Philosophy of Quantum Mechanics: Week 9

Hidden Variable Theories

- Advocates of hidden variable theories embrace the 'unitary quantum mechanics isn't everything' branch of Bell's dichotomy.
- Recall that we can't *simultaneously* know the physical properties of a system corresponding to non-commuting observables (e.g. spin with respect to the x- and z-axes at the same time; or position and momentum of some particle at the same time).
- Hidden variable theorists insist that the quantum mechanical systems of interest *really do* have determinate properties at all times—so our inability to simultaneously *determine* properties of systems such as those mentioned above is an *epistemic*, rather than *ontic*, deficiency.
- It's difficult to construct empirically adequate hidden variable theories, due to various *no-go theorems*. Two of the most important are:
  (a) Bell's theorem: No *local* hidden variable theories. (Cf. week 3.)
  (b) Kochen-Specker theorem: No *non-contextual* hidden variable theories. (Roughly.)

De Broglie-Bohm Pilot Wave Theory

The most well-known hidden variable theory is the *de Broglie-Bohm pilot wave theory* (dBB). In this theory, every particle has a determinate and deterministically-evolving position at all times (and, as we shall see, position is the *only* property that these particles have).

Ontology

1. *The wavefunction*: The pilot wave $\psi(x_1, \ldots, x_N)$ (where N is the number of particles).
2. *The corpuscles*: Point particles with determinate positions $q_1, \ldots, q_N$.

Dynamics

- The wavefunction always evolves according to the time-dependent Schrödinger equation:[1]

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + U\psi.$$

- The corpuscles obey the first-order *guidance equation*:

$$\frac{dq_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\left[\frac{\nabla_i\psi\left(q_1,\ldots,q_N\right)}{\psi\left(q_1,\ldots,q_N\right)}\right]$$

- Probabilistic hypothesis: At some arbitrary time t, the probability distribution of the corpuscle positions is given by:[2,3]

$$\Pr\left(q_1 = x_1,\ldots,q_N = x_N\right) = \left|\psi\right|^2\left(x_1,\ldots,x_N\right) \tag{0.1}$$

Summary

- The probabilistic hypothesis ensures that dBB yields the (experimentally observed) Born rule probability at some arbitrary time t. The guidance equation then ensures that the quantum mechanical probabilities are *always* given by the Born rule.

- dBB purports to solve the measurement problem as, given any state of the wavefunction that seems to involve macroscopic superpositions, the corpuscle picks out one branch as

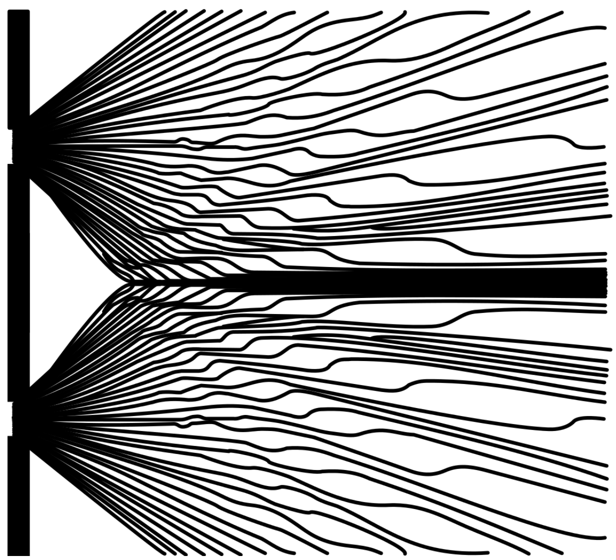
[1]Before, we wrote the time-dependent Schrödinger equation as $i\hbar\frac{\partial|\psi\rangle}{\partial t} = \hat{H}\left|\psi\right\rangle$. Whence the difference? The answer is that, above, we're dealing with the quantum state *in the position basis*—so that $\psi\left(x\right) = \langle x|\psi\rangle$. In the position basis, the Hamiltonian $\hat{H}$ decomposes into a *kinetic piece*, $-\frac{\hbar^2}{2m}\nabla^2$, and a *potential piece*, U.

