Philosophy of Quantum Mechanics: Week 7

Recap: Collapse

In orthodox quantum mechanics, the state is governed by two different kinds of dynamics:

1. Unitary, deterministic, linear Schrödinger dynamics.
2. Non-unitary, non-deterministic, non-linear collapse dynamics.

In practice, we apply collapse dynamics when we perform measurements, but nothing in our physics tells us what counts as a measurement, or why. Moreover, we don't have any idea how collapse is supposed to work!

We might seek to tackle these issues directly by modifying the Schrödinger dynamics to include collapse. The resulting theories are (unsurprisingly!) known as *dynamical collapse theories*. Any such theory must respect the following three constraints:

- Modification must have no noticeable effects on microscopic systems.
- Modification must prevent macroscopic superpositions.
- Collapse must occur in accordance with the Born rule. I.e., $\alpha \left|\uparrow\right\rangle + \beta \left|\downarrow\right\rangle$ must collapse to either $\left|\uparrow\right\rangle$ with probability $|\alpha|^2$, or $\left|\downarrow\right\rangle$ with probability $|\beta|^2$.

Dynamical Collapse Theories: Overview

Here's a toy dynamical collapse theory:

- Particles have probability τ per second of collapsing to some position eigenstate. (For example, $\alpha \left|x\right\rangle + \beta \left|y\right\rangle$ will collapse to $\left|x\right\rangle$ with probability $|\alpha|^2$, or $\left|y\right\rangle$ with probability $|\beta|^2$, where $\left|x\right\rangle$ and $\left|y\right\rangle$ represent different position eigenstates.)

- If particles are entangled, then if any one particle collapses, the quantum state for the whole system collapses.
- If τ is set correctly (e.g. $\tau = 10^{-16}\mathrm{s}^{-1}$), this will ensure that isolated particles collapse very slowly (roughly, after 100 million years), but large, *entangled* systems will collapse much faster.

A minor hitch

This model works quite well, except for a technical problem: (Albert, p. 97)

> The collapses described above leave the particles which undergo them in perfect eigenstates of the position operator, and of course that entails that the *momenta* and the *energies* of those particles (whatever their values may have been just *prior* to those collapses) will be completely uncertain just following those collapses, and *that* will give rise to a host of problems: The momenta which electrons in atoms might sometimes require in the course of such collapses, for example, would be enough to knock them right out of their orbits; and the energies which certain of the molecules of a gas might sometimes acquire in the course of such collapses would be enough to spontaneously *heat those gases up*, and those sorts of things are experimentally known *not to occur*.

The fix is to say that we don't collapse onto an *exact position*, but rather onto some *Gaussian function of position*. Albert (p. 98):

> And it turns out (and this is the punch line) that these curves can nonetheless be made *wide* enough (at the same time) so that the violations of the conservation of energy and of momentum which the multiplications by these curves will produce will be *too small to be observed*.

GRW Theory

Ghirardi-Rimini-Weber (GRW) theory models collapse via multiplication by a Gaussian function. We have collapsed to a roughly localised state, but not a position eigenstate:

- Collapse: $\psi(x) \to \mathcal{N}\psi(x) f(x-a)$.
- $f(x-a) = \exp\left(\frac{-(x-a)^2}{2L^2}\right)$ is a Gaussian function centered at $x = a$.
- a is stochastically selected in line with the Born rule: $\mathrm{Prob}(x_0 < x < x_0 + \delta x) = |\psi(x_0)|^2 \delta x$.
- $\mathcal{N}$ is a normalisation factor.
- τ and L are new constants of nature.

If we choose $\tau = 10^{-16}\mathrm{s}^{-1}$ and $L = 10^{-5}\mathrm{cm}^{-1}$, then

- Isolated particles will collapse after approximately 100 million years.
- A dust mote of $10^{-5}\mathrm{cm}^{-1}$ in diameter will collapse after about 1 second.
- A macroscopic object (e.g. a grain of sand less than $1\mathrm{mm}$ across) will collapse extremely quickly (less than $10^{-5}\mathrm{s}$ for the grain of sand).

The Problem of Tails

GRW dynamical collapse does not get rid of superpositions completely. It takes states like

$$\alpha\,|\mathrm{here}\rangle + \beta\,|\mathrm{there}\rangle \tag{1}$$

to states like

$$\sqrt{1-\epsilon}\,|\mathrm{here}\rangle + \epsilon\,|\mathrm{there}\rangle\,, \tag{2}$$

where ϵ is small. But there are obvious problems with this:

1. ***The problem of structured tails***: Suppose the system is initially in a state such as (1), but then collapses to a state such as (2). Post-collapse, the 'collapsed' part of the superposition (above, $|\text{here}\rangle$) is 'damped down', but does not disappear completely. As Wallace writes (*Measurement Problem: State of Play*, p. 43):

   > Why should the continued presence of the 'there' term in the superposition—the continued indefiniteness of the system between 'here' and 'there'—be ameliorated in any way at all just because the 'there' term has low amplitude?

2. ***The problem of bare tails***: Even if we ignore the 'there' state, the wavefunction of $|\text{here}\rangle$ is itself spatially highly delocalised. Its centre-of-mass wavefunction is no doubt a Gaussian, and Gaussians are completely delocalised in space, for all that they may be concentrated in one region or another. So how can a delocalised wave packet possibly count as a localised particle? But, as Wallace writes on this problem (*MPSOP*, p. 43):

   > *This* problem has little or nothing to do with the GRW theory. Rather, it is an unavoidable consequence of using wave packets to stand in for localised particles. For *no* wave packet evolving unitarily will remain in any finite spatial region for more than an instant.

