same as

If and only if the coin lands heads, I will do my logic work.

- This specifies *necessary and sufficient* conditions for my doing my logic work: I do the work *in exactly the same circumstances* in which the coin lands heads.
- 'If and only if' is used in definitions in mathematics and logic, because it specifies exactly when we can use a new concept.
- Often, 'if and only if' is abbreviated 'iff'.

Sets

- A *set* is a collection of objects.
- The *elements* of a set are the objects it contains.
- Sets are identical if and only if they have the same elements.
  - Corollary: Order does not matter.
- $a \in S$ means: *the object* a *is an element of the set* S.
- There exists one (and only one) set that contains no elements: the *empty set*, denoted $\emptyset$.
- Sets can be written using curly brackets
  1. E.g.: $\{$my left ear$, 0,$ the Eiffel tower$\}$.
  2. **Tests:** Is $\{0\}$ the same as $\{\{0\}\}$? Is $\{$Oxford$\}$ the same as $\{$Oxford$, \emptyset\}$? (*no*, *no*)
- Sets can be defined *extensionally*—directly in terms of their members (as above)—or *intensionally*—in terms of a rule – e.g. $\{x : x$ is a prime number$\}$. Easier to define infinite sets intensionally!

Binary relations: Intro

- Sets don't have orders built in, as we've seen. Sometimes we *want* an order. A set with two elements *with order built in* is denoted $\langle \cdot, \cdot \rangle$, and is called an *ordered pair*.

    1. E.g. can say "$\langle \text{Oxford}, \text{Cambridge} \rangle$ satisfies the *is a bigger city than* relation", but not clear what it means to say "$\{\text{Oxford}, \text{Cambridge}\}$ satisfies the *is a bigger city than* relation", because $\{\text{Oxford}, \text{Cambridge}\}$ has no order built in! ($\{\text{Oxford}, \text{Cambridge}\} = \{\text{Cambridge}, \text{Oxford}\}$).
    2. **Question:** How could one define ordered pairs in terms of sets?

- Can generalise the notion of a pair to *tuples*: $\langle \cdot, \dots, \cdot \rangle$.

- A set is a *binary relation* iff it contains only ordered pairs. E.g. the *is a bigger city than* relation:

$$\{\langle \text{London}, \text{Munich} \rangle, \langle \text{London}, \text{Oxford} \rangle, \langle \text{Oxford}, \text{Cambridge} \rangle, \dots\}$$

Binary relations: Important definitions

- A binary relation R is

    (i) *reflexive on a set* S iff for all elements $d \in S$, the pair $\langle d, d \rangle \in R$.

    (ii) *symmetric on a set* S iff for all elements $d, e \in S$: if $\langle d, e \rangle \in R$ then $\langle e, d \rangle \in R$.

    (iii) *asymmetric on a set* S iff for no elements $d, e \in S$: $\langle d, e \rangle \in R$ and $\langle e, d \rangle \in R$.

    (iv) *antisymmetric on a set* S iff for no two distinct (i.e. different) elements $d, e \in S$: $\langle d, e \rangle \in R$ and $\langle e, d \rangle \in R$.

    (v) *transitive on a set* S iff for all elements $d, e, f \in S$: if $\langle d, e \rangle \in R$ and $\langle e, f \rangle \in R$, then $\langle d, f \rangle \in R$.

- Note the restriction to a set!

    - Diagrams for each one.
    - Extra element kills reflexivity; not so symmetry etc.

- Asymmetry: no double arrows or loops. Antisymmetry: no double arrows, but loops are allowed.
- Two node transitivity example.

- A binary relation R is

  (i) symmetric iff it is symmetric on all sets.

  (ii) asymmetric iff it is asymmetric on all sets.

  (iii) antisymmetric iff it is antisymmetric on all sets.

  (iv) transitive if it is transitive on all sets.

- Points to note
  - E.g. $R = \{\langle a, b\rangle, \langle b, a\rangle\}$ is symmetric.
  - **Question:** Why is reflexivity not included in this latter list? (*answer*: then nothing would be reflexive—pointless definition)

- A binary relation R is an *equivalence relation* on a set S iff R is reflexive on S, transitive on S, and symmetric on S.
  - Equivalence relations partition sets into 'domains'.[1] **Exercise:** Prove this.

Functions

- A binary relation R is a *function* iff for all d, e, f, if $\langle d, e\rangle \in R$ and $\langle d, f\rangle \in R$, then $e = f$.

- The idea is that each 'first element' in the pairs of the relation must have at most one 'second element'. (cf. graphs in mathematics – or a machine: takes in something and tosses out a unique (possibly distinct) other thing)

- More definitions:

  (i) The domain of a function R is the set $\{d : \text{there is an } e \text{ such that } \langle d, e\rangle \in R\}$ ('x-axis')

[1] Don't confuse this sense of 'domain' with that used in the section on functions!

(ii) The range of a function R is the set $\{e : \text{there is an } d \text{ such that } \langle d, e\rangle \in R\}$ ('y-axis')

(iii) R is a function into the set S iff all elements of the range of the function are in S.

- If d is the domain of a function, one writes $R(d)$ for the unique object e such that $\langle d, e\rangle$ is in R.

Arguments, Validity, and Contradictions

- Sentences that are true or false are called *declarative sentences*.
- An *argument* consists of a set of declarative sentences (the *premisses*) and a declarative sentence (the *conclusion*) marked as the concluded sentence.
  - Here is an argument:

    Zeno is a tortoise. Therefore, Zeno is toothless.

  - This argument relies on implicit biological knowledge—an *implicit assumption* (here: that tortoises as toothless).
  - We may fill in the implicit assumption, to arrive at:

    Zeno is a tortoise. All tortoises are toothless. Therefore, Zeno is toothless.

  - Now no subject-specific knowledge is needed to see that the conclusion of the argument follows from the premisses: *if the premisses are all true, the conclusion **must** also be true*.
- An argument is *logically valid* iff there is no interpretation under which the premisses are all true and the conclusion is false.
  - Note: An *interpretation* will assign meanings to the subject-specific terms, such as 'Zeno', 'tortoise', etc.
- A set of sentences is *logically consistent* iff there is at least one interpretation under which all sentences of the set are true.

- An argument is valid iff the set obtained by adding the negation of the conclusion to the premisses is inconsistent. (**Task:** Show this.)

- Three last things:

  (i) A sentence is *logically true* iff it is true under any interpretation.
    - Sometimes, logically true sentences are called *tautologies*.
    - E.g. “Grass is green or grass is not green.”

  (ii) A sentence is a *contradiction* iff it is false under all interpretations.
    - E.g. “Tom is exactly twelve years old and also not exactly twelve years old.”

  (iii) Sentences are *logically equivalent* iff they are true under exactly the same interpretations.

Syntax, Semantics, and Pragmatics

1. *Syntax*: the bare symbols and grammar of a language, devoid of meaning.
2. *Semantics*: the meanings of the words and sentences of a language.
3. *Pragmatics*: the study of language in use (e.g. Mary replies “I’m ill” to “Are you going to work?”)

Work for Week 1

1. Halbach week 1, whole sheet
2. Peter Fritz week 1, exercises 1.5 and 1.6

Links to both sets of exercises are available at logicmanual.philosophy.ox.ac.uk/

Solutions due at noon on Thursday week 1