[2]It's important to keep in mind that, in dBB, the world can only *appear* to us to be evolving probabilistically, given that particles have determinate and deterministically-evolving positions at all times. In other words, these probabilities in dBB must always be understood as *epistemic*, rather than *ontic*.

[3]Rather than postulate that (e.g.) the initial-state probability distribution was the $\left|\psi\right|^2$ distribution, we could postulate that it was some *other* distribution, and try to show that it evolves into $\left|\psi\right|^2$ reasonably quickly. This proposal has been developed primarily by Valentini. (See Wallace, "Measurement Problem: State of Play", p. 62, and references therein.) Whether this convergence occurs in practice depends on the dynamics of the hidden variable theory in question: it is not *prima facie* obvious that an arbitrary hidden variable dynamics would have this property. One corollary of these dynamical strategies is that the universe—or at least, some subsystems of it—might not be in 'quantum equilibrium' after all. This would create *observable* violations of the predictions of quantum mechanics, and might provide a context in which hidden variable theories could be tested.

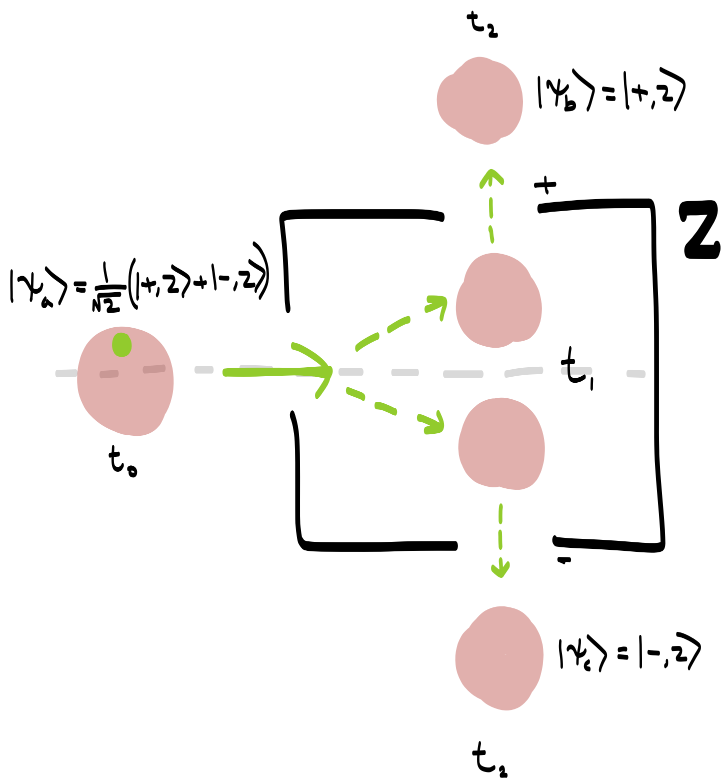
real. (We will see this in detail below.)

De Broglie-Bohm and the Double Slit Experiment



- Each particle goes through just one slit.
- The wavefunction is distributed across the whole of space.
- The wavefunction 'guides' the particle, leading to the uneven, interference-*like* distribution on the screen.

De Broglie-Bohm and Spin Experiments



- From orthodox quantum mechanics, we know that we can write the electron's state as it enters the apparatus as

$$|+, x\rangle = \frac{1}{\sqrt{2}}\left(|+, z\rangle + |-, z\rangle\right).$$

- After measurement, the electron in question will be found to have spin $+$ with respect to the z-axis half the time, and be found to have spin $-$ with respect to the z-axis half the time.

- Now consider dBB: as the electron enters the apparatus, there will be some region (shaded red) where its associated wavefunction is non-negligible;[4] let us assume that the electron (shaded green) is somewhere in this region.

[4] I do not say *non-zero* here, for technically the electron's wavefunction is nowhere-vanishing.

