LOCAL DISQUOTATION, SEMANTIC NON-CONSERVATIVITY OF TRUTH, AND MODELS OF SET THEORY

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We analyse the non-conservativeness properties of the classical locally disquotational theory of typed arithmetic truth TB and investigate its model-theoretic strength w.r.t the class of recursively saturated models of arithmetic. We first strengthen and generalise the Cieslinski-Engstrom theorem on semantic (model-theoretic) non-conservativeness of TB over PA to a new result stating that TB is not semantically conservative over any complete extension of PA, including True Arithmetic TA (= Th(N)). Cieslinski and Engstrom's proof was insufficient to justify the latter and our proof provides a new argument that can be useful in further investigations of properties of axiomatic theories of truth. Further, we transfer the characterization of models of TB over set theories, which has some philosophical implications in the debate on deflationism w.r.t. the concept of mathematical truth.

In the second part of the talk we separate the class of models of arithmetic expandable to a model of TB from the class of recursively saturated models, providing a completely new and conceptually simple proof of a result due to Łełyk and Wcisło and contributing to the research in the hierarchy of model-theoretic strength of axiomatic truth theories. The main philosophical meaning of the first result is that it also ultimately strengthens contradiction to the claim by Ketland that the phenomenon of conservativeness boils down to adding compositionality principles to a given theory of truth. However, the theorem additionally invites philosophical interpretation contributing to the debate on the conservativeness in the field of deflationary theories of truth, namely it provides a reductio argument against considering semantic conservativeness as an adequate criterion for a truth theory, since the assumption that semantic conservativeness is an adequate criterion for a deflationary theory of truth excludes TB from the class of adequate theories as a too strong one. We will extend these results to the case of set theory, i.e. to disquotational theories of truth over ZFC taken as a base theory, and demonstrate how models of richer truth theories (over set theory), such as CT^{-}, are related to the set-theoretic Multiverse.

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