COLLECTION, LEMMAS, REFLECTION

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Σ_1-collection is a basic principle of thought. In one of its forms it tells us that a recursive function on a finite set of natural numbers is bounded. The principle is so deeply ingrained in our thinking that, when working in its absence, one really has to school oneself against unconscious application.

An important application of Σ_1-collection is what I call the Lemma Principle. Consider a finite set X of theorems of a theory U. The Lemma Principle tells us that a theorem of the theory axiomatized by X is ipso facto a theorem of U. If one assumes the negation of Σ_1-collection, then one can show that every theory containing a modicum of arithmetic has a finite set of theorems that is inconsistent. So, under assumption of the negation of Σ_1-collection, the Lemma Principle fails in a rather spectacular way for consistent theories.

We will show that, under the assumption of the consistency of U, the Lemma Principle for U is equivalent to Σ_1-collection. Using the same methods it follows that under assumption of the consistency of U, Σ_1-collection is equivalent to the consistency statement of the theory axiomatized by the theorems of U. It follows that in order to verify Craig's Trick one needs Σ_1-collection. So the trick rest on a substantial insight.

Under assumption of the consistency of U, one can also show that Σ_1-collection is equivalent to $\Pi_{1,1}$-reflection for U. (The formula class $\Pi_{1,1}$ will be explained in the talk.)