

## **Paradoxes—Reading group 1**

In *The Ways of Paradox* (1966), W. V. Quine—one of the 20th century’s most influential philosophers—defined a paradox as follows:

Reasoning from apparently plausible premises, via apparently plausible steps, to a contradictory conclusion.

If we arrive at a contradiction from a set of premises, and we view contradictions as unacceptable, then it must be the case that either:

1. We should, in fact, reject one of the premises.
2. We should reject the chain of reasoning from the premises to the conclusion.
3. We should (somehow!) argue that the conclusion is, in fact, not contradictory after all.

Sainsbury, in his book *Paradoxes*, considers a number of paradoxes, and assesses which of (1)-(3) we should endorse in each case.

### **Some examples of paradoxes**

- The Liar paradox: is the following sentence true?

This sentence is false.

- The grandfather paradox: Could a time traveller go back and kill his or her grandfather?

### **Chapter 1: Zeno’s paradoxes**

There are actually several Zeno paradoxes. Following Sainsbury, we’ll focus on the following:

1. *The tortoise and Achilles*: Suppose that the tortoise and Achilles are running a race. Since the tortoise is slower, she’s given a head start. Will Achilles ever catch up with the tortoise? (p. 4)

2. *The racetrack*: How can anything begin to move at all, given that to get anywhere you must first get half-way there? (p. 5)
3. *The arrow*: How can anything truly be moving, when at every instant it is at rest? (p. 19)

On the first:

*Some have seen the paradox as produced by the assumption that space or time is infinitely divisible, and thus as genuinely proving that space or time is not infinitely divisible. Others have seen in the argument nothing more than a display of ignorance of elementary mathematics—an ignorance perhaps excusable in Zeno's time but inexcusable today.* (Sainsbury, p. 4)

## Space

Sainsbury begins by discussing a preliminary perplexity. Zeno sought to argue that there is only one object in the world ('monism')—he reasoned as follows:

**P1:** If there is more than one object, then some objects must have parts.

**P2:** If objects have parts, then they must have infinitely many parts.

**P3:** An object with infinitely many parts would be infinitely large.

**C1:** So all objects with parts must be infinitely large.

**C2:** So if there is more than one object, then there are some objects with parts, and all those objects are infinitely large.

**P4:** If there are objects with parts, then it is not the case that all such objects are infinitely large.

**C3:** So it is not the case that there is more than one object.

This argument is valid, but (arguably) unsound. That is, there is no possible way the conclusion could be false if the premises are true—but nevertheless, we do *not* think the premises are true (why not?).

## The racetrack

Sainsbury (p. 11) now turns to the racetrack paradox. He summarises the paradox here as follows:

*Let us call the starting point  $Z$  (for Zeno) and the endpoint  $Z^*$ . The argument can be analyzed into two premises and a conclusion, as follows:*

**P1:** *Going from  $Z$  to  $Z^*$  would require one to complete an infinite number of journeys: from  $Z$  to the point midway to  $Z^*$ , call it  $Z_1$ ; from  $Z_1$  to the point midway between it and  $Z^*$ , call it  $Z_2$ ; and so on.*

**P2:** *It is logically impossible for anyone (or anything) to complete an infinite number of journeys.*

**C:** *It is logically impossible for anyone to go from  $Z$  to  $Z^*$ . Since these points are arbitrary, all motion is impossible.*

Understanding ‘journey’ in an innocuous way, **P1** seems unproblematic. So: “Suspicion focuses on **P2**” (p. 12). As Sainsbury asks, “Why should one not be able to complete an infinite number of journeys in a finite time? Is that not precisely what *does* happen when anything moves?”

Many would want to say: Zeno doesn’t have a good grasp of mathematical limits. The point is that the *time* taken to undergo each step of the journey (halving the journey distance each time, and assuming constant velocity) will be less, and the sum of this infinite series can still be finite. For example, it’s now well-known that

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 1.$$

Thus, the sum of the times taken to undergo an infinite number of journeys need not be infinite. Since it is possible to complete an infinite number of journeys, so long as those journeys in total take finite time, **P2** is false.