Strangely, most of the literature on the problem of tails addresses the problem of bare tails (which is not specific to dynamical collapse theories), rather than the problem of structured tails (which *is* specific to dynamical collapse theories).

Addressing Bare Tails

We want to say that, after collapse, systems and particles have definite positions, even though the wavefunction is non-zero everywhere in space. Albert and Lower suggest we get achieve this by dropping the *eigenvector-eigenvalue link*, and replacing it with a so-called *fuzzy link*.

Eigenvector-eigenvalue link: A system in a state $|\psi\rangle$ has a definite value of the physical quantity associated to some observable $\hat{X}$, iff

$$\hat{X}\,|\psi\rangle = x\,|\psi\rangle\,.$$

Fuzzy link: A system in a state $|\psi\rangle$ has a definite value of the physical quantity associated to some observable $\hat{X}$, iff

$$\left|\hat{X}\,|\psi\rangle - x\,|\psi\rangle\right| \leq \lambda,$$

for some small λ.

The fuzzy link solves the problem of bare tails, but it leads to the so-called *counting anomaly*:

- Particle 1 is in some box (according to the fuzzy link).
- Particle 2 is in that box (according to the fuzzy link) ...
- ... Particle n is in that box (according to the fuzzy link).
- On the fuzzy link, this *does not imply*: N particles are in the box.

How is this working, in more detail? Suppose that the wavefunction of each particle is very strongly peaked inside the box, so that if $\hat{X}_i$ is the 'particle i is in the box' operator, then $\left|\hat{X}_i\,|\psi\rangle - x\,|\psi\rangle\right| \sim \epsilon$, for $\epsilon \ll \lambda$, so that each particle should be counted as being inside the box. Heuristically, $\left|\hat{X}_i\,|\psi\rangle\right| \sim 1 - \epsilon$. But now consider the proposition 'all N particles are in the box'. By definition, this is represented by the operator $\hat{X} = \prod_{i=1}^{N} \hat{X}_i$. Suppose each particle has identical state $|\psi\rangle$; i.e. suppose that each $|\psi\rangle$ is highly localised in the box, as above. Then the overall state of the N particles is $|\Psi\rangle = \bigotimes_{i=1}^{N} |\psi\rangle$. Then $\left|\hat{X}\,|\Psi\rangle\right| = \prod_{i=1}^{N} \left|\hat{X}_i\,|\psi\rangle\right| = (1-\epsilon)^N$. As Wallace says: (*MPSP* p. 44)

> this is unfortunate for the Fuzzy Link. For no matter how small ϵ may be, there will be some value of N for which $(1-\epsilon)^N < \lambda$. And for that value of N, the Fuzzy Link tells us that it is false that all N particles are in the box, even as it tells us that, for each of the N particles, it is true that *that* particle is in the box.

Given the problems for the fuzzy link, we might turn to an alternative 'link': the so-called *mass density link*.

Mass density link: A particle is in a box iff some sufficiently high fraction $1 - \epsilon$ of its mass is in the box. The meaning of 'all N particles are in the box' is 'particle 1 is in the box and particle 2 is in the box and ...', and this is true iff all the constituent propositions are true.

It's sometimes claimed that the mass density link evades the counting anomaly only at the cost of a so-called 'location anomaly'. Wallace on this (*MPSP*, pp. 46-47):

> This anomaly arises when we consider the process of *looking* at the box and physically counting the number of particles in it. The ordinary quantum theory—which the GRW theory is supposed to reproduce—then predicts that the expected number of particles found in the box will be somewhat less than N. Lewis claims that this clash between the predictions of how many particles are *found* in the box and how many are *actually* in the box "violates the entailments of ordinary language" (Lewis 2005, p. 174).

Ghirardi and Bassi, and separately Wallace, are bemused by this criticism: (*MPSP* p. 47)

> [W]e have a theory which (a) gives a perfectly well-defined description of how many particles are in the box; (b) allows a precise description, in terms acceptable to the realist, of the measurement process by which we determine how many particles are in the box; (c) predicts that if the number of particles is sufficiently (i.e., ridiculously) large there will be tiny deviations between the actual number of particles and the recorded number of particles. They, and I, fail to see what the problem is here; I leave readers to reach their own conclusions.

On Structured Tails

Consider a post-GRW collapsed state, such as

$$\sqrt{1-\epsilon}\,|\text{alive}\rangle + \epsilon\,|\text{dead}\rangle\,. \tag{3}$$

- On the Everett interpretation, a structural/functional definition of macro-ontology is given.
- Suppose we accept this. Then all there is to being a cat is being something structured like a cat.
- There's still something structured like an (alive!) cat in state (3)—it's just very low amplitude.
- On Everettian ontology, a low amplitude cat is still a cat.
- So, there are still two cats (one alive and one dead!) in the state (3).
- So is GRW just (some messy version of) Everett, in disguise?

Probability

In some ways, GRW collapse provides but the perfect example of objective chance. But what does τ *mean*? Is it only analysable in terms of propensities, or are other options available?