- According to dBB, all future positions of the electron can in principle be determined from its present position, and so the aperture through will ultimately exit—i.e. the outcome of this measurement—can in principle be determined from its initial position.
- In particular, if the electron starts out above the horizontal dotted line, it will go through the top slit; if it starts out below this dotted line, it will go through the bottom slit.
- Thus, in dBB non-position properties of particles, such as spin, are really only *effective*: they are not properties genuinely possessed by the system in question, but rather are apparent properties, reducible to the *positions* of the Bohmian particles.
- In this spin experiment, the *contextuality* of dBB can be illustrated straightforwardly: just reorient the appratus so the 'spin down' slot is at the top. We see that even though dBB is a deterministic theory, the outcome of this sort of 'measurement' will in general not be pinned down in the theory—but will rather depend upon precisely how and under what circumstances the observable in question gets measured.

Objection 1: The Dynamics

There are really two objections here:

(A) The dynamics are first-order: they depend only on the velocity of the particle; not on higher derivatives (cf. e.g. Newton's laws). This is the only case of first-order dynamics we've seen in physics—and makes the theory hard to render relativistic.

- *Response*: First order dynamics is hardly the oddest thing we've discovered in quantum mechanics... And making the theory relativistically invariant is part of an ongoing research project.

(B) The *action-reaction principle* is violated: the wavefunction acts on the corpuscles, but is not affected by them.

- *Responses*: (a) Why buy into the action-reaction principle? (b) This might be seen as a reason not to take the wavefunction to represent real physical structure, in which case the Bohmian can avoid the problem. (See below.)

Objection 2: The Ontology

What's a corpuscle? Does it really represent a particle?

- The only physical property possessed by the Bohmian corpuscle is determinate position.
- All other properties, e.g. mass, charge, spin, etc., depend upon properties of the wavefunction.
- To bring this out, consider the case of neutron interferometry discussed by Brown. (E.g. Brown, "Bohm Particles and their Detection in the Light of Neutron Interferometry".)
  - In these experiments, a magnetic field is found to alter the trajectory of a particle which is never located where the field is located.
  - If this is the case, Brown infers that the relevant properties of the particle (such as mass and charge) must be properties of the wavefunction.
  - So: Maybe everything relevant is in the wavefunction?

Objection 3: Everett in Denial?

> "Pilot-wave theories are parallel-universe theories in a state of chronic denial." —
> David Deutsch ("Comment on Lockwood", 1996.)

- The wavefunction still exists in its full form: it never collapses.
- Decoherence still happens.
- So there are still structures with all the properties of macro-objects like cats (and worlds) hanging about in the wavefunction.
- If we accept the many-worlds functionalist view on ontology, then we seem to have many worlds, but with an added corpuscle in one of the branches—isn't this just Everett with an extra unnecessary bit of ontology?

Reponse: This depends upon (a) an interpretation of the wavefunction as representing physical goings-on; and (b) Everettians' particular functionalist ontology.

The Wavefunction as Nomic

- Many people have wanted to see the wavefunction as merely reflecting our ignorance of the properties of the particle, and hence giving probability densities. Is this route available to the Bohmian?
  - No! The wavefunction here is doing much more than just reflecting our ignorance of the particle position. It plays an essential role in the particle dynamics.
  - Cf. the following quote from Bell: "*No one can understand this theory until he is willing to think of the wavefunction as a real objective field rather than just a "probability amplitude". Even though it propagates not in 3-space but 3N-space.*" ("Quantum Mechanics for Cosmologists").
  - The Bohmian having to view the wavefunction as real ties in with e.g. objection 3 above.
- Dürr, Goldstein, and Zanghì ("Bohmian Mechanics and the Meaning of the Wave Function") suggest (*pace* Bell) that we view the wavefunction as *nomic*:
  - The wavefunction acts as a guide that tells the particle where to go. In this sense, one might think of it as giving a law governing the particle's motion.
  - But this (one might claim) is problematic! The wavefunction itself is guided by the time-dependent Schrödinger equation, and (one might claim) laws shouldn't themselves be governed by other laws![5]
  - Dürr, Goldstein, and Zanghì claim that we should see the time-dependent Schrödinger equation as purely phenomenological: it arises locally, but the real solution to the wavefunction of the whole universe is just a stationary state.

[5]JR: Maybe there are interesting connections here with Lange's discussions of *meta-laws*.