## The tortoise and Achilles

*We can see this as nothing more, in essentials, than the Racetrack, but with a receding finish line. The paradoxical claim is this: Achilles can never get to  $Z^*$  because however*

*many points in the Z-series he has occupied, there are still more Z-points ahead before he gets to  $Z^*$ . Furthermore, we cannot expect him to complete an infinity of “tasks” (moving through Z-points) in a finite time. An adequate response to the Racetrack will be easily converted into an adequate response to this version of the Achilles. (Sainsbury, p. 19)*

The above discussion of mathematical limits also suffices to address the tortoise/Achilles problem.

### The arrow

The third Zeno paradox begins from consideration of a flying arrow:

*At any instant of time, the flying arrow “occupies a space equal to itself.” That is, the arrow at an instant cannot be moving, for motion takes a period of time, and a temporal instant is conceived as a point, not itself having duration. It follows that the arrow is at rest at every instant, and so does not move. What goes for arrows goes for everything: nothing moves. (Sainsbury, p. 19)*

Here’s Sainsbury’s response:

*Classical mechanics purports to make sense not only of velocity at an instant but also of various more sophisticated notions: rate of change of velocity at an instant (i.e. instantaneous acceleration or deceleration), rate of change of acceleration at an instant, and so on. (Sainsbury, p. 20)*

The point is that an object can have a velocity *at a particular time*, even if that velocity is *defined with reference to other (nearby) times*. So Zeno seemed to have an overly pointillist conception of velocity and motion (cf. the work of Butterfield for some sophisticated discussion on this).

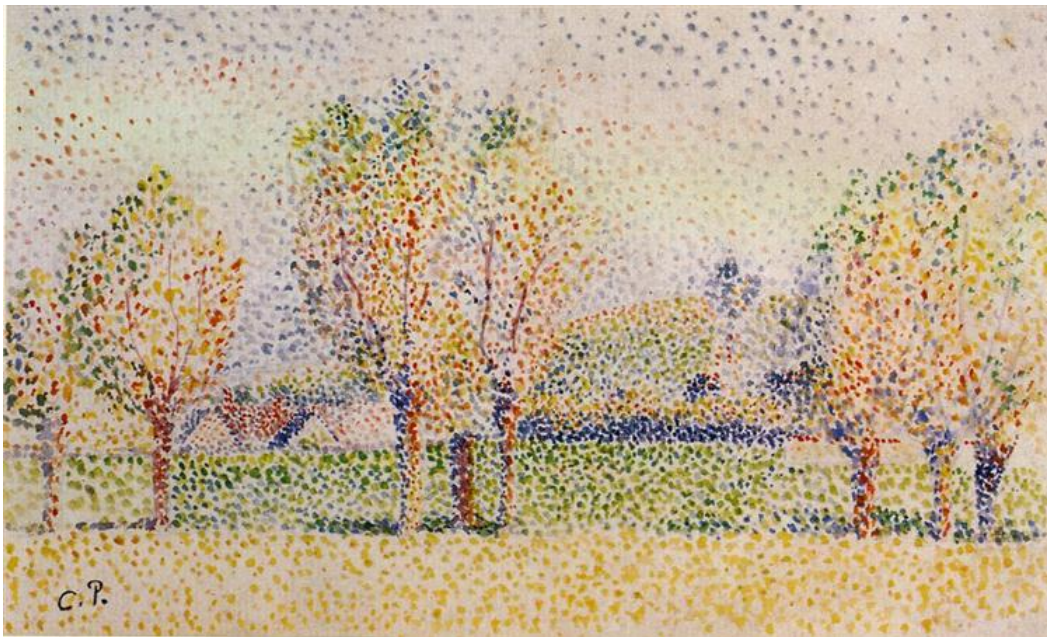


Figure 1: Eragny Landscape (c.1886) by Camille Pissarro