NATURAL FORMALISM

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The language of mathematics as found in textbooks and research articles is a variant of natural language which admits symbolic terms and formulas within argumentative natural language contexts. As it is written to be unambiguously understandable by experts it is susceptible to definite translations into (first-order) formal statements by standard (computer-)linguistic methods. Conversely those methods determine a class of accepted statement and hence implement a controlled natural mathematical language that can be seen as an enriched formal language.

Formal mathematics, which aims at complete formalisations of mathematical statements and proofs, has recently seen considerable advances using powerful software like automatic theorem provers for bridging proof steps. There is now a large selection of computer-checked proofs, including major results like the Four-Colour Theorem or the Feit-Thompson Theorem. Computer proof systems can be viewed as implementing strong formal calculi which allow sophisticated deductions known from ordinary mathematical proofs.

The combination of controlled natural language and argumention will lead to rich natural logics, which are completely formal extensions of first-order logic or some variant, but which accept proofs in the familiar language of mathematics.

In my talk I want to discuss some aspects of that development:
- relevant systems in formal mathematics and linguistics;
- associated work in the foundations of mathematics;
- implications for the practice of mathematics;
- implications for mathematical formalism.