Philosophy of Space and Time: Week 3

The Ether

Maxwell's equations

$$\nabla_a F^{ab} = J^b,$$
$$\nabla_{[a} F_{bc]} = 0,$$

—the equations of our best theory of (classical) electrodynamics—tell us that *the speed of light is a constant*, call it c. What could this mean?

- At the end of the 19th century, people took it to mean that the speed of light emitted by a source at rest with respect to the *luminiferous ether* (in which the light was supposed to move) must always be c.
- The *Michelson-Morley experiment* was designed to work out the Earth's motion with respect to the ether—but it delivered a null result!
- It could be that the Earth *just happened* to be at rest with respect to the ether, when the experiment was performed (though it seems unlikely). But when they repeated the experiment six months later, they got the same result!
- Einstein was the first to take electrodynamics' claim that the speed of light is constant by the horns.

Inertial Frames

- What is an *inertial frame*?
- Suppose that the laws of physics take their simplest form in some frame (i.e. coordinate system), F.
- The inertial frames are all those frames in uniform, rectilinear motion with respect to F.

[JR note to self: this definition seems backwards. Should start with the physics and work out what the inertial frames are, not start with a definition of the inertial frames and work out what the physics is.]

The Two Postulates of Special Relativity

1. **Relativity principle:** The laws of physics are the same in all inertial frames of reference.
2. **Light postulate:** The speed of light takes a fixed value that is independent of the speed of the source.

Together, (1) and (2) imply:

3. **Light postulate+:** The speed of light is the same for all inertial observers.

Explain how this goes...

In the rest of this seminar, we'll explore some consequences of these two postulates (the more radical of which is clearly the light postulate).

Time Dilation

See sheet.

Length Contraction

See sheet.

The Relativity of Simultaneity

See sheet:

1. Train example
2. Spacetime diagram

Coordinate Transformations

Galilean Transformations

Suppose that Newtonian mechanics is true. We have seen that the laws of Newtonian mechanics are insensitive to uniform velocity changes—this was the basis of the kinematic shift. This principle can be expressed via the principle of *Galilean relativity*: the laws of physics are the same for two (inertial—which we will take here to mean relating observers in relative motion with constant velocity) observers related by

$$
\begin{aligned}
x' &= x - vt, \\
y' &= y, \\
z' &= z, \\
t' &= t.
\end{aligned}
$$

These are the *Galilean transformations*.

Lorentz Transformations

Now suppose that the two postulates of relativity hold. Then the Galilean transformations cannot be the correct transformations relating inertial observers—for we have sen that e.g. time will dilate for a uniformly moving observer. In SR, therefore, we must use a new system of transformations to relate inertial observers:

$$
\begin{aligned}
x' &= \gamma\left(x - vt\right), \\
y' &= y, \\
z' &= z, \\
t' &= \gamma\left(t - \frac{vx}{c^2}\right).
\end{aligned}
$$

These are the *Lorentz transformations*, with

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

A derivation is beyond the scope of this course (and will invariably involve a number of subtleties), but see any good textbook. At $v = c$, γ is infinite, and faster than light ($v > c$) γ is a complex number, each of which makes the transformations unphysical.

For relative speeds much less than c, the Lorentz transformations reduce to the Galilean transformations:

$$
\begin{aligned}
x' &\approx x - vt, \\
t' &\approx t.
\end{aligned}
$$

Note that both time dilation and length contraction can be derived directly from the Lorentz transformations (see sheet).

The Spacetime Interval

- In Newtonian mechanics, absolute velocity is not an invariant notion between inertial frames.
- However, simultaneity is an invariant notion: if two events are simultaneous in one frame, they are simultaneous in all. (In fact, not just inertial frames.)
- We have seen that the simultaneity of two events is *not* an invariant notion in SR.
- However, the spacetime *interval*,

  $$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2,$$

  is invariant between all inertial frames (show—see sheet) — in fact, between *all* frames (but will only take the form in inertial frames).

The Twin Paradox

See sheet. Remember that the twin paradox (a) does not rely on GR; (b) does not rely on 'shifting between inertial frames'; etc. SR can accommodate accelerations!