Philosophy of Space and Time: Week 4

Minkowski Spacetime

Coordinate transformations

To recap: Last week, we saw that *inertial frames* (i.e. coordinate systems in which the laws of physics take their simplest forms) in Newtonian mechanics are related by the *Galilean transformations*,

$$\begin{aligned} x' &= x - vt, \\ y' &= y, \\ z' &= z, \\ t' &= t, \end{aligned}$$

whereas inertial frames in special relativity are related by the *Lorentz transformations*,

$$\begin{aligned} x' &= \gamma\left(x - vt\right), \\ y' &= y, \\ z' &= z, \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right), \end{aligned}$$

with

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Euclidean metric

- The distance D between two points in Newtonian absolute space *at a particular time* is given by

$$D^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

(this is just Pythagoras' theorem). This quantity is *invariant* under Galilean transformations (in fact, it's invariant under all transformations—but only retains the above *form* under Galilean transformations).

- We can write the vector between these two points—call them A and B—as

$$\Delta\vec{x} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

- Then the above formula for D^2 could be rewritten in the form of a *matrix equation* as

$$D^2 = (\Delta\vec{x})^T h\,(\Delta\vec{x}) = \begin{pmatrix} \Delta x, & \Delta y, & \Delta z \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix},$$

where

$$h = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is the three-dimensional *Euclidean metric*.

- Sometimes, a three-dimensional Euclidean metric is *identified* with Newton's absolute space.

Minkowski metric

- Similarly, in SR, the following quantity—the *spacetime interval* I—is invariant under Lorentz transformations (in fact, it's invariant under all transformations—but only retains the *form* below under Lorentz transformations):

$$I^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

- We can write the vector between these two points—call them A and B—as

$$\Delta \vec{x} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ c\Delta t \end{pmatrix}$$

- Then the above formula for I^2 could be rewritten in the form of a *matrix equation* as

$$I^2 = (\Delta \vec{x})^T \eta \, (\Delta \vec{x}) = \begin{pmatrix} \Delta x, & \Delta y, & \Delta z & c\Delta t \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ c\Delta t \end{pmatrix},$$

  where

$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

  is the four-dimensional *Minkowski metric*.

- Sometimes, a four-dimensional Euclidean metric is *identified* with spacetime in Einstein's special relativity.

- Note the minus sign compared with the Euclidean metric!

Bell's Rockets

- Two rockets, connected by a taut tether, start at rest with respect to a control tower and are accelerated to relativistic speeds, keeping the distance between them constant.
- Will the tether between the rockets break? *Yes!* (NB: Many physicists get this wrong.)
- *From the point of view of the control tower*, the breakage happens as a result of length contraction.
- *From the point of view of the first rocket*, the breakage happens as the second rocket moves progressively further away (due to the relativity of simultaneity—draw a spacetime diagram!).
- *From the point of view of the second rocket*, the breakage happens as the first rocket lags further behind (due to the relativity of simultaneity—draw a spacetime diagram!).

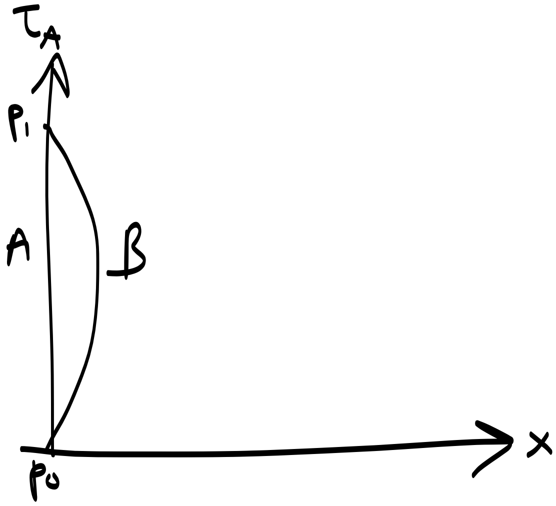
What should we make of these frame-based explanations? Bell wants us to accept contraction-based explanations as just as physical/good/real as others. Knox concurs:

> In SR, reference frames are all as good as one another—it's wrong to think of the rocket perspective as somehow offering a 'real' explanation! (2016, 4(35))

Many, however, would demur. Maudlin writes:

> The surface contradiction between these three accounts of why the thread breaks illustrates that frame-dependent narrations of events in Relativity can be misleading. There is one set of events, governed by laws that are indifferent to which coordinate system might be used to describe a situation. In each frame-dependent account, the interatomic forces in the thread play a role in determining exactly when the thread breaks. But how that role is described in a particular reference frame depends critically on which frame is chosen. (Maudlin, p. 120)

Twin Paradox, Reprise



Last week, I proffered a *geometric* explanation of the twins paradox:

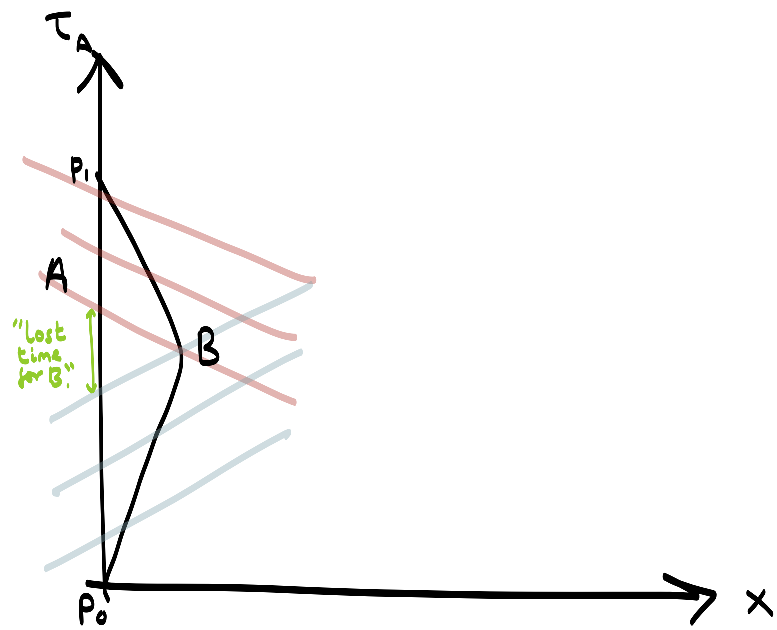
- Proper time elapsed along path A: (i.e., time elapsed on A's clock)

$$T_A = \int_{p_0}^{p_1} d\tau_A$$

- Proper time elapsed along B: (i.e., time elapsed on B's clock)

$$T_B = \int_{p_0}^{p_1} d\tau_B = \int_{p_0}^{p_1} \left(1 - \left(\frac{dx}{d\tau_A}\right)^2 - \left(\frac{dy}{d\tau_A}\right)^2 - \left(\frac{dz}{d\tau_A}\right)^2\right)^{1/2} d\tau_A < T_A$$

- This explanation *just* used the invariance of the special relativistic spacetime interval—it was not relativised to a frame.



One may also give a frame-based explanation:

- A great deal of time elapses on Earth (i.e., for A) as B's simultaneity slices shift on her turnaround.

As with the rockets, we have *frame-independent* explanations on the one hand, and *frame-dependent* explanations on the other. Bell's point is that we can do physics from the point of view of a single reference frame (this is his "Lorentzian pedagogy"). In sum:

1. The tendency to think that there's 'no physics' in choices of coordinates can lead us to think that phenomena like Lorentz contraction aren't 'real', or can't be explanatory.
2. The frame-independent approach (offering explanations in terms of Minkowski spacetime properties such as the invariance of the interval, etc.), inasmuch as it's contrasted with coordinate-based explanations, may exacerbate this tendency.
3. NB: This is *not* to say that there's anything wrong with the frame-independent approach *per se*.

Constructive and Principle Theories

Einstein famously distinguished between *constructive* and *principle* theories:

- *Constructive theories:* Theories like the kinetic theory of gases. These theories attempt to construct the phenomena we see around us 'from the bottom up', by modelling the physical world in terms of fundamental constituents and the laws that govern them.
- *Principle theories:* Theories like thermodynamics (and special relativity), which start with a handful of empirically observed principles or phenomena, elevate them to the status of postulates, and then derive a theory based on what follows from these.

One might worry that only constructive theories offer *explanations*. If this worry moves us, we should seek a constructive version of special relativity—one that not only predicts that relativistic phenomena will happen, but explains *why* they happen.

Two constructive approaches to special relativity

Two attempts to provide a constructive version of special relativity:

(A) (*Geometrical approach*) Special relativistic phenomena (in particular Lorentz invariance of the dynamical laws) are explained by matter being embedded in Minkowski spacetime. (Maudlin, Janssen)

(B) (*Dynamical approach*) Special relativistic phenomena (in particular Lorentz invariance of the dynamical laws) are explained by the fundamental microdynamics of matter. (Brown)

In addition to endorsing the dynamical approach, Brown argues that Minkowski spacetime is unnecessary in the true constructive version of special relativity:

- Once we know that the laws governing rods and clocks are Lorentz invariant, we know that they will behave in accordance with the postulates of special relativity.

- So what we think of as spacetime geometry is reducible to the symmetry properties of the dynamical laws governing matter fields. (Brown p. 133: "The appropriate structure is Minkowski geometry *precisely because* the laws of physics of the non-gravitational interactions are Lorentz covariant".)
- For Brown, Minkowski spacetime alone can't explain anything. (Brown p. 134: "It is wholly unclear how this geometrical explanation is supposed to work.")

Two Debates

It's important to keep distinct in one's mind the two debates we have discussed:

1. The debate over the legitimacy of frame-dependent/-independent explanations in spacetime theories such as special relativity.
2. The geometrical-versus-dynamical debate over the most appropriate constructive version of a given spacetime theory, here special relativity.

Roughly, Brown favours frame-dependent explanations and the dynamical approach; Maudlin and Friedman favour frame-independent explanations and the geometrical approach; Janssen favours frame-dependent explanations and the geometrical